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Scale effects on flexural wave propagation in nanoplate embedded in elastic matrix with initial stress

Yi-Ze Wang · Feng-Ming Li · Kikuo Kishimoto

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Abstract In this paper, the small-scale effects on the flexural wave in the nanoplate are studied. Based on the nonlocal continuum theory, the equation of wave motion is derived and the dispersion relation is presented. Numerical simulations are performed to investigate the influences of the scale coefficient, the surrounding elastic matrix and the initial stress on the wave propagation properties. The results show that the nonlocal model provides an appropriate method to investigate the characteristics of the flexural wave in the nanoplate. Furthermore, the direction and amplitude of the biaxial load, the stiffness of the shearing layer and the Winkler foundation can change the wave properties, significantly.

1 Introduction

Compared to the standard structural materials, carbon nano materials can present excellent electronic, chemical and mechanical characteristics. As a result, these structures hold significant promise for potential applications and have received a lot of attention [1–3]. With the difficulty and high cost to perform the controlled experiments at the nano scale and the molecular dynamics (MD) simulation for large size structures, the nonlocal continuum models have been applied to study the mechanical properties of nanostructures widely and efficiently [4, 5]. The results have shown that the dynamic properties of nanostructures are different from those of the problems with the classical continuum theory [6, 7]. Therefore, it has been considered a useful method to illustrate the small-scale effects in nanotechnology.

The carbon nanotube, which holds a hollow cylindrical geometry, is a typical nanostructure and is usually considered a beam model in numerical simulation. In order to give a better understanding and a deep insight into the dynamical properties of carbon nanotubes, some works have been carried out on the characteristics of the vibration [8, 9] and elastic wave propagation [10, 11] in nanotubes. The nanoplate is another typical structure system with the nano scale and its mechanical behavior has been the subject of numerous researches. Different from the nanotube, researches on the nanoplate are limited, which are mainly focused on the properties of the bending [12], buckling [13, 14] and vibration [15–18]. Furthermore, few works have addressed wave propagation in the nanoplate. In order to illustrate the dynamical behaviors, more attention should be paid.

It should be noted that in practical applications, due to the mismatch of the material properties and external introduced load, the initial stress in the nano devices always exists. Wang and Cai investigated the effects of the initial stress on the characteristics of vibration and wave propagation in carbon nanotubes [19, 20]. It has been shown that the initial stress can lead to significant change not only for the vibration behaviors, but also for the wave propagation characteristics of nanostructures.

In our recent work, the elastic wave properties in nanotubes with fluid have been investigated by the nonlocal continuum method [21]. The additional terms and the boundary conditions of the nanotube in the dynamic equation depend

Y.-Z. Wang · F.-M. Li (✉)
School of Astronautics, Harbin Institute of Technology,
P.O. Box 137, Harbin 150001, P.R. China
e-mail: fmli@hit.edu.cn

Y.-Z. Wang (✉) · K. Kishimoto
Department of Mechanical Sciences and Engineering, Tokyo
Institute of Technology, 2-12-1, O-okayama, Meguro-ku,
Tokyo 152-8552, Japan
e-mail: wangyize@gmail.com

on the considered problem, which are different from those of the nanoplate. In order to get more information about the nanotubes with the nonlocal continuum theory, one can refer to some related papers [22–24]. For the purpose mentioned above, the flexural wave propagation in the nanoplate embedded in elastic matrix with the initial stress is studied in this analysis. The nonlocal continuum theory is applied and the small-scale effect is discussed. Furthermore, the influences of the Winkler foundation and the initial stress on the wave propagation properties are also discussed.

2 Equations of wave motion

The nanoplate embedded in the elastic matrix with the initial stress is shown in Fig. 1. The steady flexural wave propagates along the x direction. Suppose that the initial stresses along the x and y directions are equal. Based on the nonlocal elastic theory [25, 26], which accounts for the small-scale effect by assuming the stress at a reference point as a function of the strain at every point in the body, the constitutive relation can be presented as the following integral form:

$$\sigma_{kl,k} - \rho \ddot{u}_l = 0, \tag{1a}$$

$$\sigma_{kl}(\boldsymbol{x}) = \int_V \alpha(\boldsymbol{x}, \boldsymbol{x}')\tau_{kl}(\boldsymbol{x}')\, dV(\boldsymbol{x}'), \tag{1b}$$

$$\varepsilon_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}), \tag{1c}$$

where σ_{kl} is the nonlocal stress tensor, ε_{kl} the strain tensor, ρ the mass density, u_l the displacement vector, $\tau_{kl}(\boldsymbol{x}')$ the classical (i.e. local) stress tensor, $\alpha(\boldsymbol{x}, \boldsymbol{x}')$ the kernel function which describes the influence of the strains at various location $\boldsymbol{x}'$ on the stress at a given location $\boldsymbol{x}$ and V the entire body considered. From (1b), we can observe that the spatial integrals are involved in the nonlocal constitutive relation which results in the difficulty for solving the nonlocal problem. However, these integral equations can be reduced to the partial differential forms under certain conditions with physical admissible kernels [25, 27, 28], and the nonlocal constitutive relation can be employed conveniently. For further information about the integral kernels of the nonlocal relation, one can refer to the work by Picu [29] and the related references. As a result, the nonlocal constitutive relation of the nanoplate embedded in elastic matrix with equal biaxial initial stress can be expressed as the following form:

$$\sigma_x - (e_0 a)^2 \nabla^2 \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu \varepsilon_y), \tag{2a}$$

$$\sigma_y - (e_0 a)^2 \nabla^2 \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu \varepsilon_x), \tag{2b}$$

$$\tau_{xy} - (e_0 a)^2 \nabla^2 \tau_{xy} = G\gamma_{xy}, \tag{2c}$$

where E is the Young's modulus, ν the Poisson's ratio, G the shear modulus, e_0 the material constant to be determined from experiments or other methods, a the internal characteristic length (e.g. the length of the C–C bond, the lattice spacing and granular distance) and $e_0 a$ the scale coefficient which denotes the small-scale effect. Furthermore, the accurate value of e_0 has not been reported and is still considered as a problem [5].

The stress bending moment can be given as

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_x\, dz, \qquad M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_y\, dz, \qquad M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\tau_{xy}\, dz. \tag{3}$$

Based on (2) and (3), the following relation can be obtained:

$$M_x - (e_0 a)^2 \left(\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_x}{\partial y^2}\right) = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right), \tag{4a}$$

$$M_y - (e_0 a)^2 \left(\frac{\partial^2 M_y}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2}\right) = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right), \tag{4b}$$

$$M_{xy} - (e_0 a)^2 \left(\frac{\partial^2 M_{xy}}{\partial x^2} + \frac{\partial^2 M_{xy}}{\partial y^2}\right) = -D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}, \tag{4c}$$

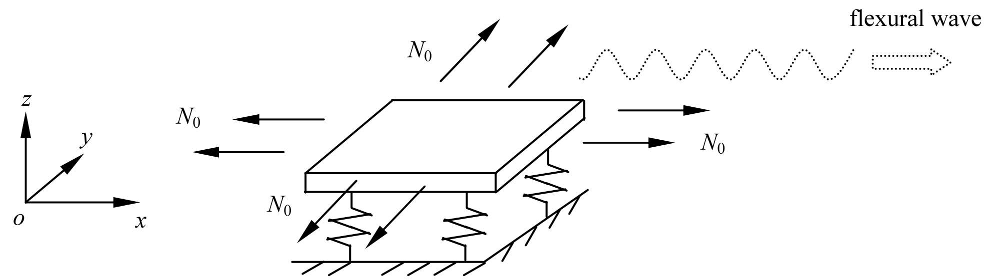
where $D = Eh^3/[12(1-\nu^2)]$ is the bending stiffness of the nanoplate and w the displacement in the z-direction.

According to the mechanical model for the nanoplate [30], the governing equation can be expressed as the following form:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_0\left(\frac{\partial^2 w}{\partial x^2} + 2\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w}{\partial y^2}\right) = \rho h \frac{\partial^2 w}{\partial t^2} + k_w w - G_b \nabla^2 w, \tag{5}$$

where h is the thickness of the nanoplate, k_w the Winkler foundation modulus, G_b the stiffness of the shearing layer, t the time, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ the Laplace operator, $N_0 = h\sigma_0$ the initial load and σ_0 the initial stress. It should be noted that the positive initial stress denotes the tension load and the negative one means the compressive case. Moreover, $e_0 a = 0$ corresponds to the classical continuum model and $\sigma_0 = 0$ denotes the nanoplate without initial stress.

Fig. 1 Flexural wave propagation in the carbon nanoplate embedded in elastic matrix with initial stress



Substituting (4) into (5), we can obtain the equation of wave motion as

$$D\nabla^2\nabla^2 w - (e_0 a)^2\left[\rho h\frac{\partial^2}{\partial t^2}\nabla^2 w + k_w\nabla^2 w - G_b\nabla^2\nabla^2 w\right] + k_w w - G_b\nabla^2 w + \rho h\frac{\partial^2 w}{\partial t^2} + N_0(e_0 a)^2\nabla^2\nabla^2 w - N_0\nabla^2 w = 0. \quad (6)$$

The harmonic solution for the flexural wave in the nanoplate can be expressed as [31]

$$w = W\exp\left[i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)\right], \quad (7)$$

where W is the amplitude, $\boldsymbol{k} = k_x\boldsymbol{i} + k_y\boldsymbol{j}$ the wave number, $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j}$ the radius vector, ω the circular frequency and $k = |\boldsymbol{k}| = \sqrt{k_x + k_y}$the absolute value of $\boldsymbol{k}$.

Substituting (7) into (6) and finding the non-trivial solutions for W, we can obtain the dispersion relation between the wave number and the circular frequency as

$$\omega = \sqrt{\frac{Dk^4 + (e_0a)^2[k_wk^2 + G_bk^4 + N_0k^4] + k_w + G_bk^2 + N_0k^2}{[(e_0a)^2k^2 + 1]\rho h}}. \quad (8)$$

Thus, the cut-off frequency for the nanoplate can be derived by setting $k = 0$ and expressed as the following form:

$$\omega_c = \sqrt{\frac{k_w}{\rho h}}. \quad (9)$$

From (9) we can observe that the cut-off frequency is independent of the scale effect. This phenomenon is similar to the flexural wave in nanotubes embedded in the elastic medium [32].

3 Numerical examples and discussions

In the following, numerical simulations are carried out to investigate the scale effect on the characteristics of the wave propagation. The material constants used in the calculation are the Young's modulus $E = 1.06$ TPa, the thickness $h = 0.34$ nm, the mass density $\rho = 2.25$ g/cm^3, the Poisson's ratio $\nu = 0.25$, the Winkler foundation modulus $k_w = 1.13 \times 10^{18}$ Pa/m, the stiffness of the shearing layer $G_b = 2$ N/m and the scale coefficient $e_0 a = 0$ to 2 nm.

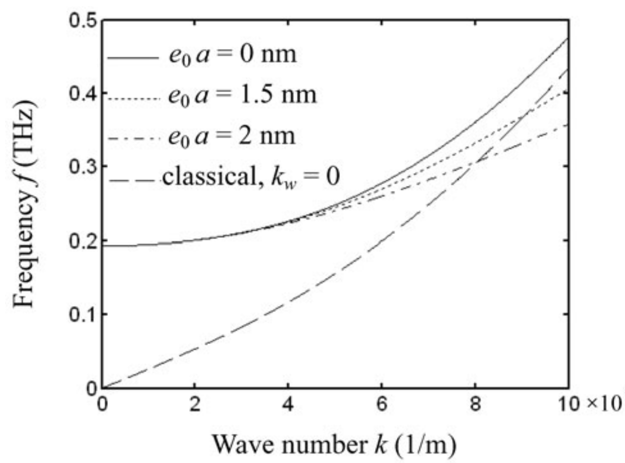


Fig. 2 Dispersion relation for flexural wave in carbon nanoplate by the nonlocal and classical models without the initial stress

In order to investigate the small-scale effect on the characteristics of the wave propagating in the nanoplate, Fig. 2 illustrates the dispersion relation between the wave number and the frequency ($f = \omega/2\pi$) for different scale coefficients without the initial stress. It can be observed that when $k_w \neq 0$, the results based on the classical model and the nonlocal continuum theory are in good agreement for small wave numbers. The difference becomes more obvious with the wave number increasing, in which the nonlocal model exhibits a lower estimation. For a fixed wave number, the larger the scale coefficient is, the smaller the frequency becomes. Furthermore, the critical frequency is reduced to zero when the Winkler foundation modulus (k_w) is neglected, which can also be seen from (9).

The relation between the scale coefficients and the frequency with the tension and compression loads are presented in Figs. 3(a) and 3(b), respectively. The most notable feature is that the frequency increases with the increase of

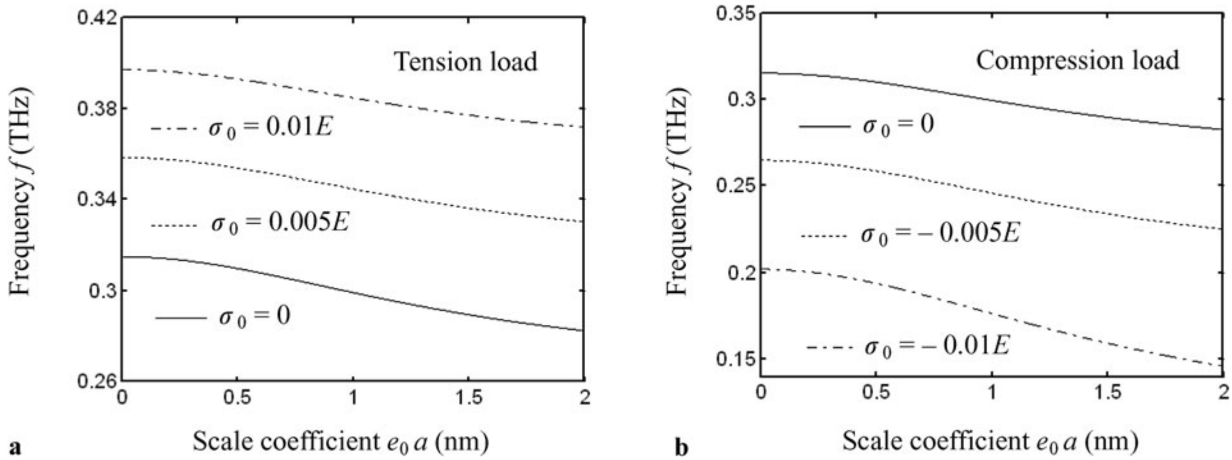


Fig. 3 Frequency f vs. scale coefficient e_0a under different biaxial loads for the wave number $k = 7 \times 10^8$ 1/m. (**a**) Tension load and (**b**) compression load

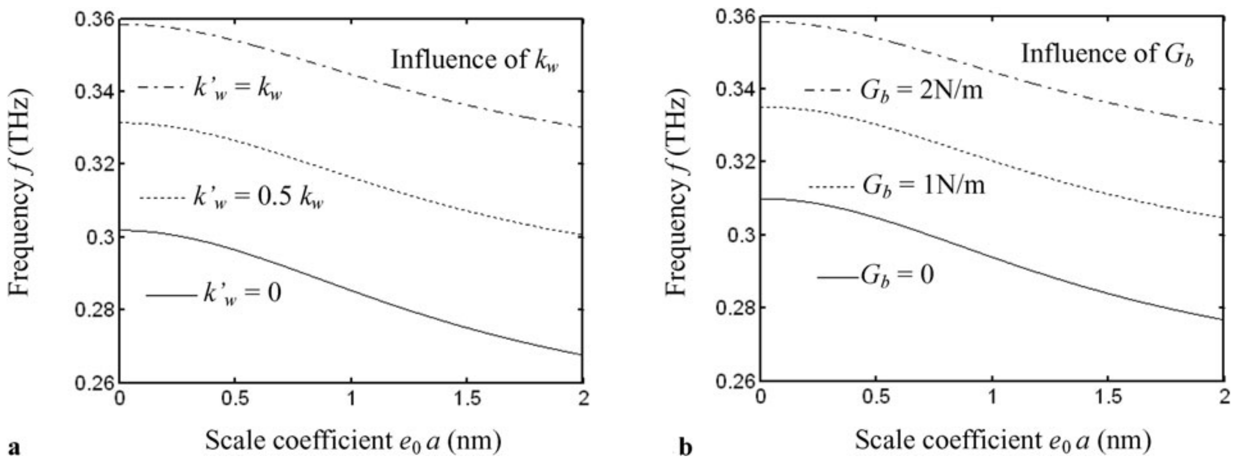


Fig. 4 Frequency f vs. scale coefficient e_0a with the effects of the Winkler foundation modulus and the stiffness of the shearing layer for the wave number $k = 7 \times 10^8$ 1/m. (**a**) Influence of the Winkler foundation modulus and (**b**) influence of the stiffness of the shearing layer

the initial stress for the tension load. However, this phenomenon is just the opposite for the compression load. It means that the direction of the initial stress is an important factor to affect the characteristics of the wave propagation in the nanoplate. Moreover, the frequency becomes smaller with the scale coefficient increasing for both tension and compression initial stress.

Figures 4(a) and 4(b) show the effects of the Winkler foundation modulus and the stiffness of the shearing layer on the relation between the scale coefficient and the frequency. The initial stress is assumed to be the tension one and $\sigma_0 = 0.005E$. It can be seen that the frequency increases with the Winkler foundation modulus becoming larger, which is similar to the result of the stiffness of the shearing layer. Further investigation shows that the influences of the k_w and G_b for the initial tension stress in Figs. 4(a) and 4(b) can also be found for the initial compressive stress. It implies that the characteristics of the wave propagation can be tuned by choosing the appropriate values of the two coefficients.

4 Conclusions

In this paper, the characteristics of the flexural wave propagating in the nanoplate are investigated by the nonlocal continuum model. We can observe that the small-scale effect plays an important role in the dispersion behaviors for larger wave numbers. It shows that the frequency becomes larger with the initial tension stress increasing, which is just the opposite for the initial compressive case. Furthermore, larger values of the Winkler foundation modulus and the stiffness of the shearing layer will result in the frequency increasing. This work is expected to be helpful in designing the nano structures in small scale physical devices.

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