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Exciton-polaritons in microcavities: present and future

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ABSTRACT An overview is given of the current research trends in the physics of exciton-polaritons in microcavities. Potential applications of the Bose–Einstein condensation and superfluidity of exciton-polaritons are discussed. The perspectives for the realization of polariton lasers, polariton spin transistors and polarization modulators are presented.

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1 Introduction

Light–matter coupling effects have stimulated development of optics during last three centuries. Nowadays, the limits of classical optics can be broken in a number of solid state systems, and quantum optics becomes an important tool for the understanding and interpretation of modern optical experiments. The rapid progress of crystal growth technology in the 20th century allowed for realization of crystal microstructures, which have unusual and extremely interesting optical properties. A large variety of intriguing optical phenomena are taking place in confined solid state structures, such as microcavities [1, 2].

Microcavities represent a unique laboratory for quantum optics and photonics. The central object of studies in this laboratory is the exciton-polariton: a half-light-half-matter quasiparticle exhibiting very specific properties and playing a key role in a number of beautiful effects including superfluidity, superradiance, entanglement, etc. Microcavities serve as building blocks for many optoelectronic devices such as light-emitting diodes, lasers and future polariton devices.

Semiconductor microcavities have attracted the special attention of physicists because of the discovery of the strong light–matter coupling regime. It was experimentally demonstrated by C. Weisbuch and co-authors in 1992 that the interaction of a confined light mode with an exciton state in a microcavity may lead to the appearance of two new eigenstates of the system that have different energies other than bare exciton and photon states [3]. These two new modes are associated with mixed light-matter quasi-particles called exciton-polaritons. At present, hundreds of research groups all over the world are working on the fabrication, optical spectroscopy, theory and applications of microcavities. The progress in this interdisciplinary field at the interface between optics and solid state physics is extremely rapid.

From a practical point of view, semiconductor microcavities appear to be a highly suitable system for realizing a new generation of optoelectronic devices, polariton devices, including polariton lasers [4], optical switches, and spin-memory elements. Apart from promising applications, the concept of polariton lasing involves several fundamental physics issues. Contrary to conventional lasers, polariton lasers emit coherent and monochromatic light without population inversion. This is achieved when the exciton-polaritons Bose-condense in a microcavity. The Bose–Einstein condensation (BEC) of polaritons is a subject of intense experimental and theoretical research at present [5, 6].

In 2000, the experimental observation of stimulated scattering of exciton-polaritons in microcavities [7] and the subsequent disentanglement of the spin dynamics of this process by the J.J. Baumberg group at the university of Southampton inspired substantial efforts on the bosonic effects in microcavities. W. Langbein in Dortmund studied the spatio-temporal dynamics and Rayleigh scattering of resonantly excited exciton-polaritons and observed the fluxes of linearly polarized polaritons propagating ballistically over 100 microns [8], which paved the way to generation of polariton spin currents. First experimental data on BEC of the exciton-polaritons appeared recently [9–11]. To date, no experimental evidence for polariton superfluidity has been reported. The difficulties come from the localization of exciton-polaritons by structure imperfections that prevent the propagation of a superfluid at least at low polariton densities. New high quality structures and a research program focused specifically on the polarization properties and spatial dynamics of exciton-polaritons would enable direct observation of the superfluid transition.

From the theoretical point of view, while the superfluidity of exciton-polaritons is usually considered as a result of the Kosterlitz–Thouless phase transition [12], the physical processes in the system and the peculiarities of the phase transition are expected to be significantly more diverse and inter-

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esting. A signature of polariton superfluidity is the appearance of a linear long-wave-length part in the dispersion of quasi-particles (i.e. a Bogolubov-like dispersion). The dispersion branches are spin-dependent for a two-dimensional polariton superfluid. This allows generation of superfluid spin-currents carried by electrically neutral exciton-polaritons. A theory of spin-entangled superfluids is far from being built, since there are important differences between polaritonic and atomic systems, which are expected to affect the dispersion and spatial dynamics of polariton superfluids, namely the:

- the strictly two-dimensional motion;
- the finite radiative lifetime of polaritons;
- non-parabolic dispersion;
- the peculiar spin-structure, with only two spin projections allowed on the axis of the cavity;
- a longitudinal-transverse (LT) splitting, which mixes spin-up and spin-down polariton states.

A recent paper by Carusotto and Ciuti [13] suggests a description of polariton superfluids through a modified Gross–Pitaevskii (GP) equation. Although an important step towards understanding polariton superfluidity, the model of Carusotto–Ciuti neglects the polarization of polaritons and their longitudinal-transverse splitting, which have previously been shown to be extremely important for understanding the dynamics of interacting polaritons [14]. The present author has theoretically and experimentally studied the spin-dynamics of exciton-polaritons over the last five years [15, 16] and predicted the optical spin Hall effect [17] that manifests itself in the spin-dependent Rayleigh scattering of light by microcavities. Our group has developed a dynamical theory of the formation of coherence in polariton lasers [18]. In a recent paper [19], we demonstrated the existence of two sound velocities in polariton superfluids. These existing models provide the basis for the construction of a full quantum theory of polariton superfluids.

In this paper a brief overview of the linear and nonlinear optical properties of semiconductor microcavities in the strong coupling regime is given. Emphasis is placed on the recently discovered Bose–Einstein condensation and the perspectives for studies of the polariton superfluidity. The paper is organized as follows: In Sect. 2 the eigenmodes of exciton-polaritons in microcavities are introduced and their angular dispersion discussed. In Sect. 3 an experiment on parametric scattering of exciton-polaritons is addressed. Sect. 4 overviews the recent experimental and theoretical work on the Bose–Einstein condensation and superfluidity of exciton-polaritons.

2 Exciton-polariton eigenmodes in semiconductor microcavities

Consider a planar semiconductor structure composed by two distributed Bragg reflectors (or Bragg mirrors) and the cavity layer sandwiched between them (Fig. 1). The reflectivity spectra of such structures are characterized by wide plateaus corresponding to the stop bands or photonic gaps of the Bragg mirrors, and sharp dips at the centres of the plateaus corresponding to the eigenmodes of the systems. In an empty microcavity (Fig. 1a), the only dip corresponds to the Fabry–Pérot mode of the resonator (further referred to as a bare cavity mode), while if quantum structures containing excitons, like quantum wells (QWs), are embedded, the characteristic double-dip structure may occur (Fig. 1b). The field profile inside the cavity is drastically different at the eigenmode frequency and away from it, as Fig. 1c and d shows: the eigenmode of the structure is localized in the cavity and exponentially decays inside both mirrors.

In the vicinity of the resonance of the excitonic transition in the quantum wells with bare cavity mode, exciton-light coupling may lead to either crossing or anticrossing of the real parts of the eigenfrequencies of the structure modes (also called exciton-polariton modes), as is schematically shown in Fig. 2. The spectrum of the system is described by the coupled-oscillator equation:

$$(\omega_0 - \omega - \mathrm{i}\gamma)\,(\omega_c - \omega - \mathrm{i}\gamma_c) = V^2\,, \tag{1}$$

where ω_0 is the exciton resonance frequency, γ is the exciton non-radiative damping rate, $\omega_c - \mathrm{i}\gamma_c$ is the complex eigenfrequency of the cavity mode in the absence of exciton-light coupling, V is the coupling constant dependent on the exciton oscillator strength and the penetration depth of the Bragg

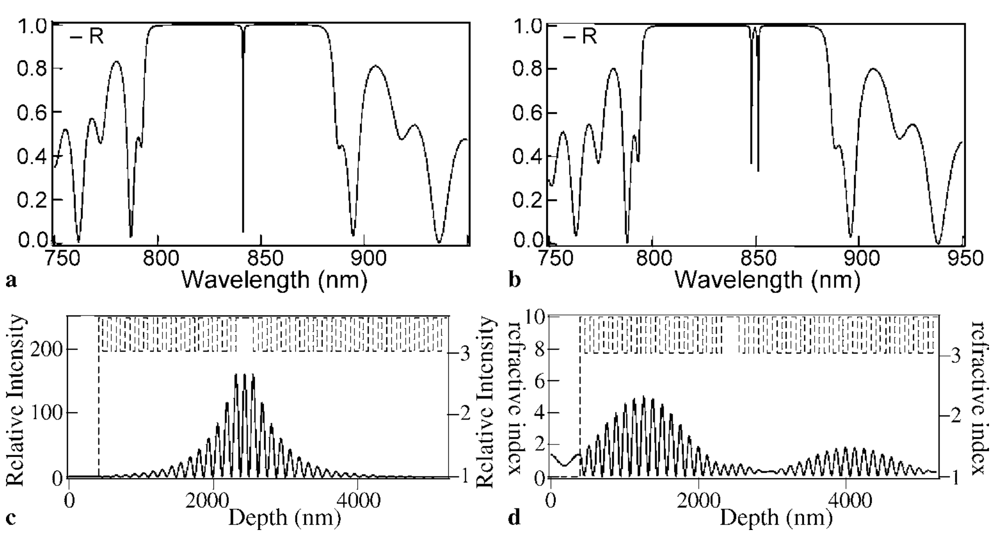

FIGURE 1 (**a**, **b**) Calculated reflectivity of a planar microcavity consisting of top and bottom Bragg mirrors with 15 and 21 pairs of GaAs/AlAs layers, respectively. In (**a**) a 240 nm bulk GaAs cavity is incorporated while in (**b**) three InGaAs quantum wells are incorporated in the same region. (**c**, **d**) show the electromagnetic field distribution for the cavity (**a**) at different wave lengths: 841 nm (**c**) and 787 nm (**d**)

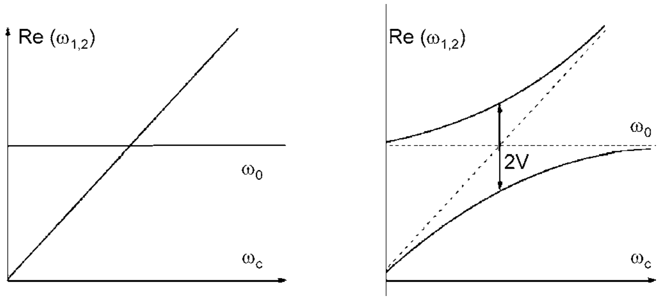

FIGURE 2 Real parts of the eigenfrequencies of the exciton-polariton modes in a microcavity in the weak coupling regime (*left*) and in the strong coupling regime (*right*)

mirrors. Equation (1) has two solutions:

$$\omega_{1,2}=\frac{\omega_0+\omega_c}{2}-\frac{i}{2}(\gamma+\gamma_c)\pm\sqrt{\left(\frac{\omega_0-\omega_c}{2}\right)^2+V^2-\left(\frac{\gamma-\gamma_c}{2}\right)^2+\frac{i}{2}(\omega_0-\omega_c)(\gamma_c-\gamma)}\,. \quad (2)$$

If $\omega_0=\omega_c$, the splitting of the two solutions is given by $2\sqrt{V^2-\left(\frac{\gamma-\gamma_c}{2}\right)^2}$. If

$$V>\left|\frac{\gamma-\gamma_c}{2}\right|\,, \quad (3)$$

the anticrossing takes place between exciton and photon modes, which is characteristic for the strong-coupling regime. In this regime, two distinct exciton-polariton branches manifest themselves as two optical resonances in the reflection or transmission spectra. The splitting between these two resonances is referred to as vacuum-field Rabi splitting. It reaches 4–15 meV in existing GaAs-based microcavities, up to 30 meV in CdTe-based microcavities, and is even larger in GaN cavities. If

$$V<\left|\frac{\gamma-\gamma_c}{2}\right|\,, \quad (4)$$

the weak-coupling regime holds, which is characterised by the crossing of the exciton and photon modes and an increase of the exciton decay rate at the resonance point. This regime is typically used in vertical cavity surface emitting lasers (VCSELs).

Equation (1) can be easily generalized for the oblique incidence case by introduction of the dependence of the cavity and exciton eigenmode frequencies ω_c and ω_0 on the in-plane wave-vector k_{xy}:

$$\omega_c=\frac{\hbar k_{xy}^2}{2m_{ph}}\,,\quad \omega_0=\frac{\hbar k_{xy}^2}{2M_{exc}}\,, \quad (5)$$

where M_{exc} is the sum of the electron and hole effective masses in the QW plane, and m_{ph} is the photonic effective mass. In an ideal λ-microcavity, the normal-to-plane component of the wave-vector of the eigenmode is given by

$$k_z=\frac{2\pi}{L_c}\,.$$

The energy of the mode is

$$\omega_c=\frac{c}{n_c}\sqrt{k_{xy}^2+k_z^2}\approx\frac{c}{n_c}k_z\left(1+\frac{k_{xy}^2}{2k_z^2}\right)\equiv\frac{2\pi c}{n_c L_c}+\frac{\hbar k_{xy}^2}{2m_{ph}}\,, \quad (6)$$

and thus $m_{ph}=\frac{hn_c}{cL_c}$. This mass is extremely light in comparison with the exciton mass, and usually amounts to 10^{-5}–10^{-4} m_0, where m_0 is the free exciton mass. Note also that the in-plane wave-vector k_{xy} is related to the angle of incidence of light illuminating the structure, φ, by the relation:

$$k_{xy}=\frac{\omega}{c}\sin\varphi\,. \quad (7)$$

By measuring the angle dependence of the resonances in the reflection or transmission spectra of microcavities, one can restore the true dispersion curves of exciton-polaritons. Figure 3 shows the angular dispersion of the exciton-polariton modes in a typical GaAs-based microcavity in the strong coupling regime and at zero detuning. The detuning of bare photon and exciton modes in a microcavity $\delta\equiv\omega_c-\omega_0$ is an important parameter, which strongly affects the shape of polariton dispersion curves in the strong coupling regime. One can see that the low-polariton dispersion branch is strongly non-parabolic. Its central part is characterized by an extremely light photon effective mass, while the outer parts have a relatively heavy exciton effective mass. This produces the so called "polariton

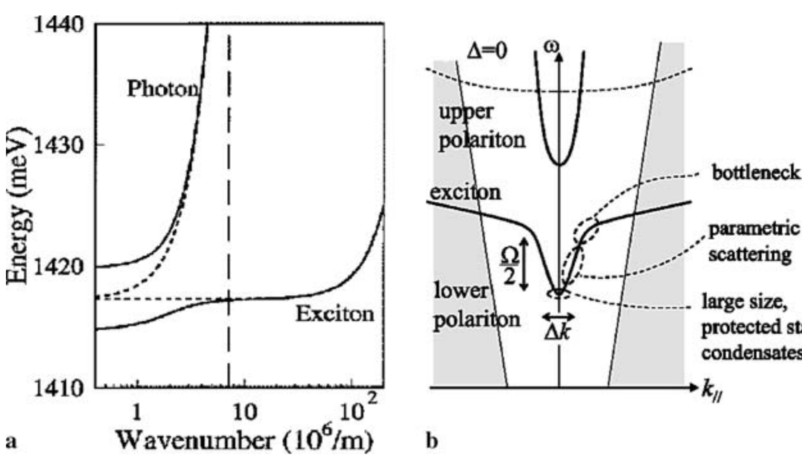

FIGURE 3 Angular dispersion of the exciton-polariton modes in a typical strong-coupling microcavity at zero detuning; (**a**) on the logarithmic scale illustrates the difference of the effective masses of the exciton and photon modes, (**b**) the critical regions around the polariton trap

trap" shown in detail in Fig. 3b: the polaritons with small in-plane wave vectors are trapped in a quasiparabolic region of the dispersion with a very light effective mass. The acoustic phonon assisted relaxation is forbidden by energy and wave-vector conservation laws in this region, so that the polaritons can thermalise only due to interactions with themselves. Being isolated from the phonon bath, the polaritons in the trap keep coherence during their life-time (typically 1–5 ps) and are subject to interesting quantum coherent effects, which are addressed in the next sections.

3 Polariton parametric scattering and amplification

First pump–probe experiments on semiconductor microcavities were performed with two beams under normal or quasi normal incidence [20–22]. The basic idea was to modify the system using an intense pump pulse and to record the resulting polarization by measuring the reflection, transmission, absorption or scattering of a weak probe pulse. The main objective of the first investigations was to elucidate the mechanism responsible for the loss of the strong coupling regime. The unexpected breakthrough came from an experiment performed in 2000 by Savvidis et al. [23] who reported a remarkable new non-linear effect. They excited resonantly with a 1 ps pump pulse in the vicinity of the lower polariton branch's inflexion point. In their microcavity this corresponded to an excitation angle of 16.5°. With a controlled delay they excited the lower branch ground state with a weak probe pulse at normal incidence and measured its reflection. This probe pulse was found to be amplified more than 70 times. The physical process involved in this amplification is the resonant scattering of two pumped polaritons into one "signal polariton" and one "idler polariton". The amplification was found to be huge because the stimulated polariton scattering is resonant, i.e. it conserves both the energy and the wave vector.

Figure 4 shows the experimental configuration, the emission spectra of pump, signal and idler polariton states and the power dependence of the signal emission. This experiment showed the bosonic behaviour of cavity polaritons. The main mechanisms governing optical non-linearity in microcavities were also shown. This discovery has given a new impulse to the whole microcavity research field. An avalanche of experimental and theoretical works followed Savvidis' work, revealing rich and deep physical phenomena.

From the point of view of classical optics, the resonant polariton–polariton scattering represents a kind of a four-wave mixing process. It can also be interpreted as a parametric amplification or parametric scattering process. Recently, a microscopic optical parametric oscillator based on triple microcavities was reported [24]. It has a remarkable practical importance: note that all commercially available optical parametric oscillators have a macroscopic size, at present.

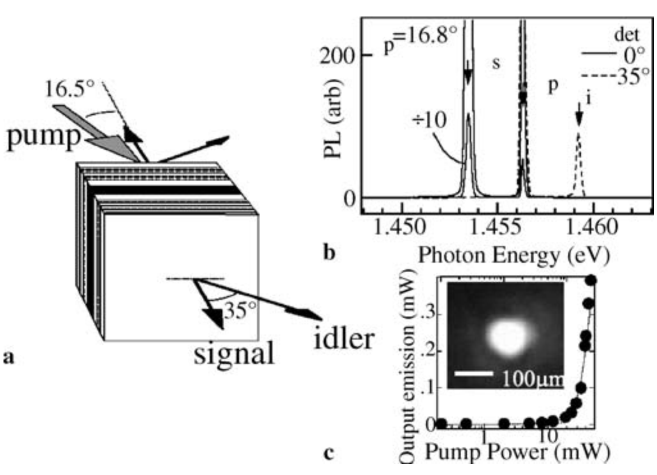

FIGURE 4 (**a**) Geometry of the microcavity based parametric oscillator, (**b**) its photoluminescence spectrum at $k = 0$, (**c**) power dependence of the main peak amplitude. From [7]

4 Polariton Bose condensation and superfluidity

Imamoglu [25] was the first, in 1996, to point out how the bosonic character of cavity polaritons can be used to create an exciton-polariton condensate that emits coherent laser light. The build-up of a ground-state coherent population from an incoherent exciton reservoir can be seen as a phase transition towards a Bose condensed state, or as a polariton-lasing effect resulting from bosonic stimulated scattering.

A polariton laser is rather different from a polariton parametric amplifier described in the previous section. In the polariton laser, the system is excited non-resonantly, optically or electronically, resulting in a cloud of electrons and holes that form excitons, which subsequently thermalise at their own temperature mainly through exciton–exciton interactions. They reduce their kinetic energy by interacting with phonons and relax along the lower polariton branch (see Fig. 3b). They finally scatter to their lower-energy state, where they accumulate because of stimulated scattering. The coherence of the condensate therefore builds up from an incoherent equilibrium reservoir and the associated phase transition can be interpreted as a BEC.

Once condensed, polaritons emit coherent monochromatic light. As the light emission by a polariton quasi-condensate is spontaneous, there is no population inversion condition required in polariton lasers, absorption of light does not play a role, and ideally there is no threshold for lasing. Concerning this latter point, the argument is that it is sufficient to have two polaritons in the ground state to create a condensate, which will result in its disintegration with the emission of two coherent photons. Moreover, because of the small polariton mass, critical temperatures larger than 300 K can be achieved. All these characteristics combined make polariton lasers ideal candidates for the next generation of laser-light emitting devices. The main hurdle to effectively obtain polariton condensates in microcavities is the too short lifetime of the particles; polaritons disappear before they have time to condense. This difficulty is exacerbated by the slow relaxation kinetic towards the ground state, the so-called bottleneck effect (see Fig. 3b). Formation of the condensate within the polariton lifetime is only possible because of the rapidity of stimulated scattering that should be able to overcome the bottleneck effect.

The evidence for BEC of exciton-polaritons in CdTe-based microcavities was recently obtained by a group from Grenoble University headed by Le Si Dang [10, 11]. They increased the pumping intensity and observed how the distribu-

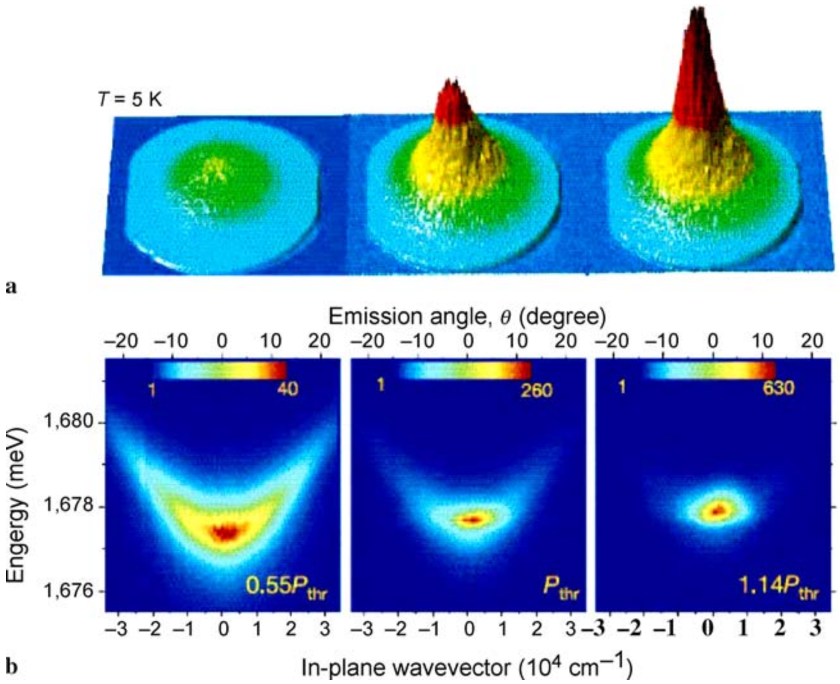

FIGURE 5 Far field angular resolved emission of a CdTe-based high quality microcavity sample below and above the threshold to the Bose–Einstein condensation of exciton polaritons. From [11]

tion of the exciton-polaritons in the reciprocal space localized at the $k = 0$ point, so most of the light emitted by the microcavity was generated by a single quantum state (Fig. 5). Furthermore, by performing a classical interferometry experiment, they observed that the light emitted by the cavity becomes coherent above the threshold pumping. Finally, the dramatic build-up of the linear polarization degree of light emitted by the cavity was detected at the threshold, which was in excellent agreement with theoretical predictions [26, 27].

Strictly speaking, the BEC observed by Le Si Dang should rather be referred to as quasi-condensation. Actually, the true BEC cannot happen in a two-dimensional system. The collective bosonic modes or condensates can be formed either by a resonant excitation (like in Baumberg's experiment), or due to the superfluid phase transition, or due to thermalisation of the Bose gas in a finite size system with a discrete spectrum of states. The observation by Le Si Dang et al. fully corresponds to the latter scenario, as they observe strong spatial localization of the condensate, which forms a kind of drops pinned at shallow potential wells created by the cavity width fluctuations.

Observation of the superfluid phase transition in two-dimensional microcavities would be a natural next step in the research on collective phenomena in microcavities. The superfluids are formed from weakly interacting bosonic gases. Due to the repulsive interactions between bosons, their spectrum changes, so that a linear part (referred to as the Bogolyubov or sound mode) appears near $k = 0$. The specifics of polariton superfluids was analysed by our group in [19]. We showed that due to the repulsion of polaritons having parallel spins the energetically preferential state of a polariton condensate (superfluid) is linearly polarized. There are two sound-like branches of the excited polariton states above the condensate (and not one, like in the original Bogolyubov theory of superfluidity [28–30]). They correspond to polaritons that are co-polarized and cross-polarized with respect to the condensate. The sound velocity of a polariton superfluid is therefore polarization-dependent, which leads us to expect the spontaneous separation of polarization in the polariton supercurrents.

Furthermore, very recently we theoretically predicted the behavior of the exciton-polariton superfluids in the presence of the magnetic field normal to the microcavity plane [31]. In the presence of the magnetic field the difference between sound velocities of two branches of the excited polariton states increases up to some critical field dependent on the concentration of polaritons and the polariton–polariton interaction constant (Fig. 6). At the critical field, one of the branches becomes parabolic, so that the superfluidity disappears (we recall that the necessary condition for superfluidity is the sound like dispersion of excitations in the vicinity of the

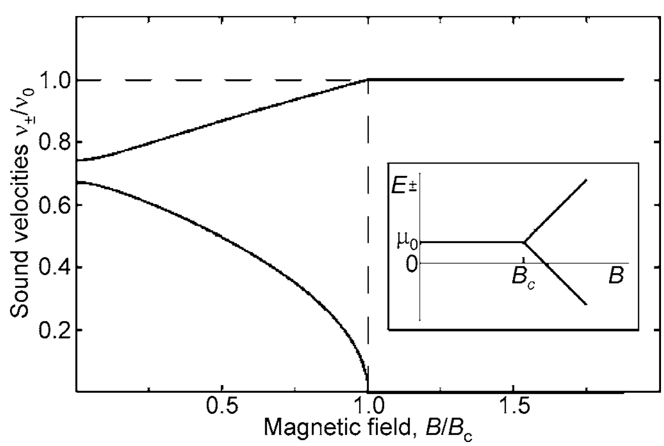

FIGURE 6 Dispersion of two sound branches of the polariton superfluid corresponding to co- and cross-linear polarizations with respect to the polarization of the condensate. The *inset* shows the Zeeman splitting of the polariton condensate: below the critical magnetic field it is fully suppressed, which manifests the full paramagnetic screening or "spin Meissner effect". From [31]

ground state of the system). Very interestingly, below the critical field, the condensate emits light at the constant energy. The red shift of the polariton energy due to the Zeeman effect is exactly compensated by an increase of the blue shift due to polarization of the condensate. Suppression of the Zeeman splitting of the condensate (see the inset in Fig. 6) is a manifestation of the full paramagnetic screening also referred to as the spin Meissner effect (in analogy to the full diamagnetic screening, or Meissner effect in superconductors). Above the critical field, Zeeman splitting is present. The lowest energy branch keeps the linear dispersion, thus the superfluidity reappears. The higher energy branch has a parabolic dispersion.

Superfluidity of the exciton-polaritons still needs to be experimentally demonstrated. This would require high-quality samples where the in-plane potential disorder does not induce localization of the condensate. On the other hand, the spin Meissner effect can be observed in localized condensates.

In conclusion, microcavities represent a rapidly developing field of solid state physics, which is extremely rich in fundamental effects. This is a unique system allowing observation of the quantum effects at high temperatures (up to room temperature). The applications of cavity polaritons in new light sources, polariton lasers, polarization switches, superfluid spintronic devices, etc. are foreseen. Very interesting potential applications might result from realization of charged polariton superfluids in microcavities containing a free electron gas.

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