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REGULAR ARTICLE

Assessing the reliability and the expected performance of a network under disaster risk

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Abstract In a disaster situation, functionality of an infrastructure network is critical for effective emergency response. We evaluate several probabilistic measures of connectivity and expected travel time/distance between critical origin–destination pairs to assess the functionality of a given transportation network in case of a disaster. The input data include the most likely disaster scenarios as well as the probability that each link of the network fails under each scenario. Unlike most studies that assume independent link failures, we model dependency among link failures and propose a novel dependency model that incorporates the impact of the disaster on the network and at the same time yields tractable cases for the computation of the probabilistic measures. We develop algorithms for the computation of the measures and utilize a Monte Carlo simulation algorithm for the intractable cases. We present a case study of the Istanbul highway system under earthquake risk, and compare different dependency structures computationally.

Keywords Network reliability · Vulnerability · Dependent link failures · Disaster risk

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1 Introduction

Increasing incidents of terrorism and natural disasters worldwide have led to heightened interest in identifying and reducing the vulnerability of infrastructure networks (Auerswald et al. 2005; Perrow 2007; Matisziw et al. 2009). The post-disaster functionality of infrastructure networks, such as a telecommunication, power, water, energy, or transportation network, is particularly critical for effective disaster response and recovery. In a disaster situation, local and central government agencies as well as civil organizations mobilize their resources immediately to rescue victims and to supply medical care, machinery, and relief commodities to the affected areas. In addition to the time-critical operations carried out by the agencies, some residents will be on the roads trying to evacuate the affected areas while others will try to reach the area to provide humanitarian aid and to help their relatives. As a result, the proper functionality of the transportation network is essential for the success of the rescue and relief operations.

It is commonly observed that a disaster may render some of the links of the transportation network non-functional, leading to the blockage of some routes and/or disconnectedness of some areas in need of aid. In the pre-disaster planning stage, it is important to assess the post-disaster performance of the network under possible disaster scenarios for the purpose of both strengthening the components of the network and for planning the post-disaster logistics activities.

Our goal in this study was to measure the reliability and the expected post-disaster performance of a network under disaster risk. An important aspect of this problem is the uncertainty in the pre-disaster stage that is due to both the magnitude and the location of the disaster to occur, and furthermore, how the infrastructure network would be affected from the disaster. Typically, several predicted disaster scenarios characterized in terms of their intensity and geographic area of influence can be identified. In addition, the vulnerability of the civil components of the network can be evaluated by engineers, mostly by structural analysis and statistical predictive methods. In this study, we assume that each link of the network will be randomly in one of two states after the disaster: (i) operational (it survives), or (ii) non-operational (it fails), as is customary in the network reliability literature. One can argue that a partially damaged link will not be available for immediate use due to the associated risk and inconvenience. Here, a more important concern for analysis purposes is whether the links would fail independently or not. When the common independence assumption in earlier studies seems to be unrealistic in the disaster context, one needs to identify the nature of dependence.

In post-disaster response, several nodes in the network act as supply and demand points, e.g., in terms of relief aid distribution and casualty transportation, thereby creating pairs of origin–destination (O–D) nodes whose connectedness carries high priority. The reliability and expected performance measures that we propose in this study vary in terms of the number of disaster scenarios and the number of O–D pairs under consideration. These measures generalize the expected network performance defined in Peeta et al. (2010) for disaster response by extending it to multiple disaster scenarios. The exact calculation of any general network reliability measure, including the basic measures of two-terminal and all-terminal connectivity, is known to be

♯P-Hard (Konak and Smith 2006). This is also the case for the measures we propose due to the exponential number of possible states. One way to overcome this computational difficulty is to reduce the state space by identifying relations among the link failures that would be pertinent to the particular disaster context. When statistical dependency exists among link failures, a joint probability distribution is needed. However, as major disasters are rare events, in most cases a sufficient amount of data do not exist to fit a joint probability distribution. Yet, a practical method is needed for quantitative analysis.

In this study, we define a novel link dependency structure that allows the existence of a polynomial-time algorithm when the number of O–D paths in the network is fixed. The proposed structure serves the practicality requisite while providing a reasonable approach to cope with insufficiency of data. First, network links are partitioned into sets by geographic proximity and the degree of disaster risk they are exposed to. Links in different sets are assumed to fail independently. Next, within each set, the vulnerability of the link components are taken into consideration to define a dependency structure that we call vulnerability-based dependency. We characterize the joint probability distribution of the network realizations under the proposed dependency structure and present efficient algorithms to compute the proposed measures. We illustrate the use of this framework by a case study related to a highly anticipated earthquake in Istanbul. We consider the Istanbul highway network whose links are likely to fail due to the collapse of structures such as bridges and viaducts in the aftermath of an earthquake. In this context, we also provide a comparison between the independent and dependent link failure cases by analyzing how the performance measures vary.

The rest of the paper is organized as follows: We review previous work related to our study and give an overview of our approach in Sect. 2. Problem definition and notation given in Sect. 3 is followed by the description of both the polynomial-time exact algorithms and the heuristic Monte Carlo simulation algorithm in Sect. 4. Computational results of the Istanbul case are reported and discussed together with a comparison between independent and dependent link failure cases in Sect. 5. Finally, concluding remarks are given in Sect. 6.

2 Literature review

Network reliability has been studied extensively, in terms of both identifying the difficulty of computing various measures under differing graph structures and developing exact and approximate algorithms (Ball et al. 1995). A main characteristic of these studies is that edge failures are component based and occur randomly over time; hence, they are not exogenous as in the case of failures due to a disaster event. As devastating natural and man-made disasters have been experienced world wide, especially within the past decade, network risk/reliability analysis gained a revived interest in the context of disaster preparedness. In this section, after a brief review of studies on network reliability measures and their computation, we focus on studies specifically in the area of network reliability and performance under a disaster, and related work on link failure dependency.

2.1 Network reliability measures

Network reliability measures have been collected under the two groups of connectivity and performability (Ball et al. 1995). Connectivity is defined as the probability that nodes of a network remain connected, whereas network performance targets the functionality of a network. Connectivity measures include (i) *Two-terminal reliability* (the probability of a path existing between two specific nodes), (ii) *All-terminal reliability* (the probability that every node is connected with every other node), and (iii) *K-terminal reliability* (the probability that every pair of nodes in a specified node subset K is connected), as described in Konak and Smith (2006). Among network performance measures, Taylor et al. (2006) define *travel-time reliability* as the probability that a trip between two specified nodes can be completed within a specified time interval. The *capacity reliability* performance index is the probability that a network can successfully accommodate a given level of travel demand. Lin evaluates the probability of the transmission of given data in a limited time from a source to a sink node over a single path (Lin 2003) and multiple paths (Lin 2010), when arc capacities are uncertain. The reader is referred to the extensive survey on network reliability by Ball et al. (1995) which includes the related definitions, and Konak and Smith (2006) for a more recent review of the reliability measures and the reliable and resilient network design problems.

When the links can be in two states, i.e., functional and non-functional, the number of possible network realizations is 2^m for a network with m links and the exact calculation of network reliability measures is known to be $\sharp$P-Hard (Konak and Smith 2006). Since algorithms that calculate reliability measures exactly in general networks take exponential time, several polynomial time exact algorithms have been developed for some restricted classes of networks (Konak and Smith 2006). In addition, heuristic methods such as sampling and Monte Carlo simulation are widely used (Chen et al. 2002; Buchsbaum and Mihail 1995) to cope with the computational complexity in general networks. Buchsbaum and Mihail (1995) presented a heuristic based on Monte Carlo and Markov Chain simulation techniques and proposed approximations and bounds on various reliability-related parameters. Matisziw et al. (2009) utilized a simulation approach to assess connectivity and flow under generated network disruption scenarios. Sumalee and Kurauchi (2006) utilized Monte Carlo simulation as well, where they generated link states and the amount of degradation in functionality of the links after the disaster randomly. Karger (1995) developed a fully polynomial randomized approximation scheme (FPRAS) to estimate the all-terminal connectedness using minimum cuts. Carey and Hendrickson (1984) provided upper and lower bounds for the expected maximum flows in capacitated networks.

2.2 Network risk assessment for disaster preparedness

The impact of different types of disasters, including flood (Sohn 2006) and earthquake (Selcuk and Yucemen 2000; Moghtaderi-zadeh and Kiureghan 1983; Chang and Nojima 2001) on transportation, water or gas pipeline networks have been analyzed for pre-disaster planning. In addition to reliability, often measures of network vulnerability, risk and accessibility have been of interest. While commonly accepted definitions are not available, many researchers use the term vulnerability closely aligned with network weaknesses and consequences of failure, in contrast to reliability that focuses on the probability of failure. Jenelius et al. (2006) argued that vulnerability appears when the network is under pressure with full capacity, and a small amount of further stress may cause a major damage that may cascade through the system. This implies that a network can be reliable, yet highly vulnerable at the same time. In the disaster context, vulnerability arises due to the weakness of its components under stress factors caused by a potential disaster rather than the everyday load.

An analysis of network vulnerability can guide strategic or tactical level decisions for risk reduction. Moghtaderi-zadeh and Kiureghan (1983) stated that their study was a first attempt at systematic methods for efficient upgrading of lifelines for post-disaster earthquake serviceability. They determined the critical components in a network that would increase the connectivity-reliability of the network the most. For a given magnitude and location of an earthquake, they determined which links will fail and which ones will survive based on a distance threshold and calculated the probability that the network is functional. They proposed to invest incrementally in the critical components to increase system reliability as a heuristic approach. In a recent application study Matisziw and Murray (2009) examined the Ohio interstate highway network to identify network facilities most vital to system flow. In a general approach, they addressed the computational challenges associated with path-based models for an uncapacitated network and developed an alternative constraint structure for establishing an upper bound on worst-case network disruptions. Selcuk and Yucemen (2000) considered the reliability of lifeline networks with multiple sources under seismic hazard and proposed a decision support system that utilizes a probabilistic model for the evaluation of the seismic reliability of a water distribution system. Peeta et al. (2010) addressed the link upgrading problem under a limited budget and a disaster scenario with the purpose of effective post-disaster response and applied their heuristic approach to the case of Istanbul for earthquake preparedness. In that study, an optimization model was formulated to make an investment decision on which links to upgrade for better survivability after a disaster. The objective was to maximize the expected performance of the network under budget constraints. The links were assumed to fail independently from each other. The previous studies support the need for quantitative measures that enable effective analysis of post-disaster network performance under disaster risk, especially with the consideration of link failures due to the disaster.

2.3 Link failure dependency

In most studies failure of network links have been assumed to be independent events. When link failures occur due to a common cause such as a disaster, it is often necessary to treat the link failures as dependent events. This was also pointed out by Garg and Smith (2008), who stated that in the context of emergency service deployment, disasters such as earthquakes may damage several roadways simultaneously, mainly due to the presence of bridges or elevated roadways. Hence, contingency plans for providing relief to affected areas need to consider the potential failure of certain vulnerable links, some of which may simultaneously fail. However, it is challenging to define the dependency relationship among the link failures.

There are a limited number of studies that take into account the dependency relationship among the links. Sumalee and Watling (2003) proposed a scenario-based model where the causes of degradation of links vary in each scenario. Each link is given a failure probability called *conditional independence* due to each specific cause. Integrating these failure probabilities with the joint probability of the causes creates link dependencies implicitly. Most probable network states are generated for the most probable cause scenarios to calculate upper and lower bounds. Taylor et al. (2006) presented a vulnerability analysis for the independent link failure case and suggested that the same procedure can be modified to a dependent failure case by considering the node failures that would lead to simultaneous failure of the links attached to them. In studies related to network interdiction, such as (Matisziw et al. 2009), network disruption scenarios, in which a number of nodes/links have been eliminated, are generated to assess network vulnerability. In this context, the simultaneous failure of nodes/links in a disruption scenario can be considered to represent link failure dependencies. Then, generation of the scenarios in a realistic setting becomes an important issue and may be tackled by simulation, as in Matisziw et al. (2009). In general, scenario-based approaches do not propose a specific dependency structure.

Selcuk and Yucemen (2000) studied the reliability of networks under seismic hazard. Although they used independent failures in their study, they proposed *spatial correlation*, where the degree of correlation between any two components depends on the distance separating them and generally decays with increasing distance. In this study, our motivation is also related to earthquakes, and seismic risk of highway networks. In fact, we also identify distance to the epicenter of the earthquake as an important risk factor. However, rather than deriving a function of risk with respect to distance, which may be sensitive to the estimated parameters, we define areas of high/low risk, in accordance with existing studies that generate zones with respect to a spatial risk analysis (JICA-IMM 2002). As an application of spatial correlation, Black (1992) suggests an approach to analyze the existence of autocorrelation in variables distributed along the arcs of a network by means of a weight matrix. For instance, the weight between two arcs may represent a measure of similarity between the flow on these arcs due to their adjacency or spatial closeness. In our approach, we do not utilize a matrix as in Black (1992). Instead, we incorporate the similarity in terms of spatial proximity and risk level by defining sets of links whose survival probability are correlated. Then, we assume that within each set the correlation is governed by the vulnerability level of the structural components on the links. Therefore, we assume a vulnerability ranking among the links in a set, as explained further in Sect. 4.

In this study, we propose an original framework for calculating the reliability and performance of a network under disaster risk by modeling a dependency structure among link failures. We use the term *reliability* as the probability that two specified nodes, an origin and destination (O–D) pair, are connected and the *performance* of a network as the expected shortest path distance between the O–D pair, including a penalty cost for disconnectedness, as in Peeta et al. (2010). In this approach, first the set of links that show dependency among each other are identified according to geographic proximity and risk factors. Links in different sets are assumed to fail independently, whereas links in a set are assumed to exhibit a linear dependency order according to their vulnerability levels. That is, the failure of a link implies the failure of the weaker links in that set. Hence, links of a set define a polynomial number of network realizations. We distinguish between tractable and intractable cases in the proposed framework and develop algorithms for exact calculation for the tractable cases and estimation via sampling for the remaining.

3 Problem definition

We are given an undirected graph $G = (V, E)$, with vertex (node) set $V = \{v_1, v_2, \ldots, v_n\}$ and edge (link) set $E = \{e_1, e_2, \ldots, e_m\}$. Each O–D pair with index $d \in C$ has a positive weight, r_d, representing either the relative importance of the O–D by setting r_d to real values in the interval [0,1], or the estimated traffic demand that would travel from the origin node $O(d)$ to the destination node $D(d)$ in a disaster by setting r_d to appropriate positive integer values. A travel time, or distance, t_{e_i} is associated with each link $e_i \in E$.

A disaster is represented by a random variable that takes values from the possible set of disaster scenarios, $\Omega = \{\omega_1, \omega_2, \ldots, \omega_{|\Omega|}\}$. The probability that scenario ω_j occurs is denoted by $\mathrm{P}(\omega_j)$. Each link, e_i, may exist in either the operational or the non-operational state after the disaster. The post-disaster state of link e_i is denoted by a random variable ξ_{e_i} that takes the value 1, if link e_i is operational after the disaster; and 0, otherwise. The vector of realizations of the random variables ξ_{e_i} over all links e_i in E, denoted by $\xi = (\xi_{e_i}), \xi \subset \{0, 1\}^{|E|}$ induces a subnetwork of G; $G(\xi) = (V, E(\xi))$ which we refer to as the *surviving network*. Here, $E(\xi) = \{e_i \in E : \xi_{e_i} = 1\}$ denotes the set of surviving edges. The set of all network realizations is denoted by $\Xi = \{1, 2, \ldots, |\Xi|\}$. The survival probability (reliability) of link e_i under disaster scenario ω_j, i.e., $\mathrm{P}(\xi_{e_i} = 1|\omega_j)$ is denoted by $p_{e_i}(\omega_j)$ and the probability of occurrence of the vector realization ξ is denoted by $p(\xi)$. The distance of a shortest path from $O(d)$ to $D(d)$ in $G(\xi)$ (with respect to the t_{e_i}) is denoted by $T_d(\xi)$. If a particular O–D pair d is disconnected in a network realization, the shortest path length is set to a penalty cost M_d for that pair in that realization. Without loss of generality, the O–D pairs are assumed to be connected when all links are functional. With this notation, we next define the reliability and performance measures to be calculated.

Natural disasters can often be predicted in terms of an area of influence and intensity, albeit with inherent uncertainty. Typically, several likely disaster scenarios are identified for a specific region, for planning and preparedness purposes (for instance, see Chang et al. (2000) for earthquake scenarios). Here, we assume the availability of such scenarios, as in our case study in Sect. 5. In assessing post-disaster network performance, one may consider the impact of only the worst-case, or the most probable disaster scenario; hence, the single disaster scenario case arises. Alternatively, the impact of all possible scenarios on the network are analyzed together, by taking an expectation or some other risk measure; hence, the multiple disaster scenario case arises. We define eight measures, each one characterized by three features. The notation (./././.) is used to represent these features, where the first entry shows the number of O–D pairs, either S for single or M for multiple O–D pairs; the second entry denotes the number of scenarios, either S for single or M for multiple scenarios; and the third entry shows the measure, either R for reliability (probability of connectedness) or P for performance (expected shortest path length).

1. O–D reliability under a single disaster scenario (S/S/R): For an O–D pair d and a disaster scenario w_j,
   $F_1 =\mathrm{P}(O(d)$ and $D(d)$ are connected in $G(\xi) \mid \omega_j)$.
2. O–D reliability under multiple disaster scenarios (S/M/R): For an O–D pair d,
   $F_2 = \sum_{j=1}^{|\Omega|} \mathrm{P}(\omega_j)\, \mathrm{P}(O(d)$ and $D(d)$ are connected in $G(\xi) \mid \omega_j)$.
3. Multi O–D reliability under a single disaster scenario (M/S/R): For a disaster scenario w_j,
   $F_3 = \sum_{d\in C} r_d\, \mathrm{P}(O(d)$ and $D(d)$ are connected in $G(\xi) \mid \omega_j)$.
4. Multi O–D reliability under multiple disaster scenarios (M/M/R):
   $F_4 = \sum_{j=1}^{|\Omega|} \mathrm{P}(\omega_j)(\sum_{d\in C} r_d \mathrm{P}(O(d)$ and $D(d)$ are connected in $G(\xi) \mid \omega_j)$.
5. O–D performance under a single disaster scenario (S/S/P): For an O–D pair d, and a disaster scenario ω_j,
   $F_5 = \sum_{\xi\in\Xi} \mathrm{P}(\xi|\omega_j)\{\mathrm{P}(O(d)$ and $D(d)$ are connected in $G(\xi) \mid \omega_j)\; T_d(\xi)$ $+ (1 - P(O(d)$ and $D(d)$ are connected in $G(\xi) \mid \omega_j))\; M_d\}$.
6. O–D performance under multiple disaster scenarios (S/M/P): For an O–D pair d,
   $F_6 = \sum_{j=1}^{|\Omega|} \{\mathrm{P}(\omega_j)\; \sum_{\xi\in\Xi} \mathrm{P}(\xi|\omega_j)\{\mathrm{P}(O(d)$ and $D(d)$ are connected in $G(\xi) \mid \omega_j)$ $T_d(\xi) + (1 - \mathrm{P}(O(d)$ and $D(d)$ are connected in $G(\xi) \mid \omega_j))\; M_d\}$.
7. Multi O–D performance under a single disaster scenario (M/S/P):
   For a scenario ω_j,
   $F_7 = \sum_{\xi\in\Xi} \mathrm{P}(\xi \mid \omega_j) \sum_{d\in D} r_d\{\mathrm{P}(O(d)$ and $D(d)$ are connected in $G(\xi) \mid \omega_j)$ $T_d(\xi) + (1 - P(O(d)$ and $D(d)$ are connected in $G(\xi) \mid \omega_j))\; M_d\}$.
8. Multi O–D performance under multiple disaster scenarios (M/M/P):
   $F_8 = \sum_{j=1}^{|\Omega|} \mathrm{P}(\omega_j) \sum_{\xi\in\Xi} \mathrm{P}(\xi|\omega_j) \sum_{d\in C} r_d\{\mathrm{P}(O(d)$ and $D(d)$ are connected in $G(\xi)|\omega_j)\; T_d(\xi) + (1 - P(O(d)$ and $D(d)$ are connected in $G(\xi)|\omega_j))\; M_d\}$.

The first four measures (F_1 to F_4) are reliability measures, while the next four (F_5 to F_8) are combined performance measures in which the reliability is also incorporated by means of a penalty cost for disconnectedness. Measures with multiple disaster scenarios are expectations taken with respect to the probability of occurrence of each scenario. If estimating these probabilities poses difficulties, one can interpret them merely as weights. Instead of expectation, other risk measures (Pflug and Romisch 2007) have also been proposed in the disaster context, such as chance and stochastic dominance constraints in Noyan (2010b), and conditional value-at-risk in Noyan (2010a).

4 Methodology

When a natural disaster affects an area enclosing the network, the causes of link failure are both internal and external to the network. Vulnerability of the components in a link, due to factors such as the strength of a bridge and the ground soil type, is internal. On the other hand, the magnitude of force, e.g., ground acceleration or wind at the link location, as well as the consequences of the disaster, such as the collapse of buildings, an explosion or fire, are external factors. The external factors are typically effective throughout an area. For example, in an earthquake scenario, a city may be divided into areas of varying risk. One can expect links in the same area to show similar behavior, creating dependency. On the other hand, the internal factors will create differences among the links exposed to the same level of risk. We propose a dependency model that aims to combine these two aspects.

4.1 Models of dependency

We first assume that in a given disaster scenario, the set of links E can be partitioned with respect to the external factors for failure.

Definition 4.1 (*Set-Based Dependency*) Given a network with edge set E subject to failure due to a disaster event, the network is said to have *Set-Based Dependency* (SB-dependency) under a disaster scenario, if E can be partitioned into mutually exclusive sets $A_l \subset E$, for $l = 1, \ldots, L$, such that links belonging to different sets fail independently, while failure of links within each set A_l might show statistical dependence.

In the above definition, if $L = 1$, then we have the *all-dependent* case, and if each A_l consists of a single link, then we have the *all-independent* case, which is the widely studied case in the network reliability literature. SB-dependency provides a sufficiently general framework because links that fail independently can be represented with singleton sets and the remaining could be grouped into dependency sets. To account for the vulnerability of the links and the internal factors for failure, we propose what we term *vulnerability-based dependency* within each set. In the remaining, for ease of presentation we simplify the notation $p_{e_i}(\omega_j)$ to p_{e_i}, and furthermore, refer to edge e_i as i and the corresponding probability as p_i.

Definition 4.2 (*Vulnerability-Based Dependency*) Given two edges i and j in a dependency set A_l with survival probabilities p_i and p_j, i and j have *Vulnerability-based dependency* (VB-dependency), if $p_i \leq p_j$ implies P(i fails | j fails)=1. A maximal set of edges with VB-dependency among each other constitute a *VB-dependency set*.

This somewhat strict model of dependency allows us to determine the joint probability distribution of the random variable ξ and the characteristics of its possible realizations. Let us denote the *joint survival probability* of links i and j by $\langle p_i, p_j \rangle$. Under VB-dependency with $p_i \leq p_j$, P(i survives, j fails) = $p_i - \langle p_i, p_j \rangle = 0$. We then have P($i$ survives, j survives)= $\langle p_i, p_j \rangle = p_i$, P($i$ fails, j survives)= $p_j - \langle p_i, p_j \rangle = p_j - p_i$ and P(i fails, j fails)= $1 - p_j - p_i + \langle p_i, p_j \rangle = 1 - p_j$. This can be generalized to more than two links as follows.

According to Definition 4.2, the failure of a particular link implies the failure of all links in the same VB-dependency set that are *weaker* than that one with respect to the link survival probabilities. Hence, a sorting of the edges in the set starting from the *strongest* to the *weakest* proves to be useful. Let [1],...,[k] denote a re-indexing of the edges of a VB-dependency set A_l such that $p_{[1]} \geq p_{[2]} \geq \cdots \geq p_{[k]}$. Then, P($e_{[i]}$ survives|$e_{[i-1]}$ fails) = 0, for $i = 2, \ldots, k$ by definition. Then, the realizations with positive probability are in the form $\xi^q = (1, 1, 1, \ldots, 1, 0, 0, \ldots, 0)$, where the first strongest q links survive and the remaining fail, for $q = 0, 1, \ldots, k$. Note that here the index q indicates the number of links that have survived in the corresponding network realization $G(\xi^q)$. If we define $p_0 = 1$ and $p_{k+1} = 0$, then for $q = 0, 1, \ldots, k$, the probability that realization ξ^q occurs is $p_{[q]} - p_{[q+1]}$, as can be seen by generalizing the argument of the two link case described above.

Proposition 4.1 *The maximum number of possible realizations for a network with m edges that all belong to a single VB-dependency set is equal to $(m + 1)$. The probability that realization $\xi^q = (1, 1, 1, \ldots, 1, 0, 0, \ldots, 0)$ occurs is $p_{[q]} - p_{[q+1]}$, for $q = 0, 1, \ldots, m$.*

Proof Only $(m + 1)$ realizations of the form ξ^q such that the first q links survive have positive probabilities when all links have distinct survival probabilities. The number of possible vector realizations decreases when there are links with equal probabilities since they behave as a single link surviving or failing together. □

When multiple VB-dependency sets exist, the maximum number of possible network realizations grows exponentially with the number of sets and as expected, an exponential number of realizations exist in the all-independent case. Let us sort and re-index the edges of a each VB-dependency set A_l such that $p^l_{[i]} \geq p^l_{[2]} \geq \cdots \geq p^l_{[m_l]}$. Then, the realizations with positive probability are such that the first strongest q^l links survive in each VB-dependency set A_l and the remaining fail, for $q^l = 0, 1, \ldots, m_l$. We can express the probability that such a realization occurs as $\prod_{l \in L} (p_{[q^l]} - p_{[q^l+1]})$, by defining $p^l_0 = 1$ and $p^l_{m_l+1} = 0$. We denote this realization as $\xi^q = (1, 1, 1, \ldots, 1, 0, 0, \ldots, 0|1, 1, 1, \ldots, 1, 0, 0, \ldots, 0| \cdots |1, 1, 1, \ldots, 1, 0, 0, \ldots, 0)$, where the elements from each set are divided by '|'. Thus, we obtain the following proposition by applying Proposition 4.1 to each dependency set and considering that the links in different sets fail independently:

Proposition 4.2 *The maximum number of possible network realizations when L VB-dependency sets exist is $(m_1 + 1)(m_2 + 1) \cdots (m_L + 1)$, where m_l is the number of links in VB-dependency set A_l. The probability that the realization in which the first strongest q^l links survive in each VB-dependency set A_l occurs is $\prod_{l \in L} (p_{[q^l]} - p_{[q^l+1]})$, for $q^l = 0, 1, \ldots, m_l$.*

4.2 Computation of the reliability and performance measures

Since the proposed measures include the connectivity of the O–D pairs, we need to concentrate on the joint probability distribution of links that lie on paths connecting the O–D pairs. For connectivity, there must be at least one surviving path between an O–D pair, and obviously, all of the links on a path should survive for a path to survive.

Proposition 4.3 *Let S denote the edge set of an O–D path. Suppose S belongs to a single VB-dependency set. Then, P(all links in S survive) $= \min_{i \in S} \{p_i\}$. If S belongs to multiple VB-dependency sets, then P(all links in S survive) $= \prod_{l=1}^{L} \min_{i \in A_l \cap S} \{p_i\}$.*

Proposition 4.3 could be utilized in a path-based analysis for the computation of any of the eight proposed measures defined above. However, this would be computationally prohibitive as conditional events related to the survival of all O–D paths have to be considered. Moreover, the paths need not be edge-disjoint, further complicating the probability calculations. Instead, we take the alternative approach of using the joint probability distribution of the random variable ξ as defined in Proposition 4.2 and check the connectivity in each network realization.

The O–D connectivity of the surviving network $G(\xi^q)$ associated with the vector realization ξ^q could be determined by sending a unit flow from the origin to the destination in $G(\xi^q)$. Alternatively, paths between the O–D pair can be checked for survival. Let Π_d be the set of all distinct paths between the origin and destination nodes, $O(d)$ and $D(d)$. Note that the number of paths between any of the O–D pairs will be small in sparse networks, but that number may be exponential in terms of the number of nodes, if the network is dense. On the other hand, many of the paths may be quite similar, or rather long and circuitous. For practical purposes, a set of appropriate paths may be selected. In cases where this is cumbersome, paths can be generated automatically in polynomial time by a k-shortest path algorithm as in Yen (1971), Lawler (1972) and Eppstein (1994). Hence, using only the k-shortest paths, with $T(\pi_i)$ denoting the total length of the kth shortest path $\pi_i \in \Pi_d$, may be a practical approach, unless k is set to a large enough number.

We give the pseudo-code to compute the reliability, Rel, and the performance, Per, of a single O–D pair in a network composed of a single VB-dependency set, A_1, under a single disaster scenario as Algorithm 1 below. The algorithm computes both of (S/S/R) and (S/S/P) measures, namely F_1 and F_5. The algorithms for the multi-scenario and the multi-O–D pair cases are similar and omitted here. We use the notation p_i for the survival probability of link i and $p_{[j]}$ for that of the jth strongest link in the VB-dependency set.

Algorithm 1: (S/S/R) and (S/S/P) when a single VB-dependency set exists

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Inputs: p_i, for all $i \in E$, $\Pi_d = \{\pi_1, \pi_2, \ldots, \pi_k\}$
Outputs: Rel, Per (Exact values of F_1 and F_5 when Π_d contains all O–D paths)
Step 1 Order and re-index all $i \in E$ such that $p_{[i]} \geq p_{[i+1]}$
Step 2 $Rel = p_{[m]}$ and $Per = p_{[m]} \cdot T(\pi_1)$ (INITIALIZE)
For $q = m - 1$ down to 0,
 Step 3 Generate ξ^q as $\xi_k^q = 1$ for $k = 1$ to q and $\xi_k^q = 0$ for $k = q + 1$ to m (GENERATE)
 Step 4 Check O–D connectivity in $G(\xi^q)$ and update Rel and Per as below (CALCULATE)
 Let $flag = 0$
 For $s = 1$ to k,
  If π_s survives in $G(\xi^q)$, then
   Set $Rel = Rel + p_{[q]} - p_{[q+1]}$, $Per = Per + (p_{[q]} - p_{[q+1]})T(\pi_s)$, $flag = 1$ and break
 If $flag = 0$, set $Per = Per + (p_{[q]} - p_{[q+1]})M_d$

Algorithm 1 computes reliability and performance measures by generating all of the $m + 1$ network realizations and checking the survival of the shortest O–D paths in each realization in order to find the shortest path length in each realization. In the first step, the links are sorted with respect to their survival probabilities. The reliability measure, F_1, which is denoted by Rel in the algorithm, is computed as the summation of the probabilities of network realizations which provide connectivity between the O–D pair. Similarly, the performance measure, F_5 (denoted by Per in the algorithm), is the summation of the product of the shortest O–D distance in each realization (or a disconnectedness penalty cost) and the probability of the corresponding realization. In the second step, to account for the realization in which all links are functional, the initial reliability is set to the survival probability of the weakest link in the set, $p_{[m]}$, due to Proposition 4.1. Similarly, Per, i.e., F_5, is set to the shortest path length between the O–D nodes since this path definitely exists when all the links are functional, by our assumption. In the third step, the next realization in which the next weakest link becomes non-functional is generated. The connectivity check is completed in Step 4 by going over the k-shortest paths starting with the shortest one. When all the k-shortest paths are checked, if none of them survives in the current realization, then the penalty cost M_d is incurred for O–D pair d. The third step is repeated m times, by failing each additional link in each iteration.

We next analyze the computational complexity of Algorithm 1, and show that it is polynomial time when k is a fixed number.

Proposition 4.4 *The computational complexity of Algorithm 1 for a single O–D pair, in a network composed of a single VB-dependency set with m links, under a single scenario is $O(knm)$, where k is the number of shortest paths between the origin and the destination nodes that are input to the algorithm.*

Proof Step 1 takes $O(mlogm)$ since m edges are sorted. For each realization, checking the survival of an O–D path in that realization can be done in $O(n)$ because an O–D path has at most $n - 1$ links, and each one would be scanned once. Since we check at most k such paths, and repeat Steps 3 and 4 m times for each realization, the complexity of the algorithm amounts to $O(knm)$. □

We can generalize Algorithm 1 for the case with multiple VB-dependency sets, A_l, for $l \in L$. Let $p^l_{[i]}$ represent the survival probability of the ith link in A_l when ranked with respect to the survival probabilities. The pseudo-code for the single O–D pair, multiple VB-dependency sets case is as follows.

Algorithm 2: (S/S/R) and (S/S/P) when multiple VB-dependency sets exist

---

Inputs: $p^l_{[i]}, \forall i \in A_l, l \in L, \Pi_d = \{\pi_1, \pi_2, \ldots, \pi_k\}$

Outputs: Rel, Per (Exact values of F_1 and F_5 when Π_d contains all O–D paths)
Step 1 Within each set A_l, order and re-index all $i \in A_l$ such that $p^l_{[i]} \geq p^l_{[i+1]}$, and $[m_l]$ is the index of the weakest link in A_l
Step 2 Set $Rel = p^1_{[m_1]} p^2_{[m_2]} \cdots p^L_{[m_L]}$ and $Per = (p^1_{[m_1]} p^2_{[m_2]} \cdots p^L_{[m_L]})\pi_1$ (INITIALIZE)

Repeat
Step 3 Generate a realization ξ^q as below (GENERATE)
Step 4 Check connectivity in $G(\xi_s)$ and update Rel and Per as below (CALCULATE)
Until all realizations have been generated
GENERATE:
For $j_1 = 0$ to m_1
Set $\xi_s^q = 1$ for all $s \in A_1$ and $\xi_s^q = 0$ for all $s \in S_1 = \{$weakest j_1 edges in $A_1\}$
For $j_2 = 0$ to m_2
Set $\xi_s^q = 1$ for all $s \in A_2$ and $\xi_s^q = 0$ for all $s \in S_2 = \{$weakest j_2 edges in $A_2\}$
…
For $j_L = 0$ to m_L
Set $\xi_s^q = 1$ for all $s \in A_L$ and $\xi_s^q = 0$ for all $s \in S_L = \{$weakest j_L edges in $A_L\}$
CALCULATE:
Let $flag = 0$
For $s = 1$ to k
If π_s survives in $G(\xi^q)$, then
Set $Rel = Rel + p(\xi^q)$, $Per = Per + p(\xi^q)T(\pi_s)$, $flag = 1$ and break
If $flag = 0$, set $Per = Per + p(\xi^q)M_d$

Algorithm 2 also starts with the full-functional realization and proceeds iteratively. In this case, F_1 is initialized to the product of the survival probabilities of the weakest links in each VB-dependency set. The initial value of F_5 is set to the product of this probability and the length of the shortest path, with the same reasoning as before. The main difference between the single VB-dependency set and the multiple VB-dependency set cases is the generation of the vector realizations. Due to Proposition 4.2, the number of possible realizations is $\prod_{l=1}^{L}(|A_l|+1)$, where the resulting realizations are combinations of the realizations generated for each single VB-dependency set as in Algorithm 1. Step 4 checks O–D connectivity and updates the measures by searching over the k-shortest paths as in Algorithm 1. We provide an example to illustrate how the algorithms work in the Appendix. Next, we analyze the computational complexity of the algorithm.

Proposition 4.5 *The computational complexity of Algorithm 2 for a single O–D pair in a network composed of L VB-dependency sets with m_l links in each set, A_l, under a single disaster scenario is $O(kn(m_{max}+1)^L)$, where m_{max} is the maximum m_l value over $l = 1, \ldots, L$ and k is the number of input O–D paths.*

The complexity can be shown as in the proof of Proposition 4.4, using Proposition 4.2 for the number of realizations. Note that as a consequence of Proposition 4.2, the complexity of Algorithm 2 depends on the number of VB-dependency sets, L.

Algorithm 2 is expected to compute the reliability and performance measures in reasonable time for most practical cases. The computational time increases when the number of shortest paths or the number of VB-dependency sets increases. In the extreme case, if every VB-dependency set consists of a single link, i.e., the all-independent link failure case, the complexity becomes exponential, $O(kn2^m)$; thus, a Monte Carlo simulation approach can be utilized to estimate these measures. The pseudo-code below summarizes such an algorithm for networks with multiple VB-dependency sets, where N is the sample size.

Algorithm 3: Monte Carlo simulation for (S/S/R) and (S/S/P) when multiple VB-dependency sets exist

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Inputs: $p^l_{[i]}$, $\forall i \in A_l, l \in L$, $\Pi_d = \{\pi_1, \pi_2, \ldots, \pi_k\}$, N

Outputs: Rel, Per (Estimates of F_1 and F_5)
Set $C = 0$, $T = 0$
For $t = 1 : N$,
    For each set A_l, $l = 1, 2, \ldots, L$
        Generate a^l_t uniformly such that $0 \leq a^l_t \leq 1$
    For each link $i \in A_l$
        If $a^l_t \leq p_i$, then $\xi_i = 1$
        Else, $\xi_i = 0$
    Let $flag = 0$
    For $s = 1$ to k
        If π_s survives, then set $C = C + 1$, $T = T + T(\pi_s)$, $flag = 1$ and break
    If $flag = 0$, set $T = T + M_d$
end for
Set $Rel = C/N$ and $Per = T/N$

In this algorithm, the reliability, F_1, and the performance, F_5, are estimated by the average values over a sample of vector realizations that are generated randomly. In each iteration, the status of the links in each set A_l are determined by the same random number to obey the definition of VB-dependency. However, the random number used for each set differs since every set is assumed to be independent from the others in the definition of SB-dependency. After generating a vector realization, its O–D connectivity and the shortest path length is found in the same way as before. This procedure is repeated for a fixed number of realizations, N, which should be determined by the decision maker.

5 Computational results from a case study

We apply our framework to a case study to analyze the expected performance of response operations and to see how the urban highway network of Istanbul would

Table 1 The k-shortest path distances and the disconnectedness penalty cost

O–D	π_1	π_2	π_3	π_4	π_5	π_6	M_d
4–8	14.0012	17.9143	18.7937	21.5110	26.7312	34.1542	35
14–7	11.1397	20.0876	25.4816	26.5750	29.0784	30.1717	31
14–20	6.6489	20.4118	29.2000	30.2664	–	–	31
12–18	9.5609	20.0476	20.2432	27.0592	–	–	28
9–7	9.4565	14.8505	16.8795	18.4473	–	–	19

perform during an anticipated major earthquake in the region, if the current risk and vulnerability factors are not reduced. As pointed out by many researchers within the past decade (such as Parsons et al. 2000; Griffiths et al. 2007), the metropolitan city of Istanbul is under a serious risk of earthquake, expected on the seismically active North Anatolian Fault line that passes around 100 km south of the city (see Fig. A1 in the Appendix). The damage caused by such an earthquake could be devastating due to population and commercial/industrial density (ISMEP 2010), as well as risky building stocks in many districts. The Municipality and the Japanese International Cooperation Agency conducted a disaster mitigation study in Istanbul in 2002 and identified several earthquake scenarios as well as the risk levels of the highway network components (JICA-IMM 2002). The resulting report is the basis for most of our input data. We constructed a network with respect to the two main highways in Istanbul. The network consists of 25 nodes and 30 links as depicted in Fig. A2 in the Appendix, where the node and edge numbers are given. Five O–D pairs have been chosen: the origin nodes are determined as the most populated districts with the worst damage expectations, and the destination nodes are chosen as the districts with high medical emergency care capacity. The selected O–D pairs are (14–7), (12–18), (4–8), (9–7) and (14–20) according to node numbers seen in Fig. A2. The travel times are difficult to assess, especially in the case of disasters, as it is not possible to forecast the behavior of people. We determined the k-shortest paths with respect to actual road distances between the O–D pairs and considered expected distances for our performance measures. The distance of each path under consideration is listed in Table 1. If an O–D pair turns out to be disconnected in the surviving network after the earthquake, then the penalty cost/distance is set to a slightly higher value than the longest shortest path distance between that pair so that it is always better to use one of the shortest paths, if they exist.

5.1 Probability of link failures

The likelihood of link failures vary with respect to the intensity and the location of the disaster experienced, as well as the condition of the network components and how the disaster impacts them. Each link that represents a road segment may contain vulnerable components such as bridges and viaducts and each component may withstand different levels of force depending on its structure. Structural civil engineers generate *fragility curves* that indicate how a particular road component responds to varying force levels and these curves guide risk analyses (Grossi et al. 2005). Under lack of sufficient past data or statistics on link failures, the factors that have been taken into account in this study for determining the link survival probabilities on the highway are the risk level of bridges and viaducts on the roads under expected disaster scenarios as reported in JICA-IMM (2002).

Scientific studies agree on four earthquake scenarios that Istanbul is mostly likely to experience, as shown in Fig. A1 in the Appendix. These scenarios are labeled as A, B, C, and D where A is characterized as the *most probable* and C as the *worst case* scenario. The earthquake is expected to take place on the fault line that goes through the Sea of Marmara, to the south of the city. The scenarios differ in terms of the magnitude of the earthquake and the fault segment to be affected (shown as bold in Fig. A1). We adjust the link survival probabilities in each of these disaster scenarios with respect to magnitude and location.

5.2 Determination of the VB-dependency sets

For the earthquake application in this study, the construction of the VB-dependency sets is guided by the *Peak Geographic Acceleration* (PGA) levels. PGA is a common measure used by earthquake engineers to evaluate the earthquake risk of a region and may be defined as the maximum acceleration experienced by an object in case of an earthquake. Four different levels of PGA values have been assigned to four regions of Istanbul in JICA-IMM (2002) with respect to the disaster scenarios mentioned above. Based on these regions, we were able to classify the links of the network into sets, $A_l, l = 1, \ldots, L$, for each PGA level under each disaster scenario. As an example, the classification for disaster scenario A is shown in Fig. A3 in the Appendix. The links belonging to each set in each disaster scenario are listed in Table A1 and their survival probabilities are given in Table A2 in the Appendix. See also Fig. A4 in the Appendix for a color coded illustration of the VB-dependency sets for scenario A.

5.3 Calculation of the measures

The single O–D measures defined for a single disaster scenario in Sect. 3 are presented in Table 2 for the selected O–D pairs and each of the four disaster scenarios A, B, C, and D. Measures (S/S/R) and (S/S/P) are given in the same row for the O–D pair specified in the second column and the earthquake scenario given in the third column. The last two columns give the reliability of the O–D pair and the performance, namely the expected shortest path distance between the O–D pair, respectively. The run time of each algorithm, implemented in Matlab 7.0, does not exceed a couple of seconds on a PC with 2×2.8 GHz Xeon processor and 5 GB RAM.

To calculate the multiple O–D measures (F_3, F_4, F_7 and F_8), we assigned equal weights to each O–D pair. For the multiple disaster scenario measures, each disaster scenario is given a weight to emphasize its relative significance rather than its probability of occurrence. Since scenario A has been labeled as the most probable disaster scenario, and C as the worst case scenario, we have chosen to give 0.4 weight for scenario C and 0.3, 0.2, 0.1 for scenarios A, B, and D, respectively. The results for multiple O–D pairs and multiple disaster scenarios are given in Table 3.

Table 2 Results for single O–D measures (F_1, and F_5)

Measure	O–D	Scenario	Rel	Per
(S/S/R), (S/S/P)	4–8	A	0.573	23.4556
		B	0.5890	23.1688
		C	0.7264	21.4638
		D	0.6359	22.1951
(S/S/R), (S/S/P)	14–7	A	0.550	20.0768
		B	0.423	23.9651
		C	0.550	20.0768
		D	0.577	19.5307
(S/S/R), (S/S/P)	14–20	A	0.7	13.9542
		B	0.757	13.9119
		C	0.7	13.9542
		D	0.735	13.1019
(S/S/R), (S/S/P)	12–18	A	0.4358	20.421
		B	0.5775	19.3215
		C	0.6	17.6330
		D	0.63	17.1147
(S/S/R), (S/S/P)	9–7	A	0.4358	15.0753
		B	0.6878	12.9097
		C	0.6	13.2739
		D	0.63	12.9876

Table 3 Results for the six measures (F_2, F_3, F_4, F_6, F_7, and F_8)

Measure	O–D	Scenario	Rel	Per
(S/M/R), (S/M/P)	4-8	multi	0.5520	23.8449
	14–7	multi	0.5270	20.7998
	14–20	multi	0.7150	13.8605
	12–18	multi	0.5492	18.7553
	9–7	multi	0.5713	13.7129
(M/S/R), (M/S/P)	multi	A	0.5389	18.5966
	multi	B	0.6071	18.6554
	multi	C	0.5894	17.9651
	multi	D	0.6417	16.9860
(M/M/R), (M/M/P)	multi	multi	0.5830	18.1947

5.4 Comparison of various dependency structures

To reach a general conclusion of whether reliability of a network is higher or lower in the independent or the VB-dependent link failure cases, we made a comparison of the results of this method with the results of the calculations in the two extreme cases, namely the all-independent and the all-dependent cases.

Since it is computationally difficult to calculate the exact reliability value for the all-independent case, we used the Monte Carlo simulation algorithm (Algorithm 3) given in the previous section, with several sample sizes. The estimations are subject to error and a confidence interval is computed for each estimated performance value by the standard formula $Per \mp z \frac{\sigma}{\sqrt{n}} = Per \mp \Delta$, where Per is taken as the estimated performance value, σ as the sample standard deviation, n as 4,000,000 for the

Table 4 Simulation results with varying sample size for O–D pair (4–8), disaster scenario C and $M_d = 35$

Sample size	Rel	Δ	Lower limit	Upper limit	Cpu time
500,000	0.6664	0.0211	22.6340	22.6762	11.71
750,000	0.6669	0.0173	22.6193	22.6539	17.62
1,000,000	0.6667	0.0149	22.6285	22.6583	23.69
4,000,000	0.6668	0.0075	22.6321	22.6471	23.53

Table 5 Simulation results with varying VB-sets for the O–D pair (4–8), disaster scenario C and $M = 35$

Link dependency structure	Rel	Per
1-set (all-dependent)	0.7000	20.7801
3-sets (PGA)	0.7264	21.4638
10-sets (PGA)	0.6031	24.0035
30-sets (all-independent)	0.6668	22.6396

sample size, and for a desired confidence of 90%, z is taken to be 1.645. With these parameters, the representative Δ values for O–D pair (4,8) are provided in Table 4, along with the confidence interval limits for Per. The results illustrate that we obtain robust accuracy with the tested sample sizes and within reasonable computation time relative to the planning context.

The results of the comparison are given in Table 5. The first column shows the link dependency structure. The case with only one VB-dependency set is given in the first row. The next two rows give the VB-dependency sets that are determined with respect to the PGA values of the region. In the all-independent case, the number of VB-dependency sets is equal to the number of links in the network.

In this comparison, it is observed that the reliability has the lowest value in the case with 10 VB-dependency sets and the all-independent case is less reliable than the all-dependent case. However, it is hard to come up with a general observation. Note that for a path to be connected, it is necessary that all the links on that path should survive. If all the links on the path belong to the same VB-dependency set, then reliability of the network is $\text{Min}\{p_i\}_{i \in E}$. If the link failures are assumed to be independent, then the reliability becomes $\prod_{i \in E} p_i$. Since $0 \leq p_i \leq 1$, $\prod_{i \in E} p_i \leq \min_{i \in E}\{p_i\}$. Therefore, the single VB-dependency set is always more reliable. On the other hand, if the links of a path with edge set S belong to more than one VB-dependency set, say k sets, then the reliability becomes $\prod_{l=1}^{k} \min \{p_i\}_{i \in A_l, i \in S}$ as stated in Proposition 4.3. Now consider the extreme case when each link belongs to a different VB-dependency set, i.e., the all-independent case. Every time a realization is generated, shortest paths are checked until the first one that exists. Thus, the effect of the survival probability of a link changes with how many different shortest paths it belongs to. Then, the reliability becomes network-specific and this prevents us from reaching a general conclusion on the comparison of reliability of the network in the independent or VB-dependent link failures. Additionally, the number of links that have the same survival probability affects these measures.

Table 6 Comparison of various dependency structures

Case Num.	p_1	p_2	p_3	p_4	p_5	$Rel_{1-\mathrm{VB}}$	$Rel_{2-\mathrm{VB}}$	Rel_{ind}
1	0.4	0.5	0.7	0.3	0.6	0.5	0.30	0.46800
2	0.4	0.4	0.7	0.7	0.6	0.4	0.28	0.53008

The simple network in Fig. A5 in the Appendix is an example that shows that there is no general behavior for the change in the reliability of the network when the link failures are assumed either to be VB-dependent or independent. For this network, the reliability in two cases is given in Table 6 where p_i shows the survival probability of link i. The seventh column, $Rel_{1-\mathrm{VB}}$ is the reliability in the case where the network consists of only one VB-dependent set. The next column, $Rel_{2-\mathrm{VB}}$, gives the reliability in the case where two VB-dependent sets are present, where the first VB-dependency set includes links 1, 2 and the second set includes the remaining links 3, 4, 5. The last column, Rel_{ind}, gives the reliability in the all-independent link failure case. In Case 1, the reliability in the all-dependent case is higher than the reliability in the all-independent case. Although this is consistent with the results in Table 5, the all-independent case has the highest reliability in Case 2. Thus, one cannot argue that the all-independent link failure structure turns out to be less reliable than the all-dependent link failure structure.

6 Concluding remarks

In this paper, a network is examined for its reliability and performance after a disaster. The paper concentrates on eight different measures in the context of post-disaster logistics, specifically considering the condition of a transportation network after an earthquake in which the links are likely to collapse. An original framework is designed for dependent link failures that can be adapted in different contexts where dependent sets can be determined by the decision maker with respect to the network and how the environment affects the dependency relationship.

In the proposed framework, in order to identify the dependencies within a set, we defined a novel vulnerability-based link dependency structure that ranks the links according to their probabilities of survival. We assumed that the failure of a stronger link, that is the link with higher probability of survival, implies the failure of weaker links (links with smaller probability of survival) with certainty. This assumption seems to be reasonable in the earthquake context where links within the same area of risk with respect to an expected earthquake scenario show a weakness/strength ranking according to the vulnerability of the components in each link, such as the structural strength of bridges. Furthermore, this assumption allows the existence of a polynomial-time algorithm when the number of paths connecting an O–D pair is fixed.

We illustrated the applicability of the proposed analysis method by means of a case study of the Istanbul highway system under earthquake risk. For the dependent link failure case, we applied our exact polynomial-time algorithm for Istanbul's sparse main highway network in the region most likely to be affected by an expected earthquake, by selecting a reasonable number of O–D pairs according to the expected earthquake scenarios. The algorithm was found to be computationally very efficient in this case. In addition, we used a Monte Carlo sampling algorithm to estimate the measures under interest for the computationally difficult case of independent link failures for purposes of comparison. These results also support that reliability and performance of a network of realistic size can be estimated with high accuracy in moderate computation time with the proposed Monte Carlo simulation method. As a result, we obtained very promising results in terms of proving the practicality of the proposed approach.

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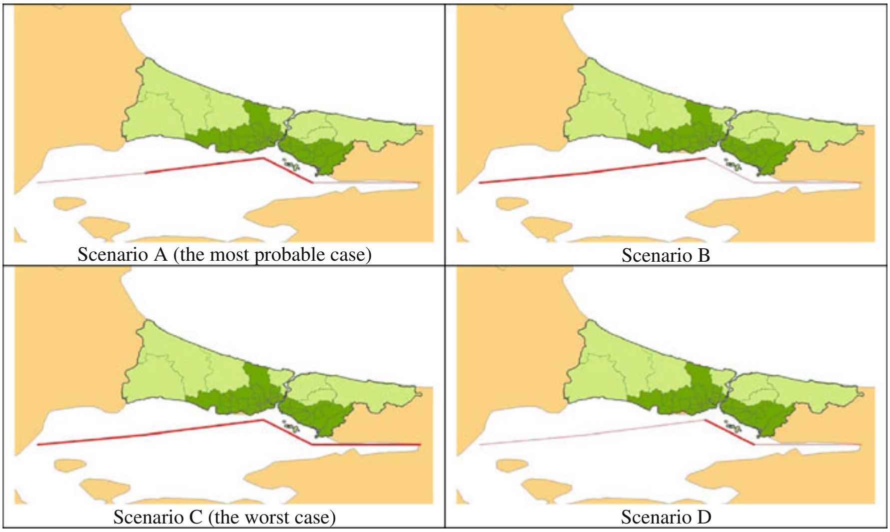
Appendix

…

Stop after ***Iteration 5*** (when all realizations have been generated.)

See Figs. A1, A2, A3, A4 and Tables A1, A2.



	Scenario A	Scenario B	Scenario C	Scenario D
Length (km)	119	108	174	37
Magnitude (Mw)	7.5	7.4	7.7	6.9

Fig. A1 The disaster scenarios for the Istanbul earthquake case

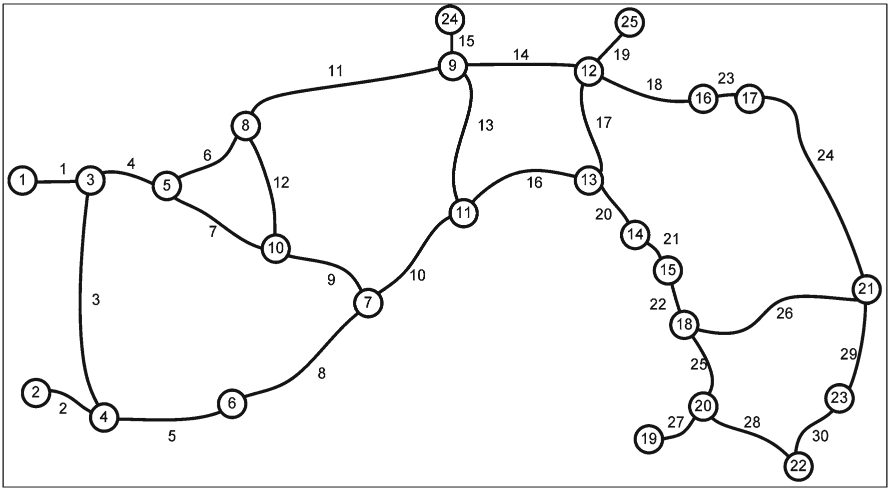


Fig. A2 Istanbul highway network

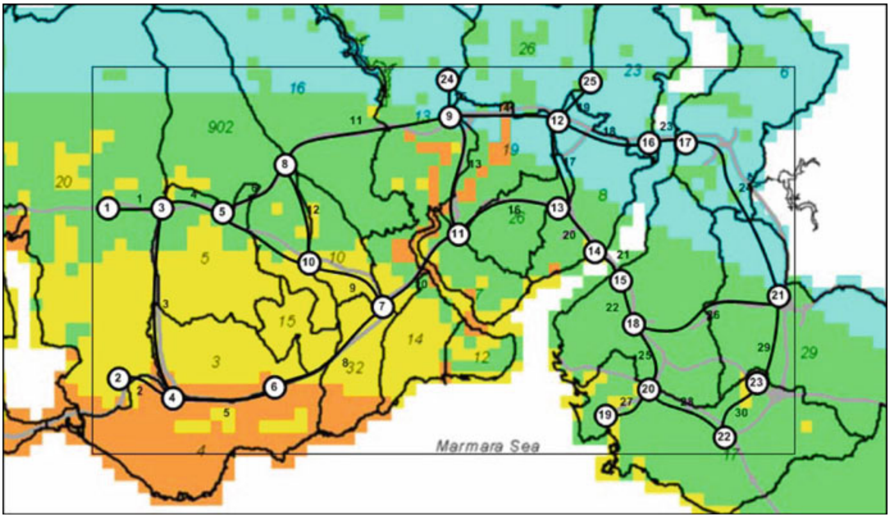


Fig. A3 Distribution of PGA for disaster scenario A

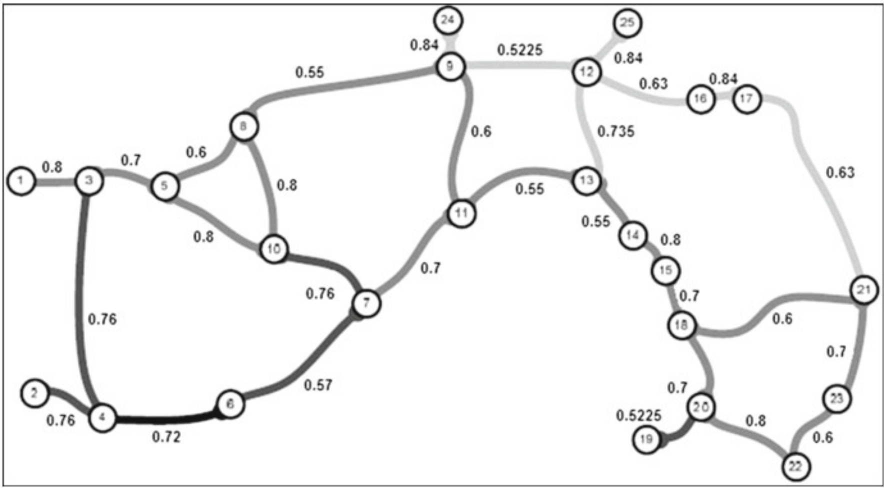


Fig. A4 Vulnerability sets for disaster scenario A

Table A1 Links included in each VB-dependency set

Set	Links
	Scenario A
A_1	5
A_2	2, 3, 8, 9, 27
A_3	1, 4, 6, 7, 10, 11, 12, 13, 16, 20, 21, 22, 25, 26, 28, 29, 30
A_4	14, 15, 17, 18, 19, 23, 24
	Scenario B
A_1	2, 3, 5, 8, 9, 27
A_2	1, 4, 6, 7, 10, 12, 13, 16, 21, 22, 25, 28
A_3	11, 14, 15, 17, 18, 19, 20, 23, 24, 26, 29, 30
	Scenario C
A_1	2, 5, 8
A_2	3, 9, 12, 27
A_3	1, 4, 6, 7, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30
	Scenario D
A_1	5
A_2	2, 3, 8, 9, 27
A_3	1, 4, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30

Table A2 Survival probabilities of the links for each scenario

Link	Disaster scenario			
	A	*B*	*C*	*D*
1	0.8	0.8	0.8	0.84
2	0.76	0.76	0.72	0.8
3	0.76	0.76	0.76	0.8
4	0.7	0.7	0.7	0.735
5	0.72	0.76	0.72	0.76
6	0.6	0.6	0.6	0.63
7	0.8	0.8	0.8	0.84

Table A2 continued

Link	Disaster scenario			
	A	*B*	*C*	*D*
8	0.57	0.57	0.54	0.6
9	0.76	0.76	0.76	0.8
10	0.7	0.7	0.7	0.735
11	0.55	0.5775	0.55	0.5775
12	0.8	0.8	0.76	0.84
13	0.6	0.6	0.6	0.63
14	0.525	0.525	0.5	0.525
15	0.84	0.84	0.8	0.84
16	0.55	0.55	0.55	0.5775
17	0.735	0.735	0.7	0.735
18	0.63	0.63	0.6	0.63
19	0.84	0.84	0.8	0.84
20	0.55	0.5775	0.55	0.5775
21	0.8	0.8	0.8	0.84
22	0.7	0.7	0.7	0.735
23	0.84	0.84	0.8	0.84
24	0.63	0.63	0.6	0.63
25	0.7	0.7	0.7	0.735
26	0.6	0.63	0.6	0.63
27	0.5225	0.5225	0.5225	0.5775
28	0.8	0.8	0.8	0.84
29	0.7	0.735	0.7	0.735
30	0.6	0.63	0.6	0.63

An illustrative example for Algorithm 2

Consider the simple network in Fig. A5. The survival probabilities of links $p_i, i \in \{1, \ldots, 5\}$ are given as 0.4, 0.4, 0.7, 0.7, and 0.6, respectively. The traversal costs are 10, 10, 5, 5, and 15 in the same order of links. The network is composed of two VB-dependency sets, namely $A_1 = \{p_1, p_2\}$ and $A_2 = \{p_3, p_4, p_5\}$. The four shortest paths are determined as $\pi_1 = 1, 4, \pi_2 = 2, 3, 4, \pi_3 = 2, 5$, and $\pi_4 = 1, 3, 5$ with travel distances between the O–D pair given as $T(\pi_1) = 15$, $T(\pi_2) = 20$, $T(\pi_3) = 25$, and $T(\pi_4) = 30$. The penalty cost is taken as $M = 31$.

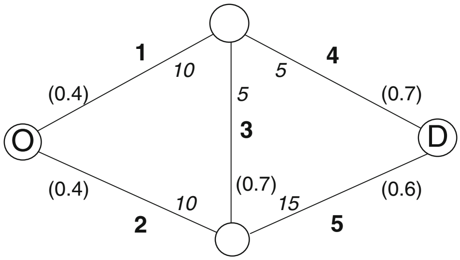


Fig. A5 A simple network

Preprocessing

Step 1 Sorting $p^1_{[1]} = 0.4,\ p^1_{[2]} = 0.4,\ p^2_{[1]} = 0.7,\ p^2_{[2]} = 0.7,\ p^2_{[3]} = 0.6$

Step 2 Start with $\xi = (1, 1, 1, 1, 1)$

$$Rel = p^1_{[2]} \cdot p^2_{[3]} = 0.4 \cdot 0.6 = 0.24$$

$$Per = (p^1_{[2]} \cdot p^2_{[3]}) \cdot T(\pi_1) = 0.24 \cdot 15 = 3.6$$

Iteration 1

Step 3 $\xi = (1, 1, 1, 1, 0)$

Step 4 $\pi_1 = (1, 0, 0, 1, 0)$. Check $\xi \geq \pi_1$ or not; TRUE

$$Rel = Rel + p(\xi) = 0.24 + (0.4 \cdot (0.7 - 0.6)) = 0.28$$

$$Per = Per + (p(\xi) \cdot T(\pi_1)) = 3.6 + (0.04 \cdot 15) = 4.2$$

Iteration 2

Step 3 $\xi = (1, 1, 0, 0, 0)$.

Step 4 $\pi_1 = (1, 0, 0, 1, 0)$. Check $\xi \geq \pi_1$ or not; FALSE

$\pi_2 = (0, 1, 1, 1, 0)$. Check $\xi \geq \pi_2$ or not; FALSE

$\pi_3 = (0, 1, 0, 0, 1)$. Check $\xi \geq \pi_3$ or not; FALSE

$\pi_4 = (1, 0, 1, 0, 1)$. Check $\xi \geq \pi_4$ or not; FALSE

$$Rel = 0.28$$

$$Per = Per + (p(\xi) \cdot M) = 4.2 + (0.12 \cdot 31) = 7.92$$

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