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REGULAR ARTICLE

A strategic and tactical model for closed-loop supply chains

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Abstract In this paper, a strategic location-allocation model is developed for the simultaneous design of forward and reverse supply chains. Strategic decisions such as network design are accounted for together with tactical decisions, namely, production, storage and distribution planning. The integration between strategic and tactical decisions is achieved by considering two interconnected time scales: a macro and a micro time. At macro level, the supply chain is designed in order to account for the existing demands and returns, whose satisfaction is planned simultaneously at the micro level where tactical decisions are taken. A Mixed Integer Linear Programming formulation is obtained which is solved to optimality using standard Branch & Bound techniques. Finally, the model accuracy and applicability is illustrated through the resolution of a case study.

Keywords Discrete time · Closed-/open-loop supply chains · Network design · Optimisation

1 Introduction

Traditionally, supply chains start with raw materials and finish with products delivered to the final customer. Several intermediate levels may be present, associated with

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different kinds of facilities, products and transportation modes, but the end user is always assumed to represent the last level in the network. However, at some point in time, products leave the final customers and must be dealt with. Exception made to organic waste, most used products still have some intrinsic value that, in a sustainable environment, should be recovered.

In the last decades, the question of how to deal with return products and how to collect and handle them in processing facilities became an important problem to both research community and society. However, most studies addressing the global supply chain only consider the traditional forward flow. The reverse supply chain that deals with the flow that leaves customers and goes back to factories or to proper disposal, has been mostly studied in an operational/tactical level. Strategic questions are yet to be answered by academia and industry (Guide et al. 2003). When dealing with the design and planning of global supply chains with reverse flows, most published papers either adopt a silo approach or are case oriented. As stated in the work of De Brito et al. (2004), a considerable number of case studies have been published in the last decade.

Amongst the most generic models proposed in the literature, the work of Jayaraman et al. (1999) should be mentioned. The authors propose a mixed integer programming model that simultaneously considers the location of remanufacturing and distribution facilities, the transhipment, as well as the production and stocking of the optimal quantities of remanufactured products and cores. The model was tested on a set of problems based on the parameters of an existing electronic equipment remanufacture firm. The authors concluded that, if on one hand demand for remanufactured products is a key decision variable as in traditional distribution networks, on the other the optimal solution depends on having enough quantities of cores to remanufacture.

Fleischmann et al. (2001) investigated whether or not to integrate collection and recovery within an existing forward distribution network. A model for the simultaneous design of forward and reverse supply chains is proposed. The chosen formulation uses a network balance constraint to integrate twowarehouse location models. The authors concluded this problem to be context dependent. Furthermore, they suggest that when the existing distribution network can no longer be used, a careful analysis of the total logistic chain should take place in order to decide whether a single network with two flows or two independent networks should be used.

Jayaraman et al. (2003) propose both a strong and a weak formulation for the reverse distribution problem. Their research addresses the modelling of independent forward and reverse networks and considers product recall, product recycling and reuse, product disposal and hazardous product return. The weak formulation, in particular, was solved using an heuristic procedure, specially developed for this model. The authors stated that this heuristic found optimal solutions for a significant proportion of tested problems within a reasonable amount of time. Moreover, it also solved a very large number of other problems that conventional MIP tools could not handle.

Later on, Fandel and Stammen (2004) propose a strategic model for the supply chain design. The authors extend the traditional supply chain to account for the recycling of products released by customers. Dynamic aspects of the network are modelled by the use of a two-step time structure. Although promising, this work did not present any case or example and, therefore, neither the adequacy nor efficiency of the proposed approach to treat real world problems was tested, nor was the model proven as solvable.

Finally, in the work developed by Salema et al. (2006), a multi-product model is proposed for the design of a reverse distribution network where both forward and reverse flows are considered simultaneously. These networks differ not only in terms of structure but also in the number of products involved. Based on a network design model, the propose MILP formulation considers binary variables to describe structural decisions (e.g. whether or not to open a factory) and continuous variables for the volume of products in transit along the network. One published case is modified and solved in order to provide a better insight into the model.

Lu and Bostel (2007) proposed a closed-loop supply chain model for a remanufacturing network. The model considered producers, remanufacturing sites, intermediate centres and customers. The intermediate centres belong exclusively to the reverse network and send their products only to remanufacturing facilities. These, together with producers, directly supply customers' demand, which means that remanufactured products are assumed as new. A lagrangean heuristic approach was developed and numerical experiments were performed on examples adapted from classical test problems.

Within the works mentioned above, one important area of research that needs further study is the simultaneous design and planning of forward and reverse networks.

The objective of this paper is to propose a multi-product and dynamic model for the design of supply chain network where production and storage planning are accounted for. A strategic locationallocation model is developed, where tactical decisions are explicitly modelled. Two interconnected time scales are considered, which allows some detail to be introduced in production, storage and distribution planning. All activities have capacity limits; maximum and minimum levels are imposed on the production on every time instance; storage maximum level is set; distribution flows, whenever they occur, must be within given lower and upper bounds.

Traditionally, the strategic nature of location models considers instantaneous flows. The proposed model allows for the establishment of travel time between network levels. Another feature modelled is the time products are being used by customers, named as "usage time". This introduces a lag time between the time that the product demands are satisfied and the availability of the products to be "remanufactured".

It may be questionable the relevance of introducing tactical planning decisions into a strategic model since uncertainty on data may exist. This option becomes however relevant since the decisions considered are taken based on a set of data aggregation with no high detail associated. In this way, the model becomes very useful to analyse the network robustness. As both kinds of decisions are integrated within a single formulation, analysis can be performed to observe the impact that changes in tactical parameters (such as products costs, distributions costs, storage costs, among others) may have on the network design. Once the locations are chosen, the tactical decisions should be reviewed considering updated information and should be analysed with the operational decisions (scheduling decisions, for instance).

This paper is organised as follows. In the next section, a detailed description of the problem is given. Then, due to its significance in the model, the time structure

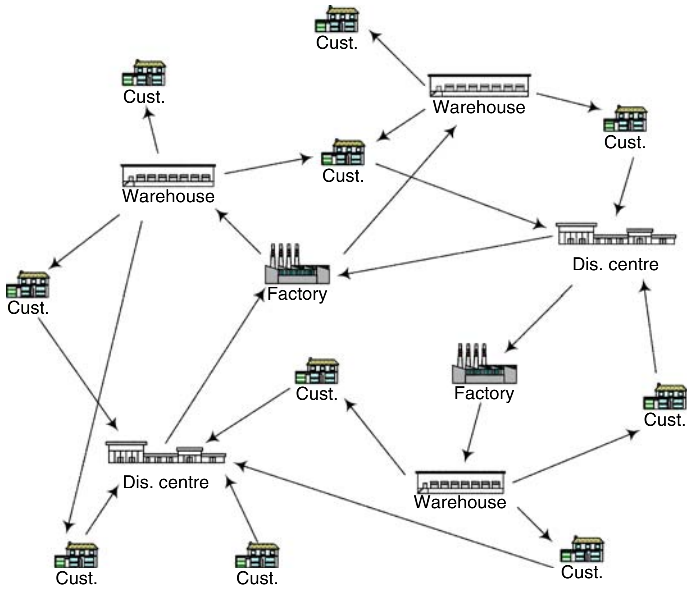

Fig. 1 Supply chain with reverse flow

is discussed in detail and two operators are defined. The mathematical formulation of the model follows, where variables, parameters and constraints are explained. The case study is then presented together with the results and a preliminary study on the model behaviour. Finally, some conclusions and future research directions are drawn.

2 Problem definition

A supply chain can be represented as a network where facilities act as nodes and links are related with direct flows between facilities. Such network may have several levels and each facility in each level can be connected to another facility in a different (not necessarily consecutive) level. In this work, three levels are considered: factories, intermediaries (warehouses and disassembly centres) and customers. Moreover, the supply chain is extended with the inclusion of reverse flows (see Fig. 1).

The forward and the reverse chains can be regarded as having a two-echelon structure each one. In the forward network, factories are connected to customers through warehouses and in the reverse network customers send their products back to factories, through disassembly centres. It is assumed that no direct connection exists between factories and customers in either direction.

The goal of the model is to maximise the global supply chain profit, assuming all prices, transfer-prices and costs to be known in advance. Several costs are considered: investment (whenever a facility is chosen), transportation, production, storage and penalty. Penalties apply to non-satisfied demands or returns.

Forward and return products are treated as independent since it is assumed that the original products may loose their identity after use (e.g. paper recycling—paper products after use are simply classified as paper). However, if required, it is also possible to track the same product in both flows.

Due to the unknown quality of return products, a disposal option is made available in the disassembly centres. Products that do not meet quality standards for remanufacturing are, thus, redirected to a different supply chain or simply to a depot. Furthermore, customers cannot introduce products coming from a different supply chain.

Using the structural options defined above, the proposed model considers two levels of decisions, which correspond to two different time scales: a "macro time" scale, where the supply network is designed, and a "micro time" scale, where planning activities are set (e.g. global production and/or storage planning). These time scales can be years/months, years/trimesters, months/days or any combination that suits the problem.

The chosen facilities will remain unchanged throughout the time horizon while the throughput may change over time.

Traditionally, strategic supply chain models consider instantaneous flows, i.e. customers have their demand satisfied at the same instance in time at which products are manufactured. However, the proposed two-time scale overcomes this limitation and the formulation enforces the definition of "travel times". Travel time is modelled as the number of micro time units that, for example, a product takes to flow from its origin to its destination.

It is considered that products before entering the reverse network must be "used" by customers. Thus for each return product a "usage time" is considered, which is defined as the minimum number of micro time units that a supplied product remains in the customer before entering the supply chain as a return product.

In short, the proposed model can be formulated as:

Given

- a possible superstructure for the location of the supply chain entities,
- the investment costs,
- the amount of returned product that will be added to the new products,
- the relation between forward and reverse products,
- the travel time between each pair of network agents,
- the minimum usage time for each return product,
- the minimum disposal fraction,

and for each macro period and product

- customers' demand values,
- the unit penalty costs for non-satisfied demand and return,

and for each micro period

- the unit transportation cost between each pair of network facilities,
- the maximum and minimum flow capacities,
- the maximum and minimum production capacities,
- the maximum storage capacities,
- the initial stock levels,
- the factory production unit costs,

- the facilities unit storage costs,
- the transfer prices between facilities,
- customers' purchase and selling prices.

Determine

- the network structure,
- the production and storage levels,
- the flow amounts and
- the non-satisfied demand and return volumes.

So as to

- maximise the global supply chain profit.

3 Time modelling characterisation

As referred above, the coexistence of two-time scales is one special feature of this model. These are closely interdependent and contemplate some aspects that should be analysed before entering the modelling detail.

Consider $T = \{1, 2, \ldots, t, \ldots, T\}$ as the set of macro periods, where the time horizon is divided in T equal size units, and $T' = \left\{0, 1, \ldots, t', \ldots, n-1\right\}$ as the set of micro periods. Let the generic elements be referred respectively as t and t'. Note that T' is a set with n elements, i.e. $|T'| = n$ and T is the time horizon.

The interconnection between these two-time scales is depicted in Fig. 2, where each $t \in T$ corresponds to n elements in T'.

Two important entities must be formulated in this approach to model time: the "previous time" and the "forward time".

3.1 Backward time operator

Previous time appears associated with the concept of travel time, which is defined between two micro period instances: origin and destination. When dealing, in the same constraint, with these two instances, two different situations must be considered: first, the origin and destination belong to the same macro period (Fig. 3a) and second, they belong to different macro periods (Fig. 3b).

In order to account for these situations an operator Υ, called "backward time operator", is defined. For $t \in T$ and $t' \in T'$, let (t, t') be the current time instance and let τ be the number of micro time units associated with a previous event, which is to be accounted for at (t, t'). Consider $\omega_1 \in \mathbb{Z}$ the smallest integer greater or equal than

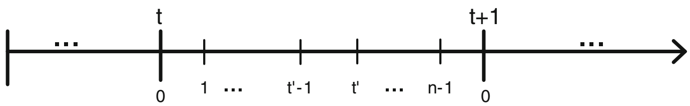

Fig. 2 Relation between macro and micro time scales

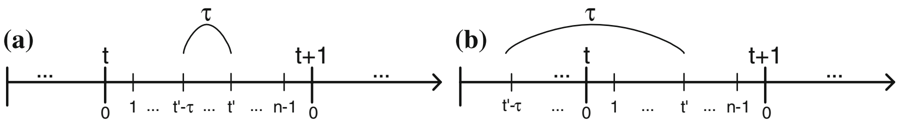

Fig. 3 **a** Both micro periods belong to the same macro period, **b** Micro periods belong to different macro periods

$\frac{\tau-t'}{n}$ (i.e. $\left\lceil \frac{\tau-t'}{n} \right\rceil = \omega_1$). The operator Υ gives the relevant time instance ("backward time") for that previous event. This operator is defined as follows:

$$\Upsilon(t, t'-\tau) = \begin{cases} (t, t'-\tau), & \text{if} \quad t'-\tau \geq 0 \\ (t-\omega_1, \omega_1 n + t' - \tau), & \text{if} \quad t'-\tau < 0 \wedge t > \omega_1 \end{cases}$$

To have a better insight into this operator let us consider an example where $n = 12$ and $(t, t') = (4, 6)$, i.e. there are 12 micro time instances and the current time instance is 4 on macro time and 6 on micro time. For a previous event characterised by $\tau = 5$, the corresponding previous time instance is given by $\Upsilon(t, t'-\tau) = \Upsilon(4, 1) = (4, 1)$ since $t' - \tau \geq 0$ (Fig. 3a). If, on the other hand, $\tau = 19$ is assumed, then $\left\lceil \frac{\tau-t'}{n} \right\rceil = \omega_1 \Leftrightarrow \left\lceil \frac{19-6}{12} \right\rceil = 2$ and the previous time instance would be $\Upsilon(t, t'-\tau) = (t-\omega_1, \omega_1 n + t' - \tau) = (2, 11)$ since $t' - \tau < 0$ (Fig. 3b).

To simplify notation and unless there is ambiguity, the generic micro time element (t, t') will be from now on denoted simply by t'.

3.2 Forward time operator

Forward time appears whenever usage time is modelled. This is the case when customers inbound and outbound flows are considered in the same constraint. One must assure that a product remains in the customer at least the predefined usage time.

As done for previous time, an operator is defined in order to allow the modelling of this other entity. The forward time operator Φ is defined as follows.

For $t \in T$ and $t' \in T'$, let (t, t') be the current time instance and let ϕ be the number of micro time units associated with an upcoming event, which is to be accounted for at (t, t'). Consider $\omega_2 \in \mathbb{Z}$ the smallest integer greater or equal than $\frac{t'+\phi-1}{n}$ (i.e. $\left\lceil \frac{t'+\phi-1}{n} \right\rceil = \omega_2$). The operator Φ gives the relevant time instance ("forward time") for that upcoming event. This operator is defined as follows:

$$\Phi(t, t'+\phi) = \begin{cases} (t, t'+\phi), & \text{if} \quad t'+\phi \leq n-1 \\ (t+\omega_2, t'+\phi-\omega_2 n - 1), & \text{if} \quad t'+\phi > n \wedge t + \omega_2 \leq \text{card}(T) \end{cases}$$

Consider where $n = 12$ and $(t, t') = (3, 8)$, i.e. there are 12 micro time instances and the current time instance is 3 on macro time and 8 on micro time. For a forward event characterised by $\phi = 2$, the corresponding forward time instance is given by $\Phi(t, t'+\phi) = \Phi(3, 10) = (3, 10)$ since $t' + \phi \leq n - 1$. If, on the other hand, $\phi = 16$ is assumed, then $\left\lceil \frac{t'+\phi-1}{n} \right\rceil = \omega_2 \Leftrightarrow \left\lceil \frac{8+16-1}{12} \right\rceil = 1$ and the forward time instance would be $\Phi(t, t'+\phi) = (t+\omega_2, t'+\phi-\omega_2 n - 1) = (4, 11)$ since $t' + \phi > n$.

4 Model formulation

In this section, the proposed model is described in detail. Indices are presented first, then the definition of sets and variables are given, followed by the constraint definition and the objective function. In the description of variables and constraints, we start by defining those that are not time dependent, followed by the ones defined over the macro time and finally the ones over the micro time.

4.1 Indices

Consider the following indices: i factories ($i = 0$, fictitious factory that allows products to leave the supply chain—disposal option), j warehouses, l disassembly centres, k customers, m_f forward products, m_r reverse products, t macro time and t' micro time.

4.2 Sets

For the model formulation, several sets are needed, which can be divided into three groups: location, products and time.

As location sets, we have:

I potential location for factories (which can be generalised as $I_0 = I \cup \{0\}$, in order to allow return products to leave the network for a different type of processing),
J potential location of warehouses,
L potential location of disassembly centres,
K potential location of customers' sites.

When dealing simultaneously with forward and reverse networks, one should be careful with the modelling of products, as they may undergo transformation while moving between networks. Thus, for each network different products were created, grouped into different sets:

M_f set of products in the forward network,
M_r set of products in the reverse network.

As mentioned above, in Sect. 3, two different time scales are defined over the time horizon:

$T = \{1, \ldots, t, \ldots, T\}$ is the broader time scale (referred as macro time), and
$T' = \left\{0, 1, \ldots, t', \ldots, n-1\right\}$ is the finer time scale (referred as micro time), noting that there are n micro intervals for each macro interval t.

4.3 Parameters

4.3.1 Time independent parameters

γ is the minimal disposal fraction for sending some return products to an external processing facility; it takes values in the range [0, 1],

$s^{p}_{m_f i0}, s^{a}_{m_f j0}, s^{r}_{m_r l0}, s^{c}_{m_r k0}$ is the initial stock of product m_f in factory i and warehouse j, and of product m_r in disassembly centre l and in the possession of customer k, respectively,

$f^{p}_{i}, f^{a}_{j}, f^{r}_{l}$ fixed cost of opening factory i, warehouse j, and disassembly centre l, respectively,

Upper and lower limits

$g^{s_p}_{i}, g^{s_a}_{j}, g^{s_r}_{l}$ maximum storage capacity of factory i, warehouse j, and disassembly centre l,

g^{p}_{i}, h^{p}_{i} maximum and minimum production of factory i,

g^{f1}_{i}, h^{f1}_{i} upper and lower bound values for flows leaving factory i,

g^{r2}_{i}, h^{r2}_{i} upper and lower bound values for flows arriving at factory i,

g^{f2}_{j} upper bound value for flows leaving warehouse j,

g^{r1}_{l} upper bound value for flows arriving at disassembly centre l,

Transfer parameters

δ_{ab} travel time between locations a and b, i.e. the number of micro time units required to travel from location a to location b,

ϕ_{m_r} usage time of product m_r, i.e. number of micro time units the product m_r spends at customers, before entering the reverse network.

Product parameter

Material balance constraints of factories and customers relate forward with reverse products. For factories, there is an inbound of return products that must accounted for together with the production of new ones. For customers, they receive forward products and use them. After use, the products are released onto the reverse chain and considered as return products.

η_{m_f} is the fraction of product m_f that is returned by customers,

$\varepsilon_{m_f m_r} = 1$ if, after use, product m_f is considered as product m_r; zero otherwise.

With these two parameters, another two, which are required in the model constraints, can be computed as follows:

$\beta_{m_f m_r} = \eta_{m_f}\varepsilon_{m_f m_r}$ fraction of forward product m_f that once used is treated as return product m_r,

$\alpha_{m_r m_f} = \dfrac{\eta_{m_f}}{\sum_{m_f \in M_f} \varepsilon_{m_f m_r}}$ fraction of product m_r that will be remanufactured as product m_f.

4.3.2 Macro time parameters

$d_{m_f kt}$ demand of product m_f to be supplied to costumer k, over macro period t,

$c^{u}_{m_f kt} (c^{w}_{m_r kt})$ unit variable cost of non-satisfied demand (return) of product $m_f (m_r)$ for customer k, for macro period t.

4.3.3 Micro time parameters

Lower and upper bounds

$h^{f_2}_{m_f kt'}$, $h^{r_1}_{m_r kt'}$ lower bound value for flows reaching and leaving customer k, at time t', respectively

Selling and transfer prices

$\pi^{f_1}_{m_f ijt'}$ unit transfer price of product m_f from factory i to warehouse j, at time t',
$\pi^{f_2}_{m_f jkt'}$ unit price of product m_f sold by warehouse j to customer k, at time t',
$\pi^{r_1}_{m_r klt'}$ unit price of product m_r bought by disassembly centre l to customer k, at time t',
$\pi^{r_2}_{m_r lit'}$ unit transfer price of product m_r from disassembly centre l to factory i, at time t'.

Costs

$c^{p}_{m_f it'}$ unit production cost of product m_f, manufactured in factory i, at time t',
$c^{s_p}_{m_f it'}$ $c^{s_a}_{m_f jt'}$ unit storage cost of product m_f kept in factory i and in warehouse j, at time t', respectively
$c^{s_r}_{m_r lt'}$ unit storage cost of product m_r kept in disassembly centre l, at time t'.
$c^{f_1}_{m_f ijt'}$ unit transportation cost of product m_f from factory i to warehouse j, at time t',
$c^{f_2}_{m_f jkt'}$ unit transportation cost of product m_f from warehouse j to customer k, at time t',
$c^{r_1}_{m_r klt'}$ unit transportation cost of product m_r from customer k to disassembly centre l, at time t',
$c^{r_2}_{m_r lit'}$ unit transportation cost of product m_r from disassembly centre l to factory $i(i \in I_0)$, at time t'.

4.4 Variables

The variables that are time independent will be described first. The macro time variables follow and finally the micro time variables will be defined.

4.4.1 Binary variables

These variables describe the choice of sites in the supply chain and if a customer should or should not be supplied.

$Y^p_i = 1$ if factory i is opened; zero otherwise, $i \in I$,
$Y^a_j = 1$ if warehouse j is opened; zero otherwise,$j \in J$,
$Y^r_l = 1$ if disassembly centre l is opened; zero otherwise, $l \in L$ and
$Y^c_k = 1$ if customer k integrates the supply chain; zero otherwise, $k \in K$.

4.4.2 Continuous variables

Two sets of variables are defined over the macro time. These describe the amount of demand/return that is not satisfied:

$U_{m_f kt}$ non-satisfied demand of product m_f of customer k, over macro period t, and
$W_{m_r kt}$ non-satisfied return of product m_r of customer k over macro period t.

All other variables are defined for the micro time. These are the flow, stock and production variables.

The flow variables are:

$X^{f_1}_{m_f ijt'}$ demand of product m_f served by factory i to warehouse j, over period t',
$X^{f_2}_{m_f jkt'}$ demand of product m_f served by warehouse j to customer k, over period t',
$X^{r_1}_{m_r klt'}$ return of product m_r of customer k to the disassembly centre l over period t', and
$X^{r_2}_{m_r lit'}$ return of product m_r of disassembly centre l to factory i ($i \in I_0$), over period t'.

In Fig. 4, the relation between flow variables and binary variables is represented, together with the different network echelons.
The stock variables are:

$S^p_{m_f it'}$ amount of product m_f stocked in factory i, over period t'.
$S^a_{m_f jt'}$ amount of product m_f stocked in warehouse j, over period t',
$S^r_{m_r lt'}$ amount of product m_r stocked in disassembly centre l, over period t'.

Lastly, the production variable is

$Z_{m_f it'}$ amount of product m_f produced by factory i, over period t'.

All continuous variables are non-negative.

4.4.3 Auxiliary variables

Auxiliary variables, which are not used as model decisions variables, are defined to ease the constraint formulation. These are the stock variables at customer sites and the dummy binary variables associated with flows. The former describe the amount of product m_r kept by customer k, over micro period t' and is represented by $S^c_{m_r kt'}$. The latter are used to define a set of disjunctive constraints.

4.5 Constraints

As before, the constraints definition is made considering first those defined over the macro times and then those over the micro times.

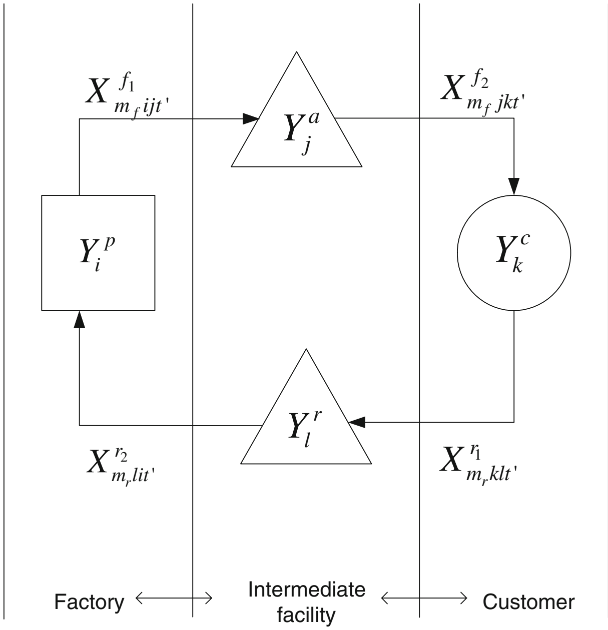

Fig. 4 Schematics of the interdependence between flow and binary variables

4.5.1 Macro time constraints

These macro time constraints relate quantities aggregated in time such as demand, return and disposal with detailed ones such as flows. For each aggregate quantity, a constraint is defined.

Demand constraint Each selected costumer k, in each macro period t, has a specific demand for product m_f that needs to be satisfied. This demand can be (totally or partially) satisfied by all the inbound flows that reach customer k, during the time interval t. Assuming that customers may or may not integrate the supply chain, this constraint is only active if the customer is chosen ($Y_k^c = 1$)

$$\sum_{j \in J} \sum_{t' \in T'} X^{f_2}_{m_f jk\Upsilon(t,t'-\delta_{jk})} + U_{m_f kt} = d_{m_f kt} Y^c_k, \quad \forall m_f, k, t \tag{1}$$

Return constraint The return volume of each product m_r for each customer k, in each macro period t is assured by constraint (2). It is considered that the total amount that is sent to any disassembly centre plus the nonsatisfied return amount (if any) must equal the total amount of used product that this customer has to send back. This total return volume is computed as the weighted sum of all forward product supplied to the customer. As above-mentioned, a usage time is taken into account by operator Φ, which does not allow products to go into reverse network before staying a least some micro time periods in customers' possession.

$$\sum_{l \in L} \sum_{t' \in T'} X^{r_1}_{m_r k l \Phi(t,t'+\phi_{m_r})} + W_{m_r k t}$$
$$= \sum_{m_f \in M_f} \sum_{j \in J} \sum_{t' \in T} \beta_{m_f m_r} X^{f_2}_{m_f j k \Upsilon(t,t'-\delta_{jk})}, \quad \forall m_r, k, t \tag{2}$$

Disposal fraction One of the features of the proposed network is that it allows products to leave the network. This is possible through the definition of a fraction of collected products that each disassembly centre sends either to recycling or to proper disposal. Thus, a part of the inbound flows of disassembly centre l can be sent to a fictitious factory, factory $i = 0$ that represents any facility outside the supply chain:

$$\gamma \sum_{k \in K} \sum_{t' \in T'} X^{r_1}_{m_r k l \Upsilon(t,t'-\delta_{kl})} \leq \sum_{t' \in T'} X^{r_2}_{m_r l 0 t'}, \quad \forall m_r, l, t \tag{3}$$

4.5.2 Micro time constraints

Having established, in the global chain, the relation between aggregated quantities and flows, it is important to relate the latter with the operational aspects, such as production, storage and transportation. As a result, the final network structure is also defined, namely sites and associated connections.

Production constraints These constraints are established for two main purposes: to assure the connection between production factories and associated inbound and outbound flows and to guarantee factory capacity production limits.

The production planning contemplated in this model is of a broad strategic nature, no consideration being given either to products' bill-of-materials or to the difference between manufacturing and remanufacturing when outbound flows are considered. However, factories have specific inbound flows that need to be handled, which are the return products sent in by disassembly centres.

- Material balance

For each micro period t', product m_f and factory i, constraint (4) assures that the amount produced plus the used products (inbound flow) and the existing inventory (which was set in the previous micro period) equals the total product delivered by the factory (outbound flow) plus the remaining stock. If $\Upsilon(t, t' - 1) = (1, 0)$, then $S^p_{m_f i \Upsilon(t,t'-1)} = s^p_{m_f i 0}$ which is the initial stock level at factory i.

$$Z_{m_f i t'} + \sum_{m_r \in M_r} \sum_{l \in L} \alpha_{m_f m_r} X^{r_2}_{m_r l i \Upsilon(t,t'-\delta_{li})} + S^p_{m_f i \Upsilon(t,t'-1)}$$
$$= \sum_{j \in J} X^{f_1}_{m_f i j t'} + S^p_{m_f i t'}, \quad \forall m_f, i \in I, (t, t') \tag{4}$$

- Capacity constraints

In any factory i, the production level is double bounded. These bounds are assumed to be constant over the model horizon and depend only on the factory. Furthermore, for a factory to be used, it must be installed. Constraints (6) and (7) model these cases:

$$\sum_{m_f \in M_f} Z_{m_f i t'} \leq g_i^p Y_i^p, \quad \forall i \in I, (t, t') \tag{5}$$

$$\sum_{m_f \in M_f} Z_{m_f i t'} \geq h_i^p Y_i^p, \quad \forall i \in I, (t, t') \tag{6}$$

In addition, as storage is allowed in factories, an upper bound is set:

$$\sum_{m_f \in M_f} S^p_{m_f i t'} \leq g_i^{s_p} Y_i^p, \quad \forall i \in I, (t, t') \tag{7}$$

Warehouse storage constraints At the warehouses, a material balance is also established where the inbound flow plus the existing stock equals the outbound flow plus the remaining stock. This must be assured in every micro period, for every existing product.

$$\sum_{i \in I} X^{f_1}_{m_f i j \Upsilon(t, t' - \delta_{ij})} + S^a_{m_f j \Upsilon(t, t'-1)} = \sum_{k \in K} X^{f_2}_{m_f j k t'} + S^a_{m_f j t'}, \quad \forall m_f, j, (t, t') \tag{8}$$

Again, if $\Upsilon(t, t'-1) = (1, 0)$, then $S^a_{m_f j \Upsilon(t, t'-1)} = s^a_{m_f j 0}$ which is the initial stock level at warehouse j. Furthermore, every warehouse j has a maximum storage capacity ($g_j^{s_a}$), if chosen (i.e. $Y_j^a = 1$).

$$\sum_{m_f \in M_f} S^a_{m_f j t'} \leq g_j^{s_a} Y_j^a, \quad \forall j, (t, t') = (1, 1) \tag{9}$$

Disassembly centre storage constraints Disassembly centres can be viewed as reverse network warehouses. Therefore, constraints are similar to constraints (8) to (9).

Equation (10) establishes the material balance, which relates storage volumes with the inbound/outbound flows. Thus, in each period t', the inbound flow and the existing storage must equal the outbound flow plus the new storage volume.

$$\sum_{k \in K} X^{r_1}_{m_r k l \Upsilon(t, t' - \delta_{kl})} + S^r_{m_r l \Upsilon(t, t'-1)} = \sum_{i \in I} X^{r_2}_{m_r l i t'} + S^r_{m_r l t'}, \quad \forall m_r, l, (t, t') \tag{10}$$

If $\Upsilon(t, t'-1) = (1, 0)$, then $S^r_{m_r l \Upsilon(t, t'-1)} = s^r_{m_r l 0}$ which is the initial stock level at disassembly centres l.

For each disassembly centre l, a maximum storage capacity ($g_l^{s_r}$) is also considered if the facility is used ($Y_l^r = 1$).

$$\sum_{m_r \in M_r} S^r_{m_r l t'} \leq g_l^{s_r} Y_l^r, \quad \forall l, (t, t') \tag{11}$$

Customer constraints Although customers are external to the company in charge of the supply chain, they are modelled as any other facility. Demand satisfaction is the criterion for customers to become part of the supply chain. In this model, we assume that there may be cases of customers, which are not chosen to be supplied on economic grounds. Thus, we introduced in the demand constraint (1), a binary decision variable that indicates whether or not a customer should be considered in this supply chain.

Constraint (12) is, then, the material balance constraint for each customer. Similarly, to factories, customers have a transformation role: they "transform" forward products into "return" ones by using them. Another feature of this constraint is the modelling of "usage time" previously defined. Usage time is introduced in the outbound term, which makes the constraint to relate the inbound at time t' with the outbound at micro time $t' + \phi$.

$$\sum_{m_f \in M_f} \sum_{j \in J} \sum_{k \in K} \beta_{m_r m_f} X^{f_2}_{m_f jk\Upsilon(t,t'-\delta_{jk})} + S^c_{m_r k\Upsilon(t,t'-1)} = \sum_{l \in L} X^{r_1}_{m_r kl\Phi(t,t'+\phi_{m_r})} + S^c_{m_r kt'}, \quad \forall m_r, k, (t, t') \tag{12}$$

Once again, for the first micro period $(t, t') = (1, 1)$, the existing volume of products at customer site is the given parameter $s^c_{m_r k0}$.

Transportation flows constraints The transportation flows constraints have a double function. They assure that flows remain within certain pre-established limits and that they only occur between opened/existing facilities.

All flows, in every micro time, must fall below a preset maximum level.

The minimum level is modelled differently among flows. Factory inbound and outbound flows are not imposed on every micro time, but when they occur they must meet the minimum limit. This is modelled using auxiliary variables, which are described next. In terms of customers, it is assumed that a minimum flow amount must be sent and collected in every micro time, for every customer.

- Factory outbound flow

$$\sum_{m_f \in M_f} X^{f_1}_{m_f ijt'} \le g^{f_1}_i E^{f_1}_{ijt'}, \quad \forall i, j, (t, t') \tag{13}$$

$$\sum_{m_f \in M_f} X^{f_1}_{m_f ijt'} \ge h^{f_1}_i E^{f_1}_{ijt'}, \quad \forall i, j, (t, t') \tag{14}$$

$$2E^{f_1}_{ijt'} \le Y^p_i + Y^a_j, \quad \forall i, j, (t, t') \tag{15}$$

where $E^{f_1}_{ijt'}$ is an auxiliary binary variable for the flow between factory i and warehouse j, at micro time t'.

- Customer inbound flow

$$\sum_{m_f \in M_f} X^{f_2}_{m_f jkt'} \le g^{f_2}_j Y^a_j, \quad \forall j, k, (t, t') \tag{16}$$

$$\sum_{m_f \in M_f} \sum_{j \in J} \sum_{t \in T} \sum_{t' \in T'} X^{f_2}_{m_f jkt'} \leq BigM_1 Y^c_k, \quad \forall k \tag{17}$$

$$\sum_{j \in J} X^{f_2}_{m_f jkt'} \geq h^{f_2}_{m_f kt'} Y^c_k, \quad \forall m_f, k, (t, t') \tag{18}$$

where $BigM_1$ is given by $BigM_1 = g^{f_2}_j \cdot |M_f| \cdot |J| \cdot |T| \cdot |T'|$

- Customer outbound flow

$$\sum_{m_r \in M_r} X^{r_1}_{m_r klt'} \leq g^{r_1}_l Y^r_l, \quad \forall l, k, (t, t') \tag{19}$$

$$\sum_{m_r \in M_r} \sum_{l \in L} \sum_{t \in T} \sum_{t' \in T'} X^{r_1}_{m_r klt'} \leq BigM_2 Y^c_k, \quad \forall k \tag{20}$$

$$\sum_{l \in L} X^{r_1}_{m_r klt'} \geq h^{r_1}_{m_r kt'} Y^c_k, \quad \forall m_r, k, (t, t') :\sim (t{=}1 \wedge t' {-} \phi_{m_r} {<} 0) \tag{21}$$

where $BigM_2$ is given by $BigM_2 = g^{r_1}_l \cdot |M_r| \cdot |L| \cdot |T| \cdot |T'|$

- Factory inbound flow

$$\sum_{m_r \in M_r} X^{r_2}_{m_r lit'} \leq g^{r_2}_i E^{r_2}_{ilt'}, \quad \forall i \in I_0, l, (t, t') \tag{22}$$

$$\sum_{m_r \in M_r} X^{r_2}_{m_r lit'} \geq h^{r_2}_i E^{r_2}_{ilt'}, \quad \forall i \in I_0, l, (t, t') \tag{23}$$

$$2E^{r_2}_{lit'} \leq Y^r_l + Y^p_i, \quad \forall i, l, (t, t') \tag{24}$$

where $E^{r_2}_{lit'}$ is an auxiliary binary variable for the flow between disassembly centre l and factory i $(i \in I_0)$, at micro time t'.

4.6 Objective function

In the proposed model, the profit is set as the objective function, where both revenue and costs are computed.

4.6.1 Revenue

The forward and reverse revenues are computed in expressions (25) and (26), respectively.

$$\sum_{m_f \in M_f} \sum_{i \in I} \sum_{j \in J} \sum_{t' \in T'} \pi^{f_1}_{m_f ijt'} X^{f_1}_{m_f ijt'} + \sum_{m_f \in M_f} \sum_{j \in J} \sum_{k \in K} \sum_{t' \in T'} \pi^{f_2}_{m_f jkt'} X^{f_2}_{m_f jkt'} \tag{25}$$

$$\sum_{m_r \in M_r} \sum_{k \in K} \sum_{l \in L} \sum_{t' \in T'} \pi^{r_1}_{m_r klt'} X^{r_1}_{m_r klt'} + \sum_{m_r \in M_r} \sum_{l \in L} \sum_{i \in I_0} \sum_{t' \in T'} \pi^{r_2}_{m_r lit'} X^{r_2}_{m_r lit'} \tag{26}$$

4.6.2 Fixed investment costs

The investment costs considered are related to the opening/usage of facilities (factories, warehouses and disassembly centres) in each possible location. Thus, we have:

$$\sum_{i \in I} f_i^p Y_i^p + \sum_{j \in J} f_j^a Y_j^a + \sum_{l \in L} f_l^r Y_l^r \tag{27}$$

4.6.3 Operational costs

The operational costs represent the costs met by each type of facility when performing its core business. So for factories, both production and storage costs are considered,

$$\sum_{m_f \in M_f} \sum_{i \in I} \sum_{t' \in T'} c^p_{m_f i t'} Z_{m_f i t'} + \sum_{m_f \in M_f} \sum_{i \in I} \sum_{t \in T'} c^{s_p}_{m_f i t'} S^p_{m_f i t'} \tag{28}$$

while storage costs are defined for warehouses and disassembly centres:

$$\sum_{m_f \in M_f} \sum_{j \in J} \sum_{t' \in T'} c^{s_a}_{m_f j t'} S^a_{m_f j t'} + \sum_{m_r \in M_r} \sum_{l \in L} \sum_{t' \in T'} c^{s_r}_{m_r l t'} S^r_{m_r l t'} \tag{29}$$

4.6.4 Transportation costs

Forward and reverse transportation costs are defined by:

$$\sum_{m_f \in M_f} \sum_{i \in I} \sum_{j \in J} \sum_{t' \in T'} c^{f_1}_{m_f i j t'} X^{f_1}_{m_f i j t'} + \sum_{m_f \in M_f} \sum_{j \in J} \sum_{k \in K} \sum_{t' \in T'} c^{f_2}_{m_f j k t'} X^{f_2}_{m_f j k t'} \tag{30}$$

$$\sum_{m_r \in M_r} \sum_{k \in K} \sum_{l \in L} \sum_{t' \in T'} c^{r_1}_{m_r k l t'} X^{r_1}_{m_r k l t'} + \sum_{m_r \in M_r} \sum_{l \in L} \sum_{i \in I_0} \sum_{t' \in T'} c^{r_2}_{m_r l i t'} X^{r_2}_{m_r l i t'} \tag{31}$$

4.6.5 Penalty costs

Extra costs, referred as penalty costs, are also considered that translate the penalty for a nonsatisfied demand and return:

$$\sum_{m_f \in M_f} \sum_{k \in K} \sum_{t \in T} c^u_{m_f k t} U_{m_f k t} + \sum_{m_r \in M_r} \sum_{k \in K} \sum_{t \in T} c^w_{m_r k t} W_{m_r k t} \tag{32}$$

5 Case study

5.1 Definition

To illustrate the applicability of the proposed model, we consider a modified case study introduced in Salema et al. (2005). In here, a company settled in the Iberian Peninsula plans a network with a forward flow of three families of products (P_1, P_2, P_3) and a reverse flow with two product families (R_1, R_2). Three possible locations for the manufacturing/remanufacturing plants are considered: Coimbra, Lisboa and Madrid. The warehouses and disassembly centres are to be located in any of the following six cities: Barcelona, Coimbra, Lisboa, Madrid, Sevilla and Zaragoza.

Regarding time, the horizon is defined as 5 years, the macro and micro time units are made equal to one year and one trimester, respectively. Note that, in this case the micro time starts at $t' = 1$ and ends at $t' = 4$. All travel times are set to zero since they are much smaller than the micro period (trimester). From now on, product families are referred simply as products.

5.1.1 Demand and return

Product demand is divided into customers' clusters, located in 16 cities: Barcelona, Bilbao, Braga, Coimbra, Coruna, Granada, Lisboa, Madrid, Malaga, Oviedo, Porto, Santander, Sevilla, Valencia, Valladolid and Zaragoza. The demand and return parameters are set as macro time parameters and, therefore, defined over the 5-year period. For the first year, demand volumes are assumed proportional to the cities' population. This is expressed as a fraction, which varies between 0.08 and 0.15. Concerning return volume, it is assumed that all three forward products are returned with a fraction of 0.8 of the total satisfied demand. Products P_1 and P_2 are collected as product R_1 and product P_3 as product R_2.

To reflect demand fluctuation throughout the remaining 4 years, a variation factor is applied over the number of inhabitants. This factor varies between 0.98 and 1.05, giving rise to an increase/decrease in the cities population. This provides a convenient way to introduce variations in demands over time.

A usage time of micro time period is set for all forward products.

5.1.2 Factories

Factories capacity limits are imposed over a 5-year period. Factories are considered having different sizes. Thus, Madrid factory may produce between 4×10^5 and 6×10^5 product units, while Lisboa and Coimbra must have their production level between 1×10^5 and 2×10^5.

The storage maximum capacity is set equal for all facilities and remains constant during the 5-year period (3×10^4 units). The initial stock level is assumed as zero.

Production and storage costs are product dependent and set on an yearly basis. They are updated by applying an actualisation rate of 3% per year.

5.1.3 Warehouses and Disassembly centres

Storage costs are considered equal for all locations and products, within facilities of the same type. Again, the actualisation rate is used to reflect changes over time. The maximum storage capacity is set to 3×10^4 units, while the initial stock level is assumed as zero. The disposal fraction γ is set to 0.3, assuming that at least 30% of all returns are not proper for remanufacturing.

Table 1 Transportation taxes

Flow from…	…Portugal to Spain	…Spain to Portugal
Tax	6%	3%

Table 2 Computational results of the case study

Total variables	Binary variables	Total constraints	Optimality margin	Iterations	CPU's (s)	Relaxation value	Obj. function value (m.u.)
14,522	871	10,151	0.04%	34,059	209	$10{,}405 \times 10^6$	$10{,}343 \times 10^6$

5.1.4 Flows

For all flows, upper and lower bounds are imposed. Flows from and to factories are set between 1×10^5 and 8×10^5, respectively. Flows to and from customers have an upper bound 6×10^5 and the lower bound is set proportional to each customer's demand.

Transportation costs are assumed to depend on the geographical distance between cities. Whenever different facilities are located in one city, a fixed short distance is assumed between them, for estimation of those costs. These are updated yearly by means of an actualisation rate.

In addition and although not explicit in the model, a tax was applied to the estimation of the flow costs, whenever the flow links cities in different countries (see Table 1).

Selling and transfer prices are also estimated and take into account any special business arrangements that the company may have with its customers.

5.1.5 Penalisation costs

These costs are assumed equal for all product and customers.

5.2 Results

The computational results for this case are summarised in Table 2. The MILP formulation was solved using GMAS/CPLEX 10 (Brooke et al. 2003), on a Pentium 4, 3.40 GHz.

From the results presented in Table 2, one can conclude that the computational effort is modest despite the relatively high number of binary variables.

The strategic and tactical results are detailed below.

5.2.1 Strategic decisions

The optimal network structure involves two factories: Coimbra and Madrid. These serve four warehouses and disassembly centres, located in Coimbra, Madrid, Sevilla and Zaragoza. Figure 5 shows both the major structure of the forward and the reverse

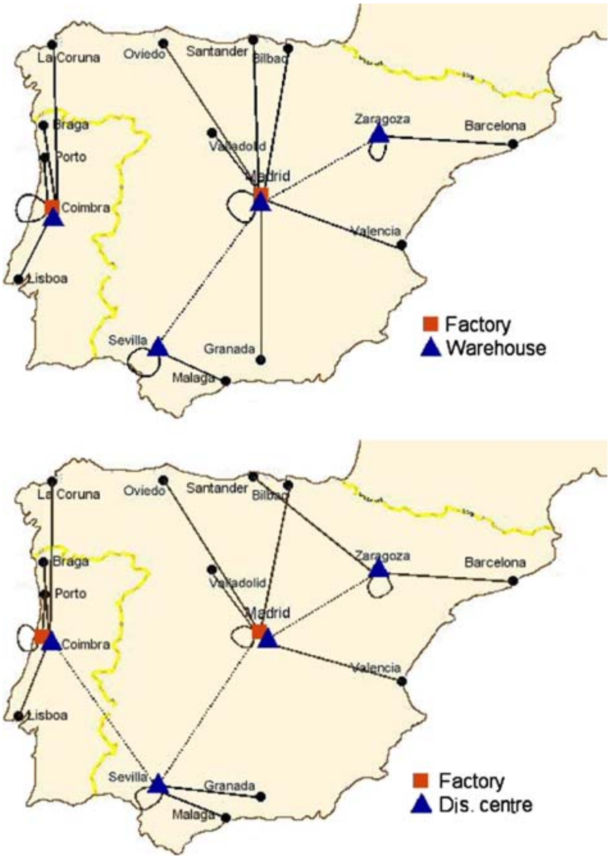

Fig. 5 Forward and reverse networks

networks. The forward network is formed by two disjunctive sub-networks: one based in Coimbra serving customers located in Portugal and one Spanish customer, and the other based in Madrid serving the remaining facilities and customers. All customers have one warehouse that satisfies their demand during most of the horizon time length. However, in some micro periods and mostly due to capacity limits, some customers require some additional supply from a different warehouse in order to be fully supplied. These one-off events are not depicted in the networks.

Forward and reverse networks are very similar. However in the reverse network, the two subnetworks are no longer disjunctive, but are linked through a connection between Sevilla and Coimbra. Disassembly centres have the same location as warehouses and serve almost the same set of customers. The major difference lies with the Santander customer that has its returns collected by Zaragoza disassembly centre. This is due to the existence of limits on flows.

It is important to notice the central role played by Coimbra and Madrid. They were chosen to locate factories, as well as warehouses and disassembly centres, which serve the majority of customers. As the transportation costs are distance based, these two

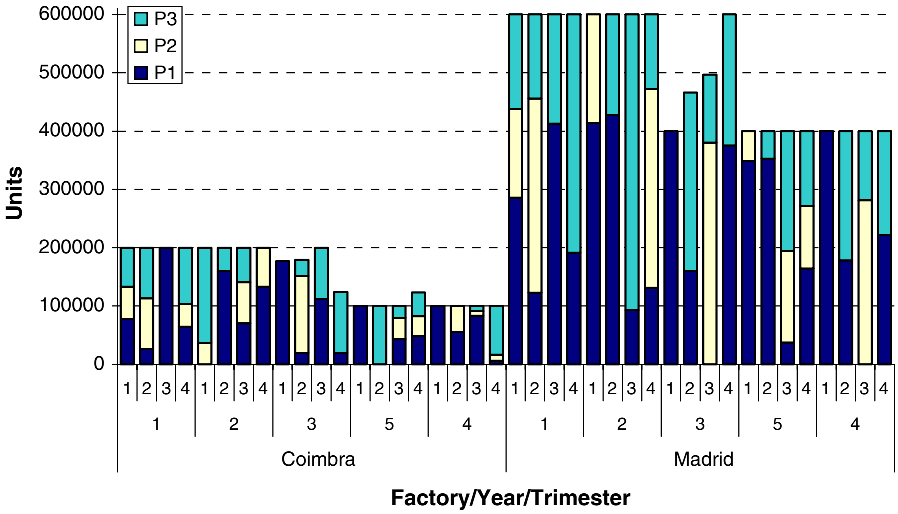

Fig. 6 The production plan

cities emerge as geographical centres. Coimbra and Madrid are the most central cities in each country, leading to a minimal cost network.

5.2.2 Tactical decisions

Concerning the tactical level of decision, three different analyses can be made, respectively for production, storage and distribution. As the model produces a large amount of information, only some of the results will be discussed.

Figure 6 shows that for most of the time, production is levelled at its maximum or minimum levels (note that factories were considered as having different sizes: Coimbra—2×10^5 and Madrid—6×10^5). One sees a predominance of product P_1, which is explained by its higher demand level. Note that, under these conditions, the total costumer demands are met and the returned products cover the difference between the level of demand and the amount produced.

In terms of storage planning, the optimal solution represents a zero stock policy.

Finally, in terms of distribution, four examples will be discussed. Each one refers to a different echelon (Figs. 7, 8, 9, 10).

Figure 7 shows the distribution plan between the factory and the warehouse located in Coimbra. Due to the minimum limit imposed on this flow, at least a volume of 1×10^5 is shipped to this facility every trimester. Moreover, all products are sent in every shipment although the minimum limit is not product dependent.

In Fig. 8, the supply plan of Sevilla customer is shown. For each product, a minimum level of supply was imposed (the values are of the order of 10,500 units for P_1, 7,000 units for P_2 and 8,400 units for P_3). In every micro time, all three products are supplied, at least at their minimum. Periods $(t, t') = (4, 4)$ and $(t, t') = (5, 4)$ exhibit this minimum supply. As mentioned above, some one-off flows occur and, in

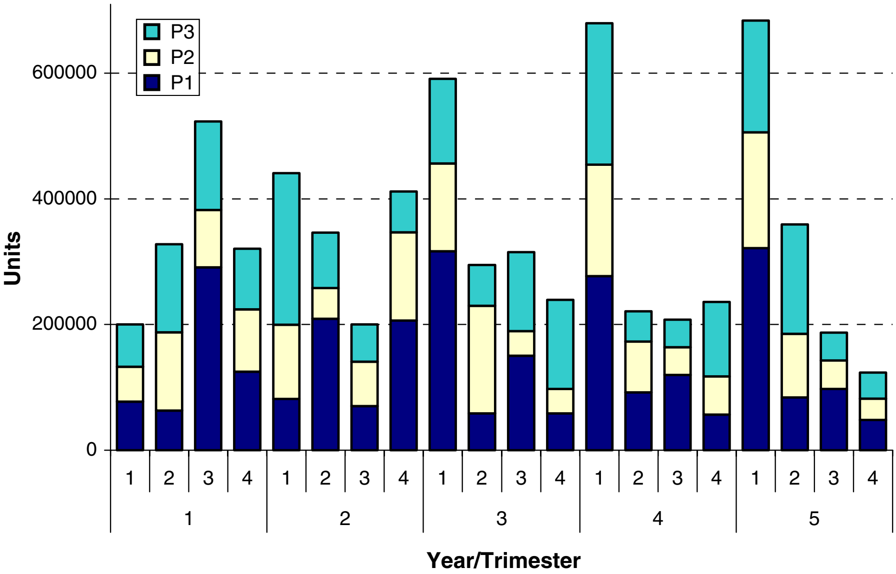

Fig. 7 Flows between factory and the warehouse located in Coimbra

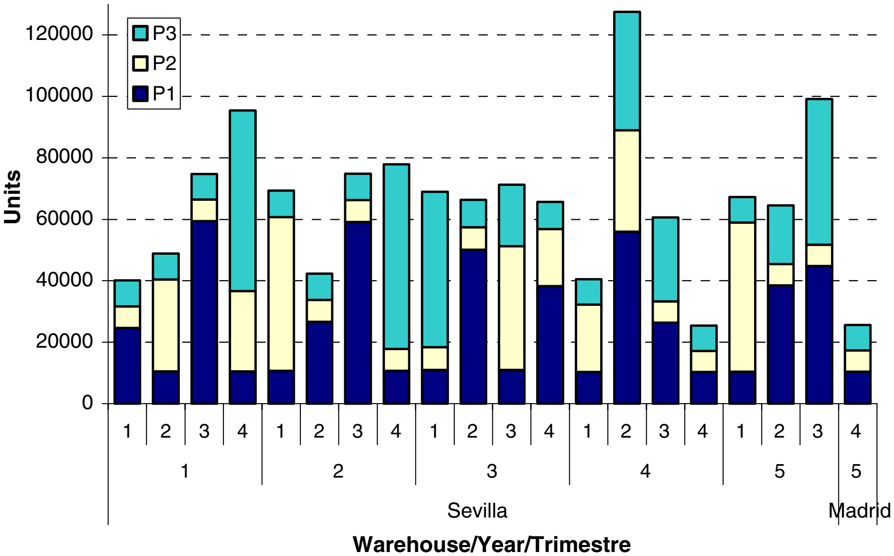

Fig. 8 Seville supply plan

this figure, one is shown: Madrid supplies Sevilla only once and in the last period of the time horizon.

Figure 9 shows the flow between customers and the disassembly centre located in Zaragoza. In this figure, three aspects are worth mentioning:

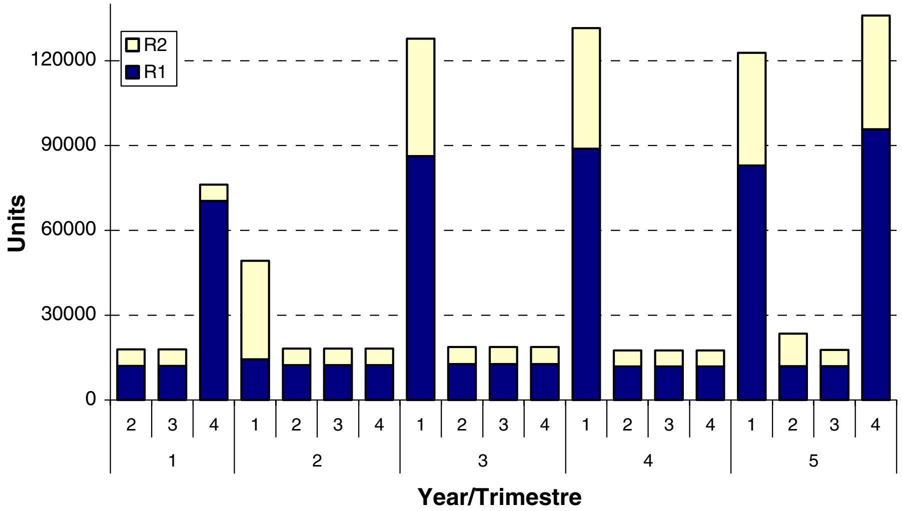

Fig. 9 Return plan for Zaragoza customer

a. there is no flow in the first micro period (1,1)—the reason behind this is that a "usage time" of one micro time unit is assumed; thus, products that are supplied in time (1,1) are only available for collection at time (1,2) onwards.
b. the predominance of product R_1—this product comes from the use of two forward products, while R_2 is related only to one product.
c. almost every year, collection runs at peak level for one trimester and at the minimum level for the remainder. This was found as the most economical plan for this customer. Other customers exist that follow a different collection pattern.

Lastly, the return of Zaragoza disassembly centre is depicted in Fig. 10. One can see that the flow between this centre and the factory is often set at its minimum level. In the fifth year, all products are sent to disposal. Again, the existence of minimum flow limits (1×10^5) is clearly shown in Fig. 10.

5.3 Some other examples

One of the main drawbacks of MILP models is their exponential growth when applied to real problem instances. In this section, we will reflect on two questions: what is a realistic problem? How does the model behaves with larger problem instances?

Looking into companies and their supply chains, we can see that the diversity of designs is extremely large. For instance, Arntzen et al. (1995) studied the supply chain of Digital Equipment Corporation, now part of the Hewlett-Packard. This was a multinational company with facilities located in seven countries. The proposed model is a MILP formulation where instances have between 2,000 and 6,000 constraints, and 5,000–20,000 variables, with about one hundred of these variables being binary. Other studies exist that deal with smaller but still realistic supply chains. Amaro and Barbosa-Povoa (2008) studied the supply chain of an industrial company that operates

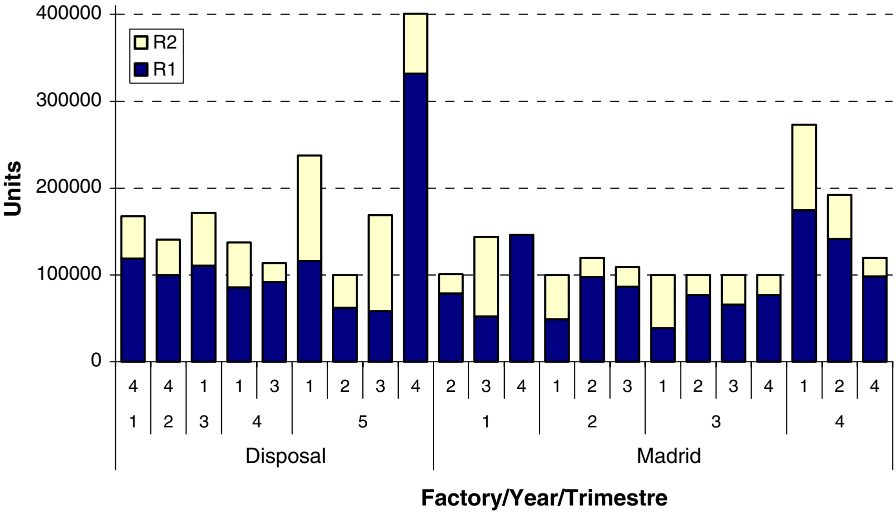

Fig. 10 Return flows of Zaragoza disassembly

Table 3 Computational results for example based on the case study

Example	Total variables	Binary variables	Total constraints	Opt gap (%)	Iterations	CPU (s)	Relaxation ($\times 10^6$)	Obj. function value ($\times 10^6$)
Base case	14,522	871	10,151	0.04	34,059	209	10,405	10,343
Example 1	42,682	2,535	29,511	0	144,056	405	10,769	10,706
Example 2	85,322	5, 055	59,011	0.01	46,442	1,095	26,656	26,589

in the Iberian Peninsula. The network is formed by two factories and ten distribution centres.

To analyse the model behaviour with larger problems, two groups of instances were generated. In the first the focus was on the time parameter, while in the second some instances were randomly generated.

The case presented above was rerun considering different time units. In the first example (example 1) the micro time unit was changed from 4 trimesters to 12 months. In the second example the time horizon was changed from 5 to 10 years, while the micro period was kept as 12 months.

The computational results are shown in Table 3 (the first line is taken from Table 1). The number of variables and constraints has grown considerably. Although the number of locations for facilities was kept unchanged, the number of binary variables has increased due to the dummy variables used to model flows. The CPU times and the optimality gap kept their low values. Finally, note that the relaxation value is very close to the objective function value.

In order to further explore the model performance when dealing with larger problem instances, some other cases were studied. These were randomly generated and six

Table 4 Some parameters of the random cases

	Number of facilities			Customers	Number of products		Time periods	
	Factories	Warehouses	Dis. centres		Forward	Reverse	Macro	Micro
Ex. 3	5	10	10	50	3	1	5 Periods	4 Periods
Ex. 4	5	10	10	50	3	1	5 Periods	**12 Periods**
Ex. 5	**10**	**20**	**20**	**150**	3	**2**	5 Periods	4 Periods
Ex. 6	10	20	20	150	3	2	5 Periods	**6 Periods**
Ex. 7	10	20	20	150	3	2	5 Periods	**12 Periods**
Ex. 8	10	20	20	150	**5**	2	5 Periods	**6 Periods**

Table 5 Computational results for random cases

	Total variables	Binary variables	Total constraints	Opt. gap(%)	Iterations Iterations	CPU (s)	Relaxation ($\times 10^6$)	Obj. function value ($\times 10^6$)
Ex. 3	49,886	2,225	34,311	0.17	73,001	520	50,285	50,028
Ex. 4	147,486	6,625	100,711	0.05	73,700	2,569	50,424	50,252
Ex. 5	342,361	8,600	173,761	0.01	197,446	2,232	109,023	107,189
Ex. 6	511,561	12,800	258,661	0.02	483,662	6,196	109,057	107,189
Ex. 7	1,019,161	25,400	513,361	Out of memory				
Ex. 8	707,461	12,800	270,961	0.04	389,327	16,354	206,001	203,705

runs were performed, each under different parameters (Table 4). The changes between runs are highlighted in bold.

The computational results are presented in Table 5. One can see that the model performs well under considerable large problem instances. However, it does not solve all instances. Memory problems appear in example 7 where there were over one million variables. This instance considered a 12-periods planning (micro time) and a five-period time horizon.

In example 3, the generated instance is larger, in terms of locations for facilities, than the case presented in Sect. 5.2. Example 4 is the same case with a different micro time unit. Comparing these two examples with regard to the number of variables and constraints one sees that the latter is three times larger than the former, and it took five times longer to be solved. Nonetheless, both examples performed well.

Examples 5–8 are the largest in terms of possible locations for facilities, since they vary in terms of both time units and number of products. With the exception of example 7, all instances took between 30 min to 5 h to be solved. Given the strategic nature of this model and the instances size, these computational times can be considered as very good.

With example 8, it is shown that an increase in the number of products flowing in the network only increases the number of continuous variables. However, this instance to be solved requires twice the CPU time found for example 6.

For all eight instances, the relaxation values can also be considered as highly satisfactory.

As a final remark, we may say that this model can solve large problems. Nevertheless, as any other MILP formulation there are instances, which are too large to be studied with a generic formulation.

6 Conclusions

The design of global supply chains is being considered by researchers as a very important field of study. However, its complexity has led to the analysis of its smaller subsystems, allowing the supply chain to be more tractable. Nevertheless, generic strategic models are needed to design and evaluate in an integrated manner, the global supply chain performance.

In this paper, we propose a model for the simultaneous design of forward and reverse networks where both strategic and tactical decisions are accounted for. Forward and reverse networks consist of twoechelon structures, creating a link between factories and customers through warehouses or disassembly centres. Strategic decisions are concerned with the design of the network while the tactical decisions enable to define production, storage and distribution planning. In order to achieve this, a two-time scale, with a fully interconnected structure, was developed. This scale involves a macro time related to the strategic decisions, and a micro time related with the tactical decisions definitions.

The final formulation leads to a MILP formulation that is fairly flexible. It considers several products in both networks, limits imposed on flows, production and storage capacities, together with product usage time at customers and different travel times along the networks.

The model was applied to a modified case study published elsewhere and the obtained results corroborate the model adequacy to real problems. As a main conclusion, it may be stated that the model appears as a useful and integrated tool to help the decision making process at the strategic and tactical levels of the supply chain management decisions.

The mathematical formulation which supports this model is, however, likely to increase significantly in complexity with the problem dimension. A first study was conducted that showed that the model performs well under large problem instances. However, there are instances that may become intractable. To overcome this possible computational burden, different solution techniques are now being explored in order to speed up the resolution. These may be general decomposition algorithms, such as the Benders Decomposition, and/or iterative algorithms based on the model time structure. Also some valid inequalities are to be tested. Further research is also being undertaken with a view to both strengthen the model formulation and to treat production planning in greater detail, with the introduction of bills of materials.

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