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REGULAR ARTICLE

Pricing and lot-sizing decisions in a two-echelon system with transportation costs

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Abstract We consider a single-buyer single-supplier system. The market demand is sensitive to the selling price set by the buyer. Both the buyer and the supplier operate with unit product costs, inventory holding costs, and order placement costs. In addition, the buyer is responsible for the freight cost. We formulate a model for determining the optimal lot-sizing and pricing decisions. Existing models for the problem do not consider the transportation costs with price sensitive market demand, and determine the optimal decisions through an exhaustive search. We propose an approximate solution procedure, and report the computational results on the effectiveness of the proposed procedure.

Keywords Inventory management · Transportation cost · Price-sensitive demand

1 Introduction

Lowering logistics costs has been of interest for enterprises. Although lot-sizing and pricing problems have been intensively studied in the literature, the effect of transportation costs has been generally neglected. Since transportation costs can be almost 50% of the total logistics cost (Swenseth and Godfrey 2002), incorporation of

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transportation costs into lot-sizing and pricing problems can have a significant value. This paper addresses this problem, and studies its impact on the buyer and the supplier.

We model a simple system consisting of a single-supplier supplying a product to a single-buyer. Demand for the product is deterministic and price-elastic. We assume an exponential demand function where the demand exponentially decreases as the price increases. The buyer buys the product from the supplier, sets the market price, and determines his order replenishment period. Meanwhile, the supplier sets the wholesale price and determines his production quantity and frequency. Both the supplier and the buyer incur procurement costs, inventory holding costs, and order placement costs. The buyer additionally incurs freight cost which depends on the size of shipments from the supplier. We assume that the production rate of the supplier is infinitely large, and there are no capacity constraints beyond the limitations of truck size in transportation.

Both parties act independently as rational agents maximizing their own profits. Given a wholesale price, the buyer determines the optimal retail price, and the order frequency and quantity. Full information is available for the supplier, and he knows the consequences of his wholesale price in the form of retail price and buyer's order frequency and quantity. Hence, the problem can be modeled as a Stackelberg game with the supplier acting as the leader and the buyer acting as the follower. The equilibrium point is determined through the solution of the Stackelberg game (an extensive review of the non-cooperative game theory can be found in Fudenberg and Tirole 1991). The objective of this paper is to develop an algorithm for approximating optimal pricing and lot-sizing decisions in this equilibrium point. We aim to develop an approximate procedure, because the profit function of the supplier is non-concave, and the freight cost function adds a discontinuity to it. In the core of this approximate solution procedure is a novel approach for modeling the response of the buyer to different values of the wholesale price. Characterizing buyer's response in turn helps us model the supplier's problem.

The remainder of the paper is organized as follows. In Sect. 2, we review the literature on pricing and lot-sizing decisions. In Sect. 3, we state the notation and provide an example revealing the contribution of the paper. In Sect. 4, we derive the optimal decisions for the buyer for a given wholesale price. In Sect. 5, we present an approximate algorithm for the optimal decisions of the supplier. A numerical illustration of the proposed procedure is presented in Sect. 6. In Sect. 7, we present a computational analysis of the proposed procedure. Finally, we present our concluding remarks in Sect. 8.

2 Literature review

Lot-sizing and pricing problems have been extensively studied for the last 3 decades. There is substantial research on the optimal lot-sizing problem with deterministic demand. Sethi (1984) develops a method for finding optimal lot sizes with price inelastic demand under quantity discounts. Kohli and Park (1989) incorporate bargaining and utility theory into the lot-sizing problem in a single-supplier single-buyer system. Chen and Chen (2005) consider a two-echelon system with multiple products and inventories at three-levels. They model major and minor setup costs for the supplier, and major transportation and minor processing costs for the buyer. They further propose a search algorithm for finding optimal production and inventory levels.

Lal and Staelin (1984) consider a single-supplier multiple-buyer system, and design the optimal quantity discounts. Lu (1995) proposes a procedure to solve this problem under information asymmetry. Wang and Wu (2000) also consider multiple price break-point discount pricing policy in this setting. Wang (2001) incorporates the power-of-two-policies, and proposes a uniform quantity discount policy to coordinate buyers' replenishments.

Kunreuther and Richard (1971) are the first to incorporate pricing decisions into the lot-sizing problem. They derive optimal pricing and lot-sizing decisions for a retailer. Abad (1988a) considers this case with all-units quantity discounted price, and Abad (1988b) considers incremental quantity discounts. Parlar and Wang (1994) model the single-supplier single-buyer system with linear demand function. They incorporate quantity discounts into lot-sizing and pricing decisions. Zahir and Sarker (1991) extend Kunreuther and Richard's (1971) setting to the single-supplier multiple-buyer model, and derive optimal production, lot-sizing, and pricing decisions under quantity discounts.

Coordination mechanisms have been of interest in single-supplier single-buyer systems with price-elastic demand functions. Weng (1995b) considers quantity discounts and pareto-efficient transactions for coordination, whereas Weng (1995a) considers coordination through quantity discounts, franchise fees, and sharing the profit according to a pre-determined ratio. Viswanathan and Wang (2003) evaluate the effectiveness of simultaneously offering quantity and volume discounts. They develop a method to determine the optimal simultaneous discount offer. Chen et al. (2001), and Wang (2005) extend the setting to a single-supplier multiple-buyer system, derive the optimal quantity and volume discounts, and study their effectiveness.

Carter and Ferrin (1996) emphasize the explicit consideration of transportation costs in lot-sizing. As we have stated earlier, Swenseth and Godfrey (2002) argue that transportation costs can be almost 50% of the total logistics cost. Therefore, incorporating transportation costs into lot-sizing decisions has been of interest. Larson (1988) introduces the economic transportation quantity model in which the transportation cost is incorporated into the optimal lot-sizing problem. Tersine and Barman (1991) propose an enumerative algorithm for a buyer confronted with quantity and freight rate discounts from the supplier. Russell and Krajewski (1991) consider a buyer having the over-declaring option, which refers to declaring a shipment amount that is larger than the real shipment size in order to take advantage of lower freight costs. They derive an optimal lot-sizing policy with freight discounts under constant demand. Carter et al. (1995) correct the algorithm of Russell and Krajewski (1991), and avoid abnormalities due to the real-life freight schedules. Hoque and Goyal (2000) develop an optimal inventory policy for a single-supplier single-buyer system with truck capacities and unequal batch sizes. Ertogral et al. (2007) incorporate production capacity into the lot-sizing problem with transportation cost.

For the lot-sizing problem, a limited number of studies offer models that include both price sensitive demand and transportation costs. Burwell et al. (1997) develop a model for determining the optimal lot-sizing and pricing policy under freight and all-units quantity discount break-points in the pricing schedule offered by the supplier.

Abad and Aggarwal (2005) extend this model, and incorporate the over-declaring option. They adopt a piecewise linear and continuous freight cost schedule. Lei et al. (2006) consider a supplier–transporter–buyer model where the transporter charges the supplier the transportation cost, which depends on shipment quantity, and the supplier charges a unit price to the buyer. The transportation cost is assumed to be linear with respect to the shipment size, and includes a fixed cost associated with each trip.

In this paper, we consider a single-supplier single-buyer system and incorporate the transportation cost into the buyer's cost structure. In order to have a realistic representation, we model transportation cost through a stepwise-linear function.

3 Notation and problem setting

We will model the problem using the following notation:

The buyer's decision variables:

Q	regular shipment size
p	selling price of the product

The supplier's decision variables:

v	wholesale price of the product
n	lot-size multiplier

Parameters:

$D(p)$	annual demand for the product, where $D(p) = ap^{-b}$,
a	demand function constant
b	elasticity parameter of the demand function, where $b > 1$
A	buyer's order placement cost
K	supplier's order placement cost
I	annual holding cost rate
m	unit manufacturing or procurement cost for the supplier
$F(Q)$	freight charge for a shipment of size Q
C	maximum load that can be placed in a truck
R	truckload charge
T	maximum number of trucks available for a single shipment
$v_{\max}$	an upper bound on the wholesale price.

Using the above notation, the buyer's (B) profit function, denoted as $\prod_B(p, Q)$, can be written as:

$$\prod_B(p, Q) = (p - v)D(p) - \left(v + \frac{F(Q)}{Q}\right) I \frac{Q}{2} - (A + F(Q)) \frac{D(p)}{Q}. \tag{1}$$

We note that the $\frac{F(Q)}{Q}$ term in the holding cost reflects the impact of the transportation cost on the value of the product (see Abad and Aggarwal 2006 for details). Likewise, the supplier's (S) profit function, denoted as $\prod_S(v, n)$, can be written as:

$$\prod_S(v, n) = (v - m)D(p^*(v)) - mI(n - 1)\frac{Q^*(v)}{2} - K\frac{D(p^*(v))}{nQ^*(v)}, \tag{2}$$

where $p^*(v)$ denotes the optimal price set by the buyer, and $Q^*(v)$ denotes the buyer's optimal replenishment quantity when the supplier sets the wholesale price as v.

To compute $F(Q)$, per truck cost R is multiplied by the number of trucks required by the order quantity Q. We assume that the transportation cost is determined in terms of truck loads, therefore freight cost is computed as:

$$F(Q) = \left\lceil \frac{Q}{C} \right\rceil R. \tag{3}$$

As a heuristic approach, the supplier's problem can be solved by ignoring the freight cost. In order to emphasize the significance of the freight cost effect, we illustrate an extreme case, and compare the results. Let us take an example with parameters $a = 100,000$, $b = 5$, $A = 125$, $K = 250$, $I = 25\%$, $m = 1$, $C = 200$, $R = 100$. If the supplier does not consider the buyer's freight cost, he optimizes his profit with wholesale price $v^* = 1.281$, and expects to achieve a profit of $\prod_S = 1494.8$ assuming that the buyer will choose his order quantity as $Q = 2,472$. With the wholesale price $v = 1.281$, the buyer does not select $Q = 2,472$ but $Q^* = 800$ since he considers the freight cost. After observing Q^*, the supplier sets the lot size multiplier $n = 2$. Finally the supplier receives a profit of $\prod_S = 49.82$. Had the supplier considered the freight cost, he would have optimized his profit at the wholesale price $v^* = 1.630$. Consequently, the buyer would have set $Q = 600$, and then the supplier would have set $n = 2$. Ultimately, the supplier would have received a profit of $\prod_S = 122.65$. Comparing 122.65 and 49.82, the supplier misses 59.38% of his optimal profit by ignoring the freight cost. Henceforth, we will refer to the approach that ignores the freight cost as the *transportation–insensitive* procedure.

4 The buyer's problem

In this section, we first discuss the solution of the pricing and lot-sizing problems for the buyer assuming that the supplier sets the wholesale price as v. We will later incorporate these results into the supplier's problem within a Stackelberg setting.

Abad and Aggarwal (2005) also derive the optimal pricing and lot-sizing decisions for a slightly different yet structurally equivalent freight cost function. For the sake of completeness, we derive the optimal decisions here again.

Incorporating the demand function, and for a fixed value of v, the profit function of the buyer can be written as

$$\prod_B(p, Q) = (p - v)ap^{-b} - \left(v + \frac{F(Q)}{Q}\right) I \frac{Q}{2} - (A + F(Q))\frac{ap^{-b}}{Q}, \tag{4}$$

or

$$\prod_B(p, Q) = (p - v)ap^{-b} - vI\frac{Q}{2} - I\frac{F(Q)}{2} - (A + F(Q))\frac{ap^{-b}}{Q}. \tag{5}$$

The first-order condition for a local maximum with respect to p is obtained as below.

$$\frac{\partial \prod_B(p,Q)}{\partial p} = ap^{-b} - \frac{(p-v)ap^{-b}b}{p} + \frac{(A+F(Q))\,ap^{-b}b}{pQ} = 0, \tag{6}$$

and we can state the price that yields the local maximum as

$$p^*(Q) = \frac{b(A+Qv+F(Q))}{Q(b-1)}. \tag{7}$$

We further take the second derivative and equate it to zero to find the p value at which the second derivative changes sign:

$$\frac{\partial^2 \prod_B(p,Q)}{\partial p^2} = -\frac{abp^{-2-b}((A+F(Q))(1+b)+Q(p(1-b)+v(1+b)))}{Q} = 0 \tag{8}$$

Solving (8) gives the price

$$p'(Q) = \frac{A+F(Q)+Qv+b(A+Qv+F(Q))}{Q(b-1)}. \tag{9}$$

Since $p'(Q) > p^*(Q)$, i.e., the second derivative changes sign after the local maximum obtained at $p^*(Q)$, and $\lim_{p\to\infty} \prod_B(p,Q) = 0$, it can be shown that $p^*(Q)$ actually yields a global optimal solution. We can then substitute p by $p^*(Q)$ in Eq. (4). We rewrite buyer's profit function as

$$\prod_B(Q) = (p^*(Q)-v)a(p^*(Q))^{-b} - vI\frac{Q}{2} - (A+F(Q))\frac{a(p^*(Q))^{-b}}{Q}. \tag{10}$$

For a fixed number of trucks t, where $(t-1)C < Q \leq tC$, we can rewrite (10) as

$$\prod_B(Q) = (p^*(Q)-v)a(p^*(Q))^{-b} - vI\frac{Q}{2} - (A+tR)\frac{a(p^*(Q))^{-b}}{Q}. \tag{11}$$

As $F(Q)$ is a stepwise function of number of trucks used, t, where $t = \left\lceil \frac{Q}{C} \right\rceil$, we write the first-order condition for a local maximum with respect to Q for a particular t value as

$$\frac{\partial \prod_B(Q)}{\partial Q} = -\frac{vI}{2} + \frac{(A+tR)\,a(p^*(Q))^{-b}}{Q^2}, \tag{12}$$

and by setting it equal to zero we obtain the following equality:

$$\left(-\frac{vI}{2} + \frac{(A+tR)a\left(\frac{b(A+Qv+tR)}{Q(b-1)}\right)^{-b}}{Q^2}\right) = 0. \tag{13}$$

Equality (13) is the local optimality condition for the buyer's order quantity when the number of trucks is fixed as t, and $(t-1)C < Q \leq tC$. Q^* value that satisfies equality (13) cannot be expressed in closed form, however it can be determined through a line search. Since T is the maximum number of trucks, the buyer can perform T line searches using (13) and obtain T many Q values. The buyer then designates the order quantity that provides the highest profit as his order quantity.

5 The supplier's problem

In this section, we study the supplier's problem of finding the optimal wholesale price. We will denote the order quantity selected by the buyer when the wholesale price is equal to v as $Q^*(v)$, and the price set by the buyer is $p^*(Q^*(v))$ or $p^*(v)$. As we have outlined in the previous section, a procedure that performs a finite number of line searches can be used to compute $Q^*(v)$. The supplier, however, wishes to determine v^*, and, to be able to solve his problem, he needs to incorporate the buyer's reaction as $Q^*(v)$ and $p^*(v)$ into his problem. Because $Q^*(v)$ is not available as an explicit function, and can be computed only through a line search, we propose an approach through which $Q^*(v)$ is approximated.

Viswanathan and Wang (2003) state that local maxima for the supplier's profit function [as given in Eq. (2)] can occur even for the case where the buyer's profit function does not involve freight cost. We have also encountered examples where the supplier's profit function is not a quasiconcave function of v. Therefore, it is difficult to obtain a global optimal solution, and we will study the structural properties of the problem in the subsequent section.

5.1 Approximating optimal order quantity response of the buyer

In this section, we analyze how the buyer's optimal order quantity, i.e., $Q^*(v)$, changes as we change v, and present an approximation of $Q^*(v)$. Let us first assume that the buyer uses only one truck, i.e., $t = 1$. Let v_1 denote the v value for which the optimality condition is satisfied with $Q = C$ and $t = 1$. For $v > v_1$, the optimal order quantity obtained from (13) will be smaller than C (see Proposition 1 in the Appendix), and using more than one truck would be more costly. Therefore, for $v > v_1$, the optimal response of the buyer would be to choose $Q^*(v)$ from (13) with $t = 1$.

Now let v_2 be the smallest value of v for which equality (13) is satisfied with $Q = 2C$ and $t = 2$. From the results of Proposition 1 in the Appendix, $v_2 < v_1$, and for $v_2 < v < v_1$, the optimal order quantity obtained from (13) with $t = 2$ would be between $2C$ and C. The buyer has now two options for a given v value, where $v_2 < v < v_1$:

1. Order one full truck-load with $Q = C$,

or

2. Use two trucks, i.e., $t = 2$, and determine the order quantity from equality (13) with $t = 2$.

It can be readily shown that the profit function of the buyer as expressed in Eq. (11) is convex with respect to v when Q is fixed. In the second option with $t = 2$, as v changes, $Q^*(v)$ is determined through a line search, and it is not possible to express the buyer's profit function (11) in closed form. Therefore, as an approximation, we assume that, in the $v_2 < v < v_1$ range, the buyer's profit function (11) will be a linear function that crosses the points $(v_2, \prod_B(Q^*(v_2)))$ and $(v_1 - \epsilon, \prod_B(Q^*(v_1 - \epsilon)))$. The profit functions of the two options (the profit function of the second option being approximated as a linear function) may intersect in the $[v_2, v_1]$ interval. Note that when $v = v_1$, one full truck-load option dominates the options with two trucks; if that is also the case for $v = v_2$, then we will assume that the one full truck-load option dominates the second option in the $[v_2, v_1]$ range, and the buyer's optimal order quantity is equal to C. If the second option (i.e., use two trucks, determine Q from (13) with $t = 2$) dominates the first option when $v = v_2$, we then find the value of v where the two options generate the same profit for the buyer. Let $b^1_{t=2}$ be the intersection point. Then, in the $v_2 < v < b^1_{t=2}$ range, the optimal order quantity is determined from (13) with $t = 2$, and for the $b^1_{t=2} < v < v_1$ range the optimal order quantity is equal to C.

By generalizing the above approach, an approximate procedure for determining the order quantity the buyer chooses can be formally stated as follows:

Step 0: Let $i = 1$. Determine the smallest value of v, v_1, for which equality (13) holds for $t = 1$ and $Q = C$. For v values greater than v_1, the buyer will choose his order quantity according to equality (13) with $t = 1$. Assuming that the optimal wholesale price can be at most $v_{\max}$, the order quantity response is linearly approximated in the interval $[v_1, v_{\max}]$ through a linear function that crosses points $(v_1, Q^*(v_1) = C)$ and $(v_{\max}, Q^*(v_{\max}))$.

Step1:

- Let $i = i + 1$.
- Determine the smallest value of v, v_i, for which (13) holds for $t = i$ and $Q = iC$.
- Define the upper envelope as the combination of the profit functions of the truck options leading to the highest profit. Determine the upper envelope of the following profit functions of the buyer in the $v_i < v < v_{i-1}$ range:
  - $\prod_B(p^*(Q), Q)$ where $Q = jC$, and $t = j, j = 1, ..., i - 1$.
  - $\prod_B(p^*(Q(v)), Q(v))$ where $t = i$ and $Q(v)$ satisfies equality (13).
- If $\prod_B(p^*(Q(v)), Q(v))$ at the point v_i is higher than any evaluated function $\prod_B(p^*(Q), Q)$ for $t = j$ where $j = 1, ..., i - 1$, first through a line search find the first indifference point where the optimal truck option changes, and then linearly approximate the order quantity between the starting point of the interval and the indifference point.
- Find the remaining indifference points, if any, where the optimal truck option changes by a line search. Within these ranges, the order quantity is a multiple of C.
- Let b^k_i, $k = 1, .., l$, be the indifference points generated by the upper envelope. Note that, from the fact that we are comparing i functions that are either convex or linear, $l < 2i$.

Step 2: If $i > T$ (maximum number of trucks) combine upper envelopes generated for intervals $[v_{j+1}, v_j]$ for $j = 1, 2, .., T - 1$, and their corresponding Q

values characterizing the buyer's optimal response, and stop; otherwise go to Step 1.

The computational complexity of the above outlined procedure lies with the generation of the upper envelope of at most T functions in Step 1. The upper envelope can be easily generated by a number of line searches under the assumption that Q linearly decreases when full truck option is not used. We provide an example to illustrate the steps of the algorithm in Sect. 6.

5.2 Derivation of the supplier's optimal wholesale price under a fixed lot size multiplier

In the previous section, we have characterized the buyer's response in terms of his order quantity. Accordingly, in wholesale price intervals that have been computed in Step 1 of the procedure presented in Sect. 5.1, the order quantity is either constant or linearly approximated. In this section, we illustrate how the supplier can determine his optimal wholesale price in each of these cases.

Case 1: If Q and n are constant for a wholesale price range $[v_{j+1}, v_j]$, then we can develop a search algorithm with respect to v in the range $[v_{j+1}, v_j]$. Replacing $p^*(v)$ with $p^*(Q)$ as given in Eq. (7), we can rewrite the supplier's profit function as follows:

$$\prod_S(v) = (v-m)a\left(\frac{b(A+Qv+F(Q))}{Q(b-1)}\right)^{-b} - \frac{mI(n-1)Q}{2} - \frac{Ka\left(\frac{b(A+Qv+F(Q))}{Q(b-1)}\right)^{-b}}{nQ}. \tag{14}$$

Taking the derivative of (14) with respect to v, we can write

$$\frac{\partial \prod_S(v)}{\partial v} = \frac{a\left(\frac{b(Qv+F(Q)+A)}{Q(b-1)}\right)^{-b}(b(K+nQ(m-v))+n(A+F(Q)+vQ)}{n(A+F(Q)+vQ)}. \tag{15}$$

Since $\frac{a\left(\frac{b(Qv+F(Q)+A)}{Q(b-1)}\right)^{-b}}{n(A+F(Q)+vQ)}$ is always positive, the sign of $\frac{\partial \prod_S(v)}{\partial v}$ changes where

$$(b(K+nQ(m-v))+n(A+F(Q)+vQ)=0, \tag{16}$$

which gives the wholesale price

$$v' = \frac{bK+An+F(Q)n+bmnQ}{nQ(b-1)}. \tag{17}$$

We further take the second derivative as follows:

$$\frac{\partial^2 \prod_S(v)}{\partial v^2} = -\frac{abQ\left(\frac{b(Qv+F(Q)+A)}{Q(b-1)}\right)^{-b}((b+1)K+n(2A+2F(Q)+Q(m+bm+v-bv)))}{n(A+F(Q)+vQ)^2}. \tag{18}$$

Since $\frac{abQ\left(\frac{b(Qv+F(Q)+A)}{Q(b-1)}\right)^{-b}}{n(A+F(Q)+vQ)^2}$ is always positive, the sign of $\frac{\partial^2 \prod_S(v)}{\partial v^2}$ changes at

$$v'' = \frac{(1+b)K + 2(A+F(Q))n + (1+b)mnQ}{(b-1)nQ}. \tag{19}$$

For the wholesale price range $[0, v'']$ the profit function $\Pi_S(v)$ is convex with respect to v. Since the sign of $\frac{\partial \prod_S(v)}{\partial v}$ is negative for the wholesale price range (v'', ∞), the optimal wholesale price v^* is given by $v^* = v'$ if $v_{j+1} \leq v' \leq v_j$. We can conclude that $v^* = v_{j+1}$ if $v' < v_{j+1}$ and $v^* = v_j$ if $v_j < v'$.

Case 2: If n is constant and Q is not constant, order quantity is approximated as a linear decreasing function of v. Let this function be $Q(v) = c - dv$ for a wholesale price range $[v_{j+1}, v_j]$. Replacing Q with $c - dv$ in Eq. (14), we approximate the best solution in this interval through a line search over v values in the $[v_{j+1}, v_j]$ range maximizing the supplier's profit.

5.3 Derivation of the optimal lot size multiplier under a fixed value of the wholesale price

After deriving the optimal wholesale price, we derive the optimal lot size multiplier in this section. The first and second derivatives of the supplier's profit function (2) with respect to n are

$$\frac{\partial \prod_S(v,n)}{\partial n} = -mI\frac{Q^*(v)}{2} + K\frac{D(p^*(v))}{n^2 Q^*(v)}, \tag{20}$$

and

$$\frac{\partial^2 \prod_S(v,n)}{\partial n^2} = -2K\frac{D(p^*(v))}{n^3 Q^*(v)}. \tag{21}$$

Since $K, v, D(p^*(v)), Q^*(v)$ are positive, $\frac{\partial^2 \prod_S(v,n)}{\partial n^2}$ is always negative. Therefore, the profit function (2) is strictly concave with respect to n. Since the optimal lot size multiplier n is an integer and the function is concave with respect to n, we can state the following two optimality conditions for n^*:

$$\prod_S(v, n^*) \geq \prod_S(v, n^*+1), \tag{22}$$

and

$$\prod_S(v, n^*) \geq \prod_S(v, n^*-1). \tag{23}$$

n^* will be denoted as n in the remaining part of this section. Expanding (22) and (23) through (2) gives

$$mI(n-1)\frac{Q^*(v)}{2}+K\frac{D(p^*(v))}{nQ^*(v)}-mI(n)\frac{Q^*(v)}{2}-K\frac{D(p^*(v))}{(n+1)Q^*(v)}\leq 0, \tag{24}$$

and

$$mI(n-1)\frac{Q^*(v)}{2}+K\frac{D(p^*(v))}{nQ^*(v)}-mI(n-2)\frac{Q^*(v)}{2}-K\frac{D(p^*(v))}{(n-1)Q^*(v)}\leq 0. \tag{25}$$

Simplifying (24) and (25), we obtain

$$-mI\frac{Q^*(v)}{2}+K\frac{D(p^*(v))}{Q^*(v)n(n+1)}\leq 0, \tag{26}$$

and

$$mI\frac{Q^*(v)}{2}-K\frac{D(p^*(v))}{Q^*(v)n(n-1)}\leq 0. \tag{27}$$

Equating inequality (26) to zero, and solving for n gives the roots $-\frac{1}{2}\left(1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)$ and $\frac{1}{2}\left(-1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)$. Inequality (26) is not satisfied in between the roots. Likewise, equating the inequality (27) to zero and solving for n gives the roots $\frac{1}{2}\left(1-\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)$ and $\frac{1}{2}\left(1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)$. Inequality (27) is satisfied in between the roots. We can then conclude that n is between $\frac{1}{2}\left(-1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)$ and $\frac{1}{2}\left(1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)$. Since the range is bounded below by 1 and n is integer, the optimal value of n can be written as

$$n^*=\left\lceil\frac{1}{2}\left(-1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rceil \text{ or } \left\lfloor\frac{1}{2}\left(1+\sqrt{1+\frac{8KD(p^*(v))}{mI[Q^*(v)]^2}}\right)\right\rfloor. \tag{28}$$

Note that above argument is valid for a fixed value of v. Since v^* depends on n from (17) and n^* depends on v from (28), an iterative procedure is required for finding the optimal decisions. We propose the following iterative process:

Step 1: Start with $n = 1$,

Step 2: Using n find v^* from (17) and check the optimality condition (28). If the condition is not met, increment n by 1 and go to Step 2. Otherwise, optimal decisions are found.

We conjecture that the procedure provides the optimal lot size multiplier, although we cannot prove its optimality.

5.4 The approximate algorithm

In this section, we present the approximate algorithm for solving the supplier's problem by combining the results that have been developed in the previous sections.

In order to solve the supplier's problem, first the buyer's response has to be characterized. Since the buyer's optimal decision cannot be expressed in closed form, the supplier's decision cannot be immediately derived. The optimal market price as a function of the order quantity can be determined by using Eq. (7). Substituting this into the profit function, we obtain Eq. (10). Using this function, we can then characterize the order quantity as the wholesale price changes (Sect. 5.1). Hence we propose a search procedure with respect to the wholesale price. The algorithm can be formally stated as follows:

Step 1: Characterize the optimal response of the buyer utilizing the procedure outlined in Sect. 5.1.

Step 2: Let the optimal profit $\prod_S^*$ be 0 with $v = 0$.

Step 3: Consider each wholesale price interval where the truck option changes in the response profile generated in Step 1, and complete the following steps for each interval:

3.1: If the order quantity is constant let $n = 1$ and go to the next step. If not, replace the linear approximation function with Q in the supplier's profit function (14). Find the optimal wholesale price maximizing the profit function through a line search over (14) as an approximation, and go to Step 3.5. (We note that, as shown in Sect. 5.3, when the wholesale price and the order quantity are fixed, the optimal lot size multiplier can be readily computed in each objective function evaluation of the line search.)

3.2: Compute the optimal wholesale price by (17).

3.3: Let the endpoints of the interval be v_{start} and v_{end}. If the optimal wholesale price of Step 3.2 is less than v_{start}, equate the wholesale price to v_{start}, whereas if the wholesale price is greater than v_{end}, equate the wholesale price to v_{end}.

3.4: If the optimality condition for the lot size multiplier holds by (28), go to Step 3.5; else, increase n by 1 and go to Step 3.2.

3.5: For the current wholesale price interval, compute the profit with the optimal wholesale price and the lot size multiplier by (14) and compare it with the optimal profit. If it is greater than the optimal profit, update the optimal profit, the wholesale price and the lot size multiplier.

6 A numerical illustration of the algorithm

We consider a randomly generated example with parameters $a = 54,496, b = 1.5166$, $A = 435.67$, $K = 222.8$, $I = 35.572\%$, $m = 1.9688$, $C = 27.523$, $R = 60$, $T = 20$. First, we find the optimal response of the buyer using the procedure outlined in Sect. 5.1.

Step 1:

- For the single-truck option, we find $v_1 = 56.58$ by solving (13) for $Q = C = 27.523$. Let $v_{\max} = 1.2v_1$, which corresponds to $v_{\max} = 67.89$. Solving the optimal

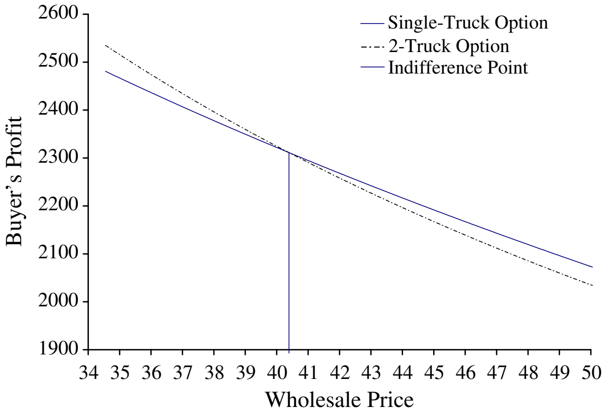

Fig. 1 The indifference point in buyer's profit function under single- and two-truck options

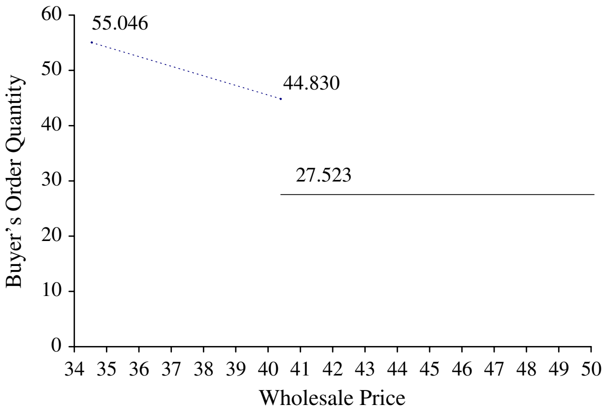

Fig. 2 Characterization of the buyer's optimal order quantity as a function of the wholesale price

order quantity for $v_{\max}$ leads to $Q = 21.64$ with a profit of 1733.98. Hence we linearly approximate the optimal order quantity between [v_1=56.68, $Q(v_1)$=27.523] and [$v_{\max} = 67.89$, $Q^*(v_{\max}) = 21.64$] as $Q(v) = 57.27 - 0.52v$.

- Next, the profit function for the 2-truck option is evaluated at the endpoints of the interval. We find $v_2 = 34.54$ for solving $Q = 2 \times C = 55.046$ with a profit of 2535.35, whereas the single-truck option at the wholesale price 34.54 provides the profit 2480.82. Hence, in this interval the best response is using the 2-truck option for small values, and, after an indifference point, using the single-truck option. Through a line search, we find the point of indifference as $v = 40.39$, where the single-truck option with $Q = 27.523$ and 2-truck option with $Q = 44.830$ both generate a profit of 2311.35. This is illustrated in Fig. 1.

  Consequently, we form the upper envelope by approximating the order quantity in the first part, and using full-truck option in the second part as it is graphically shown in Fig. 2. Note that we approximate the order quantity as a linear function between

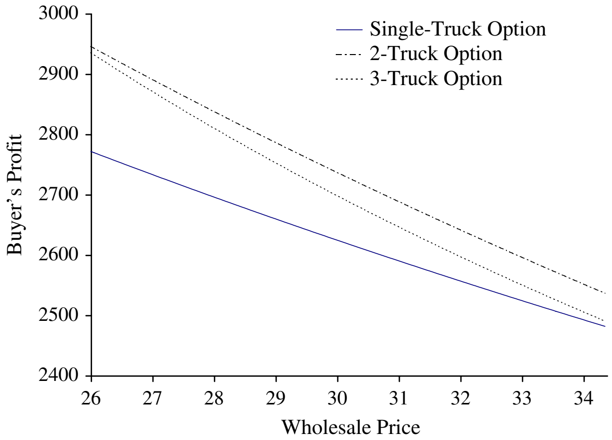

Fig. 3 Buyer's profit function under single, two-, and three-truck options

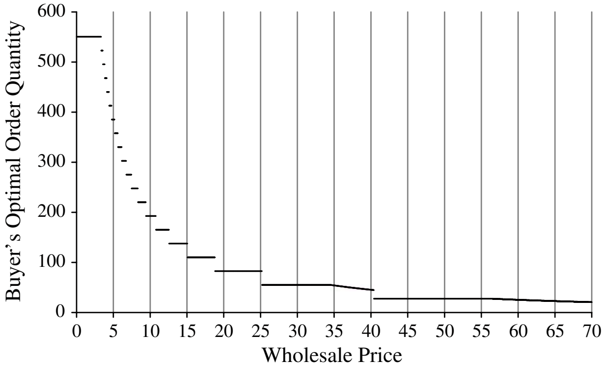

Fig. 4 Characterization of buyer's optimal order quantity response

the points [$v_2 = 34.54$, $Q(v_2) = 55.046$] and [$b_1 = 40.39$, $Q^*(b_1) = 44.830$]. Additionally, the order quantity after $v = 40.39$ is constant and equals to 27.523.

- For the 3-truck option, we find $v_3 = 26.16$ for solving $Q = 3 \times C = 82.569$ with profit 2946.08. In the interval [26.16, 34.54) we now have three functions to compare. At $v = 34.54$, the 3-truck option leads to $Q = 57.3$ with profit 2,489.0. Comparing these endpoint values with the single-truck and two-truck options, which is illustrated in Fig. 3, two-truck option dominates the other options. Since the two-truck option leads to higher profit at both of the endpoints, optimal response is $Q = 55.046$ in this interval. Note that the profit function is convex, and it decreases as the wholesale price increases. Therefore the three-truck option is dominated, and we do not need to evaluate this function within this interval.
- After evaluating v_1, v_2, v_3, and finding the response of the buyer, we similarly evaluate v_4, v_5, ..., v_{12}, and find the optimal response. Finally, the buyer's order quantity response as a function of the wholesale price is illustrated in Fig. 4.

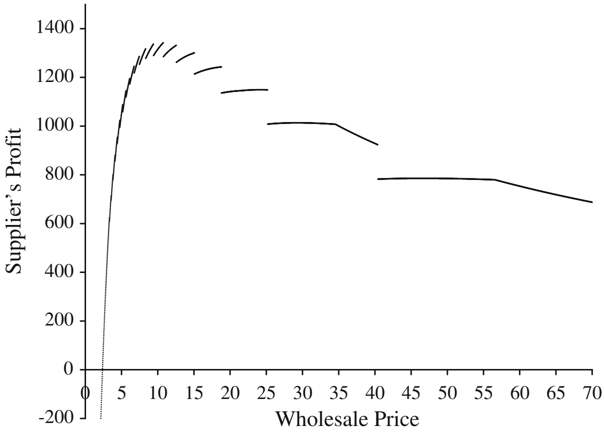

Fig. 5 Supplier's profit as a function of the wholesale

Note that the order quantity is approximated only in the intervals [34.54, 40.39] and [56.58, 67.89]. In the remaining intervals, order quantity response is available as an exact solution.

Step 2:

1. After characterizing the order quantity, we search for the optimal wholesale price. The first interval is $v \in [0, 3.33]$ where order quantity is constant as 550.46. For $n = 1$ we solve (17) and the resultant wholesale price is 12.72 satisfying (28). Since this value is greater than 3.33, optimal profit in this interval is obtained with $v = 3.33$, and $n = 1$. We update the current optimal profit as 624.10 with the optimal decisions in the [0, 3.33] range.
2. The next interval is $v \in (3.24, 3.56]$ where order quantity is constant as 522.937. For $n = 1$ we solve (17) and the resultant wholesale price is 12.86 satisfying (28). Since this value is greater than 3.56, optimal profit in this interval is obtained with $v = 3.56$ and $n = 1$. We update the optimal profit as 713.14 with the optimal decisions.
3. Note that we have not approximated the order quantity but exactly found the solution for the first two intervals. Omitting the next several interval evaluations, where the optimal profit is found to be 1,342.41 with $v = 10.77$, $n = 2$ and $Q = 192.66$, let us consider the interval $v \in (34.54, 40.39]$ where the order quantity is linearly approximated between 55.046 and 44.830. Replacing Q with the linear approximation function in the supplier's profit function (14), the optimal wholesale price is found by a line search. In this interval, the best profit is equal to $1,007.60$, generated by $v = 34.54$. Since $1,007.60 < 1,342.41$, the optimal profit is not updated.
4. After completing the evaluation of the remaining intervals, the optimal profit for the supplier is found to be 1342.41 with $v = 10.77$, $n = 2$. The supplier's profit function is presented in Fig. 5. Figure 6 illustrates the supplier's optimal profit and the buyer's optimal order quantity in the wholesale price range [5, 15]. The overall optimal profit of the supplier is also marked with a circle in Fig. 6.

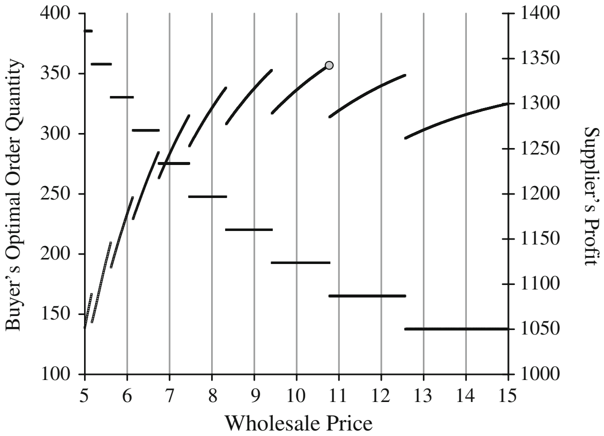

Fig. 6 Supplier's profit function and buyer's optimal order quantity in the [5, 15] wholesale price range

Note that the algorithm determines the indifference points where the order quantity changes and the supplier's problem is solved in each interval. The profit functions within these intervals are smooth, and the changes in the order quantity result in shifts in the supplier's profit function.

7 Computational results

We test the approximate algorithm's effectiveness over a set of problems that are created by considering various values of problem parameters. Initially, we fix the unit procurement cost of the supplier (m) to 1, and express all other cost and price parameters in multiples of the unit procurement cost. We let the supplier's order placement cost (K) be 10 times, 25 times, 100 times, and 250 times the unit procurement cost. The buyer's order placement cost (A) is designed to be 50, 100, and 200% of the supplier's order placement cost. The capacity of a single truck (C) is assumed to be 200, 400, and 1,000 units. The truck cost (R) is assumed to be 10, 20, 50, and 100. For the demand function, we fix the parameter a to 100,000, and select the price elasticity parameter b as 1.25, 2, 3.5, and 5. Finally, we take the annual holding cost rate as 25% and $v_{\max}$ as $1.2v_1$. The combinations of these parameters lead to a set of 576 problems.

We solve each problem with the approximate algorithm, and with an exhaustive search over all possible values of supplier's wholesale price, i.e., v. In the approximation algorithm, for finding the optimal shipment size for a given v, the line search procedure is terminated when the length of the uncertainty interval is less than 10^{-3}. We also run the exhaustive search with increments of size 10^{-3}. Under this precision scheme, the test problems have been solved in Matlab 6.5 on a computer operated under Microsoft Windows XP with an Intel Pentium M 1.60 GHz processor, and 512 MB of RAM. On average, the exhaustive search finishes in 14, 874.99 CPU time seconds. The approximate algorithm's average CPU time is 0.12 seconds.

Table 1 Computational results: average (minimum, maximum) values over the test instances

Factor	Treatment level	Centralization effect	Approximation	Transportation insensitive approach
K	10	0.75 (0.67, 0.84)	0.98 (0.54, 1.00)	0.99 (0.89, 1.00)
	25	0.74 (0.62, 0.83)	0.99 (0.68, 1.00)	0.98 (0.82, 1.00)
	100	0.73 (0.59, 0.83)	0.99 (0.79, 1.00)	0.97 (0.76, 1.00)
	250	0.71 (0.28, 0.83)	0.98 (0.55, 1.00)	0.96 (0.40, 1.00)
A	$0.5K$	0.73 (0.38, 0.83)	0.98 (0.54, 1.00)	0.98 (0.41, 1.00)
	K	0.73 (0.28, 0.83)	0.98 (0.66, 1.00)	0.97 (0.40, 1.00)
	$2K$	0.73 (0.57, 0.84)	0.99 (0.78, 1.00)	0.98 (0.76, 1.00)
b	1.25	0.80 (0.76, 0.84)	0.97 (0.54, 1.00)	0.99 (0.95, 1.00)
	2	0.73 (0.67, 0.80)	1.00 (0.97, 1.00)	0.99 (0.93, 1.00)
	3.5	0.71 (0.61, 0.81)	0.99 (0.85, 1.00)	0.97 (0.85, 1.00)
	5	0.69 (0.28, 0.83)	0.97 (0.55, 1.00)	0.95 (0.40, 1.00)
C	200	0.73 (0.28, 0.84)	0.99 (0.55, 1.00)	0.96 (0.40, 1.00)
	400	0.73 (0.52, 0.83)	0.99 (0.74, 1.00)	0.98 (0.78, 1.00)
	1,000	0.73 (0.52, 0.83)	0.98 (0.54, 1.00)	0.98 (0.83, 1.00)
R	10	0.74 (0.62, 0.84)	0.98 (0.54, 1.00)	0.99 (0.90, 1.00)
	20	0.74 (0.62, 0.83)	0.98 (0.73, 1.00)	0.99 (0.83, 1.00)
	50	0.73 (0.55, 0.83)	0.99 (0.85, 1.00)	0.98 (0.86, 1.00)
	100	0.72 (0.28, 0.83)	0.98 (0.55, 1.00)	0.95 (0.40, 1.00)

The computational results are provided in Table 1. Each cell reports the average values, and the range of minimum and maximum values for the performance measures. The centralization effect in Table 1 expresses, based on the results of the exhaustive search, the ratio of the total system profit of the decentralized case to the total system profit of the centralized system. This quantity is computed to document the potential gains that could be achieved by moving towards a more coordinated system. The approximation column reports, for the decentralized case, the ratio of the supplier's profit obtained by the approximate algorithm to the supplier's profit obtained by the exhaustive search. This is an indicator of the quality of the solutions generated by our approximation approach. Finally, we report the performance of the transportation-insensitive approach, where the supplier chooses the wholesale price by neglecting the buyer's transportation cost. We have given a detailed example illustrating this approach in Sect. 3. The figures reported under the transportation-insensitive approach column correspond to the fraction of the optimal profit (as computed by the exhaustive search) that the supplier can capture when the transportation costs are neglected.

On average, the approximate algorithm achieves 98.45% of the profit that can be achieved by the exhaustive search. The proposed algorithm works on an approximation with fixed truck options, and when the optimal shipment size is less than or equal to a single-truck load, the performance of the algorithm decreases. This is a natural outcome, because if the optimal shipment size is less than a truck load, there is no need to incorporate the transportation cost, and it can be taken as fixed. The optimal shipment size has been less than or equal to a single-truck load in 74 problem instances, and in these 74 problem instances the approximate algorithm achieves 95.15% of the profit that can be achieved by the exhaustive search. When these 74 instances are not taken into consideration, the approximate algorithm achieves 98.94% of the profit that can be achieved with the exhaustive search. We also note that the factor analysis does not point out an obvious dependence of the performance of the approximate algorithm on any of the experiment factors.

When we consider the transportation-insensitive approach, the average profit obtained deviates 2.25% from the profit obtained by the exhaustive search. However, there are instances where the deviation is about 60%, such as the example reported in Sect. 3. The computational results indicate that in the five instances where the performance is less than 80%, or the deviation is more than 20%, R is at its highest value of 100, C is equal to either 200 (in 4 instances) or 400 (in 1 instance), K is larger than 100, and b is equal to five. Similarly, in the 29 instances where the performance is less than 90%, R is at its highest value of 100 in 22 instances, C is at its lowest value of 100 in 16 instances, K is larger than 100 in 23 instances, and b is equal to 5 in 24 instances. Therefore, when the supplier's order placement cost (K) is large, sensitivity of demand to price (captured by the parameter b) is high, the truck cost (R) is large, and truck capacities are (C) small, an approach that ignores the transportation costs should be expected to result in higher profit losses.

8 Conclusion

We incorporate transportation costs into the lot-sizing and pricing problems in a two-echelon system, and present an approximate algorithm to solve the resulting optimization problem. Although the emphasis on lowering the logistics costs has been of interest, the problem stated in this paper has not been addressed in the literature, possibly due to its complexity. We have observed that our approximate algorithm has desirable computational aspects compared to an exhaustive search. The approximation delivers high quality solutions except for the cases where the order quantity is small enough to be handled in a single truck. We also investigated the effect of neglecting the transportation cost and found that, on average, this leads to a 2.25% decrease in profit. More importantly, we notice that in some extreme cases, the loss may be as high as 60%. Although these extreme cases cannot be perfectly characterized, high per-unit-capacity transportation costs as captured by R/C, and high sensitivity of demand to price as indicated by the parameter b are dominant factors. High order placement costs by the supplier also contribute to higher losses. We conclude that neglecting the transportation cost in pricing and lot-sizing decisions may cause a noticeable decrease in the profit and it is worthwhile to include transportation costs in the model, especially in the high risk cases highlighted above.

An interesting question is whether the supplier may offer a transportation cost sharing contract instead of traditional discount schemes to achieve better coordination with the buyer. In Yıldırmaz (2007), results of computational experiments in which a portion of the transportation cost is taken on by the supplier indicate that a transportation cost sharing contract is not an effective mechanism for coordination in comparison with the traditional quantity and volume discounts. Yıldırmaz (2007) also notes that when a coordination mechanism is designed by the supplier within a Stackelberg setting, a discount scheme that does not leave the buyer worse off with respect to his prior profit level is designed. However, if the supplier ignores the transportation costs in designing the discount scheme, he underestimates the buyer's total cost and overestimates his profit, and offers a scheme that is not desirable by the buyer.

We note that a single-supplier/single-buyer system may be too simplistic to capture the elements of a realistic business setting. Our approximation approach that derives a buyer's response to a wholesale price set by the supplier is general, and can be used to characterize the behavior of individual buyers in a single-supplier/multiple-buyer system. The supplier's problem then requires processing multiple response profiles simultaneously. Although the search on the wholesale price can be conducted in a similar manner in the presence of multiple buyers, the problem of determining the lot size multiplier for each buyer cannot be realistically decoupled from the search on the wholesale price. Therefore, further assumptions and approximations may be needed especially in the case where the supplier can benefit from joint setups.

Another realistic extension could consider the multiple-product case in the single-supplier/single-buyer setting. This is not a straight-forward extension, because the buyer's problem now includes a joint transportation cost component that depends on the shipment quantities. The question whether an effective joint replenishment cycle policy can be designed for the buyer is a potential research problem.

Finally, in terms of analyzing pricing and lot-sizing decisions, a system in which a third party logistics company exists as an independent decision making unit may pose another interesting question for further research.

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Appendix

Proposition 1 *For a fixed number of trucks t, as the wholesale price increases, regular shipment size decreases.*

Proof Suppose we fix the truck option as t. Although there is a feasibility consideration for the shipment size, let us first show that there are distinct optimal shipment sizes for a pair of distinct wholesale prices. Let us state the optimality condition (13) for a pair $[v_1, Q_1]$ and $[v_2, Q_2]$. Q_i corresponds to the optimal shipment quantity when wholesale price is v_i. Let $v_1 < v_2$, and using (7) we can write the demand function as follows:

$$D(p^*(v)) = a\left(\frac{b(A + Qv + F(Q))}{Q(b-1)}\right)^{-b}. \tag{29}$$

We state the optimality condition (13) for the pairs as follows:

$$-\frac{v_1 I}{2} + \frac{(A + F(Q_I))D(p^*(v_I))}{Q_1^2} = 0, \tag{30}$$

and

$$-\frac{v_2 I}{2} + \frac{(A + F(Q_2))D(p^*(v_2))}{Q_2^2} = 0. \tag{31}$$

Subtracting (31) from (30) gives the following

$$\frac{v_2 I}{2} - \frac{v_1 I}{2} + \frac{(A + F(Q_1))D(p^*(v_1))}{Q_1^2} - \frac{(A + F(Q_2))D(p^*(v_2))}{Q_2^2} = 0. \tag{32}$$

Given that $-\frac{v_1 I}{2} + \frac{v_2 I}{2} > 0$, we can claim $\frac{(A+F(Q_1))D(p^*(v_1))}{Q_1^2} - \frac{(A+F(Q_2))D(p^*(v_2))}{Q_2^2} < 0$. After simplification, we obtain

$$\frac{D(p^*(v_1))}{Q_1^2} - \frac{D(p^*(v_2))}{Q_2^2} < 0, \tag{33}$$

or

$$D(p^*(v_1))Q_2^2 < D(p^*(v_2))Q_1^2. \tag{34}$$

Since $p^*(v)$ increases as v increases, (proof in Proposition 2) $D(p^*(v))$ decreases as v increases. It is known that $D(p^*(v_1)) > D(p^*(v_2))$, hence (34) implies $Q_1 > Q_2$.

The previous argument is valid for Q satisfying $(t-1)C < Q \leq tC$. From Lemma 1, it is known that there are distinct wholesale prices v_t and v_{t-1} that solve (13) for $(t-1)C$ and tC. For the range $[v_t, v_{t-1}]$, the argument is valid and the optimal shipment size decreases as the wholesale price increases.

Now, let us take a wholesale price v' lower than v_t. Previous argument suggests that $Q' > tC$ should hold; however the feasibility consideration and the concavity of the buyer's profit force $Q' = tC$. Hence the optimal shipment size is constant in the wholesale price interval $(0, v_t)$. A similar argument is valid for the wholesale price interval (v_{t-1}, ∞) where optimal shipment size Q' equals to $(t-1)C$. □

Proposition 2 *As wholesale price v increases, optimal market price $p^*(v)$ increases.*

Proof Restating (7),

$$p^*(v) = \frac{b(A + Qv + F(Q))}{Q(b-1)}, \tag{35}$$

and taking the first derivative we obtain

$$\frac{\partial p^*(v)}{\partial v} = \frac{b}{b-1}. \tag{36}$$

Since $b > 1$, first derivative is positive. Hence $p^*(v)$ increases as v increases. □

Lemma 1 *For a full truck load shipment, there is a unique wholesale price satisfying* (13) *when $F(Q) = tR$ is fixed.*

Proof Let us restate (13) here as:

$$f(v) = -\frac{vI}{2} + \frac{(A+tR)a\left(\frac{b(A+Qv+tR)}{Q(b-1)}\right)^{-b}}{Q^2}. \tag{37}$$

Since $b > 1$, both $-\frac{vI}{2}$ and $\frac{(A+tR)a\left(\frac{b(A+Qv+tR)}{Q(b-1)}\right)^{-b}}{Q^2}$ decrease as v increases. Since $\lim_{v\to 0} f(v) = \frac{(A+tR)a\left(\frac{b(A+tR)}{Q(b-1)}\right)^{-b}}{Q^2} > 0$ and $\lim_{v\to\infty} f(v) = -\infty$, we conclude that for $Q = tC$, there is a unique v satisfying $f(v) = 0$. □

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