

## **Fleet sizing and vehicle routing for container transportation in a static environment\***

**Pyung Hoi Koo<sup>1</sup>, Woon Seek Lee<sup>1</sup>, and Dong Won Jang<sup>2</sup>**

<sup>1</sup> Department of Industrial Engineering, Pukyong National University, San100 Yongdang, Namgu Busan, 608-739 Korea (e-mail: phkoo@pknu.ac.kr)

<sup>2</sup> Home Delivery Planning Team, Chunil Cargo Transportation, Joungsanri 848-8 Mulgeum, Yangsan Gyungnam, 628-810, Korea

**Abstract.** Busan is one of the busiest seaports in the world where millions of containers are handled every year. The space of the container terminal at the port is so limited that several small container yards are scattered in the city. Containers are frequently transported between the container terminal and container yards, which may cause tremendous traffic problems. The competitiveness of the container terminal may seriously be aggravated due to the increase in logistics costs. Thus, there exist growing needs for developing an efficient fleet management tool to resolve this situation. This paper proposes a new fleet management procedure based on a heuristic tabu search algorithm in a container transportation system. The proposed procedure is aimed at simultaneously finding the minimum fleet size required and travel route for each vehicle while satisfying all the transportation requirements within the planning horizon. The transportation system under consideration is static in that all the transportation requirements are predetermined at the beginning of the planning horizon. The proposed procedure consists of two phases: In phase one, an optimization model is constructed to obtain a fleet planning with minimum vehicle travel time and to provide a lower bound on the fleet size. In phase two, a tabu search based procedure is presented to construct a vehicle routing with the least number of vehicles. The performance of the procedure is evaluated and compared with two existing methods through computational experiments.

**Keywords:** Container transportation – Vehicle routing – Fleet sizing – Tabu search

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*Correspondence to:* P. H. Koo

### 1 Introduction

Busan is one of the busiest seaports in the world, which handles about 10 million twenty-foot equivalent units (TEUs) of containers, more than 90 % of the total container volumes exported from and imported to Korea. The container terminal area in Busan is so limited that all the containers cannot be stored in on-dock container yards. Hence, a significant amount of the containers are stored and handled in off-the-dock container yards (ODCYs) located near the port container terminal. The containers are moved by container trucks between the container terminal yard and the ODCYs. The container transportation within the city causes tremendous traffic problems in the port city and increases the logistics cost which may aggravate the competitiveness of the container terminal.

Figure 1 shows a simple container transportation environment under consideration. Import containers are unloaded from a container vessel entering the port, and placed at a marshalling area in the container terminal. Then the containers are moved to on-dock container yards, ODCYs, rail container yards, inland container depots, and local coastal port yards. The flow of the export containers would be reversed. Each container to be delivered has its own destination. For example, in Figure 1, seven containers are to be moved from ODCY3 to the seaport container terminal and 21 containers from seaport container terminal to ODCY3. A container is delivered by a single vehicle and a vehicle carries only a single container at a time. Each container will not be split during the travel and, thus is considered a transportation unit load.

This paper deals with a static transportation problem in which all the transportation jobs are ready to be picked up at the beginning of a planning horizon. It is assumed that the number of containers to be moved between two locations is determined at the beginning of the planning horizon, and travel times between locations as well as loading and unloading times are deterministic and known in advance. At the beginning of the planning horizon (e.g., one shift), several identical vehicles are ready at a location. This transportation environment is referred to as

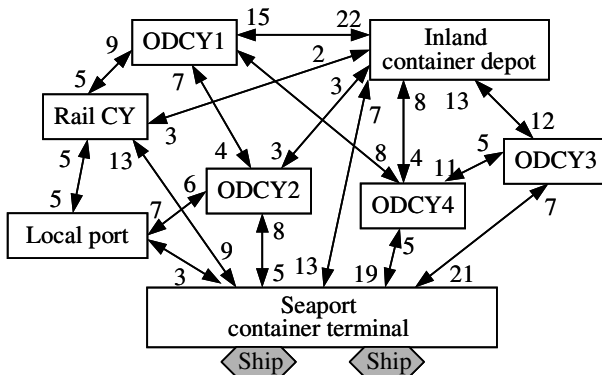


Fig. 1. An example of container transportation

a tractor-trailer transportation system (Bodin et al., 1983) or a static dial-a-ride problem with multiple vehicles of single capacity.

It is desirable to satisfy all the transportation requirements within the planning horizon with the minimum number of vehicles or fleet size. The fleet size can be found if the total vehicle travel time in the planning horizon is known. The lower bound on the required fleet size is the total vehicle travel time divided by the length of the planning horizon or time period available for a vehicle within the planning horizon. The total vehicle travel time consists of empty travel time, loading time, loaded travel time, and unloading time. Among these, the loaded travel time can be found using a from-to chart indicating transportation requirements between locations and travel time matrix. The loading and unloading times can be estimated from the number of loadings and unloadings performed in the planning horizon. The fleet size can thus be determined by the empty travel time. However, estimation of the empty travel time is a complex task since it requires information about vehicle routing.

This paper presents a two-phase fleet sizing and vehicle routing procedure. The objective of the procedure is to provide a multiple vehicle routing to complete all the transportation requirements with the minimum fleet size. Phase one uses an optimization model to produce a lower bound on the required fleet size, and phase two applies a tabu search based heuristic to generate vehicle routing along with an appropriate fleet size.

## 2 Previous research works

Many existing research works on freight transportation deal with how to determine the sequence of vehicle's visit to the request locations, which is closely related to the well-known traveling salesman problem (TSP), Multiple TSP, and general vehicle routing problems. Readers are referred to Laporte and Osman (1995), Crainic and Laporte (1998) and Chao (2002) among others. The vehicle routing problem in container transportation is slightly different from these research works in that a location may be visited as many times as the number of containers to be delivered, and a container should be delivered to a specific destination once loaded. Thus, it is similar to the single-capacity pickup-delivery transportation problem with sequence dependency. The transportation capacity constraint that a vehicle can move only a single container at a time makes the current problem more difficult to solve than the TSP which is known to be NP-hard. This implies that the optimal solution is computationally infeasible to obtain as problem sizes increase.

Most research works on container transportation systems focus on the design and operation of logistics within seaport container terminals. For vehicle operation problems in container terminals, Kim and Bae (1999) address assignment problems of container-delivery tasks to vehicles during ship operation in automated container terminals. Their work is later extended to the case of multiple quay cranes in Bae and Kim (2000). Grunow et al. (2004) present a priority-rule based dispatching procedure for a container terminal where automated guided vehicles (AGVs) with multiple-load capability are used as container transporters. They conclude through

numerical experiments that the use of dual load capabilities of the vehicles significantly improves the performance of the transportation system with respect to the total lateness and empty vehicle travel times. Kozan and Preston (1999) present a genetic algorithm based scheduling procedure for multimodal seaport container terminals to determine the optimal storage strategies and container handling schedules. They examine the effect of the number of containers, handling equipment, storage capacities and policies, and container terminal layout through simulation experiments. Kim and Kim (1999) present a routing procedure for a straddle carrier in port container terminals. An integer programming model is formulated and a heuristic is presented to solve the real world problem in an efficient way. Bish et al. (2001) develop a vehicle-scheduling-location heuristic to assign each container to a yard location and dispatch vehicles to the containers so as to minimize the time spent to download all the containers from the ship. Queueing network models (Legato and Mazza 2001) and simulation models (Gambardella et al. 1998; Yun and Choi 1999; Shabayek and Yeung 2002) are also applied to design and analyze the container terminal operations. A variety of decision problems at container terminals are classified and extensively reviewed in Vis and de Koster (2003).

Determining the fleet size is the most fundamental decision in a transportation system whose capacity is directly related to the number of available vehicles. Determining the optimal number of vehicles for a particular system requires a tradeoff between the investment costs of the vehicles and the potential penalties associated with not meeting all the demands. Beaujon and Turnquist (1991) present a nonlinear mathematical model to optimize the fleet size and vehicle allocation in a multi-period transportation planning environment. The model is transformed to a minimum cost network flow problem with a nonlinear objective function that can be solved by using yet another proposed solution procedure based on the Frank-Wolfe algorithm. Du and Hall (1997) address fleet sizing and empty vehicle redistribution for a one-to-many (or hub-and-spoke) transportation structure. Terminals are classified into surplus and shortage terminals based on the balance of the incoming and outgoing transportation requirements. A proper fleet size is determined based on the inventory control theory. It is assumed that operating costs are incurred for an excessive number of vehicles while shortage costs are charged for an insufficient number of vehicles. Vis et al. (2001) present a model and an algorithm to determine the necessary number of AGVs at an automated container terminal. A network flow based model and a polynomial time algorithm are developed to solve the problem in which containers are available for transport at known time instants. Another research arena of fleet sizing for a single-capacity transportation system is the use of AGV systems in automated manufacturing systems. Maxwell and Muckstadt (1982) propose a mathematical model to determine the minimum number of AGVs for a given number of transportation requests during a time window. Each location is associated with a net flow of vehicles which is defined as the difference between the numbers of incoming and outgoing deliveries. Flow balances of locations have to be achieved by empty vehicle movements. The model gives the lower bound on the number of vehicles needed in the system. Rajotia et al. (1998) improve the model of Maxwell and Muckstadt by imposing one more constraint that only a small portion of transportation requests from a location can be served by vehicles

being idle at the same location due to the randomness of the vehicle requests. For fleet sizing in a dynamic transportation environment, Kobza et al. (1998) present a model based on a discrete Markov chain and Koo and Suh (2002) present a queuing theory based model to estimate the vehicle waiting time and determine the fleet size in a dynamic transportation environment.

Most existing fleet sizing procedures for static transportation environments ignore vehicle routing in determining the fleet size. For fleet sizing and vehicle routing problems for container transportation, Ko et al. (2000) present a fleet sizing algorithm using an insertion algorithm. Given a planning horizon, they assign transportation orders to a vehicle one by one. When the completion time of a vehicle is larger than a predefined planning horizon, an additional vehicle is introduced and the procedure is repeated.

### 3 Two-phase fleet sizing and vehicle routing procedure

Figure 2 shows the overall procedure for fleet sizing and vehicle routing proposed in this paper. The procedure consists of two phases. In phase one, given the containers to be transported between locations and the travel times between locations, an optimization model is developed to generate a fleet planning with the minimum empty vehicle travel time. Since the model does not consider routing for each vehicle, actual empty travel times would be larger than those obtained from the optimization model. The minimum fleet size resulting from this optimization model may be regarded as a lower bound on the number of vehicles required. Given the fleet size, a tabu search based algorithm is developed to obtain the vehicle routing in phase two. Finally, the makespan (equivalent to the time taken until all the transportation jobs are finished) of the current solution is compared with the predetermined makespan limit. If not satisfactory, the procedure increases the fleet size by one and continues until the makespan constraint is satisfied. The two-phase procedure is described in more detail in the following sections.

#### 3.1 Phase one: optimization model to obtain the lower bound on fleet size

This section presents an optimization model to obtain the lower bound on the fleet size required. The fleet size depends on the total vehicle travel time required, which consists of empty travel time, loading time, loaded travel time, and unloading time. As discussed in the previous section, loading time, loaded travel time, and unloading time may easily be obtained when the transportation requirements and travel time between the locations are known. However, empty travel time is dependent on how to select the container to be delivered next when a vehicle becomes free. The lower bound on the fleet size, denoted by  $N_{min}$ , can be obtained by dividing the total vehicle travel time by the length of planning horizon or the available time of a vehicle (e.g., a shift). In order to reduce the fleet size required, the empty vehicle travel time must be minimized. The optimization model proposed by Maxwell and Muckstadt (1982) is applied to obtain the lower bound on the fleet size. Following notations will be used in the model:

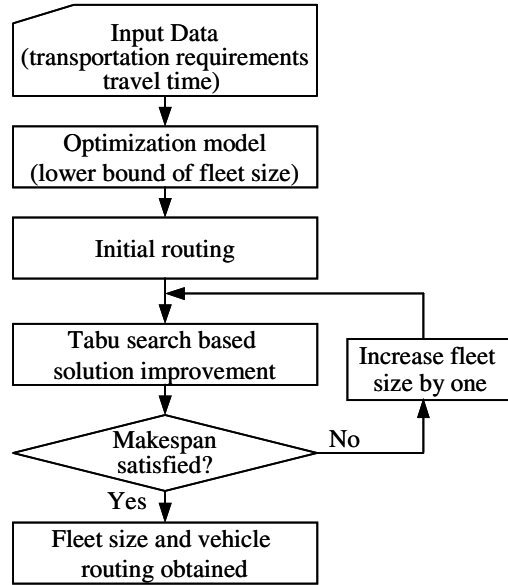


Fig. 2. Two-phase fleet sizing and vehicle routing procedure

$x_{ij}$  the number of empty vehicle trips that should be made from location  $i$  to location  $j$

$v_{ij}$  the number of containers to be delivered from location  $i$  to location  $j$  (or equivalently, the number of loaded vehicle trips that should be made from location  $i$  to location  $j$ )

$t_{ij}^a$  loaded vehicle travel time from location  $i$  to location  $j$ , which represents the time spent to load a container on a vehicle, move it from location  $i$  to location  $j$ , and unload it at location  $j$

$t_{ij}^b$  empty vehicle travel time from location  $i$  to location  $j$

Let us take location  $i$  for identifying the vehicle trip frequency during a shift. It can be observed that  $\sum_j v_{ij}$  is the number of containers to be picked up at location  $i$  during the shift. This means that  $\sum_j v_{ij}$  empty vehicles are needed at location  $i$  to move the containers. Similarly,  $\sum_i v_{ij}$  is the number of containers to be delivered to location  $j$ , and  $\sum_i v_{ij}$  vehicles will become empty after they unload the containers. For locations which do not allow overnight parking for vehicles, the total vehicle flow into the location within the shift is equal to the total vehicle flow out. The net flow for location  $i$ , denoted by  $nf(i)$ , is the difference between the total number of containers to be delivered in and the total number of containers to be picked up from there, that is,  $nf(i) = \sum_j v_{ji} - \sum_j v_{ij}$ . Since there may be requirements on the number of vehicles available at the beginning of a shift or required at the end of the shift, the net flow for location  $i$  must be adjusted to satisfy these requirements. For example, if  $f_i$  vehicles are available at the beginning of the shift and  $g_i$  vehicles are required at the end of the shift, the net flow for location  $i$  is as follows:

$$nf(i) = \left( \sum_j v_{ji} + f_i \right) - \left( \sum_j v_{ij} + g_i \right)$$

In the above equation, the first term on the right hand side indicates the number of vehicles available at location  $i$  during a shift while the second term indicates the number of vehicles required at location  $i$  during the shift. Hence, the net flow represents the number of empty vehicle trips into or out of the location. The locations with positive net flows would have empty vehicle trips available to be assigned to other locations with negative net flows. Following is the optimization model to find the number of empty vehicle trips from location  $i$  to location  $j$ :

$$\text{Min } \sum_i \sum_j x_{ij} t_{ij}^b \quad (1)$$

Subject to

$$\sum_j x_{ij} = n f(i), \text{ if the net flow for location } i \text{ is non-negative} \quad (2)$$

$$\sum_k x_{ki} = -n f(i), \text{ if the net flow for location } i \text{ is negative} \quad (3)$$

$$x_{ij} \text{ is a non-negative integer} \quad (4)$$

The objective is to minimize the total empty vehicle travel time. The total vehicle travel time, denoted by  $z$ , is then obtained by adding the total empty vehicle travel time and the total loaded travel time, that is,  $z = \sum_i \sum_j v_{ij} t_{ij}^a + \sum_i \sum_j x_{ij} t_{ij}^b$ . If  $h$  hours are available during the planning horizon per vehicle, at least  $\lceil z/h \rceil$  vehicles are required to satisfy the transportation requirements, where  $\lceil x \rceil$  is the smallest integer greater than or equal to  $x$ .

The optimization model yields the minimum number of vehicles required. However the model may not directly be applied to real world situations for the following reasons: First of all, it does not consider individual transportation requirements during the planning horizon. In addition, if the vehicle parking location at the beginning of the planning horizon is the same as the parking location at the end, the net flow of this location will be zero and there will thus be no vehicles starting from the parking location. Consequently, no vehicle trips will be made during the planning horizon, and the actual empty vehicle travel time will be underestimated. The fleet size obtained from the above optimization model will only be used as the lower bound on the fleet size in phase two, where vehicle routing and fleet sizing are solved simultaneously.

### 3.2 Phase two: tabu search based fleet sizing and vehicle routing

This section provides a vehicle routing and fleet sizing heuristic based on a tabu search (TS) algorithm. TS is a general improvement heuristic first presented by Glover (1989). It explores the solution space repeatedly moving from a solution to its best neighbor. The search process has the mechanisms that allow the objective function to deteriorate in a controlled manner and escape from local optima. Starting from an initial solution, an admissible move leads to the next solution with the minimum cost. If this solution is a local minimum, a non-improving perturbation may be accepted. To prevent cycling in the course of the search, the reverses of a certain number of moves that have recently been performed are forbidden and

recorded in a constantly updated tabu list. TS has been successfully implemented in a variety of combinatorial problems such as production scheduling (Franca et al. 1996), vehicle routing problem (Nanry and Barnes 2000, Breedam 2001, and Osman and Wassan 2002), and traveling salesman problem (Gendreau et al. 1999). See Glover (1997) and Osman and Laporte (1996) for an extensive literature review and detailed descriptions on tabu search.

Vehicles pick up containers at a yard and deliver them to another yard. They have a series of transportation jobs to be performed. The main problem is an assignment of containers to vehicles in which all the transportation requirements may be scheduled on identical vehicles with the objective of minimizing makespan. Empty vehicle travels are incurred for each transportation demand, and depend on the sequence of transportation jobs. In our TS implementation for solving fleet sizing and vehicle routing, the fleet size generated in phase one is used as the initial solution, which is then improved through the TS based improvement procedure. The procedure uses the fleet size as the primary decision criterion and makespan as the secondary decision criterion to plan the vehicle routing with the minimum number of vehicles. Note that, with the same number of vehicles, the shorter the makespan is, the more efficiently vehicles are utilized. A shorter makespan can be realized by shorter empty vehicle travel times and transportation load balance among the vehicles. A neighborhood solution is obtained by removing a transportation job from the busiest vehicle (that is, the vehicle with the longest completion time) and inserting it in a vehicle tour with the shortest completion time. As such, the makespan may further be reduced. The transportation jobs should be completed within the predetermined time limit (e.g., 480 minutes for one shift).

The initial vehicle routing follows a greedy solution procedure often used in container transportation business. When a vehicle completes a transportation job, it selects a job which is the nearest to the vehicle. The procedure inherently yields a solution with fairly short empty travel time and makespan.

#### Initial vehicle routing

- Step 0: The lower bound on fleet size  $N_{min}$ , container transportation requirements, and travel times between locations  $t_{ij}$  are determined.
- Step 1: Choose  $n$  containers at random and assign them to each vehicle.
- Step 2: Select a vehicle with the least  $C(V_i)$ , where  $C(V_i)$  is the time taken for vehicle  $V_i$  to leave a depot, perform transportation jobs for all the containers assigned, and return to the depot.
- Step 3: Select one from unassigned containers that yields the least empty vehicle travel time when it is appended to the route of the selected vehicle. Append it to the last job of the selected vehicle. Tie breaker is the longest-loaded-vehicle-time-first rule. This rule is selected because the longest processing time (LPT) rule is known to perform well in parallel machine scheduling with the objective of minimizing makespan.
- Step 4: Repeat Step 3 until all the transportation jobs are assigned.

Now we have an initial solution where each vehicle has its own route. The next step uses the concept of TS to improve the current solution.



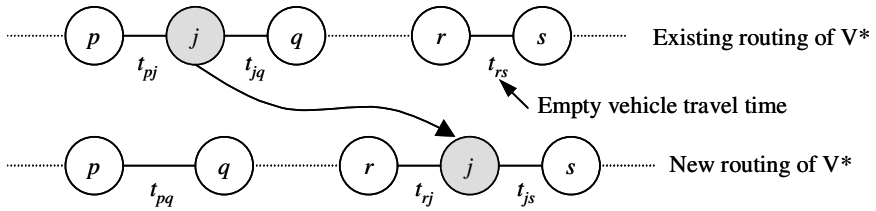


Fig. 3. Internal insertion process on  $V^*$

TS Solution improvement

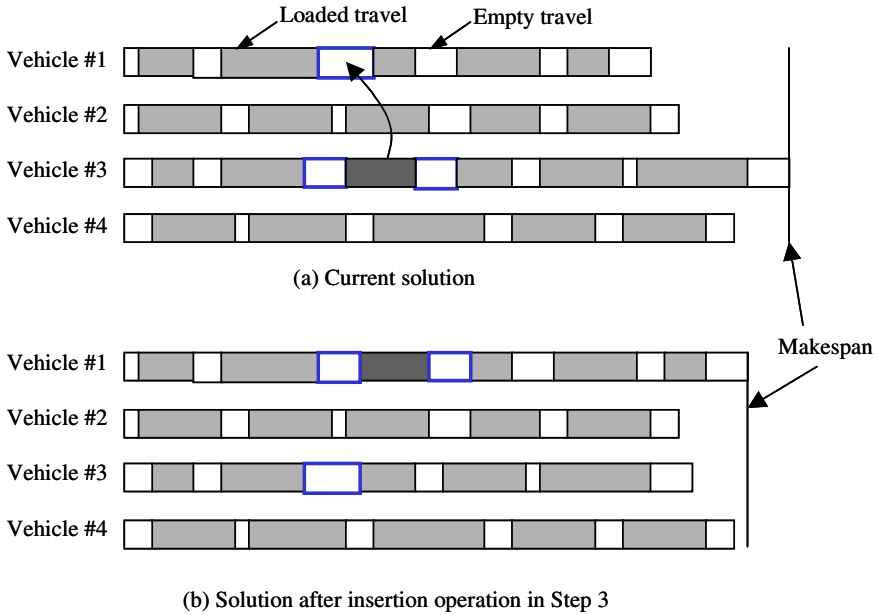
Step 0: Set the iteration counter  $c=0$ .

Step 1: Select the busiest vehicle  $V^*$  (that is, the vehicle with the largest  $C(V_i)$ ) in the current solution.

Step 2: *Internal insertion*. This step polishes the route of  $V^*$  by moving individual transportation jobs forward or backward in the same route. For each transportation job assigned to  $V^*$ , a sequence change operation is performed (i.e., a job is deleted from the sequence of  $V^*$  and inserted in a different position of the sequence of the same vehicle). For example, in Figure 3, transportation job  $j$  to be moved right after job  $p$  and right before job  $q$  is removed from the sequence and inserted back to a position between job  $r$  and job  $s$ . Then the completion time is reduced by  $(t_{pj} + t_{jq} + t_{rs}) - (t_{pq} + t_{rj} + t_{js})$ . The relocation of the job sequence resulting in the largest reduction in completion time is selected and the current solution is changed. The insertion process in this step is called internal insertion process. If we have at least one vehicle with larger  $C(V_i)$  than  $C(V^*)$ , then go to Step 1. Otherwise go to Step 3.

Step 3: *External insertion*. This step attempts to reduce the makespan by moving a transportation job of  $V^*$  to another vehicle route with the least completion time. Suppose jobs  $i, j$ , and  $k$  are consecutive jobs on the route of  $V^*$ . Calculate  $s_j = (t_{ij} + t_{jk} - t_{ik})$  for each transportation job  $j$ , where  $s_j$  is the empty vehicle travel time reduced by the removal of job  $j$  in  $V^*$ . Since less vehicle travel times are preferred, select job  $j^*$  to be removed from  $V^*$ , where  $j^* = \max(s_j)$ , unless the move is in the tabu list. The selected job  $j^*$  is inserted in the tabu list. Now identify a vehicle that has the smallest  $C(V_i)$ , and insert  $j^*$  in this vehicle tour in a way that the completion time of the selected vehicle increases least. The insertion process in Step 3 is called external insertion process. Figure 4 shows the external insertion operation, where shaded and white areas indicate loaded and empty vehicle travels, respectively. One of the transportation requirements of vehicle #3 ( $V^*$ ) is removed from the current route and inserted in the route of vehicle #1 which has the smallest completion time. The figure shows that the makespan is reduced after the external insertion process.

Step 4: Update the incumbent solution if the makespan of the current solution is less than that of all the solutions so far. Update iteration counter ( $c = c+1$ ) and tabu list. If  $c$  has not reached the predetermined iteration limit or the



**Fig. 4a,b.** External insertion operation

maximum number of iterations without improvement has not been reached, go to Step 1.

As seen in Figure 2, the makespan of the incumbent solution is then checked. If the makespan of the incumbent solution exceeds the predefined planning horizon, the fleet size is increased by one and phase two should be repeated. If the makespan lies within the planning horizon, the incumbent solution is final.

#### 4 Computational experiment

A sample test problem is adopted from Ko et al. (2000) where the container transportation environment is almost the same as in this paper. Tables 1 and 2 show from-to matrices for container transportation requirements and vehicle travel times, respectively.

The AMPL modeling tool (Fourer et al. 1993) and CPLEX solver are used to solve the optimization model. Since the optimization problem is similar to the classical transportation problem, solutions are found quite fast. The optimization model produces empty vehicle travel frequencies between locations as shown in the following:

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Loaded vehicle travel time: 4,620 minutes  
 Empty vehicle travel time: 2,550 minutes  
 Total vehicle travel time: 7,170 minutes  
 Empty vehicle travel frequency  
   E → A: 6 vehicles  
   E → B: 28 vehicles  
   E → C: 17 vehicles  
   H → A: 33 vehicles  
   I → A: 1 vehicle

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Minimum fleet size:  $\lceil 7,170/480 \rceil = \lceil 14.9 \rceil = 15$

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**Table 1.** Container transportation requirements

		To								
		A	B	C	D	E	F	G	H	I
From	A	-				15			47	2
	B		-			28				
	C			-		22			5	2
	D				-					
	E	3		10		-				1
	F						-			
	G							-		
	H	21		2					-	
	I								4	-

**Table 2.** Vehicle travel time matrix

		To								
		A	B	C	D	E	F	G	H	I
From	A	-	50	30	35	40	35	30	30	30
	B	50	-	30	35	40	35	30	35	35
	C	30	30	-	5	10	25	30	35	35
	D	35	35	5	-	5	20	25	30	30
	E	40	40	10	5	-	15	20	25	25
	F	35	35	25	20	15	-	10	15	15
	G	30	30	30	25	20	10	-	5	5
	H	30	35	35	30	25	15	5	-	5
	I	30	35	35	30	25	15	5	5	-

Suppose each vehicle can operate for 480 minutes per shift. At least 15 vehicles (the smallest integer greater than or equal to  $7,170/480$ ) are required to satisfy all the transportation requirements within the shift. If a vehicle finishes all the transportation jobs, it may not need to travel empty to somewhere else. However the optimization model counts this unnecessary empty vehicle travel, which results

in an increase in vehicle travel time. In order to tackle this problem, two nodes,  $J$  and  $K$ , are introduced where  $J$  is the location at which all the vehicles are parked at the beginning of the planning horizon while  $K$  is the location for the vehicles to be parked at the end of the time period. The new locations could be real sites such as depots or artificial locations for preventing additional empty vehicle travels. If they are artificial locations, the travel time between the existing locations and the artificial sites is set to zero. In our experiments, it is assumed that 15 vehicles are parked at an artificial location  $J$  at the beginning and they are returned to an artificial location  $K$  after they finish their transportation jobs. The optimization model again produces empty vehicle travel frequencies as follows. Now, it can be seen that the lower bound on the fleet size is 14.

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Loaded vehicle travel time: 4,620 minutes
Empty vehicle travel time: 1,950 minutes
Total vehicle travel time: 6,570 minutes
Empty vehicle travel frequency
E → B: 19 vehicles
E → C: 17 vehicles
E → K: 15 vehicles
H → A: 33 vehicles
I → A: 1 vehicle
J → A: 6 vehicles
J → B: 9 vehicles
Minimum fleet size: $\lceil 6,570/480 \rceil = \lceil 13.7 \rceil = 14$

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Based on the lower bound on the fleet size, the tabu search based algorithm yields a vehicle routing. The tabu tenure (i.e., the time period for which a move is prohibited) is set to three after some preliminary experiments. That is, the reverse move is prohibited for three periods after a move is performed. If the tabu tenure is too small, the probability of cycling increases. On the other hand, if it is too large, there is a possibility that the search space is too restricted, which may degrade the performance of the algorithm. The maximum number of iterations is set to 500. With 14 vehicles, the algorithm produces a solution with 500 minutes of the makespan and 6,965 minutes of total travel time. The total travel time is larger than the result obtained from the optimization model in phase one by 395 minutes. Since the makespan exceeds the predetermined time limit of 480 minutes, we increase the fleet size by one and run the experiment again. As a result, with 15 vehicles, total vehicle travel time and makespan are 6,740 and 460 minutes, respectively. The container transportation requirements can thus be met with 15 vehicles.

The heuristic is coded in the Visual Basic programming language. Figure 5 shows a screen capture of the experimental result when 15 vehicles are used. For example, the final solution of the tabu search based procedure generates the route of the first vehicle as follows:

Depot - 2 → 5 - 2 → 5 - 3 → 5 → 9 → 8 - 1 → 8 - 1 → 8 - 1 → 8 → 1 → 5 - 3 → 5 - 3 → 5 - Depot.

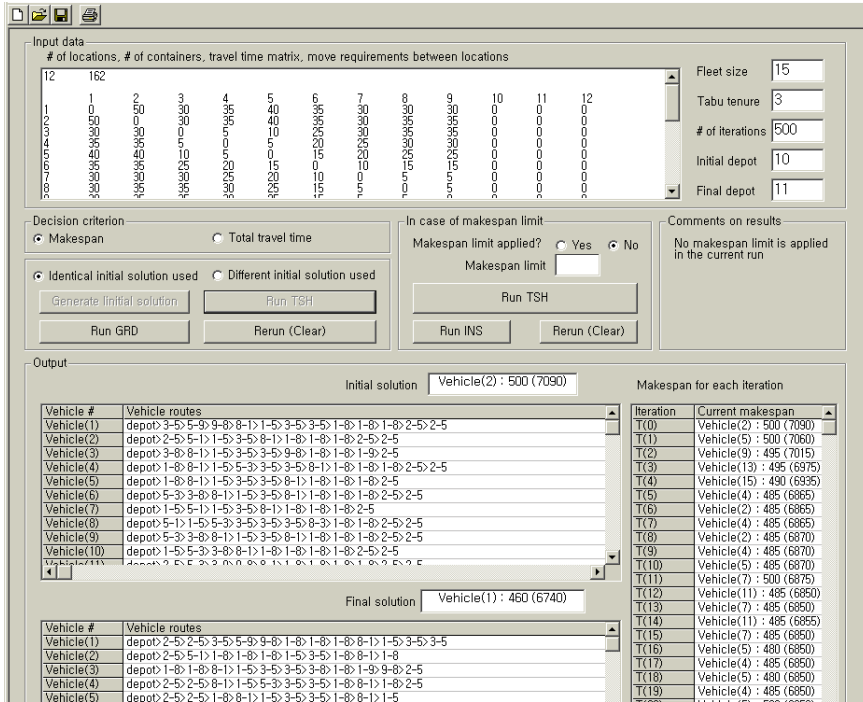


Fig. 5a,b. A screen capture of the implemented system of the tabu search based algorithm

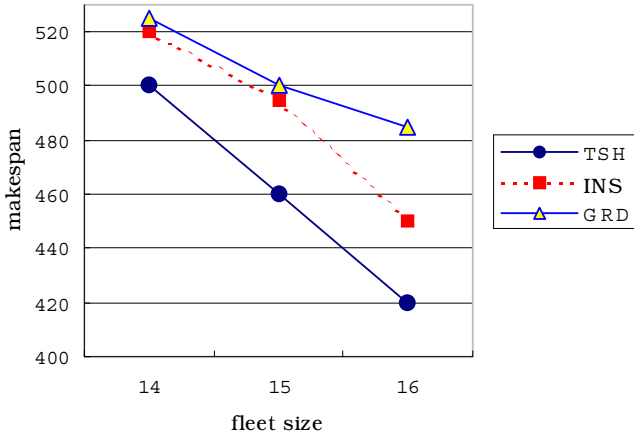
Here, ‘→’ indicates the loaded vehicle travel between two locations while ‘-’ denotes empty vehicle travel. It can be observed that the first vehicle operates 460 minutes during the planning horizon, 300 minutes for loaded travels and 160 minutes for empty travels.

The performance of the proposed two-phase heuristic (TSH) is compared with two existing methods, the insertion algorithm based heuristic (INS) of Ko et al. (2000) and a greedy procedure (GRD). In INS, given a planning horizon, transportation jobs are assigned to a vehicle on by one. INS first selects a vehicle for scheduling. A transportation job is then randomly selected and assigned to the vehicle. Among all the transportation jobs which are not assigned, a job which increases the vehicle travel time the least is selected and inserted in the route of the vehicle. The assignment procedure is repeated until the total travel time of the vehicle exceeds the planning horizon, when a new vehicle is introduced and the assignment procedure is repeated. GRD makes a myopic decision to select transportation jobs. When a vehicle becomes free, it selects the nearest job. The first stage of the proposed procedure uses GRD in order to obtain an initial solution.

Table 3 shows the makespan and total vehicle travel time when 15 vehicles are used. TSH completes all the transportation jobs within 460 minutes while INS and GRD requires 495 and 500 minutes, respectively. It is observed that TSH also produces the vehicle routing with the least total vehicle travel time.

**Table 3a,b.** Performance of the three heuristics

Method	Makespan (min.)	Total vehicle travel time (min.)
TSH	460	6740
INS	495	6860
GRD	500	7000



**Fig. 6a,b.** Change of makespan over different fleet sizes

Figure 6 compares the makespans of the three heuristics against the fleet size. As expected, with more vehicles, the makespan decreases for all the heuristics. If the makespan is restricted to only 480 minutes per shift, TSH requires 15 vehicles while INS and GRD requires more vehicles, 16 and 17, respectively. It can also be observed that, with 14 vehicles, TSH requires 20 minutes of overtime while INS and GRD needs 40 and 45 minutes, respectively.

In this paper, the travel time between two locations is assumed to be fixed. However, this assumption is often invalid in practice since the containers usually travel through congested streets. As pointed out in Park et al. (2000), a lognormal or triangular distribution may often be adequate to model the travel time between locations. The performance of the proposed procedure with stochastic travel times is investigated by assuming a triangular distribution with variations ranging from 5 % to 25 %. When the mean travel time between two locations is 50 minutes with a time variation of 20%, the travel time follows a triangular distribution with mode 50, minimum 40, and maximum 60. The experiments are repeated 10 times for each method, and the results are summarized in Table 4. As the variation increases, the makespans of TSH and INS slightly increase. The makespan of GRD seems insensitive to the travel time variations. Overall, the proposed procedure performs well even when the travel time variations are large.

The performance of the proposed procedure under various conditions is also investigated by running experiments with 40 problem instances. The average makespans of the three heuristics, TSH, INS and GRD are summarized in Table 5. TSH

**Table 4a,b.** Makespan comparison under probabilistic travel times

Method	Travel time variation					
	0%	5%	10%	15%	20%	25%
TSH	500.0	503.8	507.7	511.5	515.5	519.7
INS	540.0	543.7	547.5	551.2	555.0	558.8
GRD	580.0	577.1	577.0	577.4	578.3	577.4

**Table 5a,b.** Comparison of three methods with 40 different cases

Method	Fleet size	Makespan	Total vehicle travel time
TSH	18.2	464.0	8405.3
INS	21.3	479.1	9607.4
GRD	18.2	534.1	9105.8

requires 18.2 vehicles on average to satisfy the transportation requirements within 480 minutes while INS requires 21.3 vehicles. GRD is experimented with the same fleet size as in TSH, and it is found that its makespan is 534.1 minutes, 70.1 minutes larger than that of TSH.

### 5 Conclusions

This paper proposes a new heuristic procedure for fleet sizing and vehicle routing in a static container transportation environment. The heuristic consists of two phases. The first phase determines the lower bound on the fleet size by using an optimization model and the second phase constructs a vehicle routing by applying the concept of tabu search. The proposed procedure has been compared with two existing methods through computational experiments. It has been observed that the new procedure consistently provides good quality solutions in terms of makespan and total vehicle travel time.

A heuristic tabu search algorithm is applied in the second phase to solve the container transportation problem discussed above. It may also be meaningful to investigate the application of other meta-heuristics such as simulated annealing and genetic algorithm to this problem. Finally, a container transportation problem is investigated in a static environment in which all transportation requirements are ready to be picked up at the beginning of the planning horizon. However, this may not usually be the case in the real world since containers may arrive in the middle of the planning horizon. Then, the fleet sizing and vehicle routing problems should be addressed from different perspectives. These subjects may provide a good opportunity for further studies.

## References

- Bae J W, Kim K H (2000) A pooled dispatching strategy for automated guided vehicles in port container terminals. *International Journal of Management Science* 6: 47–67
- Beaujon G J, Turnquist M A (1991) A model for fleet sizing and vehicle allocation. *Transportation Science* 25: 19–45
- Bish E K, Leong T Y, Li C L, Ng J W C (2001) Analysis of a new vehicle scheduling and location problem. *Simchi-Levi D. Naval Research Logistics* 48: 363–386
- Bodin, L D, Golden, B L, Assad A A, Ball M O (1983) Routing and scheduling of vehicles and crews: the state of the art. *Computers and Operation Research* 10: 63–211
- Breedam A V (2001) Comparing descent heuristics and metaheuristics for the vehicle routing problem. *Computers and Operations Research* 28: 289–315
- Chao IM (2002) A tabu search method for truck and trailer routing problem. *Computers and Operations Research* 29: 33–51
- Crainic T G, Laporte G (1998) *Fleet management and logistics*. Kluwer, Amsterdam
- Du Y, Hall R (1997) Fleet sizing and empty equipment redistribution for center-terminal transportation networks. *Management Science* 43: 145–157
- Fourer R, Gay D M, Kernighan B W (1993) *AMPL a modeling language for mathematical programming*. Boyd and Fraser, Massachusetts
- Franca P M, Gendreau M, Laporte G, Muller F M (1996) A tabu search heuristic for the multiprocessor scheduling problem with sequence dependent setup times. *International Journal of Production Economics* 43: 79–89
- Gambardella L M, Rizzoli A E, Zaffalon M (1998) Simulation and planning of an intermodal container terminal. *Simulation* 71: 107–116
- Gendreau M, Laporte G, Vigo D (1999) Heuristics for the traveling salesman problem with pickup and delivery. *Computers and Operations Research* 26: 699–714
- Glover F (1989) Tabu search, part I. *ORSA Journal on Computing* 1: 190–206
- Glover F (1997) *Tabu search*. Kluwer, Boston
- Grunow M, Gunther H O, Lehmann M (2004) Dispatching multi-load AGVs in highly automated seaport container terminals. *OR Spectrum* 26: 211–235
- Kim K H, Bae J W (1999) A dispatching method for automated guided vehicles to minimize delays of containership operations. *International Journal of Management Science* 5 (1): 1–26
- Kim K Y, Kim K H (1999) A routing algorithm for a single straddle carrier to load export containers onto a containership. *International Journal of Production Economics* 59: 425–433
- Ko C S, Chung K H, Shin J Y (2000) Determination of vehicle fleet size for container shuttle service. *Korean Management Science Review* 17: 87–95
- Kobza J E, Shen Y C, Reasor R J (1998) A stochastic model of empty-vehicle travel time and load request service time in light-traffic material handling systems. *IIE Transactions* 30: 133–142
- Koo P H, Suh J D (2002) Fleet sizing under dynamic vehicle dispatching. *Journal of Korean Institute of Industrial Engineering* 28: 256–263
- Kozan E, Preston P (1999) Genetic algorithms to schedule container transfers at multimodal terminals. *International Transactions in Operational Research* 6: 311–329
- Laporte G, Osman H (1995) Routing problems: a bibliography. *Annals of Operations Research* 61: 227–262
- Legato P, Mazza R M (2001) Berth planning and resources optimization at a container terminal via discrete event simulation. *European Journal of Operational Research* 133: 537–547



- Maxwell W L, Muckstadt J A (1982) Design of automated guided vehicle systems. *IIE Transactions* 14: 114–124
- Nanry W P, Barnes J W (2000) Solving the pickup and delivery problem with time windows using reactive tabu search. *Transportation Research Part B* 34: 107–121
- Osman I H, Laporte G (1996) Metaheuristics: a bibliography. *Annals of Operations Research* 63: 513–628
- Osman I H, Wassan N A (2002) A reactive tabu search metaheuristic for the vehicle routing problem with backhauls. *Journal of Scheduling* 5: 263–285
- Park C H, Jun G S, Koh S Y, Kim D N, Kim Y C, Suh S D, Seol J H, Yun H M, Lee S M, Jang H B, Choi G J, Choe J S (2000) *Introduction to transportation engineering*. Yongji Monhwasa, Seoul
- Rajotia S, Shanker K, Batra J L (1998) Determination of optimal AGV fleet size for an FMS. *International Journal of Production Research* 36: 1177–1198
- Shabayek A A, Yeung W W (2002) A simulation model for the Kawi Chung container terminals in Hong Kong. *European Journal of Operational Research* 140: 1–11
- Vis, I F A, de Koster R, Roodbergen K J (2001) Determination of the number of automated guided vehicles required at a semi-automated container terminal. *Journal of the Operational Research Society* 52: 409–417
- Vis, I F A, de Koster R (2003) Transshipmentment of containers at a container terminal: An overview. *European Journal of Operational Research* 147: 1–16
- Yun W Y, Choi Y S (1999) A simulation model for container-terminal operation analysis using an object-oriented approach. *International Journal of Production Economics* 59: 221–230