OR Spectrum (2002) 24: 145–150

OR Spectrum

Insurance demand and the elasticity of risk aversion*

Harald L. Battermann[1], **Udo Broll**[1], **and Jack E. Wahl**[2]

[1] Department of Economics, University of Saarland, 66041 Saarbrücken, Germany
(e-mail: u.broll@mx.uni-saarland.de)
[2] Department of Finance, University of Dortmund, 44227 Dortmund, Germany

Received: January 30, 2001 / Accepted: January 14, 2002

Abstract. The aim of this study is to analyze optimum insurance demand of a risk averse agent. By introducing the concept of elasticity of risk aversion, we describe the interaction of optimum coverage and insurance risk. The agent's revision of her insurance demand is governed by a substitution effect and an income effect.

Key words: Elasticity of risk aversion – Insurance demand – Substitution effect – Income effect – (μ, σ)-Preferences

1 Introduction

Our study analyzes optimal insurance demand of a risk averse agent faced with an exogenous change in insurance risk. In contrast to the existing literature (see, e.g., Demers and Demers, 1991; Alarie, Dionne, and Eeckhoudt, 1992) we focus on the elasticity of risk aversion to characterize the relationship between a change in risk and the optimal insurance demand. Speaking in absolute terms, the elasticity of risk aversion is defined to be the percentage change in risk aversion divided by the percentage change in risk. The question how risk affects decision making is an important topic in many fields of economics and finance (see, e.g., Briys, Couchy, and Schlesinger, 1993; Broll, Wahl, and Zilcha, 1995; Broll and Eckwert, 1999)

We base our analysis on the concept of (μ, σ)-preferences. The (μ, σ)-criterion, although well-known in the literature on decision making under uncertainty, has experienced a growing attention in very recent contributions (see, e.g., Bar-Shira and Finkelshtain, 1999; Eichner, 2000; Löffler, 2001). Important new insights are provided by the (μ, σ)-framework. We use this approach to examine risk effects

* We would like to thank Bernhard Eckwert and two anonymous referees for helpful comments and suggestions which improved our paper.

Correspondence to: U. Broll

on optimum insurance demand. For a (μ, σ)-risk averse agent our paper derives a clearcut relationship between the demand for insurance and the elasticity of risk aversion.

Our investigation proceeds as follows: Section 2 presents the insurance decision problem and introduces the elasticity of risk aversion. Section 3 demonstrates the relationship between insurance risk and insurance demand by applying the elasticity measure. Our paper concludes with some brief remarks.

2 The elasticity of risk aversion

Consider a risk averse agent endowed with monetary wealth $\bar{w}$, who faces a random loss of amount $\tilde{z}$, which is insurable. Let α be the coinsurance rate and suppose the agent has the opportunity to buy an insurance contract, which pays out the indemnity $\alpha\tilde{z}$, given the insurance premium $\alpha(1+\lambda)E\tilde{z}$, where E denotes the expectation operator and λ the loading factor. It is assumed that the loading factor is positive and that the agent is risk averse (see, e.g., Mossin, 1968; Schlesinger, 1997).

Our agent is (μ, σ)-risk averse. This means that (i) the agent's preferences can be represented by a two-parameter function $\Phi(\mu, \sigma)$ defined over mean μ and standard deviation σ of the underlying random variables and (ii) that the function Φ satisfies the following properties: $\partial\Phi(\mu,\sigma)/\partial\mu = \Phi_\mu > 0$, $\partial^2\Phi(\mu,\sigma)/\partial\mu^2 = \Phi_{\mu\mu} \leq 0$, $\partial\Phi(\mu,\sigma)/\partial\sigma = \Phi_\sigma < 0,\ \sigma > 0$ and $\Phi_\sigma(\mu, 0) = 0$. Furthermore, let us assume that the partial derivatives $\partial^2\Phi(\mu,\sigma)/\partial\sigma^2$ and $\partial^2\Phi(\mu,\sigma)/\partial\mu\partial\sigma$ exist and that Φ is a strictly concave function. Hence, the indifference curves are convex in the (σ, μ)-space as often assumed in the literature (see, e.g., Bodie, Kane, and Marcus, 1993; Eichberger and Harper, 1997; Bamberg and Coenenberg, 2000).

Given (μ, σ)-risk aversion the insurance decision problem reads:

$$\max_{\alpha} \Phi(\mu_{\tilde{w}}, \sigma_{\tilde{w}}),$$

where $\tilde{w} = \bar{w} - (1-\alpha)\tilde{z} - \alpha(1+\lambda)E(\tilde{z})$ denotes uncertain end-of-period wealth. We set $\mu_{\tilde{w}} = E(\tilde{w})$ and $\sigma_{\tilde{w}} = \sqrt{E(\tilde{w} - E(\tilde{w}))^2}$.

Before analyzing the relationship between changes in risk and the optimal insurance demand α, let us define the elasticity of risk aversion. To simplify notation, in what follows we drop the subscript $\tilde{w}$.

Definition (Elasticity of risk aversion). Let $\sigma > 0$. We define the elasticity of risk aversion with respect to the standard deviation as $\varepsilon_{R,\sigma} := -R_\sigma \frac{\sigma}{R}$, where $R = -\Phi_\sigma/\Phi_\mu$ and $R_\sigma = \partial R/\partial\sigma$.

Note that R is the marginal rate of substitution between μ and σ, and, therefore, we interpret R as a measure of risk aversion in (σ, μ)-space (Lajeri and Nielsen, 2000; Löffler, 2001). The elasticity of risk aversion, $\varepsilon_{R,\sigma}$, is – in absolute value – given by the percentage change in risk aversion divided by the percentage change in risk (i.e., standard deviation). The elasticity measure allows for an intuitively appealing interpretation of the effect of risk on insurance demand.

We model a change in insurance risk as follows: $\tilde{z}(\beta) := E\tilde{x} + \beta\tilde{\eta},\ \tilde{\eta} := \tilde{x} - E\tilde{x}$, where the random variable $\tilde{x}$ has unit standard deviation and $0 < \beta < 1$. Then, increasing β models an increase in insurance risk. Substituting $\tilde{z}(\beta)$ for the random variable $\tilde{z}$ of the decision problem generates a relationship between optimal insurance demand $\alpha(\beta)$ and the risk of losses measured by the standard deviation of $\tilde{z}(\beta)$.

3 Insurance demand

Now we are ready to examine the impact of an increase in risk on the optimal coverage of the insurable risk.

Proposition (Effect of risk). Suppose that the insurance risk increases, i.e. β increases. Then optimal insurance demand will decrease, if the elasticity of risk aversion is greater than unity. Insurance demand remains unchanged, if the elasticity is unity, and increases, if the elasticity is less than unity.

Proof. Expected end-of-period wealth and standard deviation of end-of-period wealth read

$$E(\tilde{w}) = \bar{w} - (1 + \alpha\lambda)E(\tilde{z}(\beta))$$

and

$$\sigma = (1 - \alpha)\sigma_{\tilde{z}(\beta)},$$

respectively. Hence, the objective function becomes

$$\varPhi\Big(\bar{w} - (1 + \alpha\lambda)E(\tilde{z}(\beta)), (1 - \alpha)\sigma_{\tilde{z}(\beta)}\Big).$$

By using risk aversion $R = -\varPhi_\sigma/\varPhi_\mu$ and standard deviation σ the first order condition of the insurance decision problem reads:

$$\left(-\lambda E(\tilde{z}(\beta)) + \frac{\sigma R}{(1 - \alpha)}\right)\varPhi_\mu = 0.$$

This equation can be satisfied if and only if the term in brackets equals to zero, since $\varPhi_\mu > 0$. The implicit function theorem gives

$$\begin{aligned}\text{sign}\left(\frac{d\alpha(\beta)}{d\beta}\right) &= \text{sign}\left(\frac{1}{1-\alpha}\left\{\frac{\partial\sigma}{\partial\beta}R + \sigma\frac{\partial R}{\partial\sigma}\,\frac{\partial\sigma}{\partial\beta}\right\}\right)\\ &= \text{sign}\left(R + \sigma\frac{\partial R}{\partial\sigma}\right),\end{aligned}$$

since $1 - \alpha > 0$ and $\partial\sigma/\partial\beta > 0$. With the definition of the elasticity of risk aversion we get sign$[d\alpha(\beta)/d\beta] =$ sign$[1 - \varepsilon_{R,\sigma}]$. q.e.d.

Note that the two-parameter function $\varPhi(\mu, \sigma) = \mu/(\theta + \sigma)^2$, which is related to Roy's function (Roy, 1952), includes all three cases of our proposition, i.e.

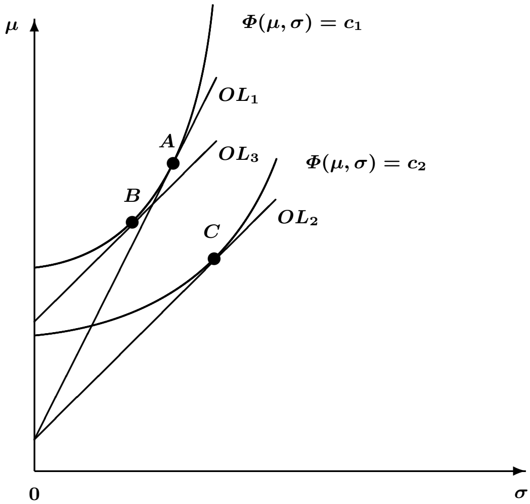

Fig. 1. Substitition and income effects under uncertainty

$\varepsilon_{R,\sigma} < (=)[>]\,1$ if $\theta > (=)[<]\,0$. Furthermore, the recent study of Eichner (2000) contains an example where the elasticity of risk aversion is greater than unity.

Adverse and nonadverse effects of insurance risk on the optimal insurance demand can be explained by Figure 1, which assumes that the optimum coinsurance rate is positive.

Let OL_1 be the initial opportunity line on the relevant range as depicted in the figure: $OL_1 := \{(\mu,\sigma) : \mu = \mu_{\tilde{w}},\ \sigma = \sigma_{\tilde{w}},\ 0 \le \alpha \le 1,$ given $\tilde{z}(\beta_1),\ \beta_1 > 0\}$. Note that a positive loading factor, i.e. $\lambda > 0$, implies an optimum $\alpha < 1$. The optimal insurance decision satisfies the tangency condition (point A). The slope of the opportunity line equals the marginal rate of substitution, where $\Phi(\mu,\sigma) =$ const. denotes an indifference curve. If the standard deviation of $\tilde{z}(\beta)$ increases, then the new opportunity line $OL_2 := \{(\mu,\sigma) : \mu = \mu_{\tilde{w}},\ \sigma = \sigma_{\tilde{w}},\ 0 \le \alpha \le 1,$ given $\tilde{z}(\beta_2),\ \beta_2 > \beta_1\}$ becomes flatter. Hence utility level decreases $(c_2 < c_1)$ and optimum moves from point A to point C.

Now suppose that expected end-of-period wealth increases for any level of risk such that the agent reaches the initial indifference curve $\Phi(\mu,\sigma) = c_1$. The resulting opportunity line OL_3 (a parallel shift of OL_2) leads to the new optimum point B. This shows the well-known negative substitution effect (see, e.g., Davis, 1989; Varian, 1992).

Let us now reduce expected end-of-period wealth for any level of risk. Changing OL_3 to OL_2 leads to the optimum point C. The movement from A to B provides the reason for the substitution effect and the movement from B to C for the income effect. In Figure 1 the income effect is dominated by the substitution effect which is saying that the elasticity of risk aversion is less than unity. If the elasticity of risk aversion is greater than unity, then the income effect dominates the substitution effect. In this case optimum insurance demand decreases although insurance risk increases. In contrast to the existing insurance literature this is a remarkably simple characterization.

Finally, note that our Proposition can also be used under the expected utility hypothesis. Our (μ, σ)-decision model is not in conflict with maximizing expected utility but has notably attractive properties. For example, it can be shown by the findings of Schneeweiß (1967), Sinn (1980), Meyer (1987), and Lajeri and Nielsen (2000) that the elasticity of risk aversion is always less than unity if preferences display decreasing absolute risk aversion in the sense of Arrow (1971) and Pratt (1964).

4 Concluding remarks

We have analyzed the revision of optimum insurance demand when insurance risk changes. The elasticity of risk aversion determines whether or not a (μ, σ)-risk averse agent (or, a risk averse expected utility maximizer) decreases/increases her optimum insurance demand when the insured risk becomes greater.

References

Alarie Y, Dionne G, Eeckhoudt L (1992) Increases in risk and the demand for insurance. In: Dionne G (ed) Contributions to insurance economics. Kluwer, Boston

Arrow K (1971) Essays in the theory of risk-bearing. North-Holland, Amsterdam

Bamberg G, AG Coenenberg (2000) Betriebswirtschaftliche Entscheidungslehre, 10. Aufl. Vahlen, München

Bodie Z, Kane A, Marcus A (1993) Investments. Irvin, Boston

Bar-Shira Z, Finkelshtain I (1999) Two-moments decision models and utility-representable preferences. Journal of Economic Behavior & Organization 38: 237–244

Briys E, Couchy M, Schlesinger H (1993) Optimal hedging in a futures market with background noise and basis risk. European Economic Review 37: 949–960

Broll U, Wahl JE, Zilcha I (1995) Indirect hedging of exchange rate risk. Journal of International Money and Finance 14: 667–678

Broll U, Eckwert B (1999) Exchange rate volatility and international trade. Southern Economic Journal 66: 178–185

Davis G (1989) Income and substitution effects for mean-preserving spreads. International Economic Review 30: 131–136

Demers F, Demers M (1991) Increases in risk and the optimal deductible. Journal of Risk and Uncertainty 51: 670–699

Eichberger J, Harper I (1997) Financial economics. Oxford University Press, Oxford New York

Eichner T (2000) A note on indifference curves in the (μ, σ)-space. OR Spektrum 22: 491–499

Lajeri F, Nielsen L (2000) Parametric characterizations of risk aversion and prudence. Economic Theory 15: 469–476

Löffler, A (2001) Ein Paradox der Portfoliotheorie und vermögensabhängige Nutzenfunktionen. Neue betriebswirtschaftliche Forschung, Bd 279. Dt. Univ.-Verl., Gabler, Wiesbaden

Meyer J (1987) Two-moment decision models and expected utility maximization. American Economic Review 77: 421–430

Mossin J (1968) Aspects of rational insurance purchasing. Journal of Political Economy 79: 553–568
Pratt J (1964) Risk aversion in the small and in the large. Econometrica 32: 122–136
Roy, AD (1952) Safety first and the holding of assets. Econometrica 20: 431–449
Schlesinger H (1997) Insurance demand without the expected-utility paradigm. Journal of Risk and Insurance 64: 19–39
Schneeweiß H (1967) Entscheidungskriterien bei Risiko. Springer, Berlin Heidelberg New York
Sinn H-W (1980) Ökonomische Entscheidungen bei Ungewißheit. Mohr, Tübingen
Varian H (1992) Microeconomic analysis. Norton, New York London