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A celebration of Fred Brauer's legacy in mathematical biology

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Abstract
Fred Brauer (1932–2021), one of the pioneers of mathematical population biology, shaped generations of researchers through his lines of research, his books which have become key references in the field, and his mentoring of junior researchers. This dedication reviews some of his work in population harvesting and epidemiological modeling, highlighting how this special collection reflects the impact of his legacy through both his research accomplishments and the formation of new researchers.

Keywords Biography · Retrospective · Predator–prey systems · Pandemic preparedness

Mathematics Subject Classification 92D30 · 92D25 · 92-06 · 34A34

1 Introduction

This collection of articles aims to honor the life and legacy of Fred Brauer (1932–2021), one of the pioneers of mathematical population biology, whose work over the past 65 years shaped generations of researchers, especially within the areas of mathematical epidemiology and of harvesting in predator–prey systems. During his long career Fred published over 170 papers on a wide array of topics, spanning the spectrum from highly theoretical papers to studies of disease outbreaks like SARS and COVID-19. He also placed great importance on the mentoring of new researchers, and several of the ten books that he published are recognized as key references, remaining

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Fig. 1 Fred speaking in 2013

in print decades (in one case over 50 years) after their initial publication. Fred gave talks (Fig. 1) in countries around the world, inspiring many young researchers, and collaborated with numerous colleagues. The papers in this collection are meant to reflect his legacy both by building (in some cases) on the work he did and by illustrating (in other cases) the work of those whose professional growth he shaped.

Fred's research can be broken into three phases. From the completion of his dissertation in 1956 through the early 1970s, Fred published about thirty studies of the mathematical properties of differential equations. During this time he wrote five books on differential equations with his colleague John Nohel, all published by W.A. Benjamin. One of these, a graduate-level text on the qualitative analysis of differential equations, has become a standard reference in the field and remains in print today, more than 50 years later.

In the 1970s, Fred became interested in using the differential equations he studied to track the growth of populations, part of the growing field of mathematical biology. For the next two decades Fred published widely in the area of population management, referred to in the literature as harvesting, and in the study of predator–prey systems. During this period his most significant collaborations were with A.C. Soudack and with David Sánchez. Some of Fred's roughly forty works from this time were published in Latin America, reflecting the widespread impact of his work as he took an especial interest in the professional development of new mathematical biologists at a moment when the field was experiencing rapid growth.

In the late 1980s, Fred's research made one final shift, into the area of mathematical epidemiology, tracing many different aspects of the spread of infectious diseases. For over three decades beginning in 1989, Fred wrote extensively about models for disease transmission, producing 100 articles and several books (some of which have become established key references in the field). Many of his most topical papers, studying then-recent epidemics such as HIV/AIDS, SARS, the 2009 H1N1 influenza pandemic, and COVID-19, were written with two or three co-authors each, leading to an incredibly diverse set of collaborations. Fred's solo papers in recent decades focused on revisiting important topics in epidemiology using simple models and techniques as elementary as possible, in order to be accessible to a wider audience, or on providing perspective on the overall direction of the field.

In the remainder of this dedication we highlight some of Fred's most important contributions within mathematical biology, which resonate in some of the contributions to this collection.

2 Population dynamical models with harvesting

In the management of renewable resources, it is important to include both economic and ecological factors in harvesting any population such as animals or plants The modeling problem is how to achieve the maximum sustainable yield (MSY) by determining the harvesting rate which keeps the population at its maximum growth rate. Fred made pioneering contributions in the mathematical modeling and analysis of population dynamics with harvesting, for both single species and predator–prey systems.

Beddington, May and colleagues had originally proposed (May et al. 1979; Beddington and May 1980) two types of harvesting: (i) *constant-effort harvesting*, described by a constant multiplication of the size of the population under harvest, and (ii) *constant-yield harvesting*, described by a constant independent of the size of the population under harvest. Lazer and Sánchez (1984) proposed a third type of harvesting: (iii) *periodic harvesting*, described by a time periodic function corresponding to seasonal harvesting such as seasonal open hunting or fishing seasons. Brauer and Sánchez first analyzed a general model for a single species under constant-yield harvesting, of the form $\frac{dx}{dt} = xf(x) - h$, establishing conditions on the population's natural growth rate under which a critical harvesting rate exists (Brauer and Sánchez 1975); harvesting yields above this threshold, or initial population sizes below a critical level, will make the population die out in finite time. (Below the critical harvesting rate, a stable equilibrium and a smaller, unstable equilibrium exist. The special case of this analysis using logistic growth is now used in many textbooks.) Brauer and Sánchez (2003) subsequently considered the analogous model with periodic harvesting and asked the following questions: If the unharvested model possesses constant equilibria, does the effect of periodic harvesting transform them to periodic solutions of the differential equations? Will the stability of equilibria be preserved and result in stable periodic solutions? If the average harvesting rate is too large, will the population(s) be driven to extinction? Brauer and Sánchez established conditions under which the results for constant-yield harvesting transfer to solutions of periodic harvesting models. Brauer and Sánchez (1975) and Brauer (1977) also studied a version of the constant-harvesting model with a time delay in the growth function f (representing, for instance, gestation time), and showed that the upper equilibrium is asymptotically stable if the harvesting rate is below the critical level and the gestation delay is small enough.

Fred went on to study the effects of age-dependent mortality and age-dependent harvesting on a managed population. Brauer (1976a) applied an age-dependent mortality rate to the single-species model with constant-yield harvesting, leading to a Volterra integral equation. His analysis identified the critical harvesting rate for this model, as well as for the corresponding model with constant-effort harvesting. Further detailed analysis was carried out in Brauer (1976b). He went on to incorporate a harvest of members with a preassigned age structure and constant total time rate into the basic age-structured model of MacKendrick-von Foerster (Brauer 1983a, b). Here he formulated the problem as a pair of equations for the total population size and the birth rate, and studied the behavior of solutions when the birth and death moduli depend on either the age or the total population size, but not both. Finally, Brauer et al. (1988) proposed a Volterra equation model to describe the effect of harvesting on populations with both delayed recruitment and age-dependent mortality, and described some numerical simulations which indicate the possible qualitative behaviors.

Fred also studied managed predator–prey systems. In an influential series of papers (Brauer et al. 1976; Brauer and Soudack 1979a, b, 1981a, b), Brauer and co-authors investigated the effect of constant-yield harvesting/stocking on either or both species in a general predator–prey model. In each possible case, Brauer and Soudack examined the system's asymptotic behavior, the domains of attraction for stable equilibrium states, and the change in the nature of those equilibria as harvesting and stocking rates change. This pioneering work attracted great attention over several decades to the effect of harvesting on the dynamics of predator–prey systems and the role of harvesting in the management of renewable resources.

One notable model featured a Holling type II predator functional response, logistic growth in the prey, and harvesting only in the predator population. Numerical studies in Brauer and Soudack (1979a) indicated the existence in this model of a homoclinic loop and a periodic orbit for some parameter values. These inspired a detailed bifurcation analysis of the model by Xiao and Ruan (1999), who demonstrated that it can also exhibit Hopf bifurcation, saddle-node bifurcation, homoclinic bifurcation, and the Bogdanov-Takens bifurcation under certain conditions. This work in turn has inspired a large body of literature on multi-parameter bifurcations in predator–prey models with harvesting.

Fred also considered harvesting in predator–prey models with gestation delay in the prey population. The first predator–prey model with delay and harvesting was proposed by Brauer (1977), which carried out a basic analysis of the existence and stability of equilibria for the system formulated with general prey growth and predator functional response, as well as examples such as logistic prey growth and Holling type II functional response. This work also inspired follow-up by others, involving more detailed bifurcation analyses (including Hopf and Bogdanov-Takens bifurcations) for other choices of prey growth and functional response, as well as other delay predator–prey models with harvesting.

3 Epidemiological modeling

Fred was a highly productive scholar for nearly 7 decades, and over half of his publications focused on epidemiological modeling. Particularly, during his last 3 years (2019–2021), he co-authored eight articles (Bai and Brauer 2021; David et al. 2020a, b; Ortigoza et al. 2020; Brauer 2019a, b; Ortigoza et al. 2019; Brauer 2019c) and one book on epidemiological modeling (Brauer et al. 2019). Many of Fred's solo articles, especially during the past 20 years, revisited key topics in the development and analysis of such models from the most elementary perspective possible, in the interest of making those topics accessible to new researchers. Some of his most highly cited solo authored articles on this topic include: (i) Backward bifurcations in simple vaccination models, 2004 (Brauer 2004); (ii) Compartmental models in epidemiology, 2008 (Brauer 2008); and (iii) Mathematical epidemiology: Past, present, and future, 2017 (Brauer 2017). In Brauer (2004), one of the simplest possible SIS models with vaccination was examined. Using elementary algebraic means, Fred showed how to analyze the existence of multiple endemic equilibria when the basic reproductive number is less than 1. The result illustrated important insights, e.g., the danger of a vaccination policy is that an unforeseen backward bifurcation may require a larger than expected vaccination fraction to control a disease. However, if a vaccine is completely effective, this possibility does not arise, and a program which decreases the contact rate can also control a disease without leading to backward bifurcations. The main focus of Brauer (2008) was to present a broad framework to describe and analyze compartmental models for disease transmission. For epidemic models, the calculations of the basic reproduction number and the final size of the epidemic were presented. For models that incorporate demographic effects and when an endemic equilibrium is present, asymptotic stability was considered. The paper also discussed age of infection models which give a unifying framework for more complicated compartmental models. In Brauer (2017), Fred provided a detailed review and discussion of past, present, and future directions for mathematical epidemiology. Particularly, he pointed out that an important broader question is the matter of what information influences the behavior of people during a disease outbreak, and how to include this in a model. This is exactly the focus of the new NSF funding opportunity on Incorporating Human Behavior in Epidemiological Models (IHBEM).

Some of Fred's influential books on this topic include: (i) *Mathematical models in population biology and epidemiology*, first published in 2001 with a second edition in 2012 (Brauer and Castillo-Chavez 2012); *Lecture notes in mathematical epidemiology*, 2008 (Brauer et al. 2008); and (iii) *Mathematical models in epidemiology*, 2019 (Brauer et al. 2019). All of these books provide helpful information for researchers in the field, especially for new investigators with an interest in entering this research area. The most recent Springer book (Brauer et al. 2019), published in the series of "Texts in Applied Mathematics," includes more than 600 pages and covers both basic epidemiological modeling and example of modeling studies for many specific infectious diseases. Simon Levin wrote in the Foreword of the book that the volume "will be a treasure trove for mathematicians and other researchers looking for an authoritative introduction to the literature and to open problems to the solutions of which they might contribute their skills".

Following the 2002–2003 SARS epidemic, Fred devoted significant effort toward pandemic preparedness initiatives, using often simple models to identify key quantities to understanding the scope and nature of an outbreak. He participated in long-term working groups including the Pandemic Influenza Outbreak Research Modelling Team (Pan-InfORM), which advised the Canadian Medical Asscoation during the 2009 H1N1 avian influenza epidemic (Pan-InfORM 2009). Fred also collaborated with researchers in the British Columbia Ministry of Health's Pandemic Preparedness Modeling Project (Davoudi et al. 2013). One 2006 study focused on the impact of the proportion of symptomatic cases (Arino et al. 2006), an issue which proved important during the early COVID-19 pandemic. Later work focused on predicting outbreak size, as influenced by both heterogeneous mixing and behavior change (Brauer 2019d), the latter a major (and complex) factor in the COVID-19 pandemic.

Fred also made great contributions to diversity in the scientific community over 1996–2020. One example is his involvement in the Mathematical and Theoretical Biology Institute (MTBI) summer program. Students of this program were primarily from national and international underrepresented populations. Fred mentored between 24–36 students every summer for over two decades through his visits to MTBI and had joint publications with some of these students.

4 Conclusion

Those of us who worked with Fred, whether as students or experienced researchers, learned tremendously from Fred throughout our collaborations with him. Christopher was Fred's last Ph.D. student at UW Madison, and over a period of several years co-wrote with him a text aimed at making elementary population modeling accessible to biology students (Brauer and Kribs 2015). Zhilan was one of the co-authors of the book (Brauer et al. 2019) published in 2019, who completed the final version during a visit to Fred's home in Vancouver in 2018. Shigui, who first met Fred in 1993 at the Fields Institute, has worked extensively in predator–prey models with harvesting, building on Fred's work in that area and collaborating. Fred has truly been a great scientist, mentor, and friend.

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