

Optimization model of an irrigation reservoir for water allocation and crop planning under various weather conditions

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Abstract This paper develops a non-linear programming optimization model with an integrated soil water balance, to determine the optimal reservoir release policies, the irrigation allocation to multiple crops and the optimal cropping pattern in irrigated agriculture. Decision variables are the cultivated area and the water allocated to each crop. The objective function of the model maximizes the total farm income, which is based on crop–water production functions, production cost and crop prices. The proposed model is solved using the simulated annealing (SA) global optimization stochastic search algorithm in combination with the stochastic gradient descent algorithm. The rainfall, evapotranspiration and inflow are considered to be stochastic and the model is run for expected values of the above parameters corresponding to different probability of exceedence. By combining various probability levels of rainfall, evapotranspiration and inflow, four weather conditions are distinguished. The model takes into account an irrigation time interval in each growth stage and gives the optimal distribution of area, the water to each crop and the total farm income. The outputs of this model were compared with the results obtained from the model in which the only decision variables are cultivated areas. The model was applied on data from a planned reservoir on the Havrias River in Northern Greece, is sufficiently general and has great potential to be applicable as a decision support tool

for cropping patterns of an irrigated area and irrigation scheduling.

Introduction

The objective of the present study is to develop a non-linear programming optimization model with an integrated soil water balance, to determine the optimal reservoir release policies, the irrigation allocation to multiple crops and the optimal cropping pattern in irrigated agriculture. Water release from the reservoir is utilized by the crops in the form of evapotranspiration. In determining the amount of release from a reservoir, it is therefore necessary to consider the crop water requirements in relation to the crop growth and its yield (Vedula and Mujumdar 1992; Vedula and Kumar 1996; Ghahraman and Sepaskhah 2002, 2004). A large body of the literature dealing with crop–water production functions focuses on the impact of water scarcity at different times of growing season to the crop yield. The most widely used relationships are Jensens (1968) and Doorenbos and Kassams (1979).

Earlier models developed for optimal reservoir operation for irrigation dealt with different aspects of the problem and with varying degrees of complexity (Vedula and Mujumdar 1992; Vedula and Kumar 1996; Ghahraman and Sepaskhah 2002). In all these papers the cropped areas are assumed to be fixed.

Also, in the literature, papers have been presented, which deal with the optimal irrigation allocation under conditions of full and deficit irrigation. The deficit irrigation is the distribution of limited amounts of irrigation water to satisfy essential water needs of plants. The water supply is reduced in less critical periods of water demand

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by the crop and supply of full amount of water during stress-sensitive periods. Deficit irrigation has been suggested as a way to increase system benefits, at the cost of individual benefits, by decreasing the crop water allocation and increasing the total irrigated land (Ganji et al. 2006). Deficit irrigation model limited to field level have been formulated mostly as a dynamic programming problem (Rao et al. 1990; Shanguan et al. 2002) but with the restriction of pre-specified crop areas, and under the assumption that the volume of water available for the entire irrigation season is fixed, and given at the beginning of this period. Wardlaw and Barnes (1999) also used pre-specified crop areas, and also assumed that the ratio of actual to maximum crop evapotranspiration is the same as the ratio of irrigation supply to demand. Sabu et al. (2000) proposed a model based on both stochastic dynamic programming and deterministic dynamic programming for computing the optimal cropping pattern and irrigation schedule. In their work the water availability depends on canal releases. Kipkorir et al. (2002) developed a dynamic programming optimization model to aid the decision making for optimal crop–area distribution and irrigation in a region where the water availability for the entire irrigation season is fixed, and given at the beginning of this period.

In the presented paper the objective function maximizes the total farm income and the decision variables are cropped areas and water releases from the reservoir to meet the corresponding crop water requirements. The initial reservoir storage is determined by inflows from the end of the previous irrigation season until the beginning of the current one. All subsequent inflow during the irrigation season is regulated and used for the irrigation. In determining the reservoir release policy for irrigation, the crop-growing season has often been used as the decision interval. In this paper instead of the crop growing season the irrigation interval is used, because the irrigation release decisions have to be made in a much shorter time interval.

To solve most of these problems in the field of reservoir operation linear, non-linear and dynamic programmings have been used (Yeh 1985; Labadie 2004). Classical optimization techniques can be useful tools for solving many of the reservoir operation problems. However the computational requirements are intractable in many instances. The computational burden and representation of the problem within an optimization solver have been consistent hurdles in solving many complex reservoir operation problems characterized by large number of decision variables (Teegavarapu and Simonovic 2002). Stochastic search techniques were used in the past in many instances for developing models for operation of water resources systems (Yeh 1985). Simulated annealing is one such stochastic search method that has proved to be useful in obtaining solutions for optimization problems

(Kirkpatrick et al. 1983; Azencott 1992; Vougioukas 1999; Webster 2001; Winston 2003). The optimization method developed in the present study is based on this technique. Recent applications of the simulated annealing technique in the area of water resources can be found in the works of Dougherty and Marrayott (1991), Kuo et al. (1992), Rizzo and Dougherty (1996), Pardo-Iguzquiza (1998), Cunha and Sousa (1999), Teegavarapu and Simonovic (2002) and Georgiou et al. (2006).

In this paper, the optimization is performed in two stages. During the first stage the simulated annealing (SA) global optimization stochastic search algorithm is used. In the second stage the solution reached by the first stage is refined by a stochastic gradient descent algorithm, which does not make use of analytically or numerically computed gradient information.

Deterministic approaches based on observed climatic or hydrological sequences are generally appropriate for water systems with seasonal regulation and short planning horizon relative to the lengths of historical records. They are not appropriate in assessments of yield, reliability or operation when the lengths of reliable, concurrent climatic and hydrological records are short relative to the planning horizon. Stochastic approaches make uncertainty explicit and allow investigation of system response against the possible range of system input and output sequences. Consequently, the rainfall, evapotranspiration and inflow which have a great amount of uncertainty are considered to be stochastic in the optimization model and synthetic series are used as an alternative to historical records.

Rainfall is the weather variable which has received the most attention from researchers interested in the stochastic interpolation and extrapolation of temporal data (Arnold and Elliot 1996; Young and Gowing 1996; Hanson et al. 1997; Nelson 2002). In this paper, rainfall generation is a two stage process whereby rainfall occurrence (i.e. wet or dry day) is based upon a first order two state Markov chain and rainfall amount is sampled from the gamma distribution. The stochastic generation of evapotranspiration has been obtained from evapotranspiration data (Tsakiris 1988; Kotsopoulos and Svehlik 1989) or meteorological data (Nicks and Harp 1980; Richardson 1981; Young 2002). In this paper meteorological data are used. Finally, the synthetic monthly inflows were generated by using an autoregressive moving average exogenous variables (ARMAX) model (Kuo et al. 1990; Bras and Rodriguez-Iturbe 1993; Hipel and McLeod 1994; Makridakis et al. 1998) which represents the relationship between inflow and precipitation.

The model is run for expected values of the above parameters corresponding to different probability of exceedence. By combining various probability levels of rainfall, evapotranspiration and inflow, four weather

conditions are distinguished for which an irrigation advice will be formulated.

Except for the case where the decision variables are cropped areas and reservoir releases, there is the problem in which the only decision variables are the cropped areas. In this case, the reservoir releases are forced to be equal to the full irrigation water requirements over the entire crop period and the relative yield is fixed to unity. The objective function is linear and the nonlinearity of the problem is due to the constraint of reservoir surface evaporation. The outputs of the developed model were compared with the results obtained from a simple model in which the only decision variables are the optimal allocation of cropped areas.

Model formulation

The problem may be considered to be one of maximizing the utilization of the available water supply when conflicts between supply and demand occur during each time interval in the irrigation season. The reservoir storage constitutes the systems state variable, whereas the system inputs—commonly referred to as decision variables—are the cultivated areas, and the water releases by the reservoir for each crop in each time interval to satisfy irrigation requirements. The reservoir inflow and the effective rainfall at each time interval are treated as uncontrolled system disturbances, and the crop water requirements and limited reservoir capacity lead to state dependent constraints for the system input.

Objective function

For application in planning, design and operation of an irrigation reservoir, it is possible to analyze the effect of water supply on crop yields. The relationships between crop yield and water supply can be determined when crop water requirements and crop water deficits, on the one hand, and maximum and actual crop yield on the other can be quantified. Water deficits in crops, and the resulting water stress on the plant, have an effect on crop evapotranspiration and crop yield. The relations which include the effects of both timing and quantities of irrigation water are called dated water production functions (Rao et al. 1988). These relationships are complex, as they must include the effects of crop water-stress in different growth stage. In the literature the most widely used relations are those that were proposed by Jensen (1968) and Doorenbos and Kassam (1979). On the basis of the production function given by Jensen (1968), the following objective function which maximizes the total farm income is considered for the optimal operation of a reservoir to irrigate n crops at

any time interval j during the irrigation season (Georgiou et al. 2006):

$$\begin{aligned} Z^* &= \max_{A_i, R_{i,j}} \sum_{i=1}^n [P_i(Y_a)_i - (B_i + C_i)] A_i \\ &= \max_{A_i, R_{i,j}} \sum_{i=1}^n \left[P_i(Y_m)_i \prod_{j=1}^k \left(\frac{(ET_a)_{i,j}}{(ET_m)_{i,j}} \right)^{\lambda_{i,j}} - (B_i + C_i) \right] A_i \end{aligned} \quad (1)$$

where Z^* = total farm income (€), P = product price (€ k g⁻¹), $R_{i,j}$ = reservoir release to cropped area i in time interval j , B = fixed cost (€ ha⁻¹), C = variable cost (€ ha⁻¹), A = cropped area (ha), Y_a = actual yield (kg ha⁻¹), Y_m = maximum crop yield under given management conditions that can be obtained when water is non-limiting (kg ha⁻¹), ET_a = actual evapotranspiration (mm), ET_m = maximum evapotranspiration (mm), λ = sensitivity index of crop to water stress, n = number of cultivation crops, k = number of time intervals, i = cultivation crop and j = time interval.

Jensens model has the advantage (Kipkorir and Raes 2002; Raes 2002) that the model can be used at time steps smaller than a growth stage, for example an irrigation interval. Doorenbos and Kassam (1979) utilized Stewart and Hagans (1974) and Stewarts et al. (1975, 1977) model and developed a methodology to quantify yield of 26 crops using crop, climatic and soil data. They derived yield response factors (k_y) for individual growth stages (i.e. establishment, vegetative, flowering, yield formation and ripening) and also for the total growing period. The sensitivity indexes of Jensens model are related to Doorenbos and Kassam (1979) yield response factors (k_y) by the polynomial function (Georgiou 2004; Georgiou et al. 2006). For the purposes of reservoir operation and irrigation scheduling the crop sensitivity during irrigation interval is much more important than during a particular stage of growth. Tsakiris (1982) provides a calculation procedure for estimating crop sensitivity to water deficiency at given time intervals, using Jensens crop water production function.

The fixed cost term (B) includes the land cost, and the variable cost term (C) is the summation of all other costs such as seed, fertilizer, pesticides, machinery, harvesting, marketing, drying, unexpected costs, etc. This variable cost (C) is independent of the quantity of irrigation water applied because currently this is the standard policy in Greek agriculture. However, this decision does not affect in anyway our optimization methodology. One can simply introduce a water depended term in the variable cost and use the same optimization methodology.

For the problem in which the only decision variables are the optimal allocation of cropped areas the actual yield is

taken to be equal to the maximum yield, and the Eq. (1) may be written as:

$$\begin{aligned} Z^* &= \max_{A_i} \sum_{i=1}^n [P_i(Y_a)_i - (B_i + C_i)] A_i \\ &= \max_{A_i} \sum_{i=1}^n [P_i(Y_m)_i - (B_i + C_i)] A_i \end{aligned} \quad (2)$$

Constraints

State equation of the reservoir

The reservoir water balance is governed by the reservoir storage continuity equation:

$$\begin{aligned} S_{j+1} &= S_j + Q_j - \sum_{i=1}^n 10A_i R_{i,j} - 0.001 E_j f(S_j) - SP_j \\ &\quad + \text{RAIN}_j \end{aligned} \quad (3)$$

where S = reservoir storage at the beginning of the time interval (m^3), Q = reservoir inflow during the time interval (m^3), A = cropped area (ha), R = reservoir release (for irrigation) in the time interval (mm), E = reservoir surface area evaporation, which is computed from the de Bruin equation (de Bruin 1978) (mm), $f(S)$ = reservoir surface area (m^2) which is computed from the function of reservoir surface area versus reservoir storage, 0.001 = coefficient of transformation from mm to m, SP = overflow loss from spillway during the time interval (m^3), RAIN = rain on the reservoir in the time interval (m^3), i = cultivation crop and j = time interval. The rain on the reservoir area is negligible and for this reason in this study it has not been included in the model.

The de Bruin equation (de Bruin 1978) calculates evaporation as a function of wind speed and saturation vapour pressure deficit and is defined by the equation:

$$E = \frac{\alpha}{\alpha - 1} \left(\frac{\gamma}{\Delta + \gamma} \right) f(u) (e_s - e_a) \quad (4)$$

where E = reservoir evaporation (mm), α = Priestley–Taylor constant and its value is usually set to 1.26 (Priestley and Taylor 1972; de Bruin 1978; Vardavas and Fountoulakis 1996), γ = psychrometric constant ($\text{kPa}/^\circ\text{C}$), Δ = derivative of the water vapour saturation pressure with respect to air temperature ($\text{kPa}/^\circ\text{C}$), $f(u) = 0.27(1 + 0.864u)$ where u = wind speed (m/s), e_s = saturation vapour pressure at temperature T (mbar) and e_a = actual vapour pressure at temperature T (mbar). The suitability of the de Bruin equation for determining time interval reservoir surface area evaporation was evaluated by using the Priestley–Taylor equation (Priestley and Taylor 1972). The results indicated that there is good agreement between the evaporation estimates computed using the de Bruin equation with $\alpha = 1.26$ and that using the Priestley–Taylor equation.

The reservoir storage at any time interval is bounded between an upper limit (full reservoir- S_{\max}) and a lower limit (dead storage- S_{\min}). Hence, it is possible to remove the overflow variable SP_j from Eq. (3) and rewrite it as a state Eq. (5), where the reservoir storage is the state.

$$S_{j+1} = \min \left(S_{\max}, S_j + Q_j - \sum_{i=1}^n 10A_i R_{i,j} - 0.001 \times E_j \times f(S_j) \right) \quad (5)$$

The state Eq. (5) is nonlinear and time varying, because of limited capacity and water evaporation from the reservoir, respectively.

Soil moisture balance

In the beginning of the irrigation season, soil moisture is assumed to be known. Here, it is assumed to be at field capacity for all soils and crops. Soil water balance equation for a given crop i and time interval j are given by Vedula and Mujumdar (1992), Ghahraman and Sepaskhah (2004), Vedula et al. (2005) and Georgiou et al. (2006):

$$\begin{aligned} (\text{SM}_{\text{in}})_{i,j+1} &= (\text{SM}_{\text{in}})_{i,j} + \text{ERAIN}_{i,j} + \text{IR}_{i,j} - (\text{ET}_a)_{i,j} \\ &\quad + (\text{TAW}_{i,j+1} - \text{TAW}_{i,j}) \end{aligned} \quad (6)$$

where SM_{in} = initial soil moisture level (mm), ERAIN = effective rainfall (mm) which is computed by the procedure described from USDA (1993) and Bos et al. (1996), IR = irrigation water allocated (mm), ET_a = actual evapotranspiration (mm), TAW = total available soil water (mm), i = cultivation crop and j = time interval.

The total available soil water (TAW) is an important soil physical characteristic that needs to be determined when formulating irrigation guidelines. TAW refers to the total amount of water available in the root zone that can be utilized by the crop. The soil water content at field capacity (FC) and permanent wilting point (PWP) are respectively the upper and lower limits of TAW. Field capacity is the quantity of water that a well-drained soil would hold against the gravitational forces. Permanent wilting point is the soil water content at which plants stop extracting water and will permanently wilt. The readily available soil water (RAW) is the amount of water that crops can extract from the root zone without experiencing any water stress (RAW). It is a fraction (p) of TAW. Allen et al. (1998) present indicative values for the soil water depletion fraction for no stress (p).

A sine function (Borg and Grimes 1986) was adopted for assessing time pattern of root growth. At the beginning of every time interval, any water added to (ERAIN and IR) is computed as if it was done instantaneously.

The maximum releases of the reservoir are given by Georgiou et al. (2006):

$$(R_{\max})_{i,j} = \frac{(IR_{\max})_{i,j}}{ME} = \frac{(1 - p_{i,j})TAW_{i,j} + (ET_m)_{i,j} - (SM_{in})_{i,j} - ERAIN_{i,j}}{ME} \tag{7}$$

where R_{\max} = maximum release from reservoir to meet irrigation requirements (mm), IR_{\max} = maximum irrigation requirements (mm), p = soil moisture depletion factor for no stress (expressed as a fraction), which depends on specific crop and maximum evapotranspiration ET_m (Allen et al. 1998), TAW = total available soil water (mm), SM_{in} = the initial soil moisture level (mm), ET_m = maximum evapotranspiration (mm), $ERAIN$ = effective rainfall (mm), ME = mean efficiency included application efficiency and conveyance efficiency, i = cultivation crop and j = time interval.

Applying suitable mean efficiency (ME) made a conversion of irrigation requirements to the total amount of reservoir release to meet irrigation requirements.

Crop evapotranspiration

Maximum evapotranspiration The maximum evapotranspiration ET_m coincides with crop evapotranspiration, which is the product of a crop factor K_c and the reference evapotranspiration which is computed from the FAO Penman–Monteith equation (Allen et al. 1998).

Actual evapotranspiration In the present model the actual rate of maximum evapotranspiration (ET_a) is given by Doorenbos and Kassam (1979), Georgiou et al. (2006):

$$ET_a(x, t) = -\frac{dx}{dt} = f(x)ET_m(t) \tag{8}$$

where ET_a = actual evapotranspiration, x = level of soil moisture in the root zone, t = time interval, $f(x)$ = soil water depletion function and ET_m = maximum evapotranspiration. In this study the soil water depletion function is formed as follows:

$$f(x) = \begin{cases} 1 & (1 - p)TAW < x \leq TAW \\ x / [(1 - p)TAW] & 0 < x \leq (1 - p)TAW \end{cases} \tag{9}$$

where x = level of soil moisture in the root zone, TAW = total available soil water and p = soil moisture depletion factor for no stress.

Bounds

In the foregoing the decision variables are the cropped areas and the irrigation releases and the state variable is

storage. Some social economic, management and market considerations restrict the model variables, such as the maximum and/or minimum areas cultivated with specific crops. The lower and upper limits for these variables are the following:

$$A_{\min} \leq A_i \leq A_{\max}, 0 \leq R_{i,j} \leq R_{i,j}^{\max}, \text{ and } S_{\min} \leq S_j \leq S_{\max} \tag{10}$$

where A = cropped area, R = reservoir release and S = reservoir storage.

In this study for the formulation of the optimization model, the relative yield (Y_a/Y_m) should be bound so that the social economic constraints which require a minimum production for each crop should be valid. For the problem in which the only decision variables are the cropped areas, the reservoir releases $R_{i,j}$ are taken to be equal to the maximum releases $R_{i,j}^{\max}$.

Stochastic generation

In optimal reservoir operation the historical data are not available for the length of operation life of reservoir. For this reason, it is necessary to simulate the historical data with mathematical models which extracting the statistical properties of historical data and using these, in combination with random numbers generators, produce synthetic series of data with the same statistical properties as that which was input.

By applying these models to the historical data, synthetic series of 50 years duration (operation life of reservoir) for rainfall, evapotranspiration and inflow were generated. Finally, 100 synthetic series of each variable were generated from which expected values corresponding to different probability of exceedence were computed with frequency analysis.

Rainfall

Rainfall is the weather variable which has received the most attention from researchers interested in the stochastic interpolation and extrapolation of temporal data (Arnold and Elliot 1996; Young and Gowing 1996; Hanson et al. 1997; Nelson 2002). As a result of this, the methodology is now fairly well established and a number of reliable simulation techniques are available.

In this paper rainfall generation is a two-stage process whereby rainfall occurrence (i.e. wet or dry day) is based upon a first order two state Markov chain and rainfall amount is sampled from the gamma distribution.

Rainfall occurrence is modelled using a first-order Markov chain approach whereby each day can be either

wet (W) or dry (D). The transitional probability matrix is given in Eq. (11) below:

$$P(t) = \begin{bmatrix} p_{DD} & p_{DW} \\ p_{WD} & p_{WW} \end{bmatrix} \quad (11)$$

where p_{DD} = conditional probability that a dry day is followed by a dry day, p_{DW} = conditional probability that a wet day is followed by a dry day, p_{WD} = conditional probability that a dry day is followed by a wet day, p_{WW} = conditional probability that a wet day is followed by a wet day.

However, by definition, $p_{WD} = 1 - p_{DD}$ and $p_{DW} = 1 - p_{WW}$ and thus, only two probabilities need be calculated from historical data, the other two being calculated from these:

$$p_{WD}(i) = \frac{\sum_{p=1}^{p=n} \{X(i+1) = W/X(i) = D\}}{\sum_{p=1}^{p=n} \{X(i) = D\}} \quad (12)$$

$$p_{WW}(i) = \frac{\sum_{p=1}^{p=n} \{X(i+1) = W/X(i) = W\}}{\sum_{p=1}^{p=n} \{X(i) = W\}} \quad (13)$$

where $p_{WD}(i)$ = the probability that if day, i is dry, then day $i + 1$ will be wet.

To fully characterize the system, therefore, a number of years of data required in which every day of the year have at least one instance of the two transitions. In many cases, the required quantity data will not be available. Even, if it is, the resulting time series of each probability will often be spiky. To account for this, the probabilities need to be smoothed and interpolated. Richardson (1981) used Fourier series, Young (2002) and Georgiou (2004) used a simple five-day moving average to achieve this.

These probabilities, in combination with a random number generator from uniform distribution, are then used to generate series of wet and dry days. Given the state of the preceding day (W or D), a random number of uniform distribution is generated and compared with the appropriate probability (p_{WD} if preceding day D and p_{WW} if preceding day W). If the number generated is greater than the probability, then the day is recorded as wet, otherwise, it is dry. The process is continued until the end of the year, the last day of one year becoming the preceding day for the start of the next.

Given that a day is wet, rainfall amounts are calculated by sampling from the gamma distribution which is assumed to represent the frequency distribution of daily rainfall amounts (Hutchinson and Ungani 1991; Young and Gowing 1996; Young 2002; Georgiou 2004). It has

two parameters which have to fitted, the shape and the scale parameters. The probability density function (pdf) of the gamma distribution is:

$$f_x(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \quad 0 < x < \infty \quad (14)$$

where α = the shape parameter ($\alpha > 0$) and β = the scale parameter ($\beta > 0$).

The great advantage of the gamma distribution over others reported in the literature for rainfall amount generation (Bogardi et al. 1988; Young and Gowing 1996; Young 2002) is that when $\alpha = 1$, the distribution is exponential in nature and that for values of 5 and above ($\alpha > 5$), it approaches the normal distribution. This means that it can be fitted to data with radically different frequency distributions using the same procedure.

The simplest way to use this equation, in combination with random numbers to generate rainfall amounts is to integrate the pdf [to give the cumulative density function (cdf)] and invert the equation to make x the dependent variable. Unfortunately, an analytical integration is not possible and therefore other methods have to be used. In this paper, two methods are used depending upon the value of α . If $\alpha < 1$ then the t distribution method is used and if $\alpha > 1$ then a switching algorithm is used (Dagpunar 1988). In the unlikely event that $\alpha = 1$, the exponential distribution is used. Both of the methods are based upon the concept of envelope rejection (Dagpunar 1988).

Reference evapotranspiration

The reference evapotranspiration is computed from the FAO Penman–Monteith equation (Allen et al. 1998). In the literature the stochastic generation of evapotranspiration is achieved by using evapotranspiration data (Tsakiris 1988; Kotsopoulos and Svehlik 1989) or meteorological data (Nicks and Harp 1980; Richardson 1981; Young 2002). In the FAO Penman–Monteith equation, the basic meteorological parameters are the mean temperature, radiation, relative humidity and wind speed. The mean temperature and radiation are generated simultaneously and relative humidity and wind speed independently.

Mean temperature and radiation

The method used for generating mean temperature and radiation is based upon that given by Richardson (1981) and Young (2002). The approach considers mean temperature and radiation to be a continuous multivariate stochastic process, with means and standard deviations conditioned upon the wet or dry status of the day. These are two stages to the procedure, parameterisation and generation.

In parameterisation there are also two stages. The two series of data are first reduced to residual elements and then various correlations are calculated both between and within the resultant residual series. The process involves reducing the time-series for each variable to a series of residual elements by removing previously smoothed means and standard deviations of dry and wet day according to:

$$x_{p,i}(j) = \frac{X_{p,i}(j) - \bar{X}_i^0(j)}{s_i^0(j)} \quad Y_{p,i} = 0 \quad (15)$$

$$x_{p,i}(j) = \frac{X_{p,i}(j) - \bar{X}_i^1(j)}{s_i^1(j)} \quad Y_{p,i} > 0 \quad (16)$$

where $x_{p,i}(j)$ = residual component of variable j , on day i year p , $x_{p,i}(j)$ = variable j , on day i year p , $\bar{X}_i^0(j)$ = mean of variable j on dry day i , $s_i^0(j)$ = standard deviation of variable j on dry day i , $\bar{X}_i^1(j)$ = mean of variable j on wet day i , $s_i^1(j)$ = standard deviation of variable j on wet day i and $Y_{p,i}$ = rainfall amount.

Richardson (1981) used Fourier series to smooth the means and standard deviations; Young (2002) used 5-day moving average with Lagrange interpolation. In this study was used the procedure by Richardson (1981). The two series of residuals are mutually dependent. Thus, the lag 1 serial correlation and lag 0 and 1 cross-correlation coefficient are used to describe the time dependence and the independence of the two variables.

In the generation, for the given status of day (wet or dry), the procedure consists of generating new residuals, multiplying these by the appropriate standard deviation (according to wet/dry status and time of year) and adding the product to the appropriate mean.

The model used by Richardson (1981) is based upon that proposed by Matalas (1967). This is a weakly stationary generating process which assumes that the residuals of mean temperature and radiation are normally distributed (mean = 0, standard deviation = 1) and can be described by a first-order linear autoregressive model. It is given by:

$$\vec{x}_{p,i}(j) = (A) \vec{x}_{p,i-1}(j) + B \varepsilon_{p,i}(j) \quad (17)$$

where $\vec{x}_{p,i}(j)$ = (2×1) matrix for day i , year p whose elements are residuals of mean temperature ($j = 1$) and radiation ($j = 2$), $\varepsilon_{p,i}(j)$ = (2×1) matrix of independent random numbers that are normally distributed (mean = 0, variance = 1). Generation of these is based upon an envelope rejection method given in Dagpunar (1988). The matrices A and B are defined such that the generated residuals have the observed serial- and cross-correlation coefficients and are given by Richardson (1981), Young (2002) and Georgiou (2004).

Relative humidity

The generation of relative humidity has received very little attention in the literature. The process used for generation and parameterisation is the same as that given in rainfall amount that two gamma distributions are used, one for wet and one for dry days. Thus, parameterisation involves the calculation two values of α and two values of β for each day.

Wind speed

Like rainfall amount, wind speed is sampled from a gamma distribution whose parameters are calculated on a daily basis. Unlike rainfall, this sampling occurs on every day of the year regardless of whether the day is wet or dry.

Inflow

Several streamflow models have been developed in the literature like SARIMA, Thomas-Fiering, etc. (Thomas and Fiering 1962; Box and Jenkins 1976; Papamichail and Georgiou 2001). Some of the forecast models are statistical models with which streamflow is estimated by equations and parameters derived from statistical characteristics of precipitation and stream flow records. In this paper, the synthetic monthly inflows were generated by using an ARMAX model (Kuo et al. 1990; Bras and Rodriguez-Iturbe 1993; Hipel and McLeod 1994; Makridakis et al. 1998) which represents the relationship between inflow and precipitation and is written as:

$$\phi(B)y_t = \omega(B)x_t + \theta(B)e_t \quad (18)$$

where t = discrete time, y_t = inflow time series, x_t = precipitation time series, e_t = normally independently distributed white noise residual with mean zero and variance σ_e^2 , $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p$ nonseasonal autoregressive (AR) operator of order p , $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$ nonseasonal moving average (MA) operator of order q and $\omega(B) = \omega_0 - \omega_1 B - \omega_2 B^2 - \dots - \omega_r B^r$ operator of order r in the numerator of the transfer function, B = backward shift operator defined by $By_t = y_{t-1}$.

The notation (p,q,r) is used to represent the ARMAX model. The application of the ARMAX models requires stationarity of time series data obtained by different transformations. In this paper the standardized transformation was used. The construction of ARMAX models involves various stages (Hipel and McLeod 1994; Makridakis et al. 1998) which are the identification, the estimation and the diagnostic checking. The purpose of the identification stage is to determine the differencing required to produce stationarity. The identification is examined by the cross correlation function. The estimation

stage involves the estimation of the time series model parameters. This estimation is obtained by the residuals squares minimization using the Marquardt algorithm (Press et al. 1992). Finally, the diagnostic checking involves examination of the residuals fitted model and this can or cannot prove the model inadequacy and it can as well inform about the model improvement. Using the identified ARMAX model with the synthetically generating precipitation and the normally independently distributed white-noise residuals, the synthetic monthly inflows have been obtained.

Model solution

The optimization of the objective function given in Eq. (1) is a problem of non-linear programming without analytical solution and with multiple local optimums (Floudas et al. 1999). Simulated annealing is a stochastic global minimization technique especially suited for this kind of problem. Simulated annealing (SA) is motivated by an analogy to physical annealing in solids, inspired from Monte Carlo methods in statistical mechanics (Kirkpatrick et al. 1983; Azencott 1992; Vougioukas 1999; Webster 2001; Winston 2003; Monem and Namdarian 2005). Kirkpatrick et al. (1983) took the idea of annealing from Metropolis algorithm and applied to combinatorial problems, e.g., the traveling salesman problem. Annealing refers to the process in which a solid material is first melted and then allowed to cool by slowly reducing the temperature. The particles of the material attempt to arrange themselves in a low energy state during the cooling process. The collective energy state of the ensemble of particles can be considered the “configuration” of the material. The probability that a particle is at any energy level can be calculated by use of the Boltzman distribution. As the temperature of the material decrease, the Boltzman distribution tends toward the particle configuration that has the lowest energy. Metropolis et al. (1953) first realized that the thermal equilibrium process could be simulated for a fixed temperature by Monte Carlo methods to generate sequences of energy state. This technique is well suited for several decision variables with different natures (discrete, continuous, real, integer) that are mostly encountered in engineering system. This method has no limitation in terms of number of decision variables, constraints and objective function, which are major limitations to classical optimization methods known as the “curse of dimensionality”.

A very important parameter in the implementation of SA is the initial temperature and the stopping criterion. In theory, one should let the temperature fall to zero. However, this would require a huge number of iterations. In some implementations of the SA algorithm the final

temperature is determined by fixing the total number of solutions to be generated, or simply the maximum number of steps. Another approach is to stop when no more progress is being made. An important realization is that at a temperature which is near zero the SA behaves essentially like a standard minimization procedure, since it only accepts solutions of lower energy. For this reason the optimization is performed in two stages. During the first stage the SA global optimization stochastic search algorithm is used and after the SA terminates at some nonzero temperature, we refine the solution reached by the SA in a second stage, using a stochastic gradient descent (SGD) algorithm. This algorithm assumes that no analytical expression exists for the constrained gradient of the cost function. The SA and stochastic gradient descent algorithm is given in Fig. 1.

In this study, both optimization stages (simulated annealing and stochastic gradient descent) have been implemented in the Matlab language (Release 6.5.1) (Mathworks 2003). After some experimentation, the initial temperature parameter for the SA was set to 30. This value was high enough to avoid getting stuck to local minima and to allow the initial exploration of the solution space without generating excessive numbers of infeasible candidate states. The termination criterion used was a simple one, i.e., a maximum number of iterations K . The value used was 4,000 iterations, and it was very conservative, in the sense that in all executions, well before this number, the SA would not discover any significantly better solutions.

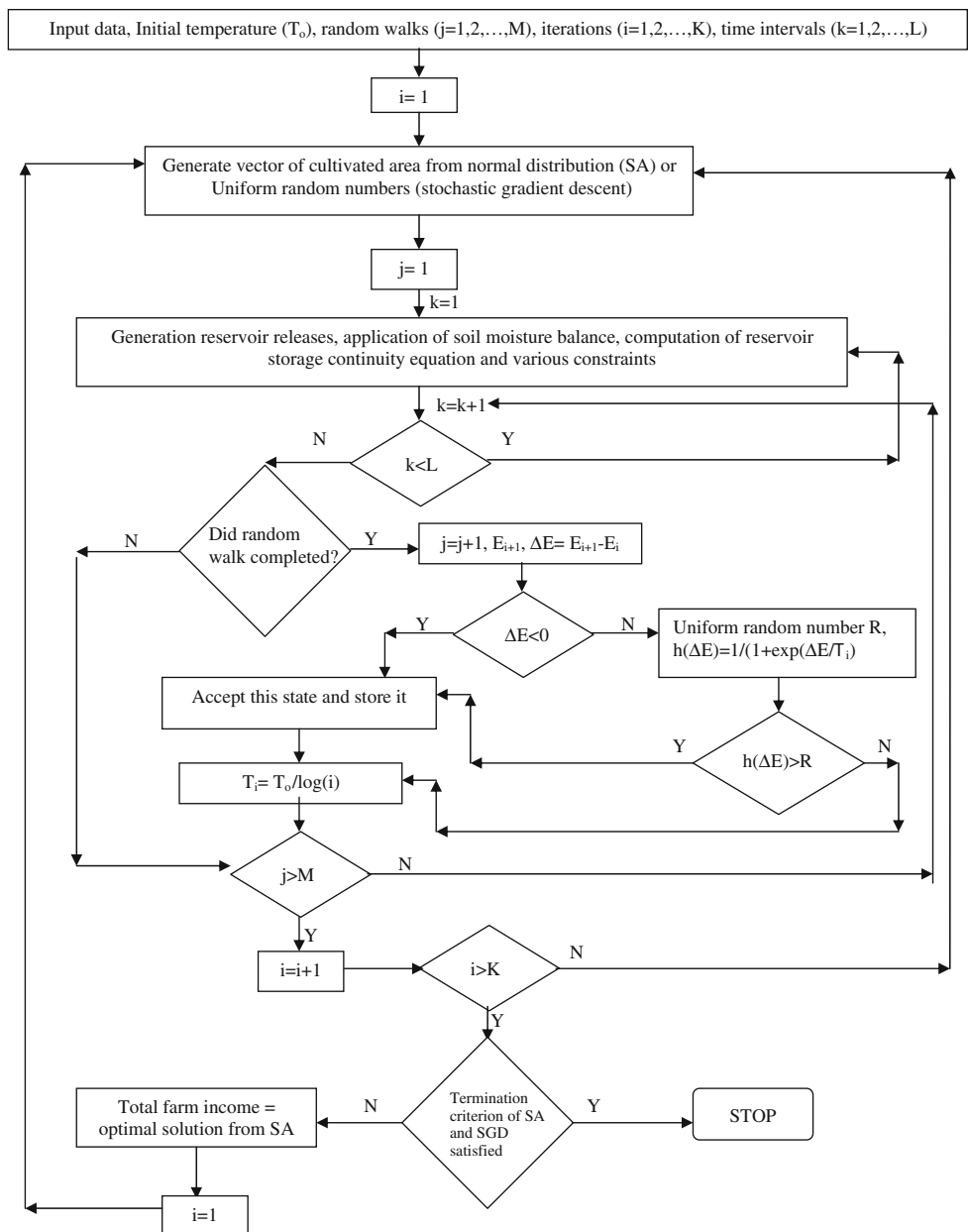
Because the optimization algorithm is stochastic, the computed optimal solution would differ slightly from execution to execution. The globally optimal solution was achieved after multiple executions of the optimization procedure for the same initial conditions and parameters. The term “globally optimal” refers to the best solution discovered among all executions.

Case study

Study region and data

The Chalkidiki region in Northern Greece was used as a case study for the optimization model. The main source of irrigation water in the region is from a planned reservoir on the Havrias river. Water availability in the reservoir at the start of irrigation season varies as a function of the winter rain and the inflow. The optimization model was used to compute the optimal cropping pattern and irrigation scheduling for six crops (corn, cotton, tomato, watermelon, olive and apricot). Determination of the optimal cropping pattern and reservoir releases to meet irrigation requirements for the region requires a good knowledge of the

Fig. 1 Flow chart of the proposed model which is solved using the simulated annealing (SA) in combination with the stochastic gradient descent algorithm



availability of the water in the reservoir during the irrigation season, meteorological conditions for the region and crops characteristics.

The main parameters required for the optimization procedure and some critical data for the indicated crops are given in Table 1 for the annual crops (corn, cotton, tomato and watermelon) and in Table 2 for olive and apricot. The yield response factors (k_y) were extracted from Doorenbos and Kassam (1979). The sensitivity indexes (λ) for Jensens model, for all growing stages were computed from a polynomial function (Georgiou 2004; Georgiou et al. 2006) by using the yield response factors (k_y) (Doorenbos and Kassam 1979) and for the time intervals by using Tsakiris (1982) procedure. Growth stage durations for different

crops were chosen based on local observations. In the cost calculations, it was assumed that farmers own the land, thus fixed cost (B) is equaled zero. The variable cost (C) for each crop was computed from data supplied by the Region of Central Macedonia, Greece (Region of Central Macedonia 2002). This variable cost (C) is independent of the quantity of irrigation water applied because currently this is the standard policy in Greek agriculture.

Tables 1 and 2 show that the irrigation season starts in April and ends in September. The year is divided into 36 periods with each month consisting of three periods. The first two periods in each month have ten days each, whereas the third period comprises the remaining days of month. For convenience, however, all the 36 periods in a year are

Table 1 Some characteristics of the annual crops under study in the Chalkidiki region

Growth stage	Parameter	Crop			
		Corn	Cotton	Tomato	Watermelon
Establishment	k_y^a	0.00	0.00	0.00	0.00
	λ	0.00	0.00	0.00	0.00
	d^b	25	15	25	15
Vegetative	k_y	0.40	0.20	0.40	0.45
	λ	0.3480	0.1664	0.3480	0.3944
	d	40	30	20	25
Flowering	k_y	1.50	0.50	1.10	0.80
	λ	1.8202	0.4415	1.1312	0.7501
	d	25	60	25	20
Yield formation	k_y	0.50	0.40	0.80	0.80
	λ	0.4415	0.3480	0.7501	0.7501
	d	45	30	25	30
Ripening	k_y	0.20	0.25	0.40	0.30
	λ	0.1664	0.2116	0.3480	0.2568
	d	15	20	20	20
Planting day ^b		21 April	21 April	26 April	21 April
Product price P (€ kg ⁻¹) ^c		0.14	0.79	0.32	0.13
Maximum yield Y_m (kg ha ⁻¹) ^c		14,500	3,900	66,600	80,000
Profit = $P \times Y_m$ (€ ha ⁻¹)		2,030	3,081	21,312	10,400
Variable cost C (€ ha ⁻¹) ^c		1,043	1,565	11,316	3,658
Net profit for maximum yield = $P \times Y_m - C$ (€ ha ⁻¹)		987	1,516	9,996	6,742

k_y Yield response factor, λ Sensitivity index of crop to water stress, d Growth stage (days)

^a Doorenbos and Kassam (1979)

^b Local observations

^c Region of Central Macedonia, Greece

Table 2 Some characteristics of the olive and apricot crops in the Chalkidiki region

Month of irrigation season	Parameter	Crop	
		Olive	Apricot
May	k_y^a	0.20	0.10
	λ	0.1664	0.0754
June	k_y	0.30	0.35
	λ	0.2568	0.3022
July	k_y	0.50	0.60
	λ	0.4415	0.5387
August	k_y	0.50	0.40
	λ	0.4415	0.3480
September	k_y	0.40	0.10
	λ	0.3480	0.0754
Product price P (€ kg ⁻¹) ^c		1.29	1.00
Maximum yield Y_m (kg ha ⁻¹) ^c		7,540	10,000
Profit = $P \times Y_m$ (€ ha ⁻¹)		9,727	10,000
Variable cost C (€ ha ⁻¹) ^c		3,724	4,706
Net profit for maximum yield = $P \times Y_m - C$ (€ ha ⁻¹)		6,003	5,294

k_y Yield response factor, λ Sensitivity index of crop to water stress

^a Doorenbos and Kassam (1979)

^c Region of Central Macedonia, Greece

referred to as ten-day period or time interval. The time interval is less or equal to the irrigation interval for the six crops and the first time interval of irrigation season corresponds to third ten-day period of April.

Although the proposed optimization model can handle heterogeneous soil, the considered soil under study was

homogenous. It consists of one layer characterized as clay loam (CL) with $FC = 0.27 \text{ cm}^3/\text{cm}^3$ and $PWP = 0.15 \text{ cm}^3/\text{cm}^3$. Due to higher rainfall during the non-irrigation season, it is assumed that soil water content at the beginning of the irrigation season is at FC and for this reason the values of k_y in the establishment stage are

assumed to be equal zero as also proposed by Tsakiris (1982) and Kotsopoulos (1989).

The reference crop evapotranspiration was derived from daily climatic data (mean temperature, radiation, relative humidity and wind speed) at Agios Mamas meteorological station (latitude 40°16'N longitude 23°20'E) by the FAO Penman–Monteith equation (Allen et al. 1998). Effective rainfall was computed from the procedure which is described by USDA (1993) where the rainfall in each time interval came from daily rainfall data. Monthly inflow was computed from a simple rainfall–runoff model which is described from Steenhuis and VanDerMolen (1986). The ten-day inflows derived by dividing the monthly inflows by three.

The mean efficiency for drip irrigation method and pressurized irrigation system is assumed to be equal 0.85. Some social economic, management and market considerations restrict the maximum and minimum areas cultivated with specific crops and the total available area.

The useful reservoir capacity is planned to be 26 hm³ and the dead capacity 4.3 hm³. Reservoir surface area evaporation is computed from de Bruin equation (de Bruin 1978) and the reservoir surface area $f(S)$ in Eq. (3) is computed by the relationship:

$$f(S) = 7.485 \times S^{0.7152} \quad (19)$$

where S = reservoir storage.

Generation of synthetic data

The available historical data of rainfall, evapotranspiration and inflow at the dam site cover a 21-year period (1977–1997). By applying the models which were described in “Stochastic generation” to the historical data, synthetic series of 50 years duration (operation life of reservoir) for rainfall, evapotranspiration and inflow were generated. Finally, 100 synthetic series of each variable were generated from which one mean synthetic series was resulted.

Figure 2 shows the comparison between the monthly means and the monthly standard deviations of the measured rainfall (1977–1997) with that as averages of 100 replicates of the generated rainfall time series (50 years each) of synthetic daily rainfall. It can be seen that the monthly means and the monthly standard deviations of the generated synthetic rainfall are close to that of the measured rainfall. This confirms the fact that the process in which the rainfall occurrence is based upon a first order two state Markov chain and rainfall amount is sampled from gamma distribution is suitable for generating synthetic rainfall.

Figure 3 shows the comparison between the monthly means and the monthly standard deviations of the computed reference evapotranspiration from the FAO Penman–

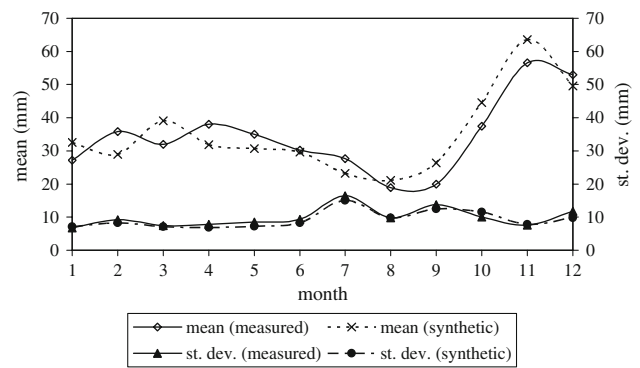


Fig. 2 Comparison between the monthly means and the monthly standard deviation of the measured rainfall (1977–1997) with that as averages of 100 replicates of the generated rainfall time series (50 years each)

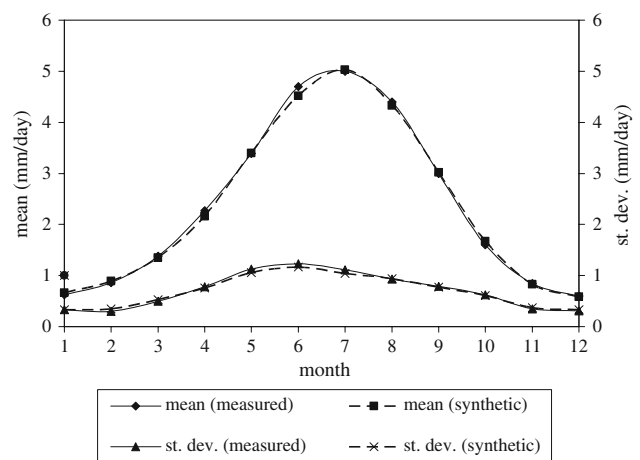


Fig. 3 Comparison between the monthly means and the monthly standard deviation of the computed reference evapotranspiration from the FAO Penman–Monteith equation (1977–1997) with that as averages of 100 replicates of the generated reference evapotranspiration time series (50 years each)

Monteith equation by using daily historical meteorological data (1977–1997) with that as averages of 100 replicates of the generated reference evapotranspiration time series (50 years each) by using generated synthetic daily mean temperature, radiation, relative humidity and wind speed. It can be seen that the monthly means and the monthly standard deviations of the generated synthetic evapotranspiration are close to that of the computed evapotranspiration. This confirms the fact that the process to generate meteorological parameters is suitable for generating synthetic reference evapotranspiration.

Figure 4 shows the comparison between the monthly means and the monthly standard deviations of the measured inflow (1977–1997) with that as averages of 100 replicates of the generated inflow time series (50 years each) of synthetic monthly inflow. It can be seen that the

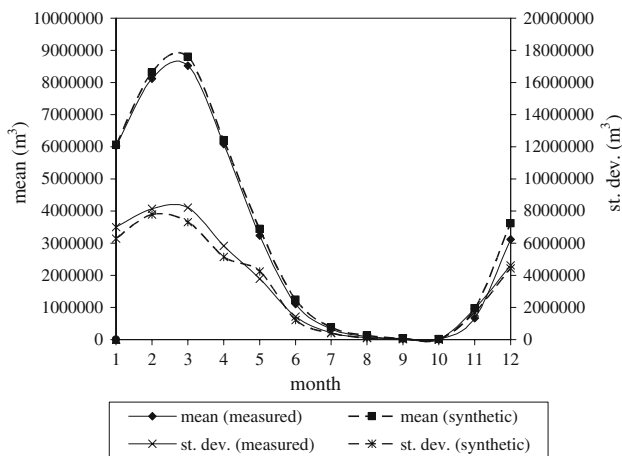


Fig. 4 Comparison between the monthly means and the monthly standard deviation of the measured inflow (1977–1997) at the reservoir with that as averages of 100 replicates of the generated inflow time series (50 years each)

monthly means and the monthly standard deviations of the generated synthetic inflow are close to that of the measured inflow. This confirms the fact that the selected ARMAX model is suitable for generating synthetic monthly inflow.

Generated values of rainfall, evapotranspiration and inflow are used to calculate frequency curves from which expected values corresponding to different probability of exceedence were received.

Since the objective of the irrigation scheduling is to give farmers guidelines for the adjustment of their irrigation calendars to the actual weather conditions, the development of the irrigation scheduling requires information on rainfall and evapotranspiration levels that can be expected with various probabilities. The probability of rainfall coincides with the probability of inflow. By combining the various probability levels of rainfall, evapotranspiration and inflow, four weather conditions are distinguished (Raes et al. 2000); hot and dry, dry, normal and wet weather conditions with the probability levels of exceedence of rainfall, evapotranspiration and inflow and are given in Table 3.

By taking to account the above probabilities, with the help of frequency curves the ten day rainfall and

evapotranspiration levels are shown in Fig. 5 and the ten day inflow levels are shown in Fig. 6.

Results and discussion

The presented optimization model (with areas and reservoir releases as decision variables) was used to compute the optimal cropping pattern and irrigation scheduling for the six crops (corn, cotton, tomato, watermelon, olive and apricot) with data from all weather conditions. The time interval used was equal to 10 days for all six crops.

For the hot and dry weather condition no feasible solution existed. The reason was that the initial reservoir storage and the inflow during the irrigation season were not enough to irrigate the crops given the imposed constraints of minimum desired area (A_{\min}) and minimum desired relative yield (Y_a/Y_m). In this study social economic constraints required a minimum production and the relative yield (Y_a/Y_m) for all crops was taken to be equal 0.70.

The optimization model was run for expected values of rainfall, evapotranspiration and inflow corresponding to different probability of exceedence (dry, normal and wet weather condition) to allocate cropped area and irrigation water to crops. The relative yield for the three weather conditions are given in Table 4 and the allocation of irrigation water from reservoir and cropped area are given in Table 5.

The allocation of irrigation water and the cropped area for each crop depend upon factors such as net profit per unit yield (price), maximum yield obtainable per unit area, profit, cost, net profit for maximum yield (Tables 1, 2), water application needed for getting the maximum yield and the total irrigation water. The net profit per unit yield (price) of cotton is very high compared to all other annual crops and the maximum yield per unit area of cropped is less than other annual crops. The net profit for maximum yield of Olive, Watermelon and Tomato is very high compared to all other crops and the Corn and Cotton is very low. According to these results the relative yields are near maximum (Table 4) for Olive, Watermelon and Tomato and the areas are greater than other crops (Table 5). Also, Corn and Cotton are allocated with the minimum relative yield and minimum areas (Tables 4, 5).

Reducing total irrigation water would impose water stress on crops. In this case, there is not only competition for water through the time intervals, but there is also competition among the crops in any time interval resulting at different relative yields (Table 4).

As the risk factor increase, the expected value of total area increase and thus the total allocation of water from reservoir also increase. At normal weather condition, the total cropped area is 7,367 ha and the corresponding water

Table 3 The four distinguished weather conditions

Weather condition	Probability level of exceedence		
	Evapotranspiration (%)	Rainfall (%)	Inflow (%)
Hot and dry	20	100 (no rain)	100
Dry	40	80	80
Normal	50	50	50
Wet	60	20	20

Fig. 5 Ten day rainfall and evapotranspiration levels with various probability levels (%) of exceedence

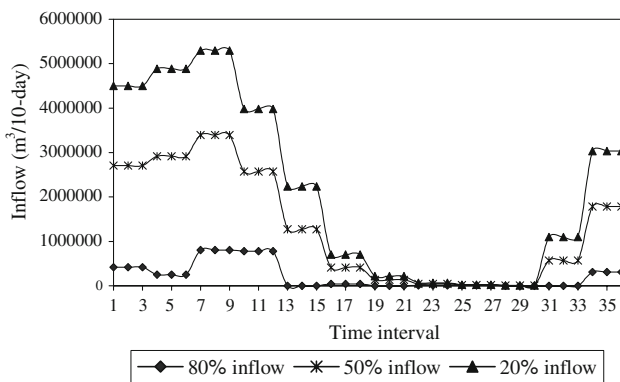
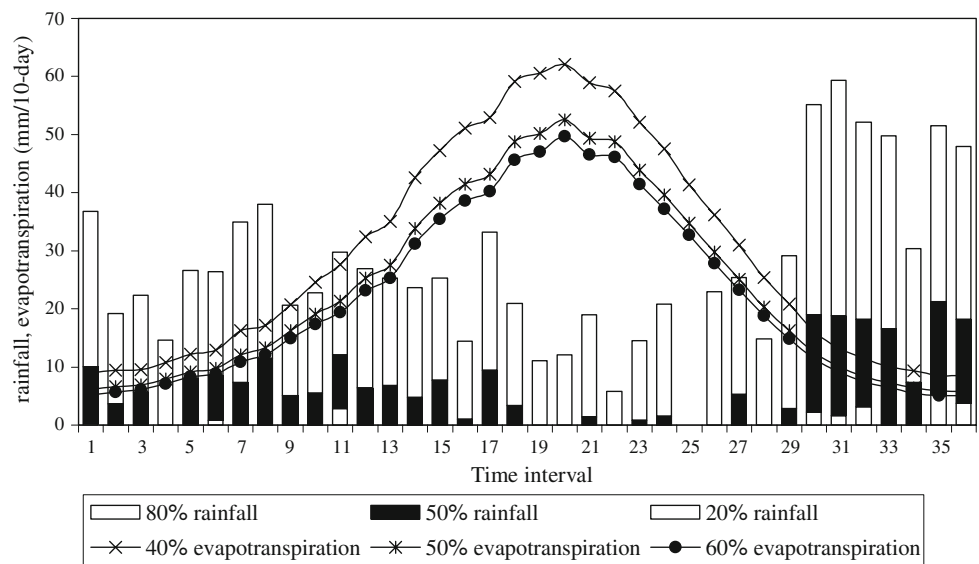


Fig. 6 Ten-day inflow levels with various probability levels (%) of exceedence

Table 4 Relative yield (Y_a/Y_m) of crops under study in the Chalkidiki region for various weather conditions

Crop	Relative yield (Y_a/Y_m)		
	Dry	Normal	Wet
Corn	0.71	0.71	0.70
Cotton	0.71	0.72	0.70
Olive	0.93	0.99	0.99
Tomato	0.99	1.00	0.99
Watermelon	0.93	0.95	0.88
Apricot	0.84	0.86	0.91

from reservoir is 26.13 hm³. At wet weather condition, the total cropped area is 12,526 ha and the corresponding water from reservoir is 27.24 hm³. The difference in the total cropped area is 70% and in the corresponding water from reservoir is 4.3%. This is owed in the fact that an important part of crop water requirement is covered by the

effective rainfall. This also it appears also in the Cotton where while the cropped area is same in the three cases of weather conditions (Table 5) the water from reservoir is decreased as long as we go from dry to the wet weather condition.

In Fig. 7 as an example are shown the input data [effective rainfall (ERAIN) and maximum evapotranspiration (ET_m)] and the outputs from developed optimization model [reservoir release to meet irrigation requirements (R) and actual evapotranspiration (ET_a)] for Cotton, Tomato and Apricot crops for each time interval during the normal and wet weather condition.

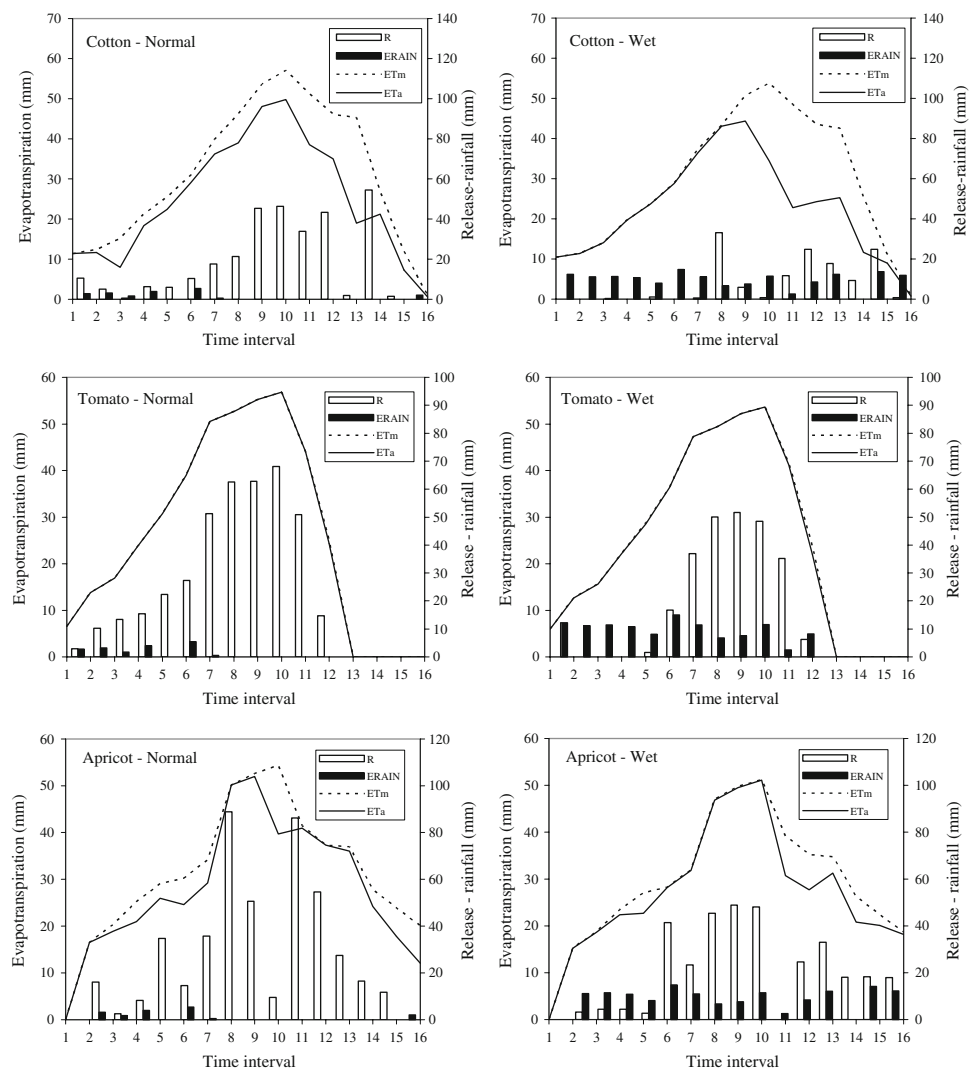
Figure 7 clearly demonstrates that the preference of the optimization model would be more pronounced with conditions of reduced full irrigation water requirements (deficit irrigation). Under these conditions the actual evapotranspiration ET_a fell below the maximum evapotranspiration ET_m . This was due to insufficient water being applied to meet ET_m for producing a full yield (Y_m). Temporal distributions of ET_a as compared with ET_m are presented for the three crops and two weather conditions in Fig. 7. As water reduction increased, ET_a decreased and fell below the ET_m values. But these declines were not the same at different time intervals, due to unequal yield sensitivity indexes (λ , Tables 1, 2). Note that for the time intervals in which the effective rainfall is non-zero, the corresponding release is adequately small or even null, as expected (Fig. 7).

Because the soil moisture is assumed to be at the field capacity (FC) of the first time interval, there will not normally be any irrigation requirement during this time interval, whereas irrigation may be required in subsequent time intervals, as shown in Fig. 7. The amount of the releases at each time interval depends on the level of soil

Table 5 Allocation of water from reservoir and cultivation area to crops for various weather conditions

Crop	Dry		Normal		Wet	
	Water (hm ³)	Area (ha)	Water (hm ³)	Area (ha)	Water (hm ³)	Area (ha)
Corn	0.167	50	0.159	50	0.151	90
Cotton	0.144	50	0.129	50	0.056	50
Olive	0.196	50	20.211	5,492	23.769	10,611
Tomato	3.305	879	3.882	1,135	2.384	1,135
Watermelon	0.151	50	1.552	590	0.738	590
Apricot	0.200	50	0.195	50	0.142	50
TOTAL	4.163	1,129	26.128	7,367	27.240	12,526

Fig. 7 Effective rainfall (ERAIN), maximum evapotranspiration (ET_m), reservoir release to meet irrigation requirements (*R*), and actual evapotranspiration (ET_a) for the normal and wet weather conditions for the three crops (cotton, tomato and apricot) under study



moisture in the root zone as determined by the soil moisture balance, the irrigation requirements and the availability of reservoir storage (*S*).

Therefore, the objective function (Eq. 1) dictated the true water allocation to the model. A complex mixture of crop sensitivity to water reduction, area of cropped, and

crop gross benefit and cost caused a non-uniform effect of water shortage on crop yield, as is governed by such an integrated optimization model.

Given the irrigation water allocated and the irrigation time interval, the irrigation scheduling can subsequently be derived by plotting the root zone depletion along the time

axis for any crop in any weather condition with the help of soil moisture balance model. In Fig. 8, as an example, is shown the simulated root zone soil moisture at the end of each time interval in normal weather condition for the tomato and corn. To avoid crop water stress, the root zone depletion should not exceed the threshold value for no stress (lower limit). If so, the depletion will be larger than RAW ($RAW = p \times TAW$) which referred as threshold in Fig. 8 and the crop will experience water stress. The resulting yield decrease depends on the severity of the stress and the sensitivity of the crop at the particular time interval. Figure 8 shows that in the crop of Tomato with relative yield equals to 1 (Table 4) the soil moisture depletion in the root zone at the end of each time interval is always above the threshold for no water stress. On the other hand, to avoid water losses, the soil water content in the root zone after an irrigation event should not exceed field capacity (upper limit).

The problem in which the only decision variables are the cropped areas is very simple as mentioned in “Model formulation” and may be solved by LINGO (Lindo Systems 2003). In this problem, the releases are forced to be equal to the full irrigation water requirements over the entire crop period and the relative yield is fixed to unity. This problem corresponds to full supply irrigation strategy and the reservoir releases are not variables but input data. The total farm income and total cropped areas for the full irrigation were computed by the LINGO for all weather conditions and are given in Table 6. Also, for the hot and dry weather condition no feasible solution existed. The reason was that the initial reservoir storage and the inflow during the irrigation season were not enough to irrigate the crops given the imposed constraints of minimum desired area (A_{min}) and maximum release from reservoir to meet irrigation requirements (R_{max}).

The initial problem, with areas and reservoir releases as decision variables, corresponds to deficit irrigation, because the relative yield of some crops is less than one. It is interesting to compare the results for the two approaches, namely, the full irrigation and the deficit irrigation. Table 6 indicates that the total farm income and the total cropped area in deficit irrigation are greater than those in full irrigation for all weather conditions. The differences in the total farm income vary from 2.2 to 8.2% and in the total cropped area vary from 7.8 to 10.7% (dry weather condition). With given reservoir conditions (reservoir storage in the beginning of the irrigation season and inflow during this period) the total farm income can be increased by extending the total cropped area, resulting to deficit irrigation for some crops compared to full irrigation.

For the investigation of effect of reservoir storage in the beginning of the irrigation season to the total farm income and total cropped area, for all weather conditions, the

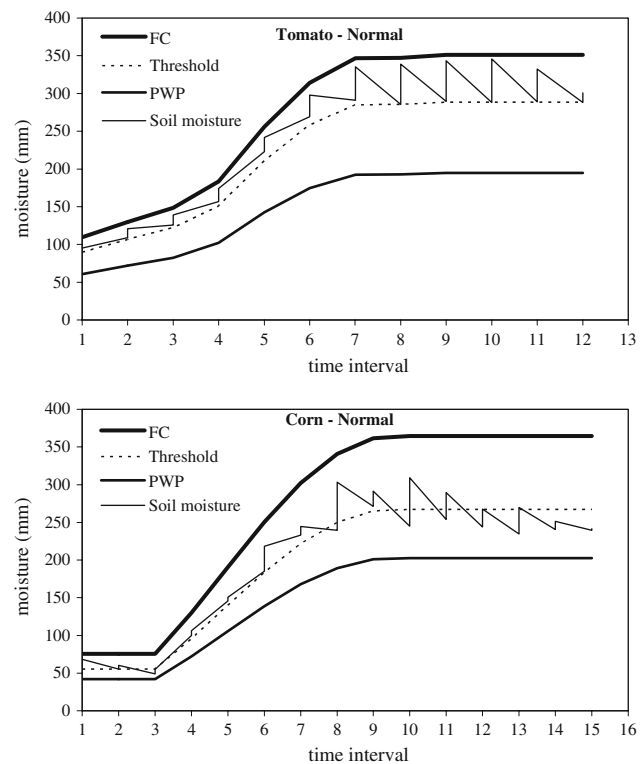


Fig. 8 Simulated root zone soil moisture at the end of each time interval in normal weather condition for the tomato and corn

initial problem with areas and reservoir releases as decision variables was applied (deficit irrigation) as well as the problem in which the only decision variables are the cropped areas (full irrigation). The available reservoir storage in the beginning of the irrigation season varied from 4.3 to 25.6 (maximum capacity) million m^3 . The total farm income and total cropped areas for the full and deficit irrigation for various reservoir storage in the beginning of the irrigation season are given in Table 7.

With a given reservoir storage, it is possible to increase the total farm income by extending the cropped area resulting to deficit irrigation in some crops compared to full supply to all crops (Table 7). As an example an increase of 1,047,610 € (i.e. 44,306,000–43,258,390) can be achieved by extending the area by 277 ha (i.e. 6,786–6,509) if 25.6 million m^3 of initial reservoir storage for normal weather condition is available. Also, in certain cases where no feasible solution existed concerning the problem of full irrigation, the initial problem (deficit irrigation) gave feasible solution.

Conclusions

This paper distinguishes two approaches to the optimal irrigation reservoir operation for water allocation and crop planning for the various crops in any given irrigation

Table 6 Total farm income and total cropped area for various weather conditions for both full and deficit irrigation

Weather condition	Total farm income (€)		Total cropped area (ha)	
	Deficit irrigation	Full irrigation	Deficit irrigation	Full irrigation
Dry	9,443,000	8,730,000	1,129	1,020
Normal	47,473,000	46,470,000	7,367	6,508
Wet	76,942,000	74,180,000	12,526	11,614

Table 7 Total farm income and total cropped area for various weather conditions and various initial reservoir storages for both full and deficit irrigation

Initial reservoir storage (m ³)	Total farm income (€)		Total cropped area (ha)	
	Deficit irrigation	Full irrigation	Deficit irrigation	Full irrigation
<i>Hot and dry</i>				
4,300,000	–	–	–	–
6,100,000	1,425,000	–	300	–
6,500,000	1,732,000	1,629,183	326	300
10,000,000	9,316,200	8,389,120	1,129	986
12,300,000	13,375,000	12,697,880	1,626	1,433
18,300,000	20,128,000	19,677,220	2,768	2,535
25,600,000	27,656,000	27,234,830	4,017	3,794
<i>Dry</i>				
4,300,000	–	–	–	–
5,100,000	1,317,000	–	300	–
5,300,000	1,620,000	1,560,474	341	303
6,300,000	4,238,300	3,647,606	609	512
12,300,000	15,899,000	15,223,970	1,925	1,808
18,300,000	22,940,000	22,473,100	3,151	3,001
25,600,000	31,672,000	31,072,040	4,683	4,434
<i>Normal</i>				
4,300,000	16,359,000	15,625,560	1,997	1,867
6,300,000	18,933,000	18,328,220	2,520	2,310
12,300,000	26,752,000	26,146,590	3,777	3,613
18,300,000	34,664,000	33,981,280	5,148	4,918
25,600,000	44,306,000	43,258,390	6,786	6,509
<i>Wet</i>				
4,300,000	34,958,000	33,602,110	5,012	4,855
6,300,000	39,457,000	37,878,820	6,057	5,567
12,300,000	52,408,000	50,729,100	8,163	7,708
18,300,000	65,324,000	63,600,900	10,288	9,852
25,600,000	76,762,000	74,180,240	12,265	11,615

season for known initial storage, inflow and initial soil moisture in the cropped area. The impact of water deficit on crop yield, the effect of soil moisture dynamics on crop water requirements and competition for water among the crops in an irrigation season are taken into account. In the first approach a multi-crop irrigation model for a single reservoir operating under water scarcity constraints (deficit irrigation) has been optimized, using the simulated annealing (SA) global optimization stochastic search algorithm in combination with the stochastic gradient

descent algorithm. In the second approach a multicrop irrigation model for a single reservoir, operating under full irrigation conditions, has been optimized using LINGO with only decision variables the cropped areas. The rainfall, evapotranspiration and inflow are considered to be stochastic and the model is run for expected values of the above parameters corresponding to different probability of exceedence. By combining the various probability levels of rainfall, evapotranspiration and inflow, four weather conditions are distinguished (hot and dry, dry, normal and

wet). The models were then applied to Chalkidiki region in Northern Greece where the source of irrigation water was the water from a planned reservoir on the Havrias river. The two approaches applied in this study under the four weather conditions give satisfactory results for deficit and full irrigation. A significant increase in a cropped area and total farm income from the irrigation system is shown to be possible when the deficit irrigation approach is adopted. In certain cases of the hot and dry weather conditions where no feasible solution existed concerning the problem of full irrigation, the initial problem (deficit irrigation) gave a feasible solution. The net profit for maximum yield of Olive, Watermelon and Tomato is very high compared to all other crops and the Corn and Cotton is very low. This resulted to relative yields being near maximum for Olive, Watermelon and Tomato and the areas being greater than other crops. Also, Corn and Cotton are allocated with the minimum relative yield and minimum areas. The results obtained from this case study suggest that the proposed optimization approach is general and can be adopted as a tool for globally optimal water allocation and crop planning of any irrigation reservoir, under various constraints and weather conditions.

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