Fracture void structure: implications for flow, transport and deformation

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Abstract This review focuses on studies of flow, transport and deformation processes at a scale of a single discontinuity. The paper provides an evaluation of: (1) various methods suggested for geometrical characterization of void structure; and (2) theoretical and practical problems arising from significant differences between the actual geometry of fracture void structure and its parallel plate representation. The use of an equivalent aperture concept is shown to be seriously misleading in: (a) evaluation of flow regime, and hence selection of appropriate flow laws; (b) correlating tracer and hydraulic tests, and assessment of solute transport properties; and (c) relating hydraulic and mechanical apertures, and predicting influence of stress perturbation and deformability.

Keywords Fracture · Flow · Transport · Deformation

Introduction

Flow, transport and deformation within rock masses are determined primarily by their network and block structures as discontinua and the hydraulic and mechanical properties of their structural elements, discontinuities. Joints, as the most ubiquitous type of discontinuity, usually form pervasive sets, each having a narrow range of relevant attributes. While this makes basic visualization of rock structure geometry and estimations of discontinuity properties a practical endeavor, the limited nature of the characterization parameters that can be measured with some accuracy and in statistically meaningful quantities introduces uncertainty in the conceptual media models and hence in their response during process simulations.

The distribution, shape and connectivity of voids/channels and contact areas, and the mismatch (or matedness) and planarity of mutual surfaces are essential parameters characterizing the void structure of a single fracture which controls its flow, transport and deformation properties. A present void structure of a fracture is initially determined by its mode of formation but is generally modified during subsequent hydrothermal, tectonic and weathering processes. Therefore methods to analyze inherited geological evolution of fracture sets, and to characterize geometry and behavior of individual fractures from such sets is the key to understanding flow, transport and deformation at rock-mass scales.

Earlier flow studies (Baker 1955; Huitt 1956; Parrish 1963; Louis 1974; Rissler 1978; Murphy 1979) idealized the void structure of a fracture as a planar opening between two rough parallel-plates (Fig. 1). These studies employed an

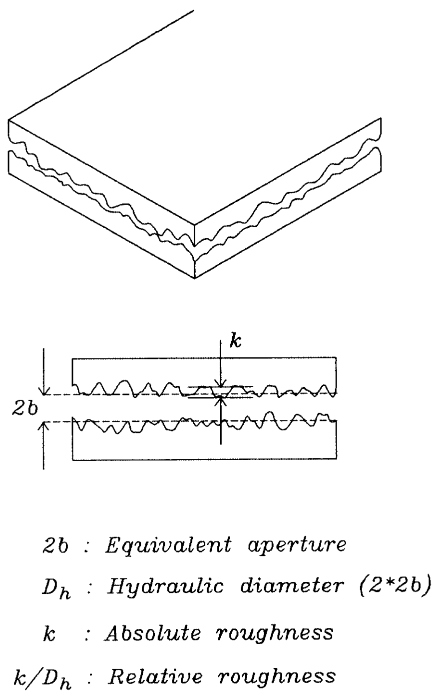

Fig. 1
Geometrical parameters characterizing the hydraulics of rough parallel plate conduits

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FLOW REGIME	LAMINAR		
RELATIVE ROUGHNESS	$k/Dh \leqq 0.033$		$k/Dh > 0.033$
HYDRAULIC BEHAVIOR	SMOOTH		
FRICTION FACTOR (λ)	$\frac{96}{Re}f$ (Poiseuille)		$\frac{96}{Re}f$ (Louis L)
CONDUCTIVITY MODIFICATION FACTOR	$f=1$		$f=1+8.8(k/Dh)^{1.5}$
FLOW REGIME	TURBULENT		
RELATIVE ROUGHNESS	$k/Dh \leqq 0.033$		$k/Dh > 0.033$
HYDRAULIC BEHAVIOR	SMOOTH	COMPLETELY ROUGH	
FRICTION FACTOR (λ)	$0.316\ Re^{-0.25}$ (Blasius)	$\left[2\log\frac{3.7}{(k/Dh)}\right]^{-2}$ (Nikuradse)	$\left[2\log\frac{1.9}{(k/Dh)}\right]^{-2}$ (Louis T)

Fig. 2
Friction and conductivity modification factors governing fracture flow in the domains delineated in Fig. 3. (After Louis 1974)

analogy with pipe flow (through hydraulic radius and friction factor concepts) to represent wall roughness and to formulate turbulent flow in fractures. Parallel-plate conceptualization, thus, reduced the laminar fracture flow problem to one of finding a modification factor, f (Fig. 2) to account for the observed deviations from the theoretical predictions using the so-called cubic law (Schlichting 1979):

$$q = \frac{1}{f}\frac{g(2b)^3}{12\nu}\frac{\partial h}{\partial x} \tag{1}$$

where q is the flow rate per unit width of the fracture, $2b$ the equivalent parallel-plate opening of the fracture, ν the kinematic viscosity of the fluid. Recent studies revealed that void structure of fractures under various stress conditions existing in-situ is not adequately described by a rough parallel-plate model.
This review focuses on studies of flow, transport and deformation processes at a scale of a single discontinuity. The paper provides an evaluation of: (1) various methods suggested for geometrical characterization of void structure; and (2) theoretical and practical problems arising from significant differences between the actual geometry of fracture void structure and its parallel plate representation.

Fracture void structure

The void structure in a single fracture is composed of branching flow routes with dead-end pockets and immobile zones isolated by contact areas as suggested by observations from transparent replicas (Sharp and Maini 1972; Hakami and Barton 1990), metal casts (Pyrak-Nolte and others 1987), aperture distribution maps generated using fractal (Thompson and Brown 1991) and statistical (Tsang and Tsang 1990) surface descriptions and video images (Gentier and others 1989; Voss and Shotwell 1990). The physical inconsistency between a parallel plate opening and effective void structure of a single fracture is manifested in differences between average aperture values derived from a deterministic evaluation of hydraulic test data using the cubic law (i.e. hydraulic aperture), and (a) from direct measurements, such as normal closure (Witherspoon and others 1980; Raven and Gale 1985; Pyrak-Nolte and others 1987), fracture volume (Schrauf and Evans 1986; Piggott and Elsworth 1990), resin thickness profiles (Gale 1990), area covered by water drops of known volume (Hakami and Barton 1990), or (b) from numerical simulations (Tsang and Tsang 1990; Thompson and Brown 1991), laboratory (Gale 1990) and field (Raven and others 1988) tracer experiments. All these approaches confirm that the equivalent aperture has no simple relationship to the actual void structure and is instead a lumped parameter like the permeability itself.
Fracture flow model studies demonstrate that the equivalent aperture obtained from the cubic law for a given flow rate and boundary pressure is not uniquely defined at the same average wall roughness and reference distance between the walls. This is shown to be due to unaccounted variations in phase difference (i.e. matedness of the wall roughness) (Sato and others 1984; Cornwell and Murphy 1985; Voss and Shotwell 1990), the spread of aperture distribution (Sharp and Maini 1972; Tsang 1984), the distribution of contact areas (Iwai 1976; Sundaram and Frink 1983) and large-scale undulations (i.e. planar angularity) (Iwai 1976; Tsang and Witherspoon 1982), all of which contribute to flow resistance to various extents. As a result, the equivalent aperture determined from the cubic law modified by a friction factor (Fig. 2), too, implicitly reflects a combined influence of the features defining the degree and pattern of tortuosity. Theoretical

studies attempting to account for the tortuosity generally idealize the void space as local equivalent apertures varying according to a statistical distribution function along the flow section (Neuzil and Tracy 1981) or in the plane (Gelhar 1987; Tsang and Tsang 1990).
Inefficiency of the parallel plate representation of fracture void space limits: (1) the applicability of the (modified) cubic law to lower flow rates; and (2) the direct use of the equivalent aperture, in two important practical problems, namely predicting the solute transport and hydromechanical performance of fractures. The following is a brief account of these implications and some suggestions for the use of the equivalent aperture in these problems.

Lower limit of transitional flow regime in fractures

Results of model experiments show that as relative roughness (Huitt 1956; Parrish 1963) and/or matedness (Cornwell and Murphy 1985) increases, the transitional flow regime is initiated at lower values of and over a wider range of Reynolds numbers. Similarly, the larger the spread of the aperture distribution and/or the planar angularity, the earlier and longer the transitional regime (Sharp and Maini 1972). During experiments with various rock fractures, the linear relationship suggested by the cubic law is observed to reasonably hold only for Reynolds numbers less than 100, given that the equivalent aperture is larger than 0.02 mm (Iwai 1976). Increasing the relative roughness from 0.25 to 1 produces a dramatic decrease in the critical Reynolds number from 2000 to less than 20 (Parrish 1963), approaching that of porous media (Bear 1972). According to Muralidhar and Long (1987), laminar range in fracture hydraulics studies is limited to $1 \leq \mathrm{Re} \leq 25$.
The critical Reynolds number, below which flow disturbances due to tortuosity of effective void structure are damped by viscous effects, is much lower than predicted by open parallel plate experiments shown in Fig. 3. Depending on the degree and pattern of tortuosity with respect to the overall gradient vector, the selection of the linear flow law and friction factors may be highly erroneous. Consequently, in the transition zone, the equivalent aperture calculated using a formula, which assumes linear flow regime, carries an additional uncertainty.

Equivalent aperture in solute transport predictions

The equivalent aperture (i.e. aperture of an open, smooth parallel plate conduit) is a generic term that refers to aperture estimates obtained from tracer as well as hydraulic tests under laminar flow conditions (Tsang 1992). The concept of equivalent aperture in the evaluation of hydraulic test data is evaluated in the previous section. It is also important to understand how well equivalent apertures derived from tracer test data reflect the influence of tortuosity and how much they deviate from the hydraulic aperture. Tracer tests yield information on the solute transport properties of a medium, and are conducted by introducing one of a variety of tracers into the medium under a certain head gradient and recording the arrival time of sampled concentrations. Different mechanisms (mainly advection, dispersion and absorption) are known to be simultaneously operating in the solute transport process and the interpretation of the data is inherently difficult.
The methods currently used to evaluate data from single fracture tracer tests produce (Smith and others 1987): (a) the volume balance aperture

$$(2b)_v = \frac{Qt_m}{A_p} \tag{2}$$

where t_m is the mean residence time of tracer transport obtained from tracer breakthrough curves, and A_p is the

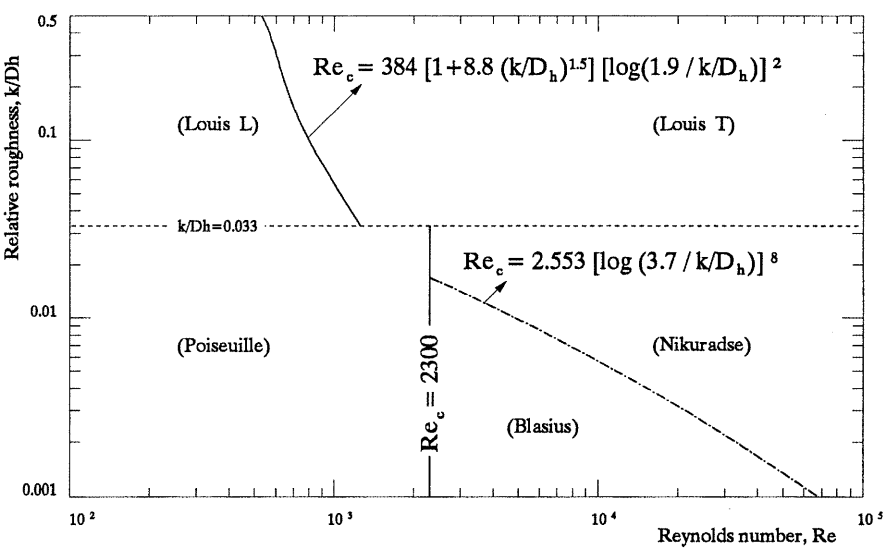

Fig. 3
Flow domains in open, rough fractures delineated by relative roughness (Fig. 1) vs. critical Reynolds number determined from friction factors (Fig. 2)

planar area of the flow domain; and (b) the tracer aperture, which for one-dimensional flow,

$$(2b)_t = L\left[\frac{12\nu}{g|\Delta h|t_m}\right]^{1/2} \tag{3}$$

where t_m is calculated from

$$t_m = \int_{l_1}^{l_2} \frac{1}{v\bar{n}}\,dl \tag{4}$$

The order of magnitude between the volume balance, hydraulic and tracer apertures is given as (Tsang 1992):

$$(2b)_v \geq (2b)_h \geq (2b)_t \tag{5}$$

Efforts to derive aperture distribution parameters from single fracture tracer tests (Gelhar 1987; Tsang and Tsang 1990) are aimed at characterizing a statistically equivalent medium in which solute transport can be studied at different spatial and temporal scales, and eventually in a network of fractures. The aperture distribution function is generally assumed to be logarithmic as suggested by direct field measurements (Bianchi and Snow 1968) and injection test results (Snow 1970).

Smith and others (1987) studied the sensitivity of all aperture estimates to perturbations in a radial flow field in a fracture with the logarithmic aperture distribution via simulated steady state pumping tests. The well bore was assumed to intersect a single horizontal fracture connected to different synthetic networks of smooth/rough, finite and intersecting parallel plate fractures. Their study revealed that the hydraulic aperture is: (1) least sensitive to the network geometry (e.g. the location of intersections of effective fractures and their areal extent); and (2) most sensitive to the aperture distribution around the well bore because of logarithmic change in head. Therefore, although very localized, hydraulic determinations relate better to the properties of the tested fracture than do tracer estimates.

A useful relation between the hydraulic and volume balance apertures of a fracture in which distribution of local apertures is logarithmic and isotropic is derived from the stochastic theory (Gelhar 1987)

$$\frac{(2b)_h}{(2b)_v} = e^{\sigma^2/2} \tag{6}$$

where σ^2 is the mean variance of the natural logarithm of the local apertures. This substantiates the possibility of using the magnitude of deviation of the hydraulic aperture from the volume balance or tracer apertures as an index of the tortuosity in the tested domain.

Hydromechanical coupling and equivalent aperture

In the design of civil engineering, resource exploitation and waste disposal projects, it is essential to consider future perturbations in the effective stress field and to predict consequent alterations in rock-mass conductivity (determined prior to and/or during the excavation, construction, withdrawal or burial) for a dependable assessment of their long-term stability and environmental impact (Sharp and Maini 1972; Louis 1974; Bandis and others 1985; Makurat 1985). The efforts to discover changes in the equivalent aperture as a function of various deformation modes (i.e. opening, normal closure and/or shear) stem from this need.

Opening of fractures can be a concern where high injection pressures are involved such as in dam foundations and water pressure tests. The net opening in zones of multiple parallel fractures are controlled by the compressibility of the intact rock blocks (Snow 1969). As reported by Carlsson and Olsson (1983), a noticeable opening may however take place when the overburden pressure is exceeded during a test.

Normal stress vs. closure curves of natural mated and unmated fractures are hyperbolic and semilogarithmic, respectively (Bandis and others 1983). As a result of a faster rate of increase in tortuosity, and hence in flow resistance, as suggested by mechanical measurements of normal closure, variations in the equivalent aperture at each incremental stress level cannot be directly predicted from these measurements (Iwai 1976; Witherspoon and others 1980). This is attributed to increasing contact area by the deformation of asperities and void space, specifically of the smallest constriction on the main flow path (Pyrak-Nolte and others 1988).

The equivalent aperture of artificial tension fractures at any stress level can be related to normal closure by (Witherspoon and others 1980)

$$(2b)_i = (2b)_d + (2b)_r \tag{7}$$

where $(2b)_d$ is the total closure from ith level and $(2b)_r$ is the residual aperture calculated at maximum stress from the cubic law. The equivalent aperture predicted by this method is a little different from that directly calculated from the cubic law and is therefore modified as (Witherspoon and others 1980)

$$(2b)^3 = \frac{(2b)_i^3}{f} \tag{8}$$

where f is an empirical modification factor which operates in the same manner as the friction factor in Eq. (1). The slight deviation probably arises from clogging/filling by crushed material and therefore larger deviations might be expected for natural fractures.

From a large-scale hydromechanical test using a natural rock fracture, Sundaram and others (1987) suggested that flow rate vs. mechanical closure (0–20 MPa) data can be fitted to the cubic law using a single modification constant $f=1.778$ if the initial aperture is taken as a reference. This constant can be compared to $f=1.49$–1.29 of Witherspoon and others (1980) for a similar rock type and the radial flow. Again, in natural fractures, Raven and Gale (1985) and Pyrak-Nolte and others (1987), with-

out applying any modification factor, observed that considerably larger apertures are predicted at high stresses (0–30 and 0–85 MPa), which, in turn, produce much lower exponents than cubic.
Contrary to these results, Schrauf and Evans (1986), from hydromechanical gas flow tests (0–20 MPa) in a natural fracture, indicated that: (1) the arithmetic average aperture (from fracture volume measurements) reduced identical to the fracture closure (from deformation measurements); and (2) the absolute wall roughness was nearly constant. These findings support theoretical models assuming elastic deformation, probably because a majority of contact areas in the tested fracture consists of touching peaks of undulations of void scale. In such fractures: (a) the empirical relationships based upon surface roughness alone (Louis 1974; Bandis and others 1985) may be sufficient to account for the combined effects of wall roughness, undulation and contact area; and (b) shear leads to more important conductivity changes (Schrauf and Evans 1986). Shear stress vs. displacement curves in pre-peak ranges are hyperbolic (Bandis and others 1983). However, conductivity of a natural rough fracture can increase 2 to 3 orders of magnitude under shear displacement of only 1 mm (Makurat 1985) indicating pre-peak dilation behavior. Coupled shear-conductivity tests substantiate gradual blockage of flow paths due to gouge production in the post-peak region (Bandis and others 1985).
A unified empirical hydromechanical model that predicts the altered equivalent aperture from a knowledge of shear/normal stress dependent variation in the mechanical aperture (i.e. shear dilation and/or normal closure) is described in Bandis and others (1985). An initial value of the equivalent aperture is used to estimate the corresponding mechanical aperture, which, in turn, is modified to predict the new equivalent aperture from

$$(2b) = \frac{(2b)_{\mathrm{m}}}{JRC^{2.5}} \tag{9}$$

where $(2b)_m$ is the mechanical aperture and JRC is the joint roughness coefficient determined from direct shear tests (Bandis and others 1985).
At great depths fracture conductivity should be nominal according to laboratory results and yet many fractured reservoirs efficiently produce at such depths (Dyke 1992). This may be mainly because of: (1) partial mineralization in open fracture planes which, while increasing contact area and stiffness, maintains sufficient large passages for flow; (2) large-scale undulations of non-mating fracture walls provide stable large voids; and (3) an arching action at rock-mass scale may reduce load transfer onto appropriate strata during effective stress changes. These points were explored by Dyke (1992) who indicated that concerns for the effects of hydromechanical coupling upon fluid withdrawal have not been justified in petroleum reservoirs, most of which sustained their production rates despite large amounts of effective stress reduction. This view is also supported by the observations that normal closure is controlled by small-scale roughness (Bandis and others 1983) while conductivity is mainly dependent on large-scale undulations (Tsang and Witherspoon 1981). Another important point is that most laboratory measurements reflect the behavior of the relaxed fractures under large stress increments, e.g. 0 to 30 MPa (Raven and Gale 1985).

Summary

The distribution, shape and connectivity of voids/channels and contact areas, and the mismatch (or matedness) and planarity of mutual surfaces are essential parameters characterizing the void structure of a single fracture which controls its flow, transport and deformation properties. Inefficiency of the parallel plate representation of fracture void structure limits: (1) the applicability of the (modified) cubic law to lower flow rates; and (2) the direct use of the equivalent aperture, in two important practical problems, namely predicting the solute transport and hydromechanical performance of fractures. A brief account of these implications and some suggestions for the use of the equivalent aperture in these problems are discussed above. The use of equivalent aperture concept is shown to be seriously misleading in: (a) evaluation of flow regime, and hence selection of appropriate flow laws; (b) correlating tracer and hydraulic tests, and assessment of solute transport properties; and (c) relating hydraulic and mechanical apertures, and predicting influence of stress perturbation and deformability.

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