Network design for groundwater monitoring – a case study

M.R. Prakash · V.S. Singh

Abstract The applicability and usefulness of Geostatistics (kriging) as a tool for optimum selection of sites for monitoring groundwater levels has been demonstrated through a case study. The criterion used is the estimation of error variance. Groundwater level data (pre-monsoon 1994) obtained from 32 observation wells of Upper Kongal basin, Nalgonda District, A.P. (India) has been stochastically analyzed. The spatial distribution of water levels and its associated error variance is computed and the locations having maximum error variance are selected as additional sites for augmenting the existing observational well network.

Key words Observation wells · Optimum location · Kriging

Introduction

Groundwater modelling needs a large quantity of geohydrological parameters like transmissivity, water levels, storage coefficient etc., at a closely spaced network of observational points. There is always an amount of uncertainty associated with these data, and from this uncertainty arises the following questions.

1. How much information is required ? (or what is the optimal sample size ?)
2. Where are the optimum locations for further sampling?

One way of arriving at the answers to the above questions is to quantify the uncertainty associated with the estimation of field values. Kriging is a geostatistical tool which provides such a measure known, as the estimation variance or the Krige variance. As a rule the estimation variance decreases with the increase in sample size. In other words, the accuracy of the estimation increases with increase in sample size. Hence, sampling procedures are designed based on maximization of the accuracy of the estimated field with budget constraints or minimizing sampling costs subject to a criterion of minimal acceptable accuracy. Earlier works on the application of optimal network analysis to groundwater include that of Rodrigueg Iturbe and Meha (1974), Carrera and others (1984), Rouhani (1985) and Bogardi and Bardossy (1985). In the present paper optimal network analysis has been carried out using variance reduction method on water-level data from the Kongal basin of Nalgonda district of A.P., (India).

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Methodology

Kriging is an interpolation technique based on the theory of regionalized variables (Matheron 1965). The Krige estimate is expressed as a weighted average of neighbouring field values falling within a certain distance called the range of influence.

The estimate of the field at an arbitrary point X based on N measured (neighbouring) values is given as

$$Z^*(x_0) = \sum_{i=1}^{N} \lambda_i Z(x_i) \quad (1)$$

The weights are so chosen that the estimate has the following properties

1. The estimate is unbiased:

$$E[Z^*(x_0) - Z(x_0)] = 0 \quad (2)$$

2. The estimate has minimum variance:

$$E[Z^*(x_0) - Z(x_0)]^2 \text{ is minimum} \quad (3)$$

where Z and Z^* are measured and estimated values respectively.

The minimum of the estimation variance given by (3) is called the Kriging variance. The Kriging variance can be utilized as a guideline in optimal sampling. For instance, a location with the highest level of estimation uncertainty indicated by high estimation variance can be targeted for further monitoring. One of the interesting features of Kriging is that the Krige variance which measures the uncertainty of the estimate can be computed before the actual measurements are available, and is purely expressed as a function of spatial coordinates. Once a model variogram is fitted, estimation at regular space intervals is carried out along with the error variance. The theory of Kriging is well documented by David (1977), Journel and Huijberts (1978) and Isacks and Srivastava (1989).

Study area

The study area is about 50 km east of Hyderabad, A.P. (India). It chiefly comprises granitic formation with dark basic dykes cut across the granitic rocks. Most of them show N-S trends. The thickness and orientation of these intrusives effect groundwater movement and its quality. Groundwater occurs in shallow-weathered and deep-fractured granite in the western and north-western parts, whereas in the remaining area it occurs in weathered gneises. The depth to water level varies from a few meters to about 25 m below ground level. The groundwater is mostly exploited through dug wells, dug–cum-bore wells and bore wells for irrigation, as well as domestic purposes.

An application of network design

A pilot area (Fig. 1) covering about 180 km^2 in the upper Kongal basin of Nalgonda district, A.P. (India), is selected for the study. Water levels (Pre-Monsoon 1994) from 32 observation wells (Fig. 2) were used for the analysis. The experimental variogram of water levels exhibited an unbounded variance in longitudinal as well as transverse directions, indicating non-stationarity nature of the variable. Hence the drift is removed by fitting a second order polynomial. The analysis of variance for the fitted model and the coefficients of the polynomial are given in Table 1. A cross-validation analysis was then performed on the residuals by varying variogram parameters using two theoretical models, for example cubic and spherical. Parameters of these models are given in Table 2. The spherical model (Fig. 3) was finally selected as it yielded less mean square error. From the fitted variogram it is found that the zone of influence is 2.5 km. Using a grid of 2.5 km, water levels and error variance were computed. The contours of kriged water levels and error variance maps are shown in Figs. 4 and 5, respectively. The selected sites were ranked using the error variance criterion. Table 3 shows the reduction in total error variance and percent gain in information with respect to the sample size and the location of the additional sites. If TV(N), TV($N+!$) represents the total estimated variance due to N and N + 1 data points respectively, then the total reduction in estimated variance due to additional measurement is given as:

$$TVR = TV(N) - TV(N+1) \qquad (4)$$

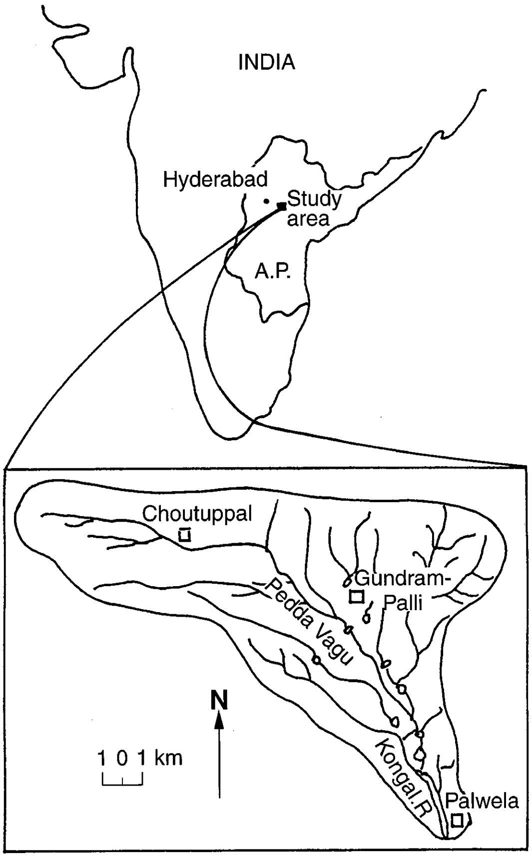

Fig. 1
Location of study area

Table 1
Anova of water levels data after quadratic fit

Source of variation	Sum of SQS	Degrees of freedom	Mean SQS
Regression	433.3936	5	86.6787
Deviation	371.5116	26	14.2889
Total	804.9052		
F-test	6.0662		
Goodness of fit	0.54		
Corr coeff	0.72		

Coefficients of the Polynomial
98.707 9.436 0.192 −16.136 −9.008 −1.673

The percent gain in precision due to additional measurement is simply the ratio of the reduction in estimated variance to the total estimated variance and is given as

$$GP = (TVR/TV(N)) \times 100 \qquad (5)$$

The basis of selecting a new site is the reduction in the total error variance due to its addition to the existing network. One site is selected at each round of Kriging. It is assumed that the new measurement does not cause any change in the parameters of the selected covariance func-

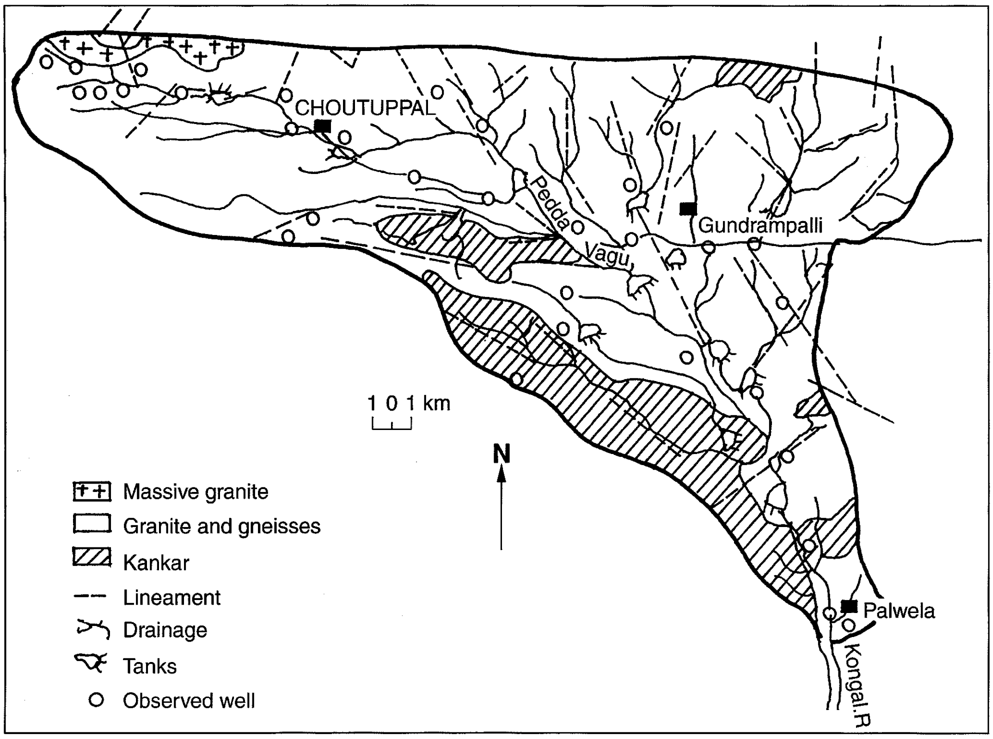

Fig. 2
Location map of observation wells

Table 2
Parameters of fitted variogram for residuals

Model type	Nugget effect (m^2)	Sill (m^2)	Range (km)	Mean square error (m^2)
Spherical	3.50	4.98	2.5	8.142
Cubic	3.50	4.77	1.3	8.410

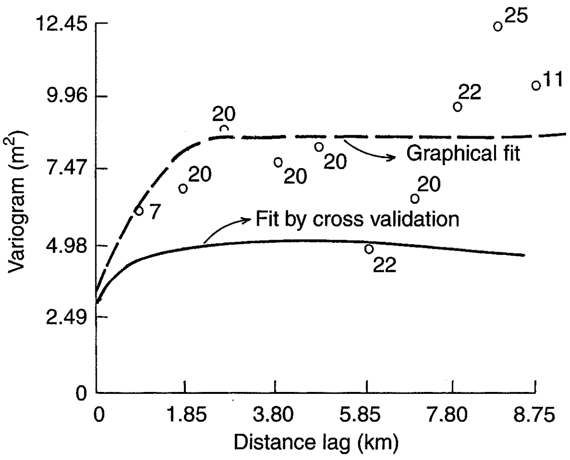

Fig. 3
Spherical model variogram

tion. Therefore, in the process of data collection no further structural analysis is conducted. Using the variance reduction criterion, the top 27 points have been ranked as the sequence of best locations for the network.

Table 3
Variance reduction due to additional sampling

Sample points	Location X-Kms	Location Y-Kms	Rank of additional points	Total est. variance	percent gain in information
32	–	–	–	203.452	–
33	17.75	7.75	1	197.852	
34	15.25	12.75	2	191.624	5.81
35	25.25	15.25	3	186.706	
36	10.25	12.75	4	181.877	10.60
37	22.75	2.75	5	176.753	
38	15.25	10.25	6	171.608	15.65
39	12.75	15.25	7	166.697	
40	17.75	15.25	8	161.667	20.54
41	20.25	7.75	9	156.673	
42	12.75	12.75	10	152.154	25.21
43	2.75	15.25	11	147.737	
44	25.25	2.75	12	143.210	29.61
45	22.75	5.25	13	138.795	
46	7.75	15.25	14	134.315	33.98
47	20.25	10.25	15	130.080	
48	15.25	15.25	16	125.818	38.16
49	17.75	10.25	17	121.873	
50	20.25	15.25	18	117.798	42.10
51	10.25	15.25	19	114.611	
52	22.75	15.25	20	111.421	45.23
53	20.25	12.75	21	108.393	
54	25.25	12.75	22	105.351	48.21
55	22.75	7.75	23	102.810	
56	22.75	10.25	24	99.940	50.88
57	17.75	12.75	25	97.689	
58	5.25	15.25	26	95.271	53.17
59	22.75	12.75	27	94.627	53.49

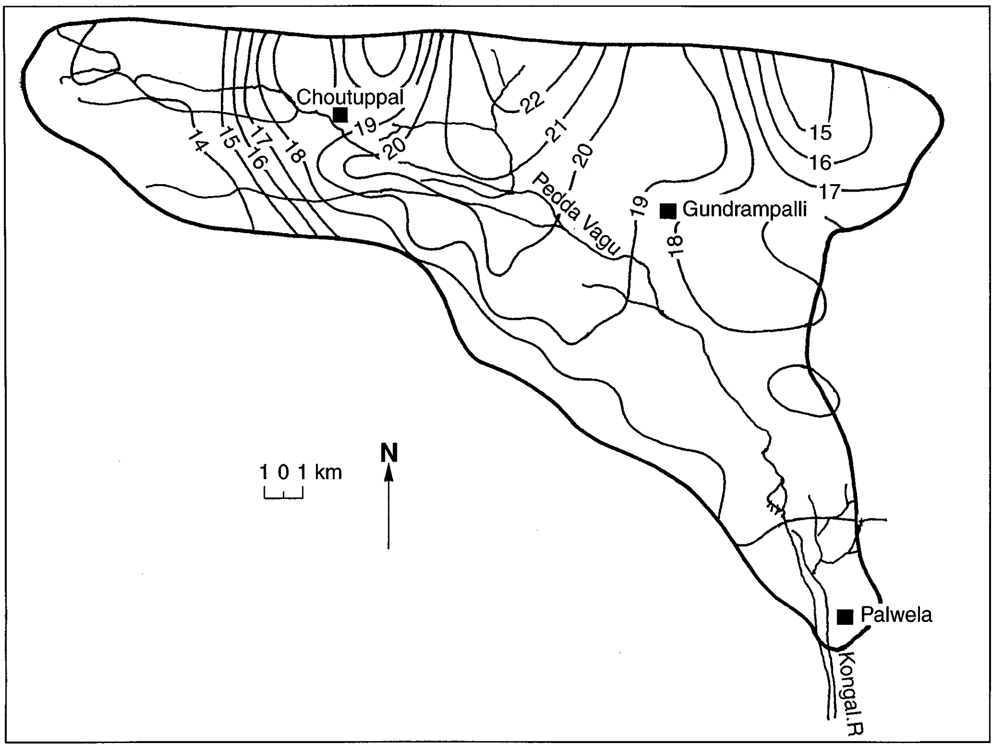

Fig. 4
Kriged water level

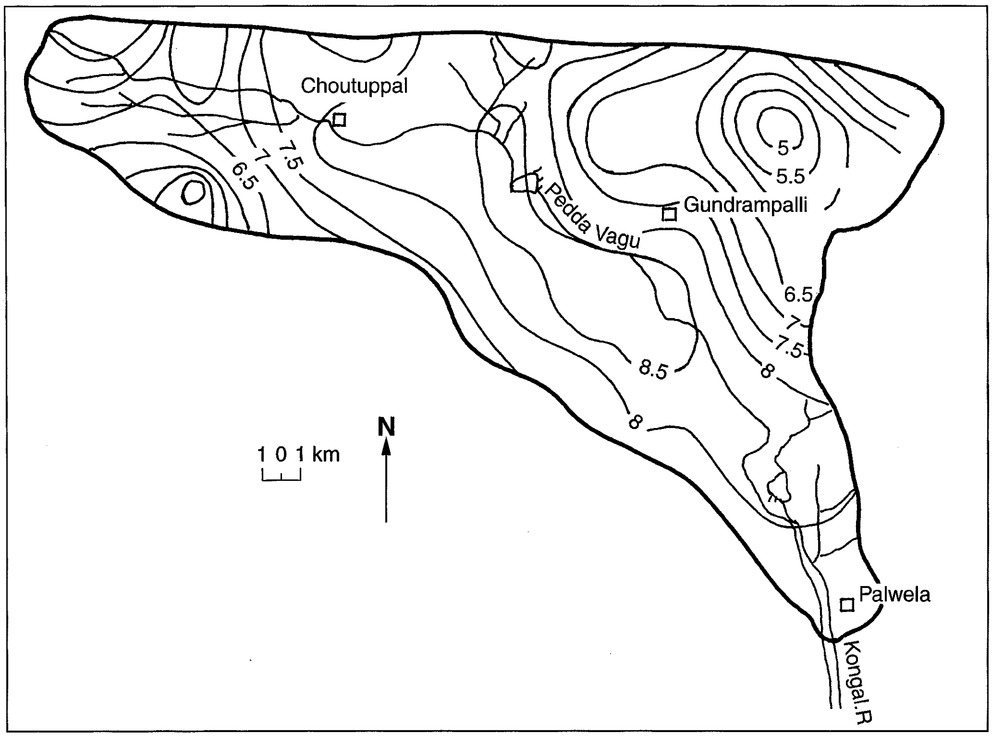

Fig. 5
Estimated error varience

Conclusion

The study presents the procedure for optimum selection of sites for augmenting the existing network. It is based on the variance reduction and ranking of prospective new sites. Most of the high values of error variance were found along the boundary of the basin. The situation can be compared with the case of stochastic steady flow in aquifers. The boundary values play a very important role in the variance of the estimated water levels. Hence, the boundary nodes are predominant choices for further measurements. Starting with 32 wells the total error variance has been reduced by 25% after augmenting ten more. It has been reduced by 50% after augmenting

24 more to the existing (32) wells. In other words the gain in information due to additional ten measuring points is 25% and it rises to 50% on adding 24 more observational wells. After this stage, the gain in information becomes almost negligible, even after adding new observational points. Thus, the optimum (maximum) number of wells that can be added to the existing network is 24, covering an area of 180 km^2 of hard rock area in the basin. Depending on the budget constraints, one can augment the network with new observational wells to be drilled in the locations obtained from the analysis.

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