ORIGINAL PAPER

Probabilistic analysis of landslide potential of an inclined uniform soil layer of infinite length: application

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Received: 22 November 2006 / Accepted: 20 June 2007 / Published online: 21 July 2007 Springer-Verlag 2007

Abstract An extended probabilistic model that is a modification of the Chen et al. (2007,) model for evaluating the failure probability of an inclined soil layer with an infinite length was developed in the present paper, and then applied to evaluate the occurrence probability of landsliderelated debris flow in Tungmen gully located in the eastern Taiwan, which occurred a devastating debris flow in 1990. The statistical properties of hydrogeological parameters were collected and summarized, and then used to evaluate the landslide-related debris-flow probabilities at various relative water depths for Tungmen gully by using the probabilistic model. Under the assumption that the soil is saturated, the soil's cohesion is negligible and the specific gravity of the solid particles of soils is a constant, a simplified probabilistic critical slope equation for the stability of an infinite slope of soils was also developed, and used to estimate the occurrence probability of debris flow. The result shows that probabilistic landslide analysis for an infinite slope could provide a suitable approximation for the risk analysis of debris flow mobilization at a given gully.

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Keywords Failure probability \cdot Probabilistic model \cdot Critical slope · Debris flow · Taiwan

Introduction

A downslope movement of soil mass is usually referred as landslide. Landslide for an infinite slope is a convenient mathematical idealization used to specify an inclined, tabular soil mass with lateral dimensions being much wider than its thick, which is frequently used to analyze shallow landslide problems and it also provides a suitable approximation for many sites of debris flow mobilization (Iverson et al. [1997;](#page-8-0) Chen et al. [2007](#page-8-0)). Mobilization hypothesis also generally assumes that the soil contains or gathers sufficient water to fully saturate the pore space (Takahashi [1978](#page-8-0), [1981;](#page-8-0) VanDine [1984;](#page-8-0) Iverson and Major [1986](#page-8-0); Gregoretti [2000\)](#page-8-0). However, these studies proposing the analysis of the inclined soil-layer stability are based on deterministic ideas without consideration of the uncertainty of the geological parameters involved in the considered soil layer. Considering the uncertainty of parameters, Chen et al. ([2007\)](#page-8-0) have developed a probability-based model for evaluating the critical bed slope of landslide, which is related to a failure probability p_f for a saturated soil layer with an infinite length, based on the first-order secondmoment method (FOSM). However, the model proposed by Chen et al. ([2007\)](#page-8-0) is limited to the condition of a saturated soil layer having the water table higher than the soillayer surface. However, in some cases, landslides could occur in an unsaturated soil layer when the water table lies below the surface of the soil layer. Thus an extended probability-based model for evaluating the stability of an inclined soil layer having either water table below or above the soil-layer surface is developed in the present study. The proposed model is used to assess the occurrence probability of landslide-related debris flows in Tungmen gully.

variables are used without consideration of variables' uncertainties.

$$
\mu_A = \frac{\mu_c / (\gamma_w \mu_H \cos \mu_\theta) + [(1 - \mu_n)(G_s - \mu_d) + \mu_s \mu_n (1 - \mu_d)] \tan \mu_\phi}{(1 - \mu_n)(G_s - \mu_d) + \mu_s \mu_n (1 - \mu_d) + \mu_m} \tag{4}
$$

Extended probability model

An inclined soil layer with an infinite length has an incline angle θ , thickness H, solid's specific gravity G_s , porosity *n*, cohesion *c* and internal friction angle ϕ , as illustrated in Fig. 1. The water table may have a depth h lower or higher than the soil-layer surface. According to the Mohr-Coulomb theory, the resistance stress τ_R that is the sum of soil's cohesion and effective shear stress can be written as

$$
\tau_R = c + [(1 - n)(G_s - d) + sn(1 - d)] \gamma_w H \cos\theta \tan\phi
$$
\n(1)

where S denotes the degree of saturation of soil in the unsaturated zone; γ_w represents the specific weight of water, and d is a parameter related to the relative water depth $m(= h/H)$. There is a relation $d = m$ when $m < 1$ and $d = 1$ when $m \ge 1$. The driving stress τ_D acting on the depth of H in the soil layer is the sum of the weight components of water and solid particles in the soil layer along the inclined plane, and can be written as

$$
\tau_D = [(1 - n)(G_s - d) + sn(1 - d) + m]\gamma_w H \sin\theta \tag{2}
$$

The state function G for the landslide of an inclined soil layer of an infinite length is defined as the difference between resistance stress τ_R and driving stress τ_D , i.e., G $= \tau_R - \tau_D$. When the parameters θ , H, h, s, n, c and ϕ in the state function G are considered as independent variables and can be described by normal distributions, an equation related to the failure probability p_f of critical slope condition for an inclined, uniform soil layer similar to that proposed by Chen et al. [\(2007](#page-8-0)), can be developed, as follows, based on FOSM.

$$
\tan \mu_{\theta} = \mu_A + BV\Phi^{-1}(1 - p_f) \tag{3}
$$

In Eq. (3), μ_{θ} denotes the mean value of incline angle θ of soil layer, function Φ represents the probability of standard normal distribution and $\Phi^{-1} (1 - p_f)$ is the inverse function of Φ at the probability $(1 - p_f)$, and μ_A is the reference slope that is the critical slope for soil-layer failure obtained by the deterministic model in which only mean values of

where μ_c , μ_H , μ_s , μ_n , μ_ϕ and μ_m represent the mean values of c, H, s, n, ϕ and m, respectively. Meanwhile, μ_d is a parameter related to μ_m , having the relation $\mu_d = \mu_m$ when μ_m < 1 and μ_d = 1 when $\mu_m \ge 1$. The parameter B and V in Eq. (3) can be respectively, expressed as follows

$$
B = \frac{-1}{(1 - \mu_n)(G_s - \mu_d) + \mu_s \mu_n (1 - \mu_d) + \mu_m}
$$
(5)

$$
V = \sqrt{\sum_{i=1}^{7} \text{CV}_{xi}^{2} (a_{i} \tan \mu_{\theta} + b_{i})^{2}}
$$
(6)

where the subindex xi (for $i = 1,2,...,7$) represents the variables n, ϕ , h,H, θ , c, and s, respectively; moreover, CV_{xi}

(a) Water surface above the surface of soil layer

(b) Water surface below the surface of soil layer

Fig. 1 Schematic diagram of an inclined soil layer having water table below or above the soil-layer surface (a) Water surface above the surface of soil layer (b) Water surface below the surface of soil layer represent the coefficients of variation, which define as the ratio of the values of standard derivation to the mean values of the corresponding variables x_i . The parameters a_i and b_i are coefficients related to the mean values μ_{vi} , as indicated in "Appendix I". When the soil layer is fully saturated, μ_d = 1, the expressions of μ_A , B and V, Eqs. (4), (5), and (6), can be reduced to the same result derived by Chen et al. [\(2007](#page-8-0)).

Rearrangement of Eq. (3) will yield the following relation (Chen et al. [2007](#page-8-0))

$$
p_f = 1 - \Phi\left(\frac{1 - S^*}{V^*}\right) \tag{7}
$$

where S^* is termed the relative slope, defined as the ratio of the soil-layer slope tan μ_{θ} to the reference slope μ_{A} , that is, $S^* = \tan \mu_\theta / \mu_A$. The uncertainty number $V^* (= -BV/\mu_A)$ is given by

Eq. (10) becomes identical to the critical slope condition of Takahashi for the occurrence of a landslide-related debris flow. Here κ is the ratio of the representative grain diameter d_s to the overland-flow depth h_o . The value of κ proposed by Takahashi is in a range from 0.7 to 0.75. If the value of κ is 0.75, which yields $(1 + \kappa^{-1})$ or μ_m of approximately 2.33 (Chen et al. [2007\)](#page-8-0).

Application (1): Tungmen debris flow gully

Many mountainous areas in Taiwan are susceptible to landslides and debris flows, as a result of heavy rains accompanying typhoons, steep topography, young and weak geological formations, strong earthquakes (25 earthquakes of magnitudes larger than 6 on the Richter scale in the twentieth century), loose soils, and land

$$
V^* = \frac{V}{\mu_c / (\gamma_w \mu_H \cos \mu_\theta) + [(1 - \mu_n)(G_s - \mu_d) + \mu_s \mu_n (1 - \mu_d)] \tan \mu_\phi}
$$
(8)

Given G_s , γ_w , μ_n, μ_s , μ_c , μ_ϕ , μ_H , μ_h , CV_{xi} , and p_f , the critical slope tan μ_{θ} can be determined from Eq. (3) using the parameters shown in Eqs. (4), (5) and (6). If all variables are invariant ($V = 0$), or if the probability equals 50% (p_f) $= 50\%$), Eq. (3) is reduced to

$$
\tan \mu_{\theta} = \mu_A \quad \text{for } V = 0, \quad \text{or} \quad p_f = 50\% \tag{9}
$$

Equation (9) is consistent with the result of the deterministic model. The variables in the deterministic model are all mean values and their variance is not considered (i.e., $x_i = \mu_{xi}$ and $CV_{xi} = 0$) in estimating landslide occurrence. If a soil layer is cohesionless ($\mu_c = 0$) and contains sufficient water to saturate the soil ($\mu_d = 1$), the condition of the soil layer is critical, and its corresponding critical slope tan μ_{θ} is written as

$$
\tan \mu_{\theta} = \frac{(1 - \mu_n)(G_s - 1)}{(1 - \mu_n)(G_s - 1) + \mu_m} \tan \mu_{\phi}
$$
\n(10)

The above equation resembles that obtained by Takah- $a\sin(1981)$ $a\sin(1981)$ $a\sin(1981)$, and is a deterministic equation in which all involved parameters are evaluated by their mean values. Takahashi [\(1978](#page-8-0)) developed a critical slope relation for that in which a saturated, inclined soil layer of uniform thickness and infinite length fails, and then is transformed into a debris flow by water flow over the soil surface. If μ_m in the denominator of Eq. (10) is replaced by $(1 + \kappa^{-1})$,

development in mountainous terrain. Among so many causes, typhoons are especially important agents of landslides and debris flows in Taiwan (Jan and Chen [2005](#page-8-0)). One of the first well documented debris-flow catastrophes in Taiwan occurred on 23 June 1990 in Tungmen village during Typhoon Ofelia. Tungmen village is located in the west of Hualien city in eastern Taiwan, as shown in Fig. [2.](#page-3-0) Typhoon Ofelia deposited a heavy precipitation in the Tungmen area on 23 June 1990, with heavy rainfall of 106 mm/h and had a maximum wind speed of 45 m/s during the period from 2:00 p.m. to 3:00 p.m. This event caused 39 deaths, 68 injuries, 24 collapsed houses and damaged to 11 buildings. The storm resulted in enormous floods, landslides, and a severe landslide-related debris flow in Tungmen gully without precaution. The Tungmen village was built at the apex of an alluvial fan. Nearly half of the village was covered by the debris flow having a volume of about $55,000 \text{ m}^3$ (NCHU [1990;](#page-8-0) Chen et al. [1999](#page-8-0); Jan and Chen [2005](#page-8-0)).

The rock types in the Tungmen area belong to the Paleozoic Tananao schist formation. It consists mainly of quartz-mica schist, phyllite, mica schist, mica quartzite and to a lesser extent, trace amounts of chlorite schist and metabasite (NCHU [1990;](#page-8-0) Chen et al. [1999](#page-8-0)). The discontinuities of the schist and joints are densely distributed on the slope surface and the bottom of the gully (Chen et al. [1999](#page-8-0)). There is a 2 mm to 4 cm spacing distributed on the

Fig. 2 Location and watershed of a debris-flow gully behind Tungmen village. (adapted from NCHU [1990\)](#page-8-0)

schistosity. The joints spacing between the discontinuities are less than 1 m. Hence, there are abundant deposited materials on the gully bed due to the geological structure containing a number of the discontinuities. The heavy rainfall resulted in a significant increase in the water depth or pore pressure on the deposited soil layer and that had considerable influence on the stability of deposited materials. For a soil layer lying on the gully bed, the erosion may occur in a gully landform. When the soil layer has steep slope, the slope surface failure by slide is more possible than erosion. The behavior of the surface water erosion is neglected and not discussed here. The extended probability model of landslides for a soil layer with an infinite length proposed in this study is used to analyze the probability of the instability of the deposited soil layer on the gully bed in Tungmen village.

Geological parameters of Tungmen gully

The 1990 Tungmen debris-flow event was a significant natural hazard in Taiwan in recent decades. In order to analyze the debris-flow occurrence condition at Tungmen gully, some field investigations and laboratory tests on geological parameters of Tungmen debris-flow site had been performed by several researchers, such as NCHU [\(1990](#page-8-0)), Ho and Chen [\(1997](#page-8-0)), Chen et al. ([1999\)](#page-8-0), among others. However, the values of the geological parameters obtained by previous researchers are not consistent, especially for soil porosity n , degree of saturation s and thickness H . The inconsistency of these parameters may result from the non-homogenous nature of soil properties, from the use of different sampling and testing methods, or from the accuracy in sampling, testing or measuring.

Tungmen debris-flow gully is a mountainous gully, and normally in a dry situation at no rain days. The gully bed contains a mixture of deposited material, including rock fragments and fine particles. The big boulders range in size from 1 to 4 m. According to a field investigation by NCHU (1990) (1990) , the average slope of the gully bed was 20° and the moving sediment layer of the gully bed was up to 2 m during the 1990 debris-flow event. A field investigation by Chen et al. ([1999\)](#page-8-0) has demonstrated that the average slope of the gully bed in the upstream area (where landslide and debris flow initiated) was about 19° and the gully bed had been deepened about 5 m after the debris-flow event. The geomaterials sampled from the gully bed and gully sideslopes were also tested by Chen et al. [\(1999](#page-8-0)), using largescale direct shear stress. Testing results indicated that the friction angle ϕ was in the range from 33.3° to 36.8° for different samples under a saturated condition, and the soil cohesion was very small and could be ignored. Ho and Chen ([1997\)](#page-8-0) took in situ samples from the deposited materials in Tungmen gully for evaluating the geomaterial properties. Their results demonstrated that the mean value of specific gravity of solid G_s was 2.74, while the void ratio of deposited material varies from 0.24 to 0.62 for different sites in the gully, which corresponds to the values of porosity n varies from 0.19 to 0.38.

Statistical properties of geological parameters

When a set of observational data is available, for example, a set of data $(x_1, x_2,...,x_N)$ with a sample of size N, common statistical estimation techniques (e.g. Ang and Tang [1975\)](#page-8-0) can be used to estimate the mean value μ_{xi} , the standard deviation σ_{xi} , and the coefficient of variation CV_{xi} (= σ_{xi} / μ_{xi}). However, in engineering, information of data is often expressed in terms of the lower and upper limits of a variable. If the specified lower and upper limits x_{li} and x_{ui} can be assumed to cover $\pm k$ standard deviations $(\pm k\sigma_{xi})$ from the mean value of an underlying normal distribution, the mean value would be (Ang and Tang [1984](#page-8-0))

$$
\mu_{xi} = (x_{ui} + x_{li})/2 \tag{11}
$$

and the corresponding coefficient of variation CV_{xi} is

$$
CV_{xi} = k^{-1}(x_{ui} - x_{li})/(x_{ui} + x_{li})
$$
\n(12)

The statistical properties of G_s , n, s, c, and ϕ are estimated by using common statistical estimation techniques (Ang and Tang [1975](#page-8-0)) with a sample of size N, respectively, while the properties of for the parameters θ and H are respectively, evaluated by Eqs. (11) and (12), using the upper and lower limits of parameters and assuming $k = 1$. The statistical values of the seven geological parameters for the Tungmen gully are shown in Table 1. Degree of soil saturation s listed in Table 1 is only for the case of the water table below the surface of soil layer. When the water table is above the surface of soil layer (i.e., $m \ge 1$), that will induce soil saturation to $s = 100\%$. By the way, the soil strength is generally affected by electric-chemical and physical properties, e.g. size distribution, shape, and density, which usually are represented by G_s , n, c and ϕ . Those four parameters may slightly relate to each other. As shown in Table 1, for simplicity, the above-mentioned four parameters are considered as independent parameters in the present analysis. Furthermore, c and ϕ may vary with h because of their dependency on the soil moisture content. However, for lack of data, parameters h,c , and ϕ are also simply regarded as independent variables in this paper.

Assessment on Tungmen gully stability

Stability analysis of Tungmen gully was conducted by using the deterministic method and the probabilistic method proposed in the present paper. In order to evaluate the stability of Tungmen gully by the deterministic model (Eq. 9), three possible geological conditions of the gully bed were considered and referred as Case1, Case2, and Case3, as shown in Table [2.](#page-5-0) The geological parameters used in these three cases were based on the data listed in Table 1. The gully bed is considered as in failure when tan $\mu_{\theta} \ge \mu_A$, while it is stable when tan $\mu_{\theta} < \mu_A$ in the deterministic model. The results of stability analysis on the bed of Tungmen gully at various relative water depths μ_m were listed in Table [3](#page-5-0). The critical condition for gully bed failure (that would be transformed into debris flow) is μ_m ≥ 1.4 for Case1, $\mu_m \geq 1.1$ for Case2, and $\mu_m \geq 1.0$ for Case3. The value of μ_m (= μ_h / μ_H) measures the quantity of water contained in the soil layer of gully bed, and also measures the driving shear stress on the soil layer. A critical relative water depth μ_m is required to initiate the soillayer failure and then to transform the failed soil into debris flow. Study of Takahashi ([1981\)](#page-8-0) showed that the critical relative water depth μ_m is in a range of $1 \leq \mu_m \leq 2.33$. Table [3](#page-5-0) shows that the higher water depth flowing over the gully bed, the higher failure potential of gully bed. The deterministic method does not provide risk information of gully bed's stability due to the uncertainty of geological parameters, but this can be overcome by using the probabilistic model as proposed in previous section.

In the gully stability analysis by using the proposed probabilistic model, the parameters' mean values μ_{xi} and

Parameter x_i	Sample number N	Range of values	Mean value μ_{vi}	Variation coefficient $CV_{vi}(\%)$
Soil's specific gravity G_s	19	$2.71 - 2.78$	2.74	1.3
Soil's porosity n	9	$0.19 - 0.38$	0.3	24
Degree of soil's saturation s	9	$0.2 - 0.5$	0.35	39
Soil's cohesion c (kPa)	9	Ω	Ω	Ω
Soil's friction angle ϕ (degree)	9	$33.3 - 36.8$	34.5	5
Gully bed slope θ (degree)	3	$19 - 21$	20	5
Soil's thickness $H(m)$	2	$2 - 5$	3.5	43

Table 1 Statistical properties of seven geological parameters at Tungmen gully

Case	Geological condition
Case 1	Taking the upper limit of ϕ , the lower limit θ , and mean values of other five parameters in the stability analysis, i.e., $\phi = 36.8^{\circ}$, $\theta = 19^{\circ}$, $G_s = 2.74$, $n = 0.3$, $s = 0.35$, $c = 0$ and $H = 3.5$ m, as listed in Table 1.
Case 2	Taking all seven parameters of their mean values in stability analysis, i.e., $\phi = 34.5^{\circ}$, $\theta = 20^{\circ}$, $G_s = 2.74$, $n = 0.3$, $s = 0.35$, $c = 0$ and $H = 3.5$ m, as listed in Table 1.
Case 3	Taking the lower limit of ϕ , the upper limit θ , and mean values of other five parameters in the stability analysis, i.e., $\phi = 33.3^{\circ}, \theta = 21^{\circ}, G_s = 2.74, n = 0.3, s = 0.35, c = 0 \text{ and } H = 3.5 \text{ m}, \text{ as listed in Table 1.}$

Table 2 Geological conditions for gully stability analysis by deterministic method

the parameters' variation coefficients CV_{xi} as listed in Table [1](#page-4-0) are used. The present probabilistic model was derived by FOSM. The results show that, the failure probability p_f of Tungmen gully increases from 0.2 to 0.75 when the relative water depth μ_m increases from 0.5 to 2.0, as shown in Table 3 and in Fig. 3. The result shows that the probabilistic model can provide more risk information than the deterministic model. The relation between p_f and μ_m obtained by the Monte Carlo simulation method (MCSM) was also plotted in Fig. 3 for comparison. The comparison shows that the failure probability analysis by FOSM and MCSM yields similar results. The probability p_f increases with the increase of relative water depth μ_m . When μ_m is larger than 1.1, the corresponding failure probability p_f for Tungmen gully bed is higher than 50%, and the failed soil would be transformed into debris flow during flood. The failure probability p_f exceeds 75% for $\mu_m > 2.0$. The

Table 3 Results of Tungmen gully stability analysis by the deterministic and probabilistic models

Relative water depth μ_m	Deterministic model	Probabilistic model Failure probability		
	Failure judgment			
	Case 1	Case 2	Case 3	p_f
0.5	NF	NF	NF	0.20
0.6	NF	NF	NF	0.26
0.7	NF	NF	NF	0.32
0.8	NF	NF	NF	0.37
0.9	NF	NF	NF	0.41
1.0	NF	NF	NF	0.46
1.1	NF	NF	F	0.49
1.2	NF	F	F	0.52
1.3	NF	F	F	0.55
1.4	NF	F	F	0.60
1.5	F	F	F	0.63
1.6	F	F	F	0.65
1.7	F	F	F	0.67
1.8	F	F	F	0.70
1.9	F	F	F	0.72
2.0	F	F	F	0.75

NF and F represent no failure and failure, respectively

relative water depth μ_m on Tungmen gully was approximately 1.2 during the 1990 debris-flow event (NCHU [1990](#page-8-0)). At $\mu_m = 1.2$, the corresponding failure probability p_f is about 52%, based on the present analysis.

Application (2): debris flow occurrence probability

Bed slopes of debris flow gullies

Landslide for an infinite slope is a convenient mathematical idealization and also provides a suitable approximation for many sites of debris flow mobilization (Iverson et al. [1997](#page-8-0); Chen et al. [2007\)](#page-8-0), as described previously. Dependence of debris-flow occurrence on the gully bed slope has been investigated by many researchers. A field investigation on debris-flow gullies conducted by Shieh [\(1993](#page-8-0)) in Hualien and Taitung counties in eastern Taiwan showed that most of debris-flow gullies (approximately 87%) had a bed slope of between 10° and 25° . Investigation of Wu ([1999\)](#page-8-0) showed that most gully slopes (approximately 84%) of debris-flow-prone gullies in southern Taiwan were between 10° and 25° . Chen et al. [\(2004](#page-8-0)) investigated debris-flow gullies in central Taiwan and found that 50% of debris flow events occurred in the gullies having slopes between 10°

Fig. 3 Evaluation of soil-layer failure probability p_f at various relative water depths μ_m in Tungmen gully by two probabilistic methods

and 20°. In Japan, Ashida et al.[\(1983](#page-8-0)) studied 34 mountainous gullies that ever suffered debris flows and found that most of these gullies had slops ranging from 17° to 22°. The probability density distribution of the bed slopes of debris-flow gullies from the data of Ashida et al. [\(1983](#page-8-0)), Shieh [\(1993\)](#page-8-0), Wu ([1999\)](#page-8-0) and Chen et al. ([2004\)](#page-8-0) were plotted in Fig. 4.

Probabilistic critical slope for debris-flow occurrence

Generally, soil cohesion reduces sharply with increasing water content in the soil, and tends to be destroyed if large soil displacements occur (Skempton [1985](#page-8-0)). When the soil layer is fully saturated, the soil's cohesion effect could be negligible. With the considerations where the soil is in fully saturation, the soil cohesion is neglected, and the specific gravity of solids is kept as a constant, a simplified critical slope relation can be yielded from Eqs. (3), (4), (5), and (6), as shown in the following

$$
\tan \mu_{\theta} = \frac{(1 - \mu_n)(G_s - 1)\tan \mu_{\phi}}{(1 - \mu_n)(G_s - 1) + \mu_m} - \frac{\Phi^{-1}(1 - p_f)}{(1 - \mu_n)(G_s - 1) + \mu_m} \sqrt{\sum_{i=1}^{5} \text{CV}_{xi}^2 (a_i \tan \mu_{\theta} + b_i)^2}
$$
\n(13)

with the coefficients $a_1 = \mu_n (G_s - 1), b_1 = -a_1 \tan \mu_\phi, a_2$ $= 0, b_2 = \mu_\phi(1 - \mu_n)(G_s - 1)(1 + \tan^2 \mu_\phi), a_3 = -\mu_m, b_3$

Fig. 4 Probability density distribution of bed slopes of debris-flow gullies based on Taiwan and Japan field data

Table 4 Hydrogeological parameters used in the analysis of debrisflow occurrence probability

Parameter x_i	Mean value μ_{vi}	Variation coefficient $CV_{vi} (\%)$
Soil's porosity n	$0.35^{\rm a}$	15 ^a
Soil's friction angle ϕ ^(°)	$35.0^{\rm a}$	15 ^a
Gully bed slope θ ^{(\circ})		$5^{\rm a}$
Relative water depth m	1.7^{b}	
Soil's thickness $H(m)$	4.0 ^a	40 ^a

^a Adopted from the medium value summarized by Chen et al. [\(2007](#page-8-0))

 b Adopted from the medium value by Takahashi ([1981\)](#page-8-0)

 $= 0, a_4 = -(1 - \mu_n)(G_s - 1), b_4 = -a_4 \tan \mu_\phi, a_5 = -\mu_\theta (1$ $-\mu_n(G_s - 1)\tan\mu_\phi$, and $b_5 = -\mu_\theta [(1 - \mu_n)(G_s - 1) + \mu_m].$

When the soil cohesion is neglected and the specific gravity of solids is kept as 2 .65, the statistical properties of other five geological parameters $(n, \phi, \theta, m \text{ and } H)$ are evaluated from the data of Chen et al. [\(2007](#page-8-0)) and Takahashi ([1981\)](#page-8-0), as shown in Table 4. Due to the lack of data, the coefficient of variation of the water depth was assumed to share the same coefficient of variation with that of soil's thickness H (i.e., $CV_h = CV_H$). Based on the above-mentioned parameters, a debris-flow occurrence probability for a considered debris-flow gully having bed slope θ can be estimated by using the simplified critical slope relation (Eq. 13).

Figure [5](#page-7-0) shows a comparison of debris-flow occurrence probabilities at various gully slopes estimated from the simplified critical slope relation (Eq. 13) and that from field data obtained by previous researchers. The comparison shows that the proposed equation can provide a reasonable description on the probability of debris-flow occurrence for a mountainous gully with a certain slope.

Conclusions

This study draws the following conclusions:

1. The extended probabilistic model that is a modification of the model proposed by Chen et al. [\(2007](#page-8-0)) for evaluating the stability of an inclined soil layer of an infinite length was developed by the first-order secondmoment method (FOSM) with the consideration of water table being either below the soil surface or above the soil surface. Applying the probabilistic model in the soil-layer stability analysis, one needs to know the statistical properties (mean values and coefficients of variation) of hydrogeological parameters in advance, such as θ , H, h, s, n, c and ϕ .

Fig. 5 Debris-flow occurrence probability for a considered gully having bed slope θ

- 2. The probabilistic model was applied to evaluate the stability of Tungmen gully where had a devastating debris flow on 23 June 1990. The statistical properties of related hydrogeological parameters of Tungmen gully were collected and analyzed. The failure probability of Tungmen gully at various relative water depths was evaluated by the proposed probabilistic model based on FOSM as well as by the probabilistic model based on MCSM for comparison. The results obtained by FOSM and MCSM were quite close to each other. The failure probability p_f increases as the relative water depth μ_m increases. The failure probability p_f of Tungmen gully exceeds 50% when μ_m is higher than 1.1.
- 3. The field investigation data from Taiwan and Japan show that most of bed slope for debris-flow-prone gullies ranged from 10° to 25° . Under the considerations where the soil is in fully saturation, the soil cohesion is neglected, and the specific gravity of solids is kept as a constant, a simplified probabilistic critical slope relation for landslide-related debris flow occurrence was developed, as shown in Eq. (13), and then used to evaluate the probabilities of a gully bed mobilized into a debris flow, based on the hydrogeological parameters listed in Table [4](#page-6-0). The theoretical result is close to the field investigation data from Taiwan and Japan. This shows that the probabilistic landslide analysis for an infinite slope could provide a suitable approximation for the risk of debris flow mobilization.

Acknowledgments The National Science Council in Taiwan is acknowledged for providing financial support in this research under Grants NSC 95-2221-E-211-001 and NSC 95-2625-Z-006-015. The writers are thankful to the anonymous reviewers for their helpful comments and suggestions. Thanks are also due to Dr. Wei-Cheng Lo for his help in grammatical check on the final draft of this paper.

Appendix I: coefficients in Eq. (6)

$$
a_1 = \mu_n[(G_s - \mu_d) - \mu_s(1 - \mu_d)]
$$

\n
$$
b_1 = -a_1 \tan \mu_{\phi}
$$

\n
$$
a_2 = 0
$$

\n
$$
b_2 = -\mu_{\phi}[(1 - \mu_n)(G_s - \mu_d) + \mu_s\mu_n(1 - \mu_d)](1 + \tan^2 \mu_{\phi})
$$

\n
$$
a_3 = \begin{cases}\n-\mu_m \mu_n (1 - \mu_s), \text{ if } \mu_m < 1 \\
-\mu_m, \text{ if } \mu_m \ge 1\n\end{cases}
$$

\n
$$
b_3 = \begin{cases}\n\mu_m[(1 - \mu_s)\mu_n - 1] \tan \mu_{\phi}, \text{ if } \mu_m < 1 \\
0, \text{ if } \mu_m \ge 1\n\end{cases}
$$

\n
$$
a_4 = \begin{cases}\n-[(1 - \mu_n)G_s + \mu_s\mu_n], \text{ if } \mu_m < 1 \\
-(1 - \mu_n)(G_s - 1), \text{ if } \mu_m \ge 1\n\end{cases}
$$

\n
$$
b_4 = -a_4 \tan \mu_{\phi}
$$

\n
$$
a_5 = -\mu_{\theta}[(1 - \mu_n)(G_s - \mu_d) + \mu_s\mu_n(1 - \mu_d)] \tan \mu_{\phi}
$$

\n
$$
b_5 = -\mu_{\theta}[(1 - \mu_n)(G_s - \mu_d) + \mu_s\mu_n(1 - \mu_d) + \mu_m]
$$

\n
$$
a_6 = \mu_c/(\gamma_w \mu_H \cos \mu_{\theta})
$$

\n
$$
b_7 = -a_7 \tan \mu_{\phi}
$$

Appendix II: notations

The following symbols are used in this paper:

- p_f = failure probability of a soil layer $s = \text{degree of soil's saturation}$
-
- S^* = tan μ_θ / μ_A
- x_i = variables
- x_{li} = lower values of variables
- x_{ui} = upper values of variables
- V = parameter relating to Eq. (3)
 V^* = parameter relating to Eq. (7)
- $=$ parameter relating to Eq. (7)
- ϕ = soil's friction angle
- γ_w = specific gravity of water
- θ = inclined angle of a soil layer
- τ_D = driving stress
- τ_R = resistance stress
- μ_A = reference slope, which is defined as Eq. (4)
- μ_c = mean value of soil cohesion c
- μ_d = parameter relating to μ_m
- μ_n = mean value of soil porosity *n*
- μ_s = mean value of degree of saturation s
- μ_{ϕ} = mean value of friction angle ϕ
- μ_{θ} = mean value of inclined angle θ
- μ_h = mean value of water depth h
- μ_H = mean value of soil thickness H
- μ_m = mean values of relative water depth m
- μ_{xi} = mean values of variables x_i
- σ_{xi} = standard deviation of variables x_i
- κ = Takahashi's parameter (κ = 0.7–0.75)

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