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OBITUARY

Gérard Lallement (1935–2006)

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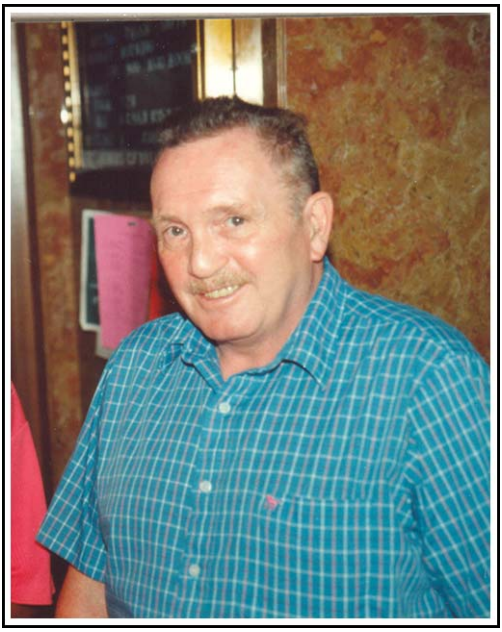

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Gérard Lallement died suddenly on January 22, 2006. He was born in 1935 in Metz, a city of Lorraine, a province disputed a long time between France and Germany. His own name reflected this duality since it means 'the German' in French.

He graduated from the University of Paris in 1961 and obtained the same year the prestigious Agrégation in mathematics. He defended his doctoral thesis prepared under the supervision of Paul Dubreil in 1966 and received the same year the Albert Châtelet medal for research from the Académie des Sciences de Paris. He was during four years a researcher at the Centre National de la Recherche Scientifique (1962–1966) and Maître de Conférences at the University of Reims two years (1967–1969). He joined the Pennsylvania State University in 1969 and remained there until 2006. He visited the University of Paris on several occasions, especially on sabbatical leave in 1975 and 1995.

His contributions in mathematics are outstanding. He has worked on both classical semigroup theory and the algebraic theory of automata. He may be considered as a major actor of the interplay between semigroup theory and theoretical computer science. In this direction, his book "Semigroups and Combinatorial Applications" [39] played an important role.

Semigroup theory Among his many results in classical semigroup theory, we single out the following ones.

His contribution to the property of *residual finiteness*. Recall that a semigroup is called residually finite if any two distinct elements can be separated by a congruence of finite index. In [21], he gave a constructive proof of the theorem of Malcev asserting that commutative semigroups are residually finite. In [23], he considered the more general case of nilpotent semigroups. Recall that a semigroup is said to be c-nilpotent if the law

$$q_c(x, y, z_1, \ldots, z_{c-1}) = q_c(y, x, z_1, \ldots, z_{c-1})$$

is satisfied for all $x, y \in S$ and $z_1, \ldots, z_{c-1} \in S^1$ where q_c is defined inductively by $q_1(x, y) = xy$ and

$$q_{i+1}(x, y, z_1, \ldots, z_i) = q_i(x, y, z_1, \ldots, z_{i-1}) z_i q_i(y, x, z_1, \ldots, z_{i-1}).$$

This notion was introduced by Neumann and Taylor in 1963 and they showed that it coincides with the usual definition of nilpotency for groups. Gérard Lallement has studied this class of semigroups and shown in particular that finitely generated regular nilpotent semigroups are residually finite. It is still not known if this statement holds without the hypothesis that the semigroup is regular.

He has worked repeatedly on finitely presented semigroups and, in particular, on *one-relator semigroups* and especially on the conjecture that all one-relator semigroups have solvable word problem. He told once that, after a lecture of Adjan on this subject, somebody in the audience asked whether a Soviet journal would publish the solution in English if somebody from the West would solve the problem. Adjan answered that before the problem was solved, everybody would publish in English in the Soviet Union. He seems to be right? Gérard Lallement published in [30] a number of results on this subject, including a characterization of one-relator semigroups containing an element of finite order, generalizing the result of Magnus, Karrass and Solitar on one-relator groups.

Algebraic automata theory Gérard Lallement has worked on the theory of decomposition of semigroups initiated by the Krohn-Rhodes theorem proved in 1965. The result describes the irreducible semigroups with respect to the operation of semidirect product. It can be stated either for abstract semigroups (or monoids) or for transformation semigroups. It can also be stated as a theorem on sequential transducers with the composition of transducers replacing the semidirect product of semigroups. The proof itself admits possible variants leading to different decomposition algorithms. Gérard Lallement has contributed to clarify this complicated domain. In [22], he gives an algebraic proof of the result using the notion of *wreath product* of monoids instead of semidirect product. The presentation given by Eilenberg in volume B of his book "Automata, Languages and Machines" in 1976 gives account of his work. The building blocks of these decompositions are either finite groups or elementary semigroups, which have been characterized by Gérard Lallement in [17].

Prefix codes and semigroups A number of his papers are devoted to a problem on which the second author has worked himself, including jointly with Gérard [43]. The general idea is to study the relation between a rational language and its syntactic semigroup. General theorems of correspondence between combinatorial properties of the language and the algebraic structure of the semigroup are known. One of the most famous is Schützenberger's theorem characterizing star-free languages as corresponding to semigroups with trivial subgroups. The general framework was set by Eilenberg as a theory of varieties of semigroups in correspondence with varieties of formal languages. In a more specific framework, Gérard Lallement has worked on the correspondence with an emphasis on the languages of the form X^* where X is a finite prefix code. He has in particular studied the case where the corresponding syntactic semigroup is a union of groups [32], an inverse semigroup [28] or a regular semigroup [33].

Gérard Lallement, as said above, played a key role in the interaction between mathematics and theoretical computer science by his commitment into the two-way fertilization between semigroup theory and automata theory.

He joined the editorial board of this journal in 1974 and became a managing editor in 1979 and a council member when the system was implemented in 1991. He remained an active member of the editorial board until the year 2000 when he became a honorary editor.

Gérard was also an outstanding professor and lecturer as outlined in the obituary published by Penn State University: "His greatest professional joy was as a teacher, as evidenced by the generation of students who remember his class fondly. He celebrated 50 years of teaching in 2005. His courses were demanding and difficult, but ultimately rewarding for all students who worked hard and persevered". He served as director of graduate studies in 1978 and had himself a number of research students including Michael Keenan, Elaine Milito, Janusz Konieczny, and the first author. He was chairman of his department from 1982 to 1985.

Let us conclude this brief account of his scientific work by a more personal touch. He was also a key person in the communication between Europe and North America. Many people, including both authors, found a splendid hospitality in his family when traveling to the United States and he was for many of us a Pygmalion welcoming

young researchers to an American campus, including memorable journeys around the countryside of Pennsylvania. The scientific community is thus missing since 2006 a prominent member as well as a cheerful, independent and deeply original personality.

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