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Convective heat transfer and pressure loss in rectangular ducts with drop-shaped pin fins

Z. Chen, Q. Li, D. Meier, H.-J. Warnecke

Abstract It has been experimentally researched that convective heat transfer and pressure loss characteristics in rectangular channels with staggered arrays of drop-shaped pin fins in crossflow of air. The effects of arrangements of pin fins on heat transfer and resistance are discussed and the row-by-row variations of the mean Nusselt numbers are presented. By means of the heat/mass transfer analogy and the naphthalene sublimation technique, the heat transfer coefficients on pin fins and on endwall (base plate) of the channel have been achieved respectively. The total mean heat transfer coefficients of pin fin channels are calculated and the resistance coefficients are also investigated. The experimental results show that heat transfer of a channel with drop-shaped pin fins is higher than that with circular pin fins while the resistance of the former is much lower than that of the latter in the Reynolds number range from 900 to 9000.

List of symbols

a relative transverse pitch [–]
a_t thermal diffusivity [m^2/s]
A mass transfer area [m^2]
A_p mass transfer area of a pin fin [m^2]
A_w mass transfer area on the endwall [m^2]
b relative streamwise pitch [–]
c_w concentration of naphthelene vapour on surface [kg/m^3]
c_i concentration of naphthelene vapour in air ahead of the test section [kg/m^3]
c_e concentration of naphthelene vapour in air behind the test section [kg/m^3]
D diameter [m]
D_π equal circumference diameter ($D_\pi = U/\pi$) [m]
D_{AB} mass diffusion coefficient [m^2/m]
E fin efficiency [–]
H pin fin length [m]
k constant [–]
Δm mass loss [kg]
N number of pin fin rows [–]
Δ_p pressure drop [N/m^2]
Q volume flow rate [m^3/s]
$S1$ spanwise pitch [m]
$S2$ streamwise pitch [m]
S_p cross section area of a pin fin [m^2]
u maximum flow velocity [m^2/s]
U circumference of pin fins [m]
Z total number of pin fin rows [–]

Greek letters

α total heat transfer coefficient [$W\,m^{-2}\,K^{-1}$]
α_p heat transfer coefficient on pin fins [$W\,m^{-2}\,K^{-1}$]
α_w heat transfer coefficient on the endwall [$W\,m^{-2}\,K^{-1}$]
β mass transfer coefficient [m/s]
β_p mass transfer coefficient on pin fins [m/s]
β_w mass transfer coefficient on endwall [m/s]
λ thermal conductivity of air [$W\,m^{-1}\,K^{-1}$]
λ_p thermal conductivity of pin fin material [$W\,m^{-1}\,K^{-1}$]
ρ density of air [kg/m^3]
τ sublimating time [s]
μ dynamic viscosity [Pa s]
ν kinematic viscosity [m^2/s]

Dimensionless numbers

Eu Euler number ($\mathrm{Eu} = \Delta p/(\rho u^2 Z)$)
Nu Nusselt number ($\mathrm{Nu} = \alpha D_\pi/\lambda$)
Pr Prandtl number ($\mathrm{Pr} = \nu/a_t$)
Re Reynolds number ($\mathrm{Re} = u D_\pi/\nu$)
Sc Schmidt number ($\mathrm{Sc} = \nu/D_{AB}$)
Sh Sherwood number ($\mathrm{Sh} = \beta D_\pi/D_{AB}$

1 Introduction

Many investigations have been recently reported for heat transfer and pressure drop of narrow channels with short pin fins [1, 2, 3], because of their wide applications in the trailing edges of gas turbine blades, in some modern electronic systems and in aerospace industry. The most important findings of the prior research works are that heat transfer characteristics of short cylinders are quite different both from those of long cylinders (with large length-to-diameter ratio H/D, as used in crossflow heat exchangers) and from those of channels with very short cylinders (with very small H/D, as the plate-pin-and-tube heat exchangers). Generally to say, the overall heat transfer rate of channels with short pin fins is lower than that of channels with long cylinders.

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It is well known that tubes or cylinders with streamline-shaped cross section have much less flow resistance than circular ones. On the other hand, it is by no means clear that circular pin fins are the best for heat transfer. But most of the recent researches are restricted to pin fins with circular cross section. The research on cylinders or tubes with other cross sections is relatively little. Ota et al. [4] studied the heat transfer and flow around a long elliptic cylinder of axis ratio 1:3. Their experimental results show that heat transfer coefficients of the elliptic cylinder is higher than that of a circular one with equal circumference and the pressure drag coefficients of the former are much lower than that of the latter. Li et al. [5] have recently investigated the convective heat transfer and pressure drop for arrays of long drop-shaped cylinders in crossflow. Their conlusions show that the mean heat transfer coefficients of drop-shaped cylinder arrays are about 8%$\sim$29% higher than those of the corresponding circular cylinder arrays, and the pressure drop of the former is only about 1/2 of the latter. Metzger et al. [6] studied the heat transfer and pressure drop characteristics for an array of oblong pin fins at various attack angles. Their experiments show that heat transfer of oblong pin fin arrays increases of approximately 20% over that of corresponding arrays with round pin fins, but these increases are offset by increases in pressure loss of about 100%.

All the above experimental results proved that cylinder shapes affect greatly the heat transfer characteristics and the resistance of channels with cylinder arrays. Therefore, it is very necessary to investigate further the pin fins with other cross sections in order to enhance heat transfer and to save energy.

The present experiment is to investigate the heat transfer and pressure drop characteristics in a narrow duct with staggered arrays of drop-shaped pin fins. By employing the heat/mass transfer analogy and the naphthalene sublimation technique, the heat transfer coefficients on pin fins and on the endwall (base plate) were measured respectively in order to find out their own contribution to the total heat transfer of the pin fin channel. The comparison with circular pin fins is made to find out the advantages of the drop-shaped pin fins.

2 Experimental apparatus and technique

2.1 Wind tunnel

The suction-mode wind tunnel used in this experiment is shown in Fig. 1. The test section has a cross section 240 mm wide and 20 mm high and was made from 10 mm thick plexiglass. The blower is connected to the wind tunnel with flexible tube in order to reduce the mechanical vibration of the system.

2.2 Equipment for the measurements

The flow velocity in the wind tunnel is measured with a rotor flow meter (see Fig. 1). The pressure drop has been achieved through measuring the static pressure ahead of and behind the test section by means of pressure taps

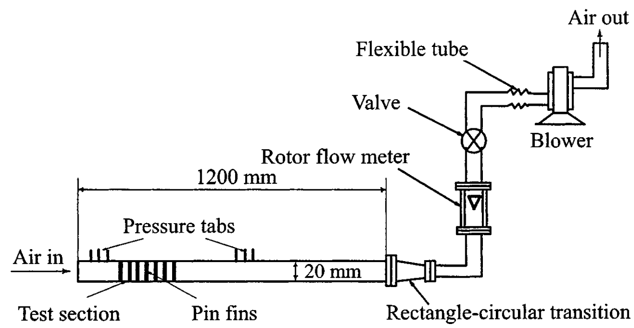

Fig. 1. Wind tunnel system

together with micromanometer. An analytical balance was used to measure the mass loss Δ m during the test run and air temperature in the wind tunnel is measured both with thermocouples and with standard glass thermometers.

2.3 Experimental specimens

The configuration of the drop-shaped pin fins and their arrangement are as shown in Fig. 2. Its major diameter is $d1 = 8$ mm and the minor diameter $d2 = 3$ mm. The distance between centers of the two circles is $l = 9.5$ mm. Thus the equal-circumference-diameter of pin fins is $D_\pi = 11.5$ mm (see Eq. (1)). The height of pin fins is 20 mm (equal to the height of the wind tunnel). The relative spanwise pitches are $a = S1/D_\pi = 1.4 \sim 3.0$ and in flow direction $b = S2/D_\pi = 1.5 \sim 3.0$. In the experiments only one pin fin is made of naphthalene for each run and the others are made of wax.

By manufacturing the pin fins, the melted analytical naphthalene (or wax) was poured into a specially designed steel mould that had been painstakingly fabricated in order to provide casting with very smooth surfaces. The pin fin specimens are formed when they cold down. The test specimens for heat/mass transfer on endwall are also manufactured by casting. The mould for the naphthalene specimens is shown in Fig. 3.

2.4 Experimental technique and procedures

The naphthalene sublimation technique is employed in the present research. That is, by means of measuring the mass loss before and after a test run the mean heat transfer coefficients can be achieved by heat/mass transfer analogy.

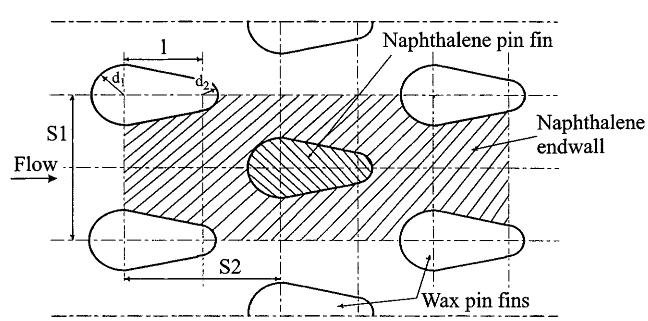

Fig. 2. Configuration and arrangement of drop-shaped pin fins

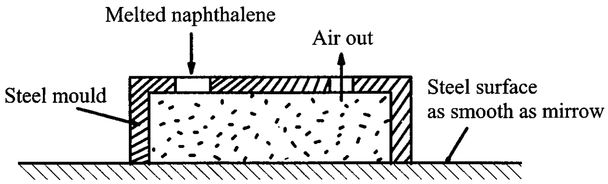

Fig. 3. Mould for casting specimens of endwall

According to its symmetry, the local modeling method is adopted in the experiment and the mass transfer is confined to the oblique line area of Fig. 2. The pin fin made of naphthalene is always placed in the middle of the 6th row in 10-rows-pin fin-arrays for every test run for mean Nu. This means, the experimental results can be used for fully developed flow cases and for the arrays with only a few pin fin rows must be modified by using the row-by-row variation characteristics of heat transfer (see Fig. 4).

3
Data reduction

All Reynolds numbers in this paper are based on the equal-circumference-diameter D_π of the elliptic pin fins and velocities u calculated using the minimum flow area in pin fin arrrays, therefore

$$D_\pi = U/\pi \tag{1}$$

$$\mathrm{Re} = D_\pi u/\nu \tag{2}$$

here U is the circumference of an elliptic cylinder.

The mean mass transfer coefficient is defined as

$$\beta = \frac{\Delta m}{A\tau\Delta c} \tag{3}$$

where Δm is the sublimating mass during the test time τ on the sublimating surface area A and the logarithmic-mean concentration difference is

$$\Delta c = \frac{(c_w - c_i) - (c_w - c_e)}{\ln[(c_w - c_i)/(c_w - c_e)]} \tag{4}$$

here c_w is the naphthalene vapour concentration on sublimating surface and can be obtained by naphthalane vapour pressure equation [9] together with the ideal gas law. The c_i and c_e are the naphthalene vapour concentrations in the air ahead of and behind the test section in the wind tunnel. In the present work we have

$$c_i = 0 \tag{5}$$

$$c_e = \frac{\Delta m}{\tau Q} \tag{6}$$

Thus the average Sherwood number of the pin fins is

$$\mathrm{Sh}_p = \beta_p D_\pi/D_{AB} \tag{7}$$

and the Sherwood number on endwall is

$$\mathrm{Sh}_w = \beta_w D_\pi/D_{AB} \tag{8}$$

According to heat/mass transfer analogy, we have

$$\mathrm{Nu}_p = \frac{\alpha_p D_\pi}{\lambda} = \left(\frac{\mathrm{Pr}}{\mathrm{Sc}}\right)^n \mathrm{Sh}_p \tag{9}$$

and

$$\mathrm{Nu}_w = \frac{\alpha_w D_\pi}{\lambda} = \left(\frac{\mathrm{Pr}}{\mathrm{Sc}}\right)^n \mathrm{Sh}_w \tag{10}$$

The total mean Nusselt number of the duct with pin fins can be calculated as

$$\mathrm{Nu} = \frac{\alpha D_\pi}{\lambda} = \frac{(\mathrm{Nu}_p A_p E + \mathrm{Nu}_w A_w)}{A_p + A_w} \tag{11}$$

where A_p and A_w are the effective heat transfer surface area on pin fins and on endwall respectively. E is the fin efficiency

$$E = \frac{\tanh(mh)}{mh} \tag{12}$$

here h is the height of pin fins and m can be calculated as

$$m = \left(\frac{\alpha_p U}{\lambda_p S_p}\right)^{1/2} \tag{13}$$

here λ_p is the heat conduction coefficient of pin fins and S_p is the cross section area of a pin fin.

The Euler number for representing the pressure drop is defined as

$$\mathrm{Eu} = \frac{\Delta p}{\rho u^2 Z} \tag{14}$$

where Δp is the overall pressure drop of the test section and Z is total row number of pin fin arrays.

4
Experimental results and discussion

4.1
Heat transfer characteristics of pin fins in flow direction

For channels with pin fins or tube banks, the convective heat transfer varies in the flow direction with the developing of flow. It is essential to know the Nusselt number Nu distribution along both the streamwise and the spanwise direction of the channel when experimental results are applied in design of heat exchangers in practice, especially in the cases of only a few rows.

The row-by-row variation of Nu_p for drop-shaped pin fin arrays is shown in Fig. 4. As expected, the 1st row has lower heat transfer rate than inner rows, but the difference is only about 19% $\sim$ 26%, much less than that for arrays of long circular cylinders (30% $\sim$ 100% generally)[8]. The Nu_p of 2nd and 3rd row is almost the same as that of inner rows at lower Re, but less than others at higher Reynolds numbers. The unexpectable characteristic is that the Nusselt numbers of 4th row are somewhat higher at all three Reynolds numbers than those of inner rows. The Nu_p tends to keep constant from the 5th row. Therefore, it can be said that the flow and mass transfer is from 5th row fully developed.

4.2
Effect of the sidewalls

It is clear that the boundary layer of side walls will influence on flow and heat/mass transfer near the wall. In order to detect the extent of this influence, a special test run was performed at 6th row with 11 cylinders. It can be seen from Fig. 5 that only two pins near the sidewall have somewhat lower Nu_p than those in the middle of the row. The others have almost the same heat transfer rate. Therefore, in order to avoid the influence of entrance and the sidewalls, the test specimens for measuring the mean Nu_p are always placed at 6th row and in the middle in spanwise direction.

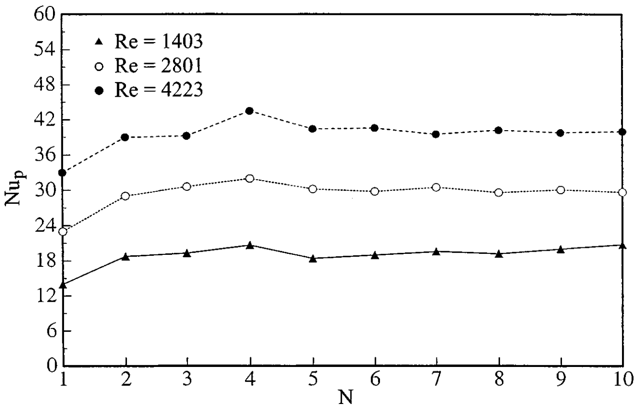

Fig. 4. Nu_p development for drop-shaped pin fin arrays

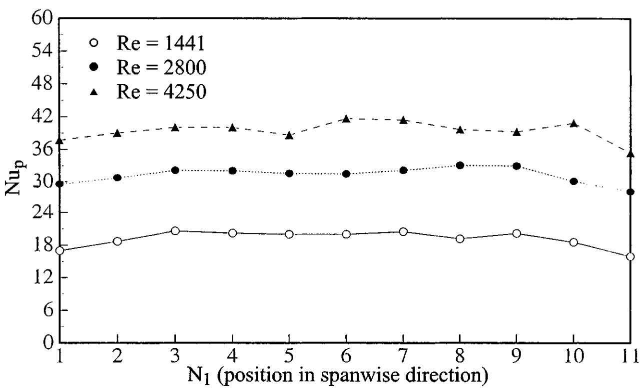

Fig. 5. Nu_p development for drop-shaped pin fin arrays in the spanwise direction

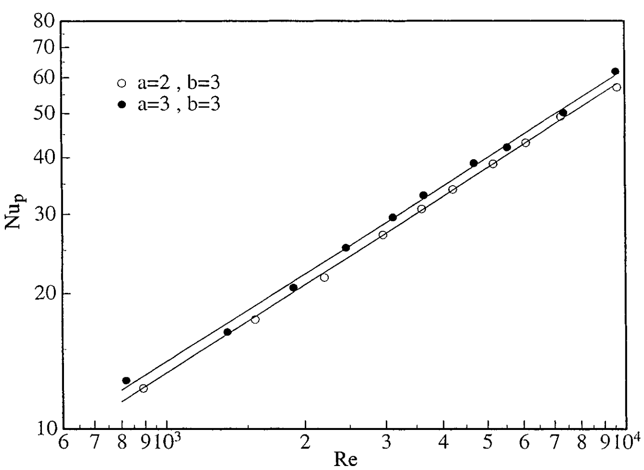

Fig. 6. Nu_p versus Re for $b = 3.0$

4.3
Heat transfer on pin fins of inner rows

Heat transfer on pin fins of inner rows at various arrangements is shown in Fig. 6 and Fig. 7. From the figures we can see that Nu_p gets higher with the increasing of spanwise pitches a when the longitudinal pitch b keeps constant, but it decreases with increasing of the streamwise spacing between pin fins when the transverse pitch is constant. This regularity is the same as that in the cases of circular tube banks [8]. But the differences of Nu_p for various arrangements are not so great as in the latter case.

4.4
Heat transfer on the endwall

Fig. 8 displays the effect of arrangements on heat transfer rate of the endwall. It is obvious that Nu_w is higher when the pin fin array is more compact. Nevertheless, the influence of a and b appears to be not regular.

The comparison of Nu_p with Nu_w has been presented in Fig. 9. At lower Reynolds numbers, heat transfer on pin fins is distinctively greater than that of the endwall. With increasing of Re the difference becomes less and less and finally Nu_p tends to be equal to Nu_w at higher Reynolds number.

4.5
Overall mean heat transfer

The overall mean Nusselt numbers calculated by Eq. (11) are displayed in Fig. 10 and Fig. 11. Here the fin efficiency

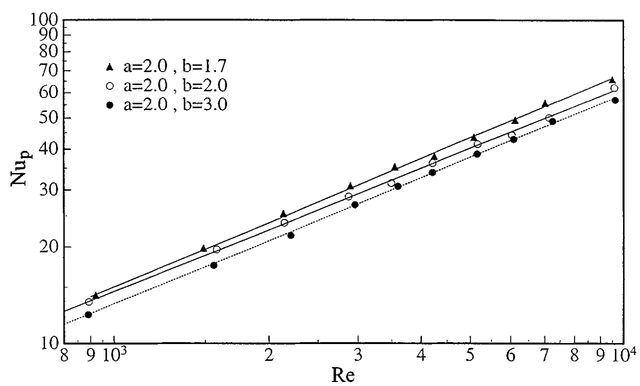

Fig. 7. Nu_p versus Re for $a = 2.0$

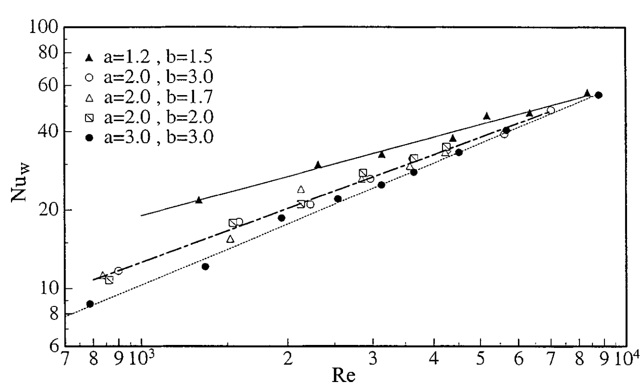

Fig. 8. Effect of arrangements on the heat transfer rate of the endwall

is considered for steel heat exchangers in practice. The effect of arrangements is similar to that on pin fins, i.e., Nu becomes higher with increasing of a and gets lower with increasing of b.

The heat transfer results for long circular cylinders [8] and for channel with round pin fins [9] are also presented in Fig. 11 for comparison. The mean Nu of circular cylinders (inner rows) with large H/D is considerably higher than that of corresponding arrays of short circular or drop-shaped pin fins. The length-to-diameter ratio of round pin fins in reference [9] is only $H/D = 1.0$ (as shown in Fig. 11). But the prior research work [3] show that mean heat transfer of channels with pin fins is not dependent on length-to-diameter ratio when $H/D < 2.0$. It can be seen from Fig. 11 that the Nusselt numbers of a channel with drop-shaped pin fins are slightly higher than those of circular ones.

The experimental results for overall mean heat transfer can be expressed by the minimum squares fits as the following

$$Nu = K_1 \ Re^{k_2} \qquad (15)$$

The constants k_1 and k_2 for various arrangements are listed in Table 1.

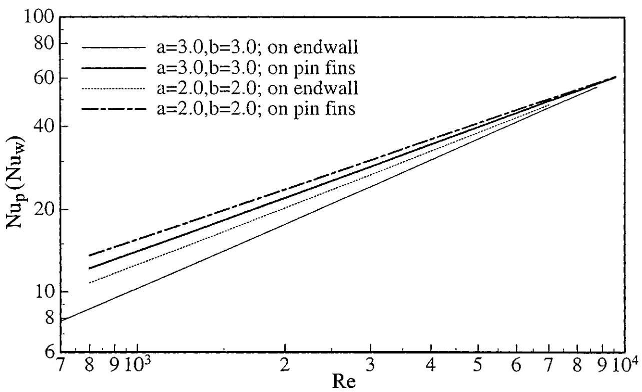

Fig. 9. Comparison of Nu_p with Nu_w

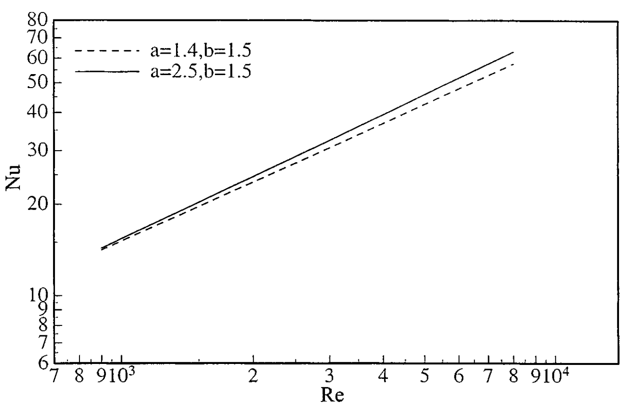

Fig. 10. Mean heat transfer versus Reynolds number for $b = 1.5$

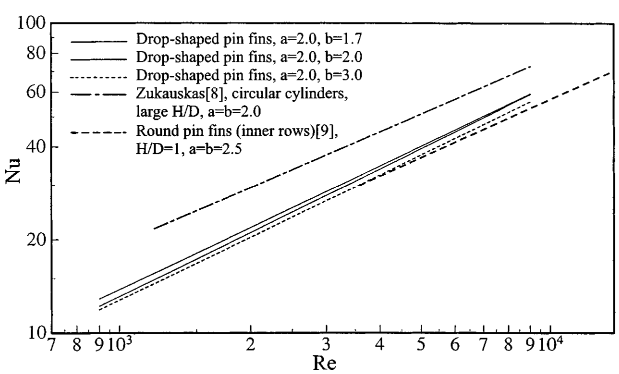

Fig. 11. Mean heat transfer versus Reynolds number

4.6 Flow resistance

The flow resistance characteristics of channels with drop-shaped pin fins are shown in Fig. 12. The mean Euler numbers per row are calculated by Eq. (14). The results show that arrays with only a few rows of pin fins have higher mean Eu per row. With increasing of total number rows Z the average Euler numbers decrease at first rapidly and then (when $Z > 10$) tend to keep constant. In the following experiments for flow resistance we use always pin fin arrays with 14 rows. This means, the experiments can be used in practice for any corresponding drop-shaped pin fin arrays with more than 10 rows, for those with less than 10 rows the results must be modified by using Fig. 12.

The influence of arrangements on resistance is shown in Fig. 13 and 14. It is obvious that Eu decreases with increasing of transverse pitches because the degree of blockage becomes lower. When streamwise pitches get larger, the mean Euler numbers increase quite slightly.

The flow resistance of channels with round pin fins [9] or with long circular cylinders [8] is also displayed in Fig. 14. It can be seen that the pressure drop of drop-

Table 1. k_1 and k_2 in Eq. (15)

$a \times b$	H/D	k_1	k_2
1.4 × 1.5	1.7	0.176	0.645
2.5 × 1.5	1.7	0.142	0.679
2.0 × 1.7	1.7	0.139	0.665
2.0 × 2.0	1.7	0.115	0.685
2.0 × 3.0	1.7	0.121	0.674

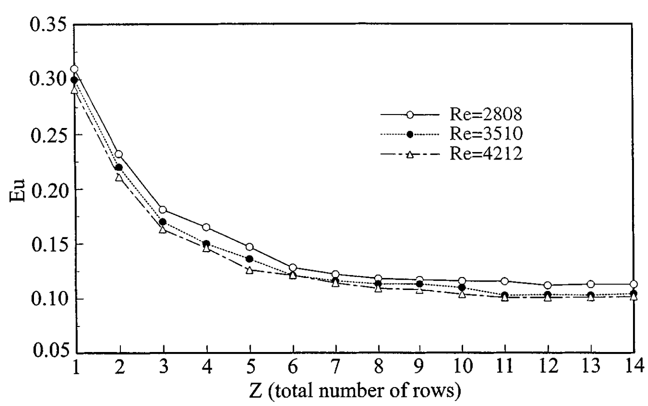

Fig. 12. Eu versus Z

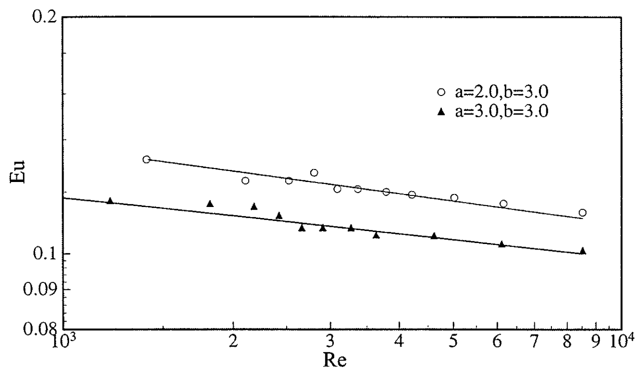

Fig. 13. Effect of pin fin arrangement on the resistance of pin fin channels

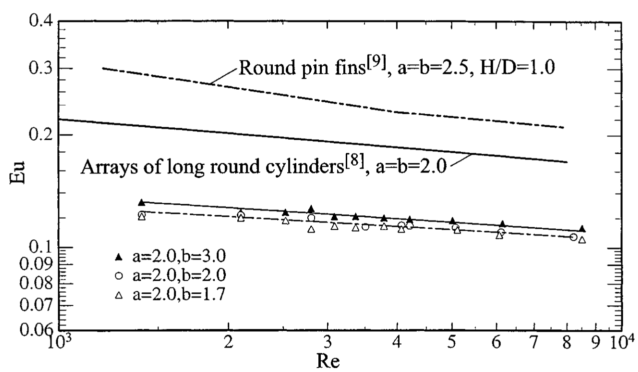

Fig. 14. Comparison of Eu with Ref. [8] and [9]

shaped pin fins is much less than that of round ones. For example, the former is only about 42% and 51% of the latter at Re = 1200 and Re = 8000 respectively.

5 Conclusions

1. The channel with drop-shaped pin fins has better heat transfer behaviours than that with circular ones in the tested Reynolds number range.
2. In the whole Reynolds number range of interest, the drop-shaped pin fin channel has much lower flow resistance than that with circular pin fins. The Euler numbers of the former are only about 42% $\sim$ 51% of the latter when Re changes from 1200 to 8000.
3. The heat/mass transfer coefficients on pin fins and on the endwall can be easily measured respectively with the naphthalene sublimation method.
4. The Nusselt numbers of pin fins are generally higher than those of the endwall in the Re range from 700 to 7000, but they are almost the same in the Re range from 7000 to 10 000.
5. The arrangement of pin fins affects both on heat transfer and on the flow resistance of the pin fin channel.
6. For rectangular ducts with short drop-shaped pin fins, the flow and mass transfer are generally from 5th row fully developed and the effect of sidewalls is restricted to one or two cylinders near the walls.

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