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Thermal dispersion effects on non-Darcy natural convection with lateral mass flux

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Abstract The method of similarity solution is used to study the influence of lateral mass flux and thermal dispersion on non-Darcy natural convection over a vertical flat plate in a fluid saturated porous medium. Forchheimer extension is considered in the flow equations and the coefficient of thermal diffusivity has been assumed to be the sum of molecular diffusivity and the dispersion thermal diffusivity due to mechanical dispersion. The suction/injection velocity distribution has been assumed to have power function form $A\,x^l$, where x is the distance from the leading edge and the wall temperature distribution is assumed to be uniform. When $l = -1/2$, similarity solution is possible, and the results indicate that the boundary layer thickness decreases where as the heat transfer rate increases as the mass flux parameter passes from injection domain to the suction domain. The increase in the thermal dispersion parameter is observed to enhance the heat transfer. The combined effect of thermal dispersion and fluid suction/injection on the heat transfer rate is discussed.

List of symbols

- A constant
- C empirical constant
- d pore diameter
- f non-dimensional streamfunction
- f_w non-dimensional mass flux parameter
- g gravitational constant
- K permeability
- k molecular thermal conductivity
- k_d dispersion thermal conductivity
- k_e effective thermal conductivity
- l real constant
- p pressure
- q local heat flux
- T temperature
- u, v velocity components in the x and y directions
- v_w dimensional mass flux parameter
- x, y Cartesian coordinates

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Greek symbols

- ρ fluid density
- μ viscosity of the fluid
- v fluid kinematic viscosity
- α_x, α_y thermal diffusion coefficients in x and y directions respectively
- α molecular thermal diffusivity
- α_d dispersion diffusivity
- β thermal expansion coefficient
- γ mechanical dispersion coefficient
- δ_T boundary layer thickness evaluated at $\theta(\eta) = 0.01$
- η similarity parameter
- ψ dimensional streamfunction
- θ non-dimensional temperature

Subscripts

- w evaluated on the wall
- ∞ evaluated at the outer edge of the boundary layer

1 Introduction

Study of convective heat transfer in porous media has been the interest of several researches owing to its wide applicability in engineering and geophysical problems such as in oil recovery technology, in the use of fibrous materials for thermal insulations, in the design of aquifer as an energy storage system, in utilization of porous layers for transpiration cooling by water for fire fighting, and also in Resin Transfer Molding process in which fibre reinforced polymeric parts are produced in final shape.

Most of the works dealing with convective heat transfer in porous media have been motivated by geothermal applications. Understanding the formation of geothermal reservoirs and its utilization for energy extraction needs thorough understanding of the convection in porous media. Inspite of the fact many of the geothermal reservoirs are known to be fracture dominated, the studies based on the idealization of geothermal reservoir as a saturated porous medium can provide considerable insight into the physical process involved. To understand the convection phenomena in the geothermal reservoir, fluid is injected from the side walls and at the same time, the response of the fluid reservoir resulting from the sudden heating is observed. A nice review about heat transfer in geothermal systems has been presented in Cheng [1].

In this direction, the study of the effect of injection and suction of fluid along the vertical and horizontal surfaces has been the subject of numerous researches. Cheng [2] studied the effect of fluid suction and injection the natural convection heat transfer from a vertical wall in a Darcian fluid saturated porous medium. He obtained similarity solution for different wall temperature variations and for different injection/suction velocity distributions. But unfortunately, when both the wall temperature and injection/suction velocity remain constant, similarity solution is not possible. This realistic case has been dealt by Merkin [3]. A series solution method has been adopted. The asymptotic analysis which is valid at large distances from the wall has also been done. It was found that for the case of withdrawal, the boundary layer remains very thin and settles down quickly to one of constant thickness.

When the pore diameter dependent Reynolds number is high enough for the Darcy model to break down, Forchheimer extension has been used by Plumb and Huenefeld [4], Bejan and Poulikakos [5], and Nakayama et al. [6], to study the non-Darcy natural convection from the vertical wall. All these studies assume that the thermal diffusivity is constant. But under the conditions at which the inertial effects are prevalent, the thermal dispersion effect become significant as observed in Plumb [7], Hong and Tien [8], Nield and Bejan [9]. In the present paper, we aim at studying the effect of lateral mass flux on the Forchheimer free convection over a vertical wall when thermal dispersion effects are considered and neglected. It has been observed that the similarity solution is possible only for uniformly heated hot wall with injection/suction velocity varying as $A\,x^{-1/2}$. Results indicate the general trend that the heat transfer rate increases as the mass flux parameter passes from injection domain to suction domain, and the increase in thermal dispersion parameter further increases the heat transfer.

2
Governing equations

Consider the problem of non-Darcy natural convection flow and heat transfer over a semi infinite vertical surface in a fluid saturated porous medium as shown in the Fig. 1. The isothermal hot wall is assumed to be permeable with a lateral mass flux in the form $v_w(x) = A\,x^l$. $x = 0$ represents the leading edge of the hot wall. Here it is worth noting that $v_w = 0$ corresponds to the impermeable wall case. The wall temperature T_w is considered greater than the ambient temperature T_∞. The governing equations for the flow and heat transfer from the wall $y = 0$ into the fluid saturated porous medium $x \geq 0$ and $y > 0$ in this case are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u + \frac{C\sqrt{K}}{\nu}u^2 = -\frac{K}{\mu}\left(\frac{\partial p}{\partial x} + \rho g\right) \tag{2}$$

$$v + \frac{C\sqrt{K}}{\nu}v^2 = -\frac{K}{\mu}\left(\frac{\partial p}{\partial x}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial x}\left(\alpha_x\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\alpha_y\frac{\partial T}{\partial y}\right) \tag{4}$$

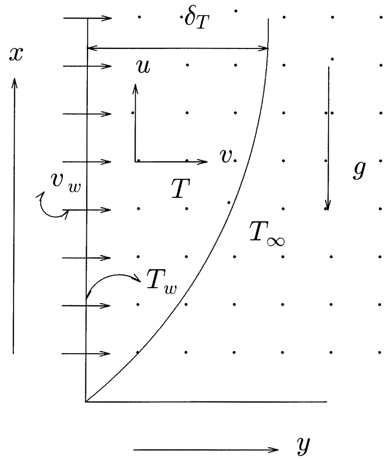

Fig. 1. External natural convection over a vertical wall in a fluid saturated porous medium

$$\rho = \rho_\infty[1 - \beta(T - T_\infty)] \tag{5}$$

along with the boundary conditions

$$\left.\begin{aligned} y = 0, v_w(x) = Ax^l, T_w = \text{const} \\ y \to \infty, u = 0, T \to T_\infty \end{aligned}\right\} . \tag{6}$$

Here x and y are the Cartesian coordinates, u and v are the Darcian velocity components in x and y directions, p is the pressure, T is the temperature, K is the permeability constant, C is an empirical constant, β is the coefficient of thermal expansion, μ is the viscosity of the fluid, ν is the kinematic viscosity, ρ is the density, g is the acceleration due to gravity, α_x and α_y are the components of thermal diffusivity in x and y directions. The suffix w and ∞ indicate the conditions at the wall and at the outer edge of the boundary layer respectively.

Experimental and numerical studies on convective heat transfer in a porous medium show that thermal boundary layers exist adjacent to the heated or cooled walls. When the thermal boundary layer is thin (i.e., $x \gg y \sim \delta_T, \delta_T$ is the boundary layer thickness), boundary layer approximations analogous to classical boundary layer theory can be applied [9]. Near the boundary, the normal component of seepage velocity is small compared with the other component of the seepage velocity and the derivatives of any quantity in the normal direction are large compared with derivatives of the quantity in the direction of the wall. Under these assumptions, the Eqs. (1)–(5) become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$u + \frac{C\sqrt{K}}{\nu}u^2 = -\frac{K}{\mu}\left(\frac{\partial p}{\partial x} + \rho g\right) \tag{8}$$

$$\frac{\partial p}{\partial y} = 0 \tag{9}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha_y\frac{\partial T}{\partial y}\right) \tag{10}$$

Eliminating the pressure and invoking the Boussinesq approximation, the Eqs. (8)–(10) become

$$\frac{\partial u}{\partial y}+\frac{C\sqrt{K}}{v}\frac{\partial u^2}{\partial y}=\left(\frac{Kg\beta}{v}\right)\frac{\partial T}{\partial y} \tag{11}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha_y\frac{\partial T}{\partial y}\right) \tag{12}$$

Here α_y is a variable quantity which is the sum of molecular thermal diffusivity α and dispersion thermal diffusivity α_d. Following Plumb [7], the expression for dispersion thermal diffusivity will be $\alpha_d = \gamma\, d\, u$, where γ is the mechanical dispersion coefficient whose value depends on the experiments and d is the pore diameter.

3
Similarity solution

First we represent the governing Eqs. (11) and (12) in terms of streamfunction and temperature formulation. The velocity components u and v can be written in terms of streamfunction ψ as: $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$. This representation is valid since the expressions for velocity components clearly satisfy the continuity equation. Now the resulting equations are

$$\frac{\partial^2\psi}{\partial y^2}+\frac{C\sqrt{K}}{v}\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right)^2=\frac{Kg\beta}{v}\frac{\partial T}{\partial y} \tag{13}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left([\alpha+\alpha_d]\frac{\partial T}{\partial y}\right) \tag{14}$$

Comparing the order magnitudes of Darcy and buoyancy terms in the momentum equation, we get the order magnitude estimate for ψ as

$$\psi \sim Ra_x\,\alpha\frac{\delta_T}{x} \tag{15}$$

where Ra_x is the modified Rayleigh number, $Ra_x=\frac{Kg\beta(T_w-T_\infty)x}{\alpha v}$. The energy equation gives the order magnitude estimate for ψ as

$$\psi \sim \frac{\alpha x}{\delta_T} \tag{16}$$

From the above estimates for ψ, we get an estimate for the boundary layer thickness δ_T as

$$\delta_T \sim xRa_x^{-1/2}\ . \tag{17}$$

Now, the similarity variable η which is defined as

$$\eta=\frac{y}{\delta_T} \tag{18}$$

will become

$$\eta=\frac{y}{x}Ra_x^{1/2}\ . \tag{19}$$

Then from the above expressions, we obtain the non-dimensional streamfunction as

$$f(\eta)=\frac{\psi}{\alpha Ra_x^{1/2}} \tag{20}$$

and write the non-dimensional temperature distribution as

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\ . \tag{21}$$

Now Eqs. (19), (20) and (21) constitute the similarity transformation if this set transforms the governing partial differential Eqs. (13) and (14) into ordinary differential equations with x being eliminated completely explicitly and also from the boundary conditions. Then from the definition of the streamfunction, the velocity components become

$$u=\frac{\alpha}{x}Ra_xf'(\eta)\ , \tag{22}$$

$$v=-\frac{\alpha}{2x}Ra_x^{1/2}[f-\eta f']\ . \tag{23}$$

Applying the similarity transformation to the governing Eqs. (11) and (12), we get

$$f''+2F_oRa_df'f''-\theta'=0 \tag{24}$$

$$\theta''+\frac{1}{2}f\theta'+\gamma Ra_d(f'\theta')'=0 \tag{25}$$

and the boundary conditions (6) are transformed as

$$\left.\begin{array}{l}\eta=0, f=f_w, \theta=1\\ \eta\to\infty, f'=0, \theta=0\end{array}\right\}\ . \tag{26}$$

On the wall $(\eta=0)$ Eq. (23) becomes

$$v_w(x)=-\frac{\alpha}{2x}Ra_x^{1/2}f_w \tag{27}$$

and the particular value of l for which v_w will be free from x is $l=-1/2$. So with this value of l, $f=f_w$ will become constant and the boundary conditions also become free from x. Thus the resulting ordinary differential equations with the boundary conditions can be solved using the generalized techniques for solving ordinary differential equations. The negative power distribution for injection/suction will lead to infinite injection/suction at the leading edge, which is unrealistic, but the method of similarity solution will still give accurate results sufficiently far from the leading edge.

The governing parameters are identified as F_o, Ra_d and Ds. The parameter $F_o=\frac{C\sqrt{K}\alpha}{vd}$ represents the structural and thermophysical properties of the porous medium, $Ra_d=\frac{kg\beta(T_w-T_\infty)d}{\alpha v}$ is the pore diameter dependent Rayleigh number which describes the relative intensity of the buoyancy force, and the dispersion parameter $Ds=\gamma Ra_d$ represents the thermal dispersion effects, γ is the mechanical dispersion coefficient. In general γ should be found out from the experiment, and it has been observed from the previous experimental results that its value lies between 1/7 and 1/3 [7]. For all calculations, γ is assigned a value 0.3 in the present study. Note that $F_o=0$ correspond to the Darcian free convection and $\gamma=0$ represents the case where the thermal dispersion effects neglected.

4
Results and discussion

The resulting ordinary differential Eqs. (24) and (25) with the corresponding boundary conditions (26) are solved by numerical integration using the fourth-order Runge-Kutta method and Newton-Raphson technique by giving proper guess values for $f'(0)$ and $\theta'(0)$. The present results are accurate upto the sixth decimal place. The present analysis consists of two related problems. These are (a) the effect of lateral mass flux on the Forchheimer free convection and (b) the combined effect of thermal dispersion and lateral mass flux on Forchheimer free convection in a fluid saturated porous medium. To understand the effect of various parameters on the free convection process, in the present analysis, the parameter F_o is varied from 0 to 1.0, the Rayleigh number Ra_d is varied from 1.0 to 10. The mass flux parameter f_w is varied from -1.0 to 1.0. It is clear from the analysis that $f_w = 0$ corresponds to the impermeable surface, $f_w > 0$ corresponds to suction and $f_w < 0$ corresponds to injection of the fluid into the porous medium.

The flow field and the temperature distribution are presented in terms of the non-dimensional velocity component in the x-direction $f'(\eta)$ and non-dimensional temperature distribution $\theta(\eta)$. Figs. 2 and 3 correspond to $f'(\eta)$ and $\theta(\eta)$ versus the similarity variable η by considering and neglecting the thermal dispersion effects for three different values of the non-dimensional mass flux parameter f_w, for fixed value of $F_o = 0.1$ and $Ra_d = 5.0$. In both the cases, the velocity and temperature profiles thicken as the mass flux parameter passes from the suction domain to the injection domain. Also it is worth noting here that the increase in the value of the parameter F_o will thicken these profiles, because the Forchheimer term accounts for the form drag in the porous medium.

The boundary layer thickness δ_T as a function of the mass flux parameter is plotted in the Fig. 4 for varying values of the dispersion parameter. The value of the similarity variable at which $\theta(\eta)$ becomes equal to 0.01 is noted as the boundary layer thickness. From (17), it is noted that the boundary layer thickness varies inversely as 1/2 power of the Rayleigh number. From the definition of the dispersion parameter it is clear that Ds varies directly linearly with the Rayleigh number. By fixing $F_o = 0.1$ and $Ra_d = 5.0$, the increase in the value of $\gamma(0 - 0.3)$ is observed to increase the boundary layer thickness as seen from the Fig. 4. Also, the boundary layer thickness decreases as the mass flux parameter moves from the injection domain to the suction domain, and favors the heat transfer in the suction domain.

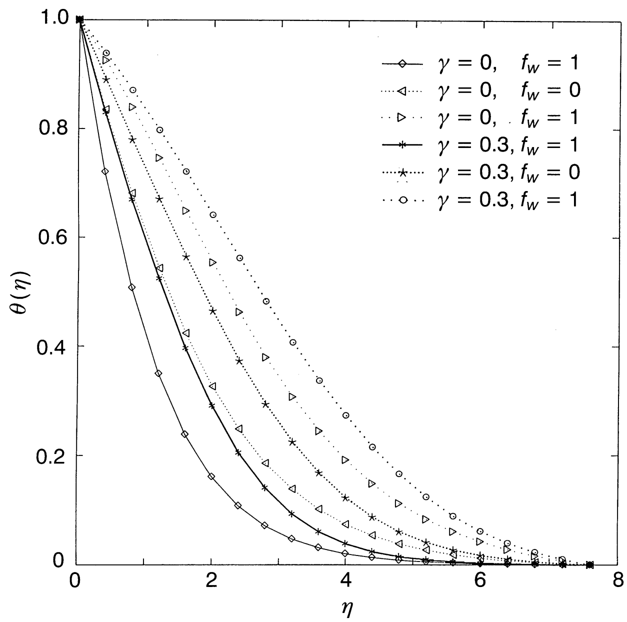

Fig. 3. Variation of $\theta(\eta)$ with similarity variable η for $F_o = 0.1$, $Ra_d = 5.0$, for varying dispersion and mass flux parameter values

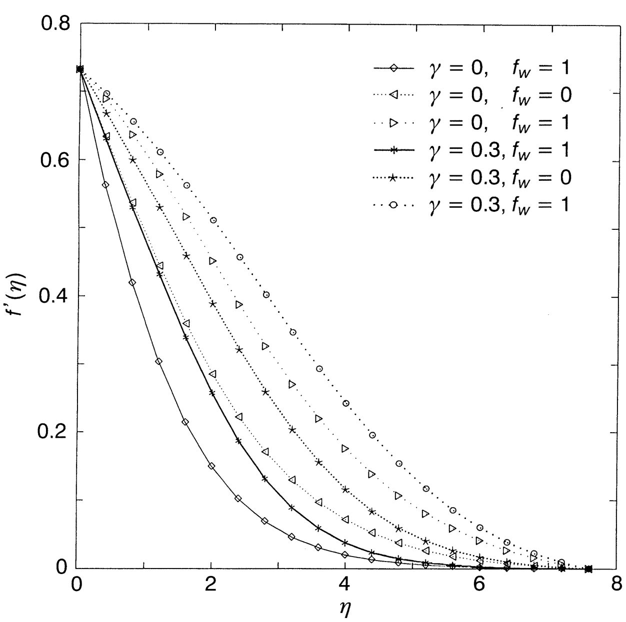

Fig. 2. Variation of $f'(\eta)$ with similarity variable η for $F_o = 0.1$, $Ra_d = 5.0$, for varying dispersion and mass flux parameter values

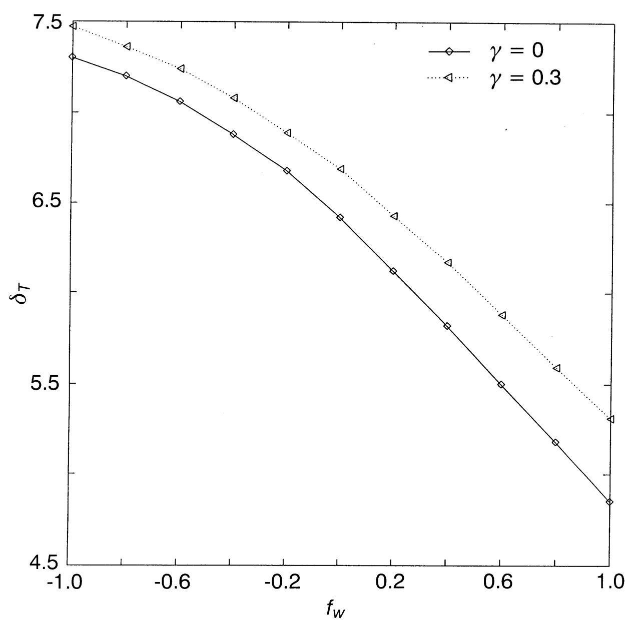

Fig. 4. Variation of boundary layer thickness δ_T with mass flux parameter in the non-Darcy ($F_o = 0.1, Ra_d = 5.0$) porous medium for varying values of dispersion parameter

Table. 1. Combined effect of thermal dispersion and surface mass flux on Nusselt number results

Ra_d	$\gamma = 0$			$\gamma = 0.3$		
	$f_w = -1$	$f_w = 0$	$f_w = 1$	$f_w = -1$	$f_w = 0$	$f_w = 1$
1.0	0.1909	0.4295	0.7740	0.2293	0.4709	0.8103
2.0	0.1804	0.4180	0.7641	0.2479	0.4911	0.8284
5.0	0.1579	0.3929	0.7424	0.2847	0.5308	0.8646

The local heat transfer rate which is the primary interest of the study is given by

$$q = -k_e \frac{dT}{dy}\Big|_{y=0} = -[k + k_d]\frac{dT}{dy}\Big|_{y=0} \tag{28}$$

where k_e is the effective thermal conductivity of the porous medium which is the sum of the molecular thermal conductivity k and the dispersion thermal conductivity k_d.

The heat transfer coefficient in terms of Nusselt number is given by

$$\frac{Nu}{Ra_x^{1/2}} = [1 + Dsf'(0)][-\theta'(0)] \tag{29}$$

Nusselt number results for varying values of the F_o and Ra_d are presented in Table 1 for $\gamma = 0$ and $\gamma = 0.3$. Since the value of $f'(0)$ is always positive, it can be noticed from Eq. (29) that dispersion always enhances the heat transfer coefficient. In Fig. 5 the Nusselt number results are plotted as a function of mass flux parameter for varying Ra_d, fixing γ at 0.3. From this figure, it is clear that the value of the Nusselt number increases as the non-dimensional mass flux parameter moves from the injection domain to suction domain. Moreover, it has been observed that the increase in the value of the dispersion parameter enhances the Nusselt number. Also, the increase in the value of the parameter F_o decreases the heat transfer rate. The combined effect of thermal dispersion and surface mass flux on natural convection heat transfer over the vertical wall in porous medium is that the Nusselt number increases as the mass flux parameter moves from injection domain to suction domain. But the relative increase in the Nusselt number values of injection, no injection/no suction and suction domains is observed to be enhanced with the increase in the value of Ra_d.

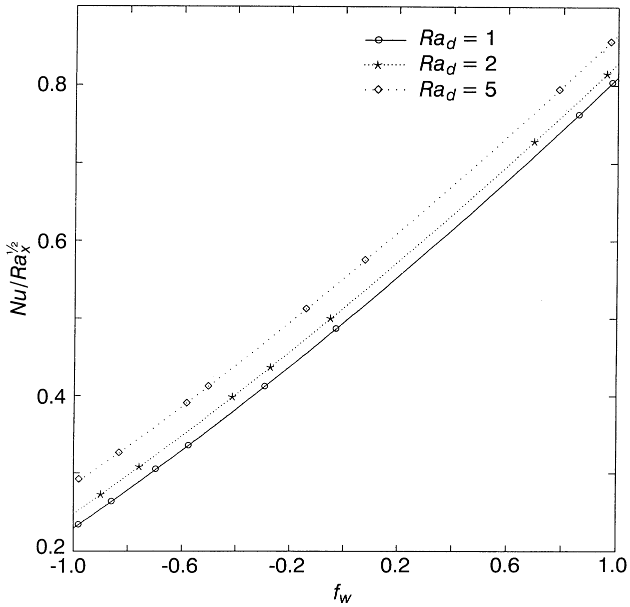

Fig. 5. Variation of Nusselt number results with mass flux parameter for fixed $F_o = 0.1$, $\gamma = 0.3$ and varying values of the Rayleigh number Ra_d

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