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Modelling for heat and mass transfer parameters in deep-frying of products

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Abstract In this paper, the development of new models to determine heat and mass transfer parameters (HMTPs) in terms of the thermal diffusivity, heat transfer coefficient, moisture diffusivity and moisture transfer coefficient for slab, cylindrical, and spherical products being deep-fried was presented. In the model development, two cases of the Biot number, i.e., $0 < Bi < 100$, and $Bi \geq 100$ in the transient heat and mass transfer were considered. In order to verify the models, frying experiments were performed using the cylindrically shaped potatoes as test samples, and these samples were fried in a deep fryer at 180°C. The lag factor and frying coefficient for a frying process, which affect heat and mass transfer parameters, were first-defined and considered most important process parameters. By using the temperature and moisture measurements, the frying coefficient and lag factor were determined and incorporated into the models. The HMTPs were determined in a simple and effective manner. In this respect, we can conclude that the present models are useful tools for determining the HMTPs for the products during frying and will be beneficial to the practical applications.

Modell zur Ermittlung von Wärme- und Stoffübergangsparametern beim Tauchgaren von Lebensmitteln

Zusammenfassung In der Arbeit werden Modelle vorgestellt, über die sich Wärme- und Stoffübergangsparameter als Funktionen der Temperatur- und Feuchteleitfähigkeit und der Wärme- und Stoffübergangskoeffizienten für platten-, zylinder- und kugelförmige Nahrungsmittel beim Tauchgaren bestimmen lassen. Im instationären Wärme- und Stoffaustauschmodell werden zwei Bereiche der Biot-Zahl betrachtet: $0 \leq Bi \leq 100$ und $Bi \geq 100$. Die Modellvorstellungen konnten durch Garexperimente bei 180°C an zylindrisch vorgeformten Kartoffelstücken verifiziert werden. Als wichtigste Prozeßparameter wurden über Temperature- und Feuchtegehaltsmessungen ein Verzögerungsfaktor und ein Garkoeffizient ermittelt und hieraus, in sehr einfacher und effektiver Weise, die interessierenden Wärme- und Stoffaustauschparameter. Es zeigte sich, daß die vorgeschlagenen Modelle recht gut geeignet sind, um die relevanten Wärme- und Stoffübergangsparameter für zum Tauchgaren bestimmte Produkte ermitteln zu helfen.

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Nomenclature

a	thermal diffusivity or moisture diffusivity in Eqs. (1) and (33)
A, B	constants
Bi	Biot number
c	constant representing thermal conductivity or moisture diffusivity in Eq. (4)
D	moisture diffusivity, m^2/s
f	frying coefficient, 1/s
Fo	Fourier number
g	lag factor
$J_0(\mu_1)$	zeroth order Bessel function of the first kind
$J_1(\mu_1)$	first order Bessel function of the first kind
h	heat transfer coefficient, $W/m^2 K$
k	moisture transfer coefficient, m/s
L	half thickness, m
Le	Lewis number
m	constant in Eq. (1)
M	moisture content, kg water/kg total matter
r	radial coordinate
r^2	correlation coefficient
R	radius, m
s	constant representing heat transfer coefficient or moisture transfer coefficient in Eq. (4)
S	drying coefficient, 1/s
t	time, s
T	temperature, °C or K
y	coordinate
Y	half thickness or radius
z	cartesian coordinate

Greek symbols

α	thermal diffusivity, m^2/s
ϕ	temperature difference or moisture content difference, °C or kg/kg
θ	dimensionless temperature or moisture content

μ	root of the transcendental characteristic equation
λ	thermal conductivity, W/mK
π	pi number

Subscripts

a	frying medium
e	equilibrium
f	final
i	initial
m	MTC
n	nth characteristic value
t	HTC
1	1st characteristic value

1 Introduction

Frying, which is a convenient method of cooking where a high temperature and rapid cooking are desired, has well-known food processing operation for hundreds of years and finds a large possibility of application in the industry. For example, deep-fried foods are one of the major items of the North American diet. Recent figures for the USA indicate that a major portion of the edible fats and oils consumed yearly are used in deep-fried foods. Over 100 million kg of fats and oils are used yearly for doughnut production and over 250 million kg for potato chip production. It is considered that the frying is one of the most energy consumed application. The popularity of deep-fried products is due in part to the basic structure imposed on them by the way in which they are cooked. In deep-frying, the food is totally immersed in the hot fat or oil as the heat transfer medium to provide the required high temperature. Cooking in this way is more efficient than the dry heat of an oven and more rapid than boiling in water since the higher temperatures possible with deep-frying result in more rapid penetration of heat into the product being cooked [1].

Therefore, frying is not simply the study of fats and oils, foods, or even processing equipment and conditions. For a product-frying process, there are two sides; first one related to the physical (such as, color, texture, flavor, etc.) and chemical (such as, oxidation, polymerization, hydrolysis, etc.) changes in the product, as well as nutritional changes and second one related to the energy technology (such as, the analysis of the heat and mass transfer, energy consumption etc.). In the relevant literature, despite many studies on the effects of the medium temperatures on the composition and nutritional aspects of fats and oils, and on the physical and chemical changes, a few studies have been done to analyze heat and mass transfer only considering the case of $Bi \geq 100$, during deepfrying of products (see, for example, [2–6]). In the energy related hand, heat and mass transfer during frying occurs from the outside to the inside of the food, as shown in Fig. 1 [7]. At the same time, moisture from the product vaporizes inside as well as outside the product and migrate through the frying media. An exact analysis of the heat and mass (moisture) transfer during a frying application is very important to provide optimum processing conditions for the physical, chemical and nutritional changes, and hence efficient frying operation, as well as leading energy saving. In this respect, there is a need to develop simple and effective models

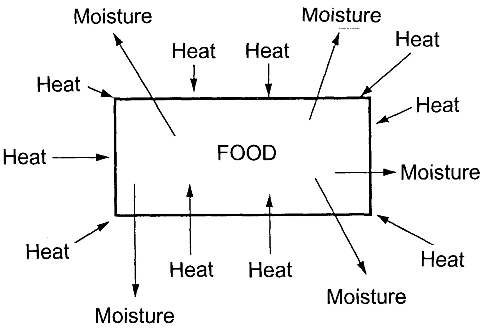

Fig. 1. Representation of heat and mass transfer during food-frying [7]

for the HMTPs for practical frying applications. The main objective of the present study is to develop such models to determine the HMTPs for slab, cylindrical and spherical products exposed to deep-frying.

2 Model development

The transient moisture diffusion process observed in frying of solid foods is similar in form to the process of transient heat conduction in these objects. The governing Fickian equation is exactly in the form of the Fourier equation of heat transfer, in which temperature and thermal diffusivity are replaced with concentration and moisture diffusivity. Therefore, one can consider similarly three different situations for the unsteady heat and mass transfer, namely as cases where the Biot number takes values $Bi \approx 0$, $0 < Bi < 100$, and $Bi \geq 100$. However, the first case corresponding to situation (i.e., lumped capacitance method) where $Bi \approx 0$ implies the negligible internal resistance to the heat and moisture transfer within the solid product. On the other hand, the remaining cases which are the most practical, i.e., $Bi \geq 100$ including the negligible surface resistance at the solid product, and $0 < Bi < 100$ leading to the inclusion of the finite internal and surface resistances. In the modelling, all cases of $0 < Bi < 100$ and $Bi \geq 100$ are investigated for both heat and moisture transfer.

Consider that an infinite slab product (ISP) (or an infinite cylindrical product (ICP) or a spherical product (SP)) is being fried in an oil medium. Assume that the heat and mass transfer parameters, and physical properties of the soild products are constant; the products are homogeneous and isotropic; the effect of heat transfer on the moisture transfer and moisture transfer on the heat transfer is negligible; initial temperatures and moisture contents of the products are uniform, and the medium temperature is constant; and the effects of the internal heat generation and oil-uptake on the temperature distribution and moisture distribution are negligible. Therefore, the transient governing equation for heat conduction and moisture diffusion in one-dimensional in one-dimensional rectangular, cylindrical, and spherical coordinates can be written in terms of the excess temperature or moisture (ϕ) compactly as follows:

$$a(1/y^m)(\partial/\partial y)[y^m(\partial\phi/\partial y)] = (\partial\phi/\partial t) \quad (1)$$

where $m = 0$, 1 and 2 for ISP, ICP and SP; $y = z$ for ISP and $y = r$ for ICP and SP; $\phi = (T - T_a)$ and $a = \alpha$ for the heat transfer case (HTC); and $\phi = (M - M_e)$ and $a = D$ for the moisture transfer case (MTC).

The following initial and boundary conditions exist:

$$\phi(y, 0) = \phi_i \quad \text{for } 0 < Bi < 100 \text{ and } Bi \geq 100 \tag{2}$$

where $\phi_i = (T_i - T_a)$ for the HTC and $\phi_i = (M_i - M_e)$ for the MTC.

$$(\partial\phi(0, t)/\partial y) = 0 \quad \text{for } 0 < Bi < 100 \text{ and } Bi \geq 100 \tag{3}$$

$$-c(\partial\phi(Y, t)/\partial y) = s\phi(Y, t) \quad \text{for } 0 < Bi < 100 \text{ and } \phi(Y, t) = 0 \quad \text{for } Bi \geq 100 \tag{4}$$

where $s = h$ for the HTC and $s = k$ for the MTC; $c = \lambda$ for the HTC and $c = D$ for the MTC; and $Y = L$ or R.

Solution to the governing equation given in Eq. (1) under the relevant conditions, i.e., Eqs. (2)–(4) with $r = 0$ yield dimensionless center temperature and moisture content of the corresponding products, i.e., ISP, ICP and SP in the following form (For further details, see [8–10].

$$\theta = \sum_{n=1}^{\infty} A_n B_n \quad \text{for } 0 < Bi < 100 \text{ and } Bi \geq 100 \tag{5}$$

where for ISP:

$$A_n = (2 \sin \mu_n)/(\mu_n) + (\sin \mu_n \cos \mu_n) \quad \text{for } 0 < Bi < 100 \text{ and}$$

$$A_n = 2(-1)^{n+1}/\mu_n \quad \text{for } Bi \geq 100 \tag{6}$$

for ICP:

$$A_n = 2Bi/(\mu_n^2 + Bi^2) + J_0(\mu_n) \quad \text{for } 0 < Bi < 100 \text{ and}$$

$$A_n = 2/(\mu_n) J_1(\mu_n) \quad \text{for } Bi \geq 100 \tag{7}$$

and for SP:

$$A_n = (2Bi \sin \mu_n)/(\mu_n) - (\sin \mu_n \cos \mu_n) \quad \text{for } 0 < Bi < 100 \text{ and}$$

$$A_n = 2(-1)^{n+1} \quad \text{for } Bi \geq 100 \tag{8}$$

and for all products:

$$B_n = \exp(-\mu_n^2 Fo) \quad \text{for } 0 < Bi < 100 \text{ and } Bi \geq 100 \tag{9}$$

Also, the following dimensionless parameters are written:

$$\theta = (T - T_a)(T_i - T_a) \quad \text{for the HTC and}$$

$$\theta = (M - M_e)/(M_i - M_e) \text{for the MTC} \tag{10}$$

$$Bi = hY/\lambda \quad \text{for the HTC and } Bi = kY/D \text{ for the MTC} \tag{11}$$

$$Fo = \alpha t/Y^2 \quad \text{for the HTC and } Fo = Dt/Y^2 \quad \text{for the MTC} \tag{12}$$

We introduce a simplification in Eq. (5) by taking $(\mu_1^2 Fo) > 1.2$. Thus, the infinite sum in Eq. (5) is well approximated by the first term only, i.e.,

$$\theta = A_1 B_1 \tag{13}$$

where for ISP:

$$A_1 = (2 \sin \mu_1)/(\mu_1) + (\sin \mu_1 \cos \mu_1) \quad \text{for } 0 < Bi < 100 \text{ and}$$

$$A_1 = 2/\mu_1 \quad \text{for } Bi \geq 100 \tag{14}$$

for ICP:

$$A_1 = (2Bi)/(\mu_1^2 + Bi^2) + J_0(\mu_1) \quad \text{for } 0 < Bi < 100 \text{ and}$$

$$A_1 = 2/(\mu_1) J_1(\mu_1) \quad \text{for } Bi \geq 100 \tag{15}$$

and for SP:

$$A_1 = (2Bi \sin \mu_1)/(\mu_1) - (\sin \mu_1 \cos \mu_1) \quad \text{for } 0 < Bi < 100 \text{ and}$$

$$A_1 = 2 \quad \text{for } Bi \geq 100 \tag{16}$$

And for all products:

$$B_1 = \exp(-\mu_1^2 Fo) \quad \text{for } 0 < Bi < 100 \text{ and } Bi \geq 100 \tag{17}$$

The above equations for A_1, i.e., Eqs. (14)–(16) for $0 < Bi < 100$ are simplified as follows [11]: for ISP:

$$A_1 = g = \exp((0.2533Bi)/(1.3 + Bi)) \tag{18}$$

for ICP:

$$A_1 = g = \exp((0.5066Bi)/(1.7 + Bi)) \tag{19}$$

and for SP:

$$A_1 = g = \exp((0.7599Bi)/(2.1 + Bi)) \tag{20}$$

After these simplifications, the characteristic equations of ISO, ICO and SO can be written, respectively, as follows: For $0 < Bi < 100$:

$$(\cot \mu_1) = (1/Bi)(\mu_1) \tag{21}$$

$$(J_0(\mu_1)/J_1(\mu_1)) = (1/Bi)(\mu_1) \tag{22}$$

$$(\cot \mu_1) = (1 - Bi)/(\mu_1) \tag{23}$$

and for $Bi \geq 100$:

$$\mu_1 = (\pi/2) \tag{24}$$

$$J_0(\mu_1) = 1 - (\mu_1^2/2^2) + (\mu_1^4/2^2 \times 4^2) - (\mu_1^6/2^2 \times 4^2 \times 6^2) \ldots \tag{25}$$

$$\mu_1 = \pi \tag{26}$$

The following simple expressions were developed previously, instead of Eqs. (21)–(23) for the roots characteristic equations [12]:

for ISP:

$$\mu_1 = \text{atan}(0.64Bi + 0.38) \quad \text{for } 0 < Bi < 100 \tag{27}$$

for ICP:

$$\mu_1 = ((0.72)\ln(6.8Bi + 1))^{1/1.4} \quad \text{for } 0 < Bi < 10 \tag{28}$$

$$\mu_1 = (\ln(1.74Bi + 147.3))^{1/1.2} \quad \text{for } 10 < Bi < 100 \tag{29}$$

for SP:

$$\mu_1 = ((1.12)\ln(4.9Bi + 1))^{1/1.4} \quad \text{for } 0 < Bi < 10 \tag{30}$$

$$\mu_1 = ((1.66)\ln(2.2Bi + 152.4))^{1/1.2} \quad \text{for } 10 < Bi < 100 \tag{31}$$

where for the case of $Bi \geq 100$, the root values in Eqs. (24)–(26) can be used directly.

It is possible to define the dimensionless temperature and moisture content in terms of the lag factor and frying coefficient, similar to what has been done for a cooling profile earlier [13], i.e.,

$$\theta = g \exp(-ft) \tag{32}$$

where $g=g_t$ and $f=f_t$ for the HTC and $g=g_m$ and $f=f_m$ for the MTC.

After equating Eqs. (13) and (32) with $A_1=1$, and hence we find: $(\mu_1^2 Fo)=ft$. After making required substitutions in this term, the following model is developed for the thermal diffusivities for the HTC and moisture diffusivities for the MTC:

$$a=(fY^2/\mu_1^2) \quad (33)$$

where $a=\alpha$ for the HTC and $a=D$ for the MTC.

The models for the heat transfer coefficients and moisture transfer coefficients are obtained in terms of the lag factor after extracting the Biot numbers from Eqs. (18)–(20), as follows:

For the heat transfer coefficients:

$$h=(\lambda/L)\,[(1.3\,\ln g_t)/(0.2533-\ln g_t)] \quad \text{for ISP} \quad (34)$$

$$h=(\lambda/R)\,[(1.7\,\ln g_t)/(0.5066-\ln g_t)] \quad \text{for ICP} \quad (35)$$

$$h=(\lambda/R)\,[(2.1\,\ln g_t)/(0.7599-\ln g_t)] \quad \text{for SP} \quad (36)$$

For the moisture transfer coefficients:

$$k=(D/L)\,[(1.3\,\ln g_m)/(0.2533-\ln g_m)] \quad \text{for ISP} \quad (37)$$

$$k=(D/R)\,[(1.7\,\ln g_m)/(0.5066-\ln g_m)] \quad \text{for ICP} \quad (38)$$

$$k=(D/R)\,[(2.1\,\ln g_m)/(0.7599-\ln g_m)] \quad \text{for SP} \quad (39)$$

A detailed information about how to implement the above HMTPs for an object of irregular shape can be found in Keey [14].

3
Experimental

In the experimental investigation, nine cylindrical potatoes (0.075 ± 0.0001 m long and 0.01 ± 0.0001 m in diameter) were trimmed and cut from the larger pieces of potatoes. Then, these were brought to same size and weight and used as test samples. A deep fryer with thermostatic control was used and had one-litre sunflower oil as frying medium. Three thermocouples connected to a multi-channel microprocessor device (Ellab Instruments, Copenhagen) were well-inserted at the centers of three samples, and two thermocouples were placed in the oil medium. Initially, the oil medium was heated to 180°C and the eight samples were placed in a wire basket fitted with a mesh lid. After the oil temperature reached 180°C, the basket was dipped into the oil medium. One of the samples was taken to measure initial moisture content, weight and oil content before frying. During frying experiment, five samples which were not contained the thermocouples were taken from the oil medium at every two minute intervals in the frying time of 10 min for determining the changes in the moisture content, weight and oil content. During the experiment, the total five samples were taken and the other remaining three samples in the oil medium were used to measure the center temperatures throughout the experiment. The temperatures of these three samples were averaged and used for data analysis to minimize temperature measurement errors. The fried samples were removed from the oil, and drained (approx. 2 min) as well as put onto an absorbent paper. For the analyses, the sample was finely ground and mixed using a Waring blender. Triplicate samples (2 g) were dried around 18 hr at 102 °C in an air convection oven. The oil contents of the samples were determined on duplicate 4 g samples by drying to constant weight in an air convection oven 6 h at 102 °C and by soxhlet extraction (2 h) with petroleum ether respectively. Initial and final temperatures and moisture contents, as well as equilibrium moisture content were measured as follows: $T_i=15$°C, $T_f=106$°C, $M_i=0.70$, $M_f=0.63$ and $M_e=0.01$. In the end of experiment, the oil content of the sample was 7.5%. A detailed description on the experimental measurements and procedure can be found in Yildiz and Dincer [6].

4
Results and discussion

In the verification of the present models, the experimental temperature and moisture content data belonging to an individual cylindrical potato were used. The following steps were conducted. Firstly, the measured temperatures and moisture contents were transformed into the dimensionless form by means of Eq. (10). Then, these dimensionless temperature and moisture content distributions in the form of Eq. (32) were exponentially well regressed using the least squares method. The lag factors and frying coefficients for temperature and moisture content distribution were obtained as: $g_t=1.04$, $g_m=1.0027$, and $f_t=0.00166$ 1/s, $f_m=0.000202$ 1/s with a high accuracy (i.e., $r^2>0.99$).

Here we define the frying process parameters, i.e., lag factor and cooling coefficient. The lag factor represents the magnitude of both the internal and external resistance to the heat transfer or moisture transfer to/from the solid product and has a direct effect on the heat transfer coefficient and moisture transfer coefficient as a function of the Biot number. On the other hand, the frying coefficient indicates the frying capability of the solid product being fried in any medium and has a direct effect on the thermal and moisture diffusivities as seen in Eq. (33).

The Biot number were calculated as $Bi=0.15$ for the HTC and $Bi=0.01$ for the MTC after substituting the lag factors into Eq. (19). The internal resistance to the heat transfer in the sample became higher. The roots of the characteristic equation ($\mu_1 s$) for the HTC and MTC were calculated to be 0.57 and 0.13 from Eq. (28). The thermal diffusivity and moisture diffusivity were determined as $\alpha=1.27\times10^{-7}$ m^2/s and $D=2.99\times10^{-7}$ m^2/s via Eq. (33). The thermal conductivity of the sample was measured 0.47 W/mK. The heat transfer coefficient and moisture transfer coefficient were found as $h=14.1$ W/m^{2}K and $k=5.98\times10^{-7}$ m/s. Here, we can calculate the Lewis number, which is the ratio of the thermal diffusivity divided by the moisture diffusivity ($Le=\alpha/D$), to give an idea about the magnitude of heat transfer and moisture transfer. Its value is 0.42 and this indicates that the moisture transfer realized 2.35 times faster in the cylindrical sample.

The measured and regressed dimensionless center temperature distributions and moisture distributions of an individual cylindrical product during deep-frying versus the frying time are given in Figs. 2 and 3. As can be seen from these figures, there is a remarkably good agreement between measured and regressed temperature and moisture content profiles: maximum difference between the measured and regressed

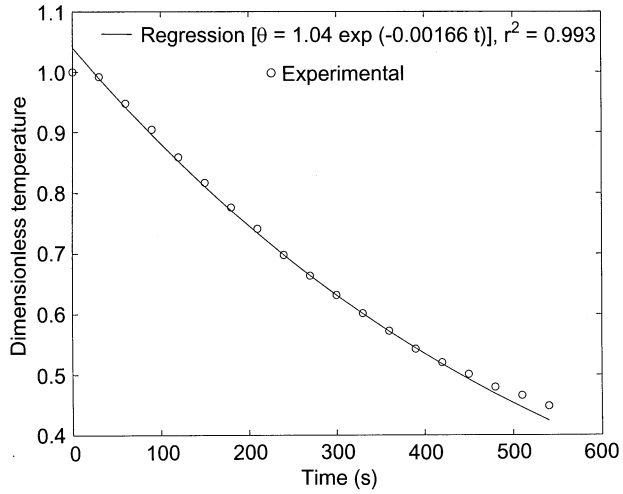

Fig. 2. Measured and regressed dimensionless temperature distributions of a cylindrical potato subject to frying

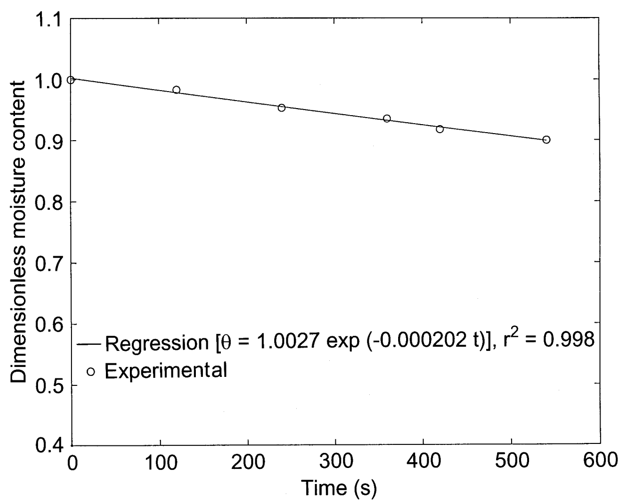

Fig. 3. Measured and regressed dimensionless moisture content distributions of a cylindrical potato subject to frying

temperature and moisture content values are 4.0% and 0.47%. The results of this study indicates that the present models are capable of determining the HMTPs for the products subject to deep-frying in a simple and effective manner.

5 Conclusions

Transient heat transfer and mass transfer during deep-frying of an individual ISP, ICP and SP were analyzed under the conditions of $0 < Bi < 100$, and $Bi \geq 100$. Simple analytical models were developed to determine the HMTPs for such products being fried. In order to verify the present models, the results of an experimental investigation were used. The frying process parameters, i.e., frying coefficient and lag factor were defined and incorporated into the models. We can conclude that the present models in the compact form are simple, but efficient tools to determine the HMTPs for such solid products exposed to frying, and could be of benefit to the related frying industry.

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