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Numerical study of natural convection characteristics of nanofluids in an enclosure using multiphase model

Yan-Jun Chen[1] · Ping-Yang Wang[1] · Zhen-Hua Liu[1]

Abstract The natural convective heat transfer and flow characteristics of nanofluids in an enclosure are numerically simulated using the multiphase-flow model and single phase model respectively. The simulated results are compared with the experimental results from the published papers to investigate the applicability of these models for nanofluids from a macro standpoint. The effects of Rayleigh number, Grashof number and volume concentration of nanoparticles on the heat transfer and flow characteristics are investigated and discussed. Comparisons of the horizontal and vertical central dimensionless velocity profiles between nanofluid and water for various Grashof numbers are studied. In addition, both streamline contours and isotherms lines for different volume concentrations of nanofluids are analyzed as well. The study results show that a great deviation exists between the simulated result of the single phase model and the experimental data on the relation of Nusselt number and Rayleigh number, which indicates that the single phase model cannot reflect the heat transfer characteristic of nanofluid. While the simulated results using the multiphase-flow model show a good agreement with the experimental data of nanofluid, which means that the multiphase-flow model is more suitable for the numerical study of nanofluid. For the natural convection, the present study holds the point that using Grashof numbers as the benchmark would be more appropriate to describe the heat transfer characteristics of nanofluid. Moreover, the simulated results demonstrate that adding nanoparticles into the base fluid can enhance both the motion of fluid and convection in the enclosure significantly.

Abbreviations
H Height of enclosure
W Width of enclosure
q Heat flux (W/m^2)
D Diameter of tube (m)
u Velocity of x direction (m/s)
v Velocity of y direction (m/s)
U Dimensionless velocity of x direction (m/s)
V Dimensionless velocity of y direction (m/s)
h Heat transfer coefficient (W/K/m^2)
Ra Rayleigh number
Gr Grashof number
Nu Nusselt number
T Temperature (k)

Greek alphabets
ρ Density (kg/m^3)
φ Volume fraction (vol%)
μ Viscosity of fluid (Pa s)
λ Thermal conductivity (W/m K)

Subscripts
np Nanoparticles
bf Base fluid
nf Nanofluid
k The k phase
l Liquid
s Solid
m The mixture fluid

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1 Introduction

Nanotechnology is an emerging technology that uses single atom or molecule to fabricate material, which is studied in a scale of 0.1–100 nm. Masuda [1] added the nanoparticles (Al_2O_3, TiO_2) into the water firstly and showed that the thermal conductivity could be enhanced by 32 % with volume concentration of 4.3 % compared with water. While viscosity of nanoparticles colloid did not increase evidently. Choi [2] proposed the concept of nanofluid for the first time, which is defined as a kind of new heat transfer working fluid that added nanoparticles into base fluid. Lee and Choi [3] applied nanofluid into cooling system for the cooling of crystal silicon lenses in a high intensity X-ray source. Water, liquid nitrogen and nanofluids were employed; and the nanofluids contained nanoparticles of CuO and γ-Al_2O_3. The experimental results indicate that heat transfer coefficients of nanofluids are three times higher than that of water, while the heat flux could reach to 1500 W/cm^2. Besides, the heat resistance of nanofluids is about half of water with the lowest value of 0.04 °C/W/cm^2. Due to the high thermal conductivity, nanofluid was employed to different heat transfer applications. Saripella [4] and Leong [5] studied the performance of nanofluid cooling the engine of automotive car and truck as working fluid respectively. Buongiorno [6] applied the nanofluid to the advanced nuclear power plants. Pantzali [7] investigated the efficacy of nanofluids as coolants in plate heat exchangers. Nguyen [8] researched the heat transfer enhancement using Al_2O_3-water nanofluid for an electronic liquid cooling system. From the above literature, nanofluids show a significant practical value.

There are abundant of study about the natural convection heat transfer of nanofluid in enclosures, most of study are about numerical calculation using simple flow models; while the literature about experimental research are not much. Ho [9] studied the natural convection heat transfer of water-based nanofluid in a vertical square enclosure with different sizes. The nanofluid formulated in the experiment is water dispersed with the alumina (Al_2O_3) nanoparticles ranging from 0.1 to 4 vol%. The Rayleigh number varies in the range of 6.21×10^5–2.56×10^8. The experimental results showed that the physical properties of nanofluids, like density, viscosity and thermal conductivity, were increased compared to those of water; and the trend went up with the increase of the nanofluid concentration. Besides, the Nu–Ra curves showed that heat transfer was deteriorated for the nanofluids with volume concentration of $c_v \geq 2$ vol% over the entire range of the Rayleigh number. Such degradation cannot be explained simply based on the net influence due to relative changes in thermophysical properties of the nanofluid contained such low particle fraction, thus strongly suggesting that other factors may come into play.

Putra [10] carried out the natural convection heat transfer characteristics of nanofluids inside a horizontal cylinder. Unlike conduction or forced convection heat transfer, a systematic and definite Nusselt number deterioration in natural convective heat transfer has been found to occur. The deterioration is dependent on particle volume concentration as well as the aspect ratio of the cylinder. The reason for this effect is also unclear in Putra [10]. However, the role of particle–fluid slip and sedimentation seems to be important which requires to be investigated more closely in the future.

Recent years, there appeared a lot of numerical studies for investigating natural convection heat transfer of nanofluid in enclosures [11–32] as shown in Table 1. Most of papers regarded the nanofluid as single phase fluids, and used various traditional concepts to calculate the physical parameters of nanofluids such as thermal conductivity, viscosity [11–30]. However, the single phase model may not reveal the mechanism of heat transfer and flow characteristic of nanofluid. Khanafer [11] investigated the heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids for various pertinent parameters. A single phase model is developed to analyze heat transfer performance of nanofluids inside an enclosure. The effect of suspended metallic nanoparticles on the fluid flow and heat transfer processes within the enclosure is analyzed and the effective thermal conductivity enhancement is studied for various parameters. However, the comparison between their numerical results and the experimental literatures is not included. The nanoparticle effect was taken into consideration using a thermal conductivity correlation, which means the nanofluid was regarded as a kind of single phase. For literatures that regarded nanofluid as single phase fluid, the effective thermal conductivity of the nanofluid is calculated by many kinds of models which combine nanoparticle concentration, thermal conductivities of nanoparticles and basefluids. More complex model may contain the nanoparticle shape factor or the effects of Brownian movement or micro convection of boundary layer. The purpose of these complex thermal conductivity models is to calculate the thermal conductivity as precise as that of nanofluid.

The other simulation work [31, 32] employed the multiphase model to study the natural convection heat transfer characteristics of nanofluid. Moghari et al. [31] studied the effect of nanoparticle mean diameter on laminar mixed convection of nanofluid flow consisting of Al_2O_3-water in an annulus. The two phase mixture model has been employed for modeling of nanoparticles' mean diameter effects on thermal and hydrodynamics characteristics. The results demonstrate that for a given solid volume fraction, increasing the diameter of nanoparticles causes to increase non-uniformity of nanoparticle distribution at the any cross section of annulus. With using nanoparticles with smaller

Table 1 Summary of numerical studies on natural convection of nanofluids

Authors	Physical model	Mathematic model	Nanofluids	Properties and results
Khanafer et al. [11], Eiyad et al. [12]	Square cavity heated partially	Single phase flow $10^3 \leq Ra \leq 10^5$	Cu/water	The simulation results of nanofluid did not compare to the experiment of nano-fluid. The nanofluid can increase the heat transfer rate at any given Grashof number, which also increases with an increase in the nanoparticles volume fraction. Inclination angle of the enclosure was found that lower heat transfer is formed for 90°. Effects of inclination angle on percentage of heat transfer enhancement become insignificant at low Rayleigh number but it decreases the enhancement of heat transfer with nanofluid
Eiyad et al. [13]	Horizontal annuli	Single phase flow $10^3 \leq Ra \leq 10^5$	Al_2O_3/water	For $Ra > 10^4$ and $\varphi > 5$ %, the average Nusselt number is reduced by increasing the volume fraction of nanoparticles. For $Ra \geq 10^4$, the Nusselt number is deteriorated everywhere around the cylinder surface especially at high expansion ratio, the difference in MG and Chon et al. model prediction is small
Ho et al. [14]	Square cavity heated partially	Single phase flow $10^3 \leq Ra \leq 10^6$	Al_2O_3/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. The uncertainties with different formulas for the thermal conductivity and viscosity of nanofluid have a strong bearing on the natural convection heat transfer characteristics in the enclosure
Abu-Nada et al. [15]	Horizontal annuli	Single phase flow $10^3 \leq Ra \leq 10^5$	Cu Ag Al_2O_3 TiO_2/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. For high values of Ra and L/D ratio, an enhancement in heat transfer is achieved around the inner cylinder surface due to the presence of Al_2O_3 nanoparticles except in the plume region

Table 1 continued

Authors	Physical model	Mathematic model	Nanofluids	Properties and results
Aminossadati et al. [16]	Square cavity heated vertically	Single phase flow $10^3 \leq Ra \leq 10^6$	Cu Ag Al_2O_3 TiO_2/water	The simulation results of nanofluid did not compare to the experiment of nano-fluid. Cu and Ag nanoparticles have the highest cooling performance at low Ra, the addition of 20 % nanoparticles results in 42.8 % reduction of maximum temperature. The influences of Al_2O_3 and TiO_2 are not as significant as that for Cu and Ag
Abu-Nada et al. [17]	Square cavity heated partially	Single phase flow $10^3 \leq Ra \leq 10^5$	CuO Al_2O_3/water	The heat transfer enhancement in the enclosure using variable thermal conductivity and variable viscosity of Al_2O_3–water and CuO–water nanofluids is investigated. The Nu of Al2O3–water nanofluid at high Ra was reduced by increasing the volume fraction of nano-particles above 5 %. At low Ra, the Nu was slightly enhanced by increasing the volume fraction of nanoparticles
Oztop et al. [18]	Rectangular cavity heated partially	Single phase flow $10^3 \leq Ra \leq 10^5$	Cu Al_2O_3 TiO_2/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. The type of nanofluid is a key factor for heat transfer enhancement. The highest values are obtained when using Cu nano-particles
Arefmanesh et al. [19]	Square cavity with bottom wall heat-ing and moving	Single phase flow $Gr = 10^4$	Al_2O_3/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. Effects of the uncertainties in viscosity formula relied on Richardson number and fraction of nanoparticles
Nabavitabatabayi et al. [20]	Rectangular cavity heated vertically	Single phase flow MRT LBM $10^3 \leq Ra \leq 10^5$	Cu Ag Al_2O_3 TiO_2/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. Adding nanoparticles to base liquid causes the maximum temperature decrease on account of the irregular motion of nanofluids. Ag and Cu with higher thermal conductivity are slightly better for using
Yurong He et al. [21]	Cavity heated partially	Single phase flow LBM $8 \times 10^3 \leq Ra \leq 8 \times 10^5$	Al_2O_3/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. The average Nusselt number is reduced with increasing volume fraction of nanoparticles

Table 1 continued

| Authors	Physical model	Mathematic model	Nanofluids	Properties and results
Nemati et al. [22]	Lid-driven cavity heated partially	Single phase flow LBM $1 \leq Re \leq 100$	Cu CuO Al_2O_3/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. The type of nanofluid is a key factor for heat transfer enhancement and the highest and lowest values of Nu number are obtained when using Cu and Al2O3 nanoparticles
Fattahi et al. [23]	Cavity heated partially	Single phase flow LBM $10^3 \leq Ra \leq 10^6$	Cu Al_2O_3/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. The higher values of Nusselt number are obtained when using Cu nanoparticles. Heat transfer increases with increase in Ra and volume fraction of nanoparticles
Lai et al. [24]	Cavity heated partially	Single phase flow LBM $10^3 \leq Ra \leq 10^6$	Al_2O_3/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. The average Nu increases with the increase of Ra and nanoparticle volume concentration
Kefayati et al. [25]	Cavity heated partially	Single phase flow LBM $10^3 \leq Ra \leq 10^5$	SiO_2/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. A critical value of Rayleigh number exist for the effect of nanoparticles
Guo et al. [26]	Cavity heated partially	Single phase flow LBM $10^3 \leq Ra \leq 10^5$	Al_2O_3/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. The critical Ra is lower for nanofluid than that for base fluid. With the increase of Ra, the nanofluid thermal boundary layer near the vertical wall turns thin
Haddad et al. [27]	Rayleigh- Bénard problem	Single phase flow $10^3 \leq Ra \leq 10^6$	CuO/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. Because of nanoparticles thermophoresis and Brownian motion, there exist an enhancement in heat transfer. The enhancement is more pronounced at low volume fraction of nanoparticles
Alloui et al. [28]	Cavity heated vertically	Single phase flow $10^3 \leq Ra \leq 6 \times 10^4$	Cu CuO Ag Al_2O_3 TiO_2/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. The presence of nanoparticles is found to reduce the strength of flow field, this behavior being more pronounced at low Rayleigh number

Table 1 continued

Authors	Physical model	Mathematic model	Nanofluids	Properties and results
Mahmoodi [29]	L-shaped cavities	Single phase flow $10^3 \le Ra \le 10^6$	Cu/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. At all Ra considered, the Nu increases when the aspect ratio of the cavity and the volume fraction of the nanofluid increase
Sheikholeslami et al. [30]	Cavity with sinusoidal wall	Single phase flow $10^3 \le Ra \le 10^5$	Unkonwn	The simulation results of nanofluid did not compare to the experiment of nanofluid. The Nu decreases as Nr increases until it reaches a minimum value and then starts increasing
Moghari et al. [31, 32]	Annulus with mixed convection	Two phase mixture model $Re = 500$ $3 \times 10^5 \le Gr \le 6 \times 10^5$	Al_2O_3/water	The simulation results of nanofluid did not compare to the experiment of nanofluid. For a given volume fraction, increasing the diameter of nanoparticles causes to increase non-uniformity of nanoparticle distribution at any cross section of annulus

diameter, the secondary flow in an annulus becomes stronger. Also reducing nanoparticles' mean diameter enhances the Nusselt number while it does not have any significant effect on friction factor and contours of nondimensional axial velocity. Heat transfer enhancement of a mixed convection laminar Al_2O_3-water nanofluid flow in an annulus with constant heat flux boundary condition has been studied in Moghari et al. [32]. employing two phase mixture model and effective expressions of nanofluid properties. The Brownian motions of nanoparticles have been considered to determine the effective thermal conductivity and the effective dynamic viscosity of Al_2O_3-water nanofluid, which depend on temperature. Numerical simulations have been presented for the nanoparticles volume fraction between 0 and 0.05 and different values of the Grashof and Reynolds numbers. The calculated results show that at a given *Re* and *Gr*, increasing nanoparticles volume fraction increases the Nusselt number at the inner and outer walls while it does not have any significant effect on the friction factor. Both the Nusselt number and the friction coefficient at the inner wall are more than their corresponding values at the outer wall. These simulation results in literatures [31, 32] indicate that two phase model shows a better performance than the single phase. However, their simulation results did not compare by the experimental values of natural convection of nanofluid. All the simulations are in annulus, and the natural convection in cavity did not studied. For paper [31], the volume concentration did not vary in simulation.

Among various kinds of convective heat transfer modes, the natural convection heat transfer shows the lowest heat flux. However, it has the advantages of safety, low cost, noise-free, etc. Natural convection heat transfer is widely used in electronic device with low power density, cooling of nuclear power plants under accident condition, etc. Besides, natural convection in an enclosure is also a classic case of CFD and numerical heat transfer research.

The present work focuses on the numerical study of the natural convective heat transfer and flow characteristics of nanofluids in an enclosure using both the single phase model and the solid–liquid mixture model. The effects of nanoparticle concentration, Rayleigh number and Grashof number on the average Nusselt numbers has been investigated systematically. Moreover, the simulated results have been compared to the experimental data, which can assess the applicability of the single phase model and multiphase-flow model to simulate the heat transfer characteristics of nanofluid. In addition, comparisons of the horizontal and vertical central velocity profiles between nanofluid and water for various Gr numbers are studied as well. Moreover, streamline contours and isotherms for different volume concentration are also analyzed.

Fig. 1 Schematic of physical model **a** case **b** case 2
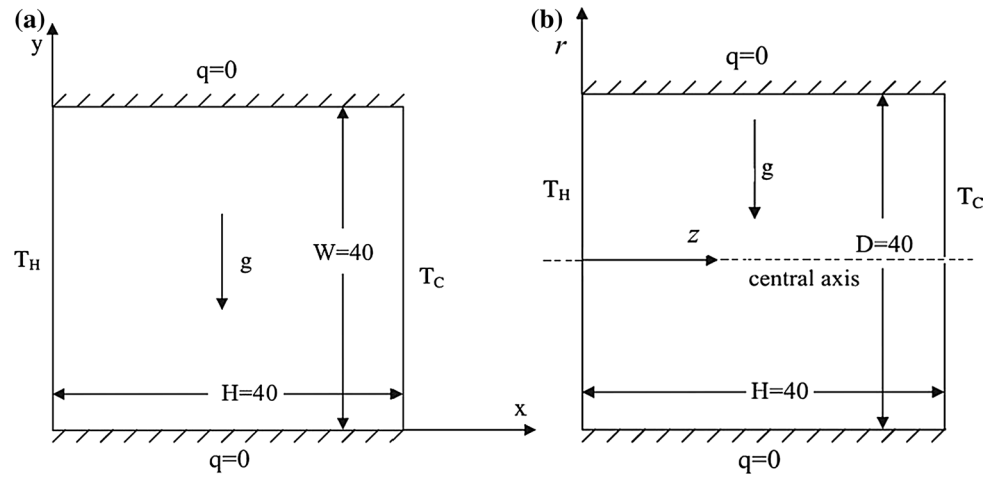
Table 2 Properties of the nanoparticle, water and nanofluids

Property	Al_2O_3	Water	1 vol% nanofluid	3 vol% nanofluid	4 vol% nanofluid
ρ (kg/m^3)	3600	998	1024	1076	1102
c_p (J/kg K)	765	4182	4059	3836	3732
μ (Pa s)	–	10^{-3}	1.025×10^{-3}	1.075×10^{-3}	1.1×10^{-3}
λ (W/m K)	30	0.6	0.621	0.68	0.698
β (1/K)	8.5×10^{-6}	2.08×10^{-4}	2.06×10^{-4}	2.02×10^{-4}	2.00×10^{-4}

2 Mathematical formulation and numerical procedure

2.1 Geometric models and boundary conditions

The present simulation study carried out two cases of which geometric dimensions, boundary conditions and nanofluids are originated from the experimental literatures with detailed experimental conditions [9, 10], respectively. Case 1 was compared to the experimental results of Ho [9]. The schematic of physical model of case 1 was shown in Fig. 1a. The flow and heat transfer in a quadrate enclosure could be simplified to a two dimensional model with the width of 40 mm and height of 40 mm. On the other hand, case 2 was compared to the experimental results of Putra [10]. The experimental facility of Putra [10] is a cylinder enclosure with the diameter of 40 mm and height of 40 mm. As shown in Fig. 1b, the flow and heat transfer in an enclosure could also be simplified to a 2 dimension model; and the horizontal central line could be an axis of the cylinder. In both cases, the top and bottom wall are heat insulated. The left wall is the heating wall with the constant temperature of T_H, while the right wall is the cooling wall with the constant temperature of T_C. The boundary conditions for case 1 and case 2 have been listed in Eqs. (1) and (2) respectively.

$$\left\{ \begin{array}{lll} u = v = \frac{\partial T}{\partial y} = 0 & y = 0, W & 0 \leq x \leq H \\ u = v = 0, T = T_H & x = 0 & 0 \leq y \leq W \\ u = v = 0, T = T_C & x = H & 0 \leq y \leq W \end{array} \right\} \quad (1)$$

$$\left\{ \begin{array}{lll} u = v = \frac{\partial T}{\partial r} = 0 & r = \frac{D}{2}, \frac{D}{2} & 0 \leq z \leq H \\ u = v = 0, T = T_H & z = 0 & \frac{D}{2} \leq r \leq \frac{D}{2} \\ u = v = 0, T = T_C & z = H & \frac{D}{2} \leq r \leq \frac{D}{2} \end{array} \right\} \quad (2)$$

2.2 Nanofluids

The working fluid in the enclosure is water and water based Al_2O_3 nanofluids. Three different volume concentrations were considered. As the analysis in introduction, physical properties included thermal conductivity are the key in the single phase simulation. The simulation using the single phase model in present research employed the experimental values of nanofluid properties instead of the various physical property model of the nanofluid. The properties of nanoparticle and base fluid are used in the multiphase-flow model calculation, respectively. In addition, the properties of the nanoparticle, water and nanofluids originated from literature [9, 10] has been shown in Table 2. For case 1, the mean diameter of nanoparticles is 33 nm, and case 2 is 60 nm.

In the present study, the physical properties of nanofluids are assumed not to change with temperature variations. However, the density variation due to the effect of buoyancy is considered by using Boussinesq approximation.

2.3 The basic theory of multiphase solid–liquid mixture model

The mixture model is employed in the present study. It can be used to simulate by solving the momentum, continuity, and energy equations for the mixture, while the volume fraction equations for the secondary phases, and algebraic expressions for the relative velocities [33].

The continuity equation for the mixture model is:

$$\nabla \cdot (\rho_m v_m) = 0 \tag{3}$$

where v_m is the averaged velocity.

$$v_m = \frac{\varphi_s \rho_s v_s + \varphi_l \rho_l v_l}{\rho_m} \tag{4}$$

And ρ_m is the mixture density:

$$\rho_m = \varphi_s \rho_s + \varphi_l \rho_l \tag{5}$$

The momentum equation can be expressed as:

$$\nabla \cdot (\rho_m v_m v_m) = -\nabla p + \nabla \cdot [\mu_m (\nabla v_m + \nabla v_m^T)] + \rho_m g \\ + \nabla \cdot (\varphi_s \rho_s v_{dr,s} v_{dr,s} + \varphi_l \rho_l v_{dr,l} v_{dr,l}) \tag{6}$$

where μ_m is the viscosity of the mixture:

$$\mu_m = \varphi_s \mu_s + \varphi_l \mu_l \tag{7}$$

$v_{dr,k}$ is the drift velocity for phase k (k represents s or l):

$$v_{dr,k} = v_k - v_m \tag{8}$$

From the continuity equation for solid phase s, the volume fraction equation for secondary phase can be obtained:

$$\nabla \cdot (\varphi_k \rho_k v_m) = -\nabla \cdot (\varphi_k \rho_k v_{dr,k}) \tag{9}$$

The energy equation for the mixture takes the following form:

$$\frac{\partial}{\partial t} \sum_{k=1}^{2} \varphi_k \rho_k E_k + \nabla \cdot \sum_{k=1}^{2} \varphi_k v_k (\rho_k E_k + p) = \nabla \cdot (k_{eff} \nabla T) \tag{10}$$

where k_{eff} is the effective conductivity ($\sum \alpha_k k_k$). The first term on the right-hand side of Eq. (12) represents energy transfer due to conduction.

$$E_k = h_k - \frac{p}{\rho_k} + \frac{v_k^2}{2} \tag{11}$$

where h_k is the sensible enthalpy for phase k. The slip velocity is defined as the velocity of a secondary phase relative to the velocity of the primary phase:

$$v_{sl} = v_s - v_l \tag{12}$$

The drift velocity and the slip velocity (v_{sl}) are connected by the following expression:

$$v_{dr,s} = v_{sl} - \sum_{k=1}^{2} \frac{\varphi_k \rho_k}{\rho_m} v_{lk} \tag{13}$$

Following Manninen et al. [34], the form of the relative velocity is given by:

$$v_{sl} = \frac{\tau_s}{f_{drag}} \frac{(\rho_s - \rho_m)}{\rho_s} a \tag{14}$$

where τ_s is the particle relaxation time

$$\tau_s = \frac{\rho_s}{18 \mu_l} d_s^2 \tag{15}$$

d is the diameter of the nanoparticles, a is the particle's acceleration. The drag function f_{drag} is taken from Schiller and Naumann [35]

$$f_{drag} = \begin{cases} 1 + 0.15 Re_s^{0.687} & Re_s \le 1000 \\ 0.0183 & Re_s > 1000 \end{cases} \tag{16}$$

where the definition of Re_s is as Eq. (16).

$$Re_s = \frac{\rho_l d_s |v_s - v_l|}{\mu_l} \tag{17}$$

The acceleration a is of the form:

$$a = g - (v_m \cdot \nabla) v_m \tag{18}$$

An effective solid viscosity model in terms of solid volume fraction was obtained from

$$\mu_s = -0.188 + 537.42 \varphi \tag{19}$$

2.4 Numerical procedure

The present study was carried out by FLUENT 6.3.26 based on finite volume method. Specifically, SIMPLE is used to couple the pressure and velocity field. Then, second order upwind is used to solve the momentum, energy and volume fraction equations of multiphase flow. In addition, PRESTO is used to solve pressure correction equation.

2.5 Dimensionless parameter arrangement

The basic parameters can be cast in non-dimensional form by incorporating the following dimensionless parameters [11].

$$\left\{ \begin{array}{c} X = \frac{x}{H}, Y = \frac{y}{H} \\ U = \frac{u}{\sqrt{(g\beta \Delta T H^3)}}, V = \frac{v}{\sqrt{(g\beta \Delta T H^3)}}, \theta = \frac{T - T_C}{T_H - T_C} \end{array} \right\} \tag{20}$$

Grashof number

$$Gr = \frac{g\rho^2 \beta \Delta T H^3}{\mu^2} \qquad (21)$$

Rayleigh number

$$Ra = \frac{g\rho^2 c_p \beta \Delta T H^3}{\lambda \mu} \qquad (22)$$

Heat transfer coefficient

$$h = \frac{q}{\Delta T} \qquad (23)$$

Nusselt number

$$Nu = \frac{qH}{\Delta T \lambda} \qquad (24)$$

where, ρ is the density of fluid, c_p the specific heat, β the expansion coefficient, ΔT the temperature difference of left and right wall, H the characteristic length, λ the thermal conductivity of fluid, μ the viscosity of fluid and q is the heat flux.

3 Results and discussion

3.1 Numerical validation

Quadrangle uniform grid is employed. The mesh is divided along the direction of y and x in case 1 and the direction of r and z in case 2. Table 3 shows the influence of mesh quantity on the Nusselt number of case 1 which corresponds to the experimental conditions in Ho [9] with Rayleigh number of 10^6 using water as working fluid. As is shown, with the increase of mesh quantity, the variation of results is decreasing. When the mesh is 80×80, the effect of mesh quantity on the simulated results is infinitesimally small and can be neglected. Therefore, 80×80 is employed as the mesh quantity. Since the physical model scale of case 2 is the same with that of case1, 80×80 is also employed in case 2.

Before the numerical study of nanofluids, heat transfer characteristics of water are calculated using single phase model and mixture model in which the concentration of solid phase is 0. The calculated results are compared with experimental data from Ho [9]. As shown in Fig. 2, the maximum error between the simulated results and the experimental values is 6.9 %, which represents a desirable accuracy for both the single phase and multiphase models.

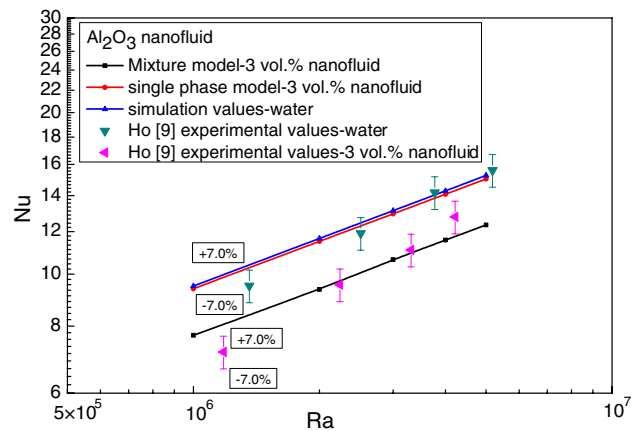**Table 3** Validation of mesh quantity

Mesh quantity	60×60	70×70	80×80	90×90
Nu	9.63	9.50	9.51	9.52
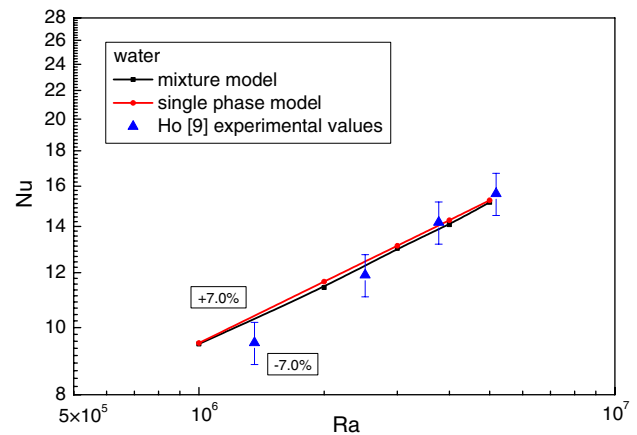
Fig. 2 The comparison between simulation and experimental values using water
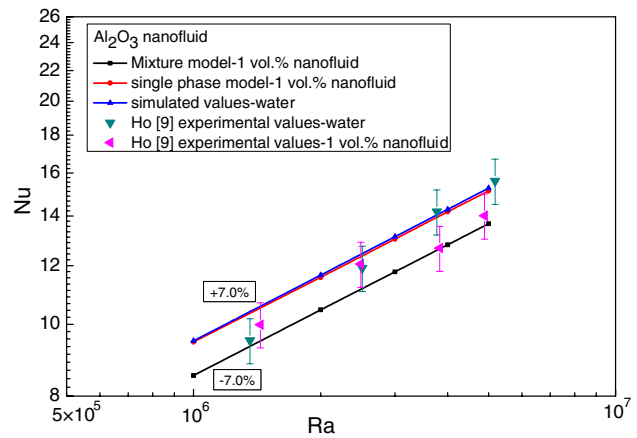
Fig. 3 Relation of Ra and Nu-case 1 (1 vol%)

Fig. 4 Relation of Ra and Nu-case 1 (3 vol%)

3.2 Comparisons between the simulated results and experimental data of Ho [9]

The present simulated results are compared with the experimental results of Ho [9]. The simulated curves of Nu and Ra based on single phase and mixture models are compared with the experimental curves of Nu and Ra, respectively.

Figures 3 and 4 show the effect of Rayleigh number on Nusselt number for nanofluid with the volume concentration of 1 and 3 vol% respectively for case 1. As shown, for a given Rayleigh number, the experimental value of Nusselt number of nanofluid with 1 vol% is smaller than that of water. The difference is increasing at higher concentration. As for the nanofluid with volume concentration of 3 vol%, the Nu number is decreased by about 20 % compared to that of water.

For the single phase model simulation, the measured physical properties of nanofluids are employed and the nanofluids are still regarded as single phase fluid. It can be seen from Figs. 3 and 4 that the simulated result of nanofluid is very close to that of water, while the change of concentration does not have obvious effect on the trend in the curve of Nu and Ra. From the comparison, it can also be seen that there appears large deviations between single phase model simulated results and experimental values. The deviation is worsening with the increasing of nanofluid concentration. The results indicate that the single phase model is not suitable to simulate the natural convection heat transfer of nanofluid in an enclosure.

For the multiphase-flow model computations, Figs. 3 and 4 show that the simulated Nu–Ra curves of nanofluid agree well with the experimental data for different concentrations. Therefore, the multiphase flow model is a better model to simulate the natural convective heat transfer characteristic of nanofluid in enclosures from a perspective of macroscopic scale, which shows the essence of nanofluid as a kind of solid–liquid mixture.

3.3 The comparisons between the simulated results and experimental data of Putra [10]

In order to further validate the applicability of multiphase flow model to nanofluid, case 2 was carried out and compared to the experimental data from Putra [10]. Although the Rayleigh number in case 2 is much bigger than that in case 1, while the flow is in laminar as well.

Figures 5 and 6 show the effect of Rayleigh number on Nusselt number of nanofluid with the volume concentration of 1 and 4 vol% respectively for case 2. For a given Rayleigh number, the Nusselt number of nanofluid is smaller than that of water, while the value is decreasing with the
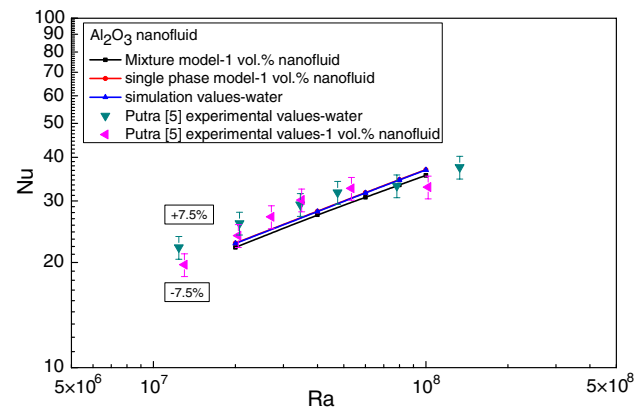
**Fig. 5** Relation of Ra and Nu-case 2 (1 vol%)
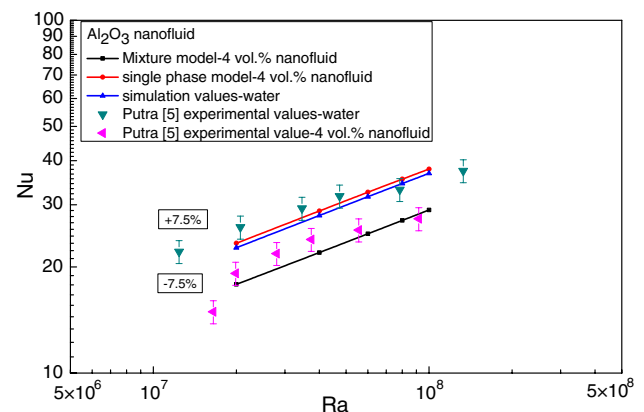
Fig. 6 Relation of Ra and Nu-case 2 (4 vol%)

increase of nanofluid concentration. For 1 vol% nanofluids in case 2, due to the low nanoparticle concentration, the experimental results between nanofluids and water are very similar. The simulated results using single phase model and mixture model also have similar values, which is close to the experimental data. The simulation results of nanofluids using single phase model even overlap with the curve for water. While for the nanofluid with volume concentration of 4 vol%, the experimental data indicates that the Nusselt number is decreased by about 30 % compared to that of water. The calculated values of nanofluid using multiphase model have good agreement with the experimental data, while obvious deviation occurs between single phase simulation values and experimental data. From the comparison above, compared to the experimental data of water, the nanofluid with high concentration deteriorates the Nusselt number of natural convection in the enclosure, which can be well simulated by multiphase model rather than single phase model.

Table 4 The calculated results basing Grashof number

Gr	Water		1 vol% Al$_2$O$_3$ nanofluid		3 vol% Al$_2$O$_3$ nanofluid	
	Temperature difference (K)	Heat transfer coefficient (W/K)	Temperature difference (K)	Heat transfer coefficient (W/K)	Temperature difference (K)	Heat transfer coefficient (W/K)
1×10^4	0.077	64.68	0.078	68.21	0.079	80.51
1×10^5	0.770	128.48	0.776	136.68	0.788	161.54
1×10^6	7.696	252.81	7.755	269.52	7.878	320.61
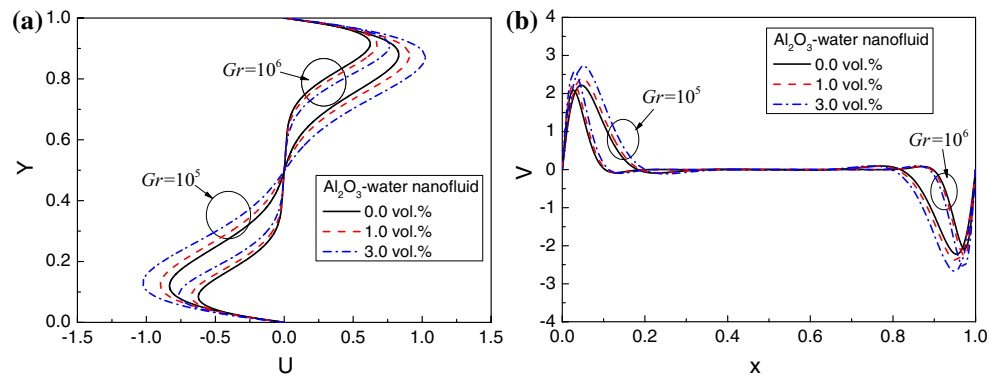
Fig. 7 Comparison of the central velocity profiles between nanofluid and water **a** vertical direction ($X = 0.5$) **b** horizontal direction ($Y = 0.5$)

### 3.4 The effect of nanofluids on the temperature and velocity in the enclosure

From the analysis above, it can be concluded that nanofluids deteriorate the Nusselt number of the enclosure using Rayleigh number as the benchmark. However, according to the Eqs. (22) and (24), the thermal conductivity affects the Rayleigh number and Nusselt number simultaneously. Therefore, the effect of nanofluid on heat transfer coefficient cannot be well presented by the relation between Rayleigh number and Nusselt number. Moreover, for natural convection, the heat transfer in an enclosure does not have the external driving force, like pump power in the forced convection. The sole heat transfer driving force in an enclosure is the temperature difference of heating or cooling wall. Therefore, the temperature difference should be the benchmark to compare the heat transfer enhancement. In order to reduce the effect of thermal conductivity, the present study brings in Grashof number as the benchmark to simulate the effect of nanofluid concentration on the velocity profiles, streamlines contour, isotherm and heat transfer coefficient of the enclosure by using multiphase flow model. The following study is based on case 1.

Table 4 gives out three simulation cases for the given Grashof numbers to show the effect of Gr and concentrations on the heat transfer coefficients of nanofluid in case 1. It can be seen from Table 3 that the temperature differences between the heating and cooling wall for three kinds of fluids vary little for a given Grashof number. Hence, the heat transfer enhancement can be shown clearly by comparison of the heat transfer coefficient (HTC) under the same Grashof number. According to Table 3, the simulated results show that nanofluid can strengthen the HTC in the enclosure; the enhancement goes up with the increase of nanofluid concentration. As for 3 vol% nanofluid, the HTC can be enhanced from 24 to 27 % which is relatively significant. Besides, the enhancement is basically independent of Grashof number but depends on the concentration of nanoparticle, which shows a characteristic of continuity.

Figure 7 shows the velocity profiles in the central line along the horizontal and vertical direction. The dimensionless coordinates of X and Y is in the range between 0 and 1. Therefore, the dimensionless coordinates of central line along the horizontal direction is $Y = 0.5$, $0 \leq X \leq 1$; the dimensionless coordinates of central line along the vertical direction is $X = 0.5$, $0 \leq Y \leq 1$. The dimensionless velocities are defined by Eq. (20). For the same Grashof number, the dimensionless velocities have the same denominator. As shown in Fig. 7, the central dimensionless velocities of horizontal and vertical direction increase with the increasing of nanofluid concentration, which is due to the convective effect and energy transfer enhancement in fluid.

Moreover, Fig. 7 also shows that the velocity of fluid is accelerated near the heating and cooling wall and decelerated in the central region of the enclosure. For different Grashof number, the velocity in central region is
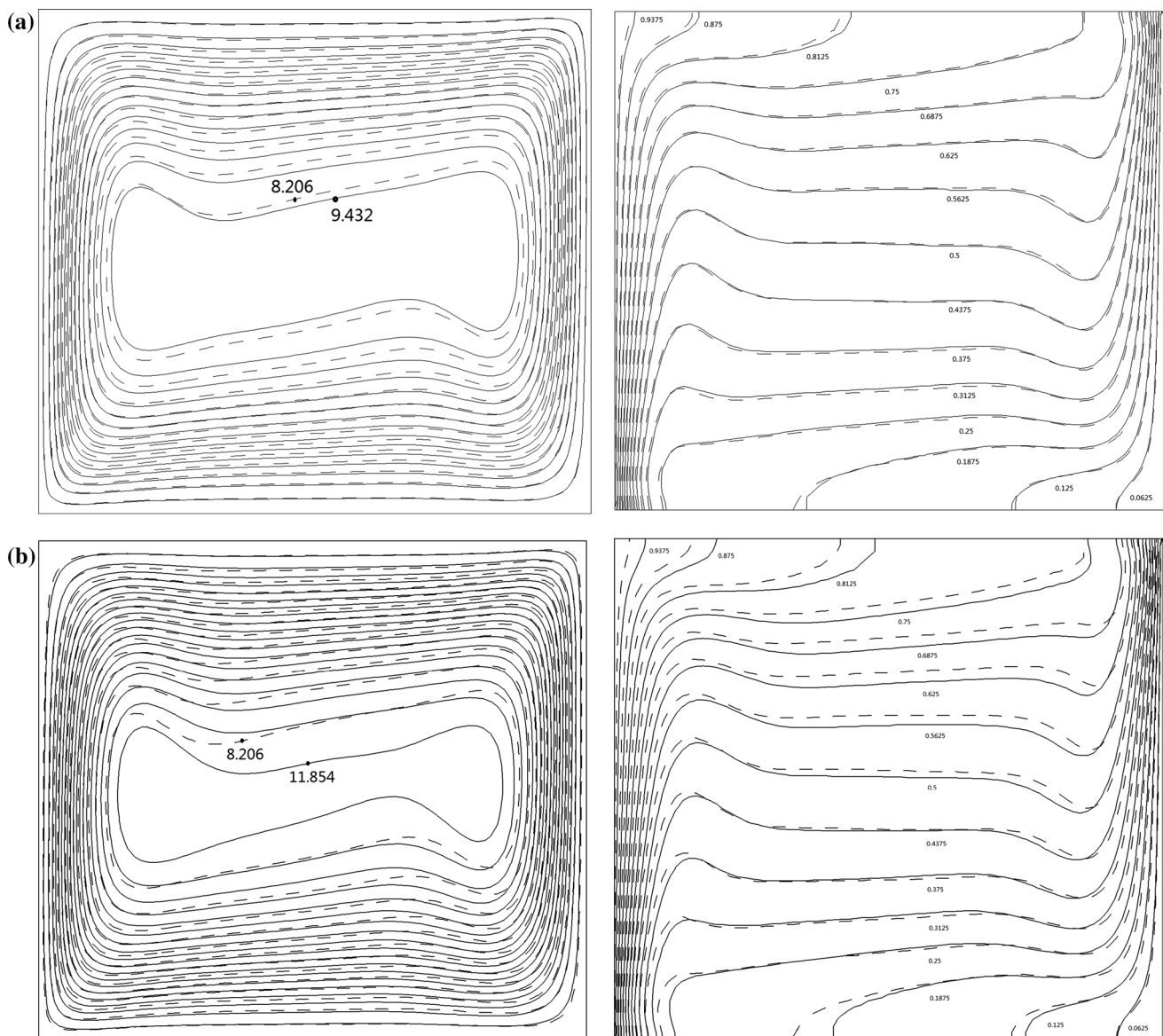
Fig. 8 Streamlines contours (*left*) and isotherms (*right*) at various volume fractions ($Gr = 10^5$). **a** 1.0 vol% Al_2O_3 nanofluid (*left* streamlines contours, *right* isotherms). **b** 3.0 vol% Al_2O_3 nanofluid (*left* streamlines contours, *right* isotherms)

much smaller than the velocity near the wall. The velocity changes due to the temperature results in the buoyancy differences which can accelerate or decelerate the fluid. This phenomenon appears when the working fluid is water. Therefore, the nanofluid behaves close to fluid compared to the suspension with millimeter or micrometer particles.

Figure 8 show the streamlines contours (left figure) and isotherms (right figure) for water and nanofluids based on case 1 under the Grashof number of 10^5. The streamlines and isotherms of nanofluids are showed by solid line and those of water are expressed by dashed line. As aforementioned that the left is hot wall and the right is cold wall. The fluid in the enclosure is heated by the left wall, which

results in the density reduction of fluid. Therefore, the fluid floats upwards along the left wall. While the right cold wall cools down the heated fluid. As a result, the fluid in the enclosure is actuated by the temperature difference to move. By the effect of thermal conduction, the isotherms in the enclosure are parallel to the heating wall. However, due to the effect of convection, the heated fluid has a trend to move to the upper region. Therefore, it can be seen from Fig. 8 that the isotherms near left wall show a trend in the upper of enclosure that flutter to the right wall, which actually results from the convective effect. Moreover, it shows that 1 vol% nanofluid also has an effect of convective enhancement but the trend is weaker than that of 3 vol%

nanofluid, which demonstrates that the convection enhancement is increasing at a higher nanoparticle concentration.

In terms of the streamlines contours in Fig. 8, the streamlines are more intensive for nanofuilds than those of water, while the streamlines with higher concentration is more intensive than the lower concentration case. The results indicate that the irregular movement of nanoparticles and the interaction between nanoparticles and base fluid accelerate the fluid motion and the energy transfer.

The analyses of the velocity profiles along horizontal and vertical direction, streamlines contours and isotherms show that nanofluid can intensify the motion of fluid and the energy transfer under various temperature differences due to the existing of nanoparticles.

4 Conclusions

The natural convective heat transfer of nanofluids in an enclosure have been numerically simulated using multiphase-flow model and single phase model respectively. The effects of Rayleigh number, Grashof number and volume concentration of nanoparticles were investigated and discussed. Comparisons of the central velocity profiles along the horizontal and vertical direction, streamlines contours and isotherms among nanofluids and water were studied. The following results were obtained:

1. When comparison between nanofluids and water is based on the relation of Nusselt number and Rayleigh number, the results calculated by the multiphase flow model for nanofluids have a good agreement with the experimental values. While, the results calculated by the single phase model for nanofluids are close to the values of water. It can be concluded that the multiphase flow model is a better way to simulate natural convective heat transfer characteristics of nanofluid from perspective of macroscopic scale.

2. When the relation of Nusselt number and Rayleigh number is employed to study the heat transfer characteristics, the calculated results using the multiphase flow model and the experimental values all demonstrates that the nanofluid will deteriorate Nusselt number. It cannot, however, reflect the change of heat transfer coefficient in the natural convection. Grashof number should be a reasonable benchmark for comparison in order to eliminate the effect of thermal conductivity. The study shows that nanofluid can enhance the heat transfer coefficient of the fluids under the same Grashof number. Moreover, the effect is enhanced with the increasing of nanofluid concentration.

3. The motion in the enclosure can be strengthened by adding nanoparticles into the fluid. The central veloci-

ties of horizontal and vertical direction increase with the increase of nanofluid concentration, which is in favor of strengthening the energy transfer in fluid. The enhancing mechanism mainly results from the irregular movement of nanoparticles and the interaction between nanoparticles and base fluid.

4. For nanofluids and water, by increasing the Grashof number, the stream lines become denser; the relative vertical isotherms will incline to the cold wall, which demonstrates that the convection effect is strengthened. In addition, the incline trend is more obvious for nanofluids comparing to water, which indicates the convection effect has been enhanced by nanoparticles from another perspective.

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