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Prediction of the outlet temperatures in triple concentric—tube heat exchangers in laminar flow regime: case study

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Abstract In this study there has been developed a numerical model of predicting outlet temperatures in a triple concentric-tube heat exchanger. For the model elaboration there have been used the equations of heat transfer and of fluid-dynamics, as well as a numerical algorithm to solve systems of non-linear equations. Based on experimental data, the obtained model has been practically tested to cool a petroleum product with water in a triple concentric-tube heat exchanger. The results obtained using the numerical simulation have been compared with the experimental data and data from literature in order to validate the proposed numerical model.

List of symbols

A	Heat transfer area, $A = \pi dL$ (m^2)
c_p	Specific heat (J/kg °C)
d	Diameter (m)
h	Local heat transfer coefficient (W/m^2 °C)
L	Length (m)
m	Mass flow rate (kg/s)
N	Number of the data points
Nu	Number Nusselt
Pr	Number Prandtl
Re	Number Reynolds
T	Temperature (°C)
U	Overall heat transfer coefficient (W/m^2°C)
w	Linear average velocity (m/s)

Greek letters

ρ	Density (kg/m^3)
μ	Dynamic viscosity (kg/ms)
ϑ	Kinematic viscosity (m^2/s)
λ	Thermal conductivity (W/m °C)

Subscripts

e	Equivalent
i	Inner
in	Inlet
o	Outer
out	Outlet
w	Wall
1	Central tube
2	Intermediate tube (inner annular)
3	External tube (outer annular)

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1 Introduction

Applications of the triple concentric-tube heat exchangers include heating, cooling, pasteurizing/sterilizing, freezing, preparation and concentration processes used in food and dairy industries. It is known that they are ideal for higher viscosity fluids with medium to small particulates and fluids with large viscosity changes and that they are proficient for direct or indirect product regeneration.

In a triple concentric-tube heat exchange, the heat transfer is enhanced in comparison with a double tube heat exchanger, due to the additional passage that improves the

heat transfer and provides a larger surface area for the heat transfer per unit length [1].

The triple concentric-tube heat exchangers have been theoretically analyzed from several points of view: the model scope, the assumptions of steady state and dynamic mathematical models. Zuritz has developed a set of analytical equations for the fluid temperatures and has performed a case study for the counter-current arrangement [1]. Based on a simplified physical model, Unal has developed a fully analytical expression for the variations of the bulk temperatures of the three fluids streams along the heat exchanger [2]. The same author published a series of case studies for counter-current arrangement and proved that the heat exchanger performance or size is dependent on the relative sizes of the three tubes [3]. The previously temperature distribution expressions obtained by Unal has been completed with a fully analytical expression for the effectiveness of triple concentric-tube heat exchangers with both the counter-current and co-current arrangements [4].

Garcia-Valladares [5] has developed a numerical criteria which allow the simulation, in both transient and steady state, of the thermal and fluid-dynamic behavior of a triple concentric-tube heat exchanger.

Batmaz and Sandeep [6] have developed a new method to determine the overall heat transfer coefficients and distribution of fluids in a triple concentric-tube heat exchanger with both the counter-current and co-current arrangements, by using the energy balance on a control volume.

A double tube heat exchanger mathematical model, where the heat exchanger is considered a lumped parameter system, was presented in [7]. The literature data analyze has been identified the particular aspects of the mathematical modeling of the heat transfer in the triple concentric-tube heat exchangers, as follows:

- The type of the tube surface;
- The existence/absence of the phase exchange;
- Newtonian/non-Newtonian fluids;
- Nusselt number correlations used to calculate local heat transfer coefficients;
- Steady state or dynamic models;
- Models used for design or within process simulation;
- Models with lumped/distributed parameters.

Triple concentric-tube heat exchangers are widely used in the food industry. A new practical application of this type of heat exchanger is the petroleum processing industry. In this industrial field, it is necessary to recover the heat energy from the secondary energetic flows, characterized by lower flow rates and high temperatures. As studies on the use of triple concentric-tube heat exchangers in the petroleum refineries are not found, our research has been focused on elaborating a robust mathematical model, applicable for the simulation of the triple concentric-tube

heat exchangers in this field. The researches have been materialized both in the experimental study of the petroleum product-water heat transfer and in the elaboration of a mathematical model to verify this heat exchanger type. The mathematical model allows the prediction of the outlet temperatures of the three fluids and the estimation of the local heat transfer coefficients and of the effective overall heat transfer coefficient.

2 The structure of the triple concentric-tube heat exchanger

The mathematical model proposed in this study involves cooling a product in counter-current arrangement in a triple concentric-tube heat exchanger with straight and smooth tubes. The cross and longitudinal sections of the triple concentric-tube heat exchanger are shown in Fig. 1.

The cold fluid in the inner tube (Fluid 1) and the outer annulus (between the outer two tubes, Fluid 3) enters the heat exchanger at a temperature of $T_{1,in} = T_{3,in}$ and exits at temperatures $T_{1,out}$ and $T_{3,out}$ in the inner tube and outer tube, respectively. The hot product (Fluid 2) enters in the inner annulus (between the inner two tubes) of the heat exchanger at the temperature $T_{2,in}$ and exits at temperature $T_{2,out}$.

The triple concentric-tube heat exchanger system is characterized by six inlet variables and three outlet variables, Fig. 2. This figure shows the input–output causality of heat exchanger system and it does not describe a mass or heat balance.

## 3 The mathematical model of the triple concentric-tube heat exchanger

There have been considered the following assumptions, for simplicity:

- The heat exchanger is at a steady state regime;
- The heat exchanger is a lumped parameters system;
- The conduction resistance of the tube wall is neglected in the thin tube;
- Both fluids are incompressible;
- Fluid properties are constant;
- Phase change does not take place;
- The exchanger is perfectly insulated against the surroundings.

For the triple concentric-tube heat exchanger, the heat balance equation can be written as:

$$m_2 c_{p2}\left(T_{2,in} - T_{2,out}\right) = m_1 c_{p1}\left(T_{1,out} - T_{1,in}\right) \\ + m_3 c_{p3}\left(T_{3,out} - T_{3,in}\right) \tag{1}$$

Fig. 1 The cross and longitudinal sections of the heat exchanger
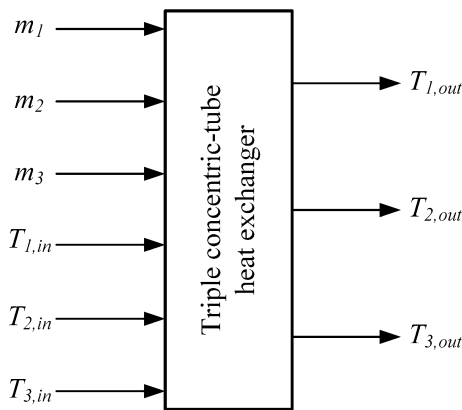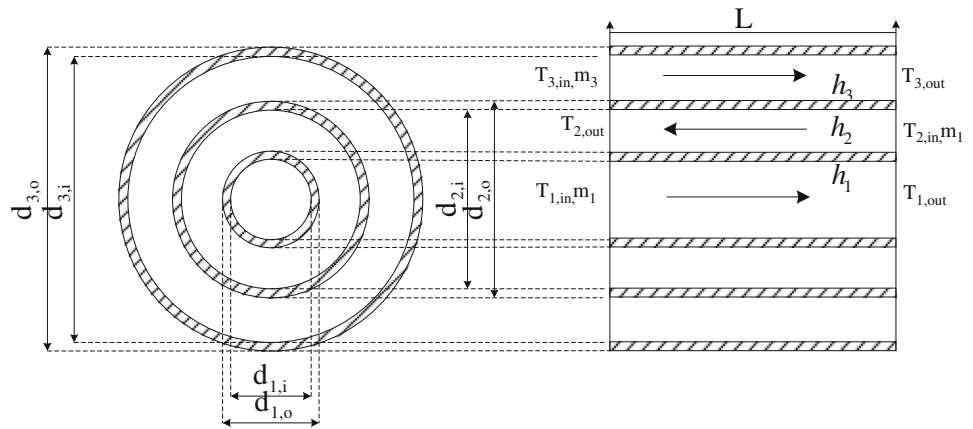
Fig. 2 The structure of the triple concentric-tube heat exchanger

In the heat exchanger, the energy of the product is transferred in two opposite directions; therefore, two different overall heat transfer coefficients (U_1 and U_2) can be defined. For the heat exchanger one effective overall heat transfer coefficient can be defined. After processing different design and experimental data presented in [1, 8], we have obtained various values of the effective overall heat transfer coefficient, classified in the following types: $U_1 < U < U_2$; $U_2 < U < U_1$; $U \approx \frac{U_1 + U_2}{2}$. Based on these observations, we have proposed to use an effective overall heat transfer coefficient U in the heat triple concentric tube exchanger mathematical model. In these conditions, the heat balance between the fluids can also be written as follows:

$$A_{1o}U \frac{(T_{2,in} - T_{1,out}) - (T_{2,out} - T_{1,in})}{\ln \frac{(T_{2,in} - T_{1,out})}{(T_{2,out} - T_{1,in})}} = m_1 c_{p1}(T_{1,out} - T_{1,in}) \tag{2}$$

$$A_{2i} U \frac{(T_{2,in} - T_{3,out}) - (T_{2,out} - T_{3,in})}{\ln \frac{(T_{2,in} - T_{3,out})}{(T_{2,out} - T_{3,in})}} = m_3 c_{p3}(T_{3,out} - T_{3,in}) \tag{3}$$

Equations (1), (2) and (3) form a system of non-linear equations with three unknown variables, $T_{1,out}$, $T_{2,out}$ and $T_{3,out}$, that are to be further solved.

4 Solving the mathematical model

The system consist the Eqs. (1), (2) and (3) has the form

$$\begin{cases} f_1(T_{1,out}, T_{2,out}, T_{3,out}) = 0 \\ f_2(T_{1,out}, T_{2,out}, T_{3,out}) = 0 \\ f_3(T_{1,out}, T_{2,out}, T_{3,out}) = 0 \end{cases} \tag{4}$$

Functions f_1, f_2 and f_3 of the equations system (4) have the following expressions:

$$f_1 = m_2 c_{p2}(T_{2,in} - T_{2,out}) - m_1 c_{p1}(T_{1,out} - T_{1,in}) - m_3 c_{p3}(T_{3,out} - T_{3,in}) \tag{5}$$

$$f_2 = A_{1o}U \frac{(T_{2,in} - T_{1,out}) - (T_{2,out} - T_{1,in})}{\ln \frac{(T_{2,in} - T_{1,out})}{(T_{2,out} - T_{1,in})}} - m_1 c_{p1}(T_{1,out} - T_{1,in}) \tag{6}$$

$$f_3 = A_{2i} U \frac{(T_{2,in} - T_{3,out}) - (T_{2,out} - T_{3,in})}{\ln \frac{(T_{2,in} - T_{3,out})}{(T_{2,out} - T_{3,in})}} - m_3 c_{p3}(T_{3,out} - T_{3,in}) \tag{7}$$

The Newton–Raphson algorithm has been used to solve the non-linear equations system (4). The Jacobean matrix of this equations system can be written as follows:

$$J(X) = \begin{bmatrix} \dfrac{\partial f_1}{\partial T_{1,out}} & \dfrac{\partial f_1}{\partial T_{2,out}} & \dfrac{\partial f_1}{\partial T_{3,out}} \\ \dfrac{\partial f_2}{\partial T_{1,out}} & \dfrac{\partial f_2}{\partial T_{2,out}} & \dfrac{\partial f_2}{\partial T_{3,out}} \\ \dfrac{\partial f_3}{\partial T_{1,out}} & \dfrac{\partial f_3}{\partial T_{2,out}} & \dfrac{\partial f_3}{\partial T_{3,out}} \end{bmatrix} \qquad (8)$$

In order to estimate the Jacobean matrix partial derivates [9, 10], the following general form has been tested:

$$\frac{\partial f_i}{\partial x_j} = \frac{f_i\left(x_1^{(k)}, \ldots, \left(x_j^{(k)} + h_j^{(k)}\right), \ldots, x_n^{(k)}\right) - f_i\left(\mathbf{X}^{(k-1)}\right)}{h_j^{(k)}},$$

$$i = 1, \ldots, n; \quad j = 1, \ldots, n \qquad (9)$$

5 The adaptation of the mathematical model

The adaptation of the heat exchanger mathematical model consists of the following steps:

(1) The determination of the approximation functions of the physical properties;
(2) The specification of the heat exchanger dimensions;
(3) The selection of the Nusselt number correlations for the calculation of the heat transfer coefficients;
(4) The determination of the effective overall heat transfer coefficient.

5.1 The calculus of the physical properties of the fluids

The relations used for calculating local heat transfer coefficients take into consideration the following physical properties of the fluids: density, specific heat, dynamic/kinematic viscosity and heat conductivity. The model developed has been adapted for an experimental triple tube-concentric heat exchanger, where the hot fluid is a petroleum product and the cold fluid is water. The petroleum product used is a mineral oil with $d_4^{20} = 0.881$. The physical properties of water have been estimated from literature data [11, 12] and by using PRO II® simulator and

they can be calculated by polynomial approximation functions in the following form:

$$Property_{water} = a_{0,i} + a_{1,i}T + a_{2,i}T^2 + a_{3,i}T^3 + a_{4,i}T^4 \qquad (10)$$

The approximation functions coefficients for Eq. (10), determined by using the polynomial regression, are listed in Table 1.

The physical properties of oil have been determined by using the following relations [11, 13]:

$$\rho_{oil} = 1000\, d_4^{20} \left(1 - \frac{T - 20}{2290 - 6340 d_4^{20} + 5965\left(d_4^{20}\right)^2}\right) [\text{kg/m}^3] \qquad (11)$$

$$c_{p,oil} = 4186.8\, \frac{1}{d_4^{20}}\, (0.403 + 0.00008\, T)[\text{J/kg°C}] \qquad (12)$$

$$\vartheta_{oil} = 0.034 T^{-1.8722}[\text{m}^2/\text{s}] \qquad (13)$$

$$\lambda_{oil} = 0.1344 \times 10^{-4}\, \frac{1 - 0.0054\, T}{d_4^{20}} [\text{W/m°C}] \qquad (14)$$

5.2 Specifying the dimensions of the heat exchanger

The dimensions of the experimental heat exchanger are listed in Table 2 [8].

5.3 The calculus of the convective heat transfer coefficients

The calculation of the heat transfer coefficients involves the following steps:

1. Defining the flow space;
2. Specifying the characteristic length;
3. Calculating the linear average velocity;
4. Calculating Reynolds number and identifying of the flow regime;
5. Calculating the Prandtl number;
6. Calculating the Nusselt number;

Table 1 The numerical coefficients of the water physical property functions

Physical property	Property index	$a_{0,i}$	$a_{1,i}$	$a_{2,i}$	$a_{3,i}$	$a_{4,i}$	R^2
Density, ρ_{water} (kg/m³)	1	999.91	0.0446	−0.0072	4e−5	−1e−7	1
Heat capacity, $c_{p,water}$ (J/kg °C)	2	4,209.8	−2.1041	0.0328	−1e−5	–	0.9966
Dynamic viscosity, $\mu_{water} \times 10^4$ (kg/ms)	3	17.83	0.5624	0.0103	−1e−4	4e−7	0.9997
Thermal conductivity, λ_{water} (W/m² °C)	4	0.5507	0.0027	−1e−5	5e−9	9e−11	0.9998

Table 2 Dimensions of the heat exchanger

Parameters	Notation	Dimensions
Inner diameter of inner tube (m)	$d_{1,i}$	0.012
Outer diameter of inner tube (m)	$d_{1,o}$	0.014
Inner diameter of intermediate tube (m)	$d_{2,i}$	0.026
Outer diameter of intermediate tube (m)	$d_{2,o}$	0.028
Inner diameter of outer tube (m)	$d_{3,i}$	0.040
Outer diameter of outer tube (m)	$d_{3,o}$	0.042
Length of inner tube (m)	L_1	1.193
Length of intermediate tube (m)	L_2	1.193
Length of outer tube (m)	L_3	0.935

Fig. 3 Schematic diagram of experimental setup of oil–water heat transfer

7. Calculating the local heat transfer coefficient.

The relations for steps (3)–(8) are listed in Table 3 where the subscripts '1', '2' and '3' are used to denote the variable associated with each fluid.
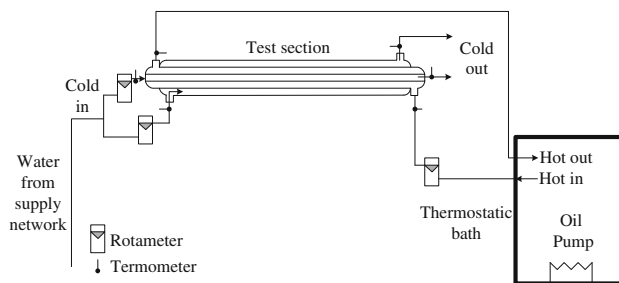
It should be mentioned that in order to simulate the mathematical model, the physical properties of each fluid have been calculated at an arithmetic average temperature between the inlet and outlet temperatures.

In order to calculate heat transfer coefficients, there have been used Nusselt number correlations specific to the flow regime and to flow space.

The heat transfer coefficient for the inside surface of the inner tube, h_1, has been calculated starting from the following correlations [12, 14]:

$$Nu_1 = \begin{cases} Sieder-Tate : 1.86 \left(\mathrm{Re}_1 \, \mathrm{Pr}_1 \frac{d_{1,i}}{L_1}\right)^{0.33}\left(\frac{\mu_1}{\mu_{w1}}\right)^{0.14}, & \mathrm{Re}_1 < 2300 \\ Hausen : 0.116\left(\mathrm{Re}_1^{2/3} - 125\right)\mathrm{Pr}_1^{0.4}\left(1 + \left(\frac{d_{1,i}}{L_1}\right)^{2/3}\right)\left(\frac{\mu}{\mu_{w1}}\right)^{0.14}, & 2300 \leq \mathrm{Re}_1 < 10^4 \\ Colburn : 0.023\mathrm{Re}_1^{0.8}\,\mathrm{Pr}_1^{0.4}, & \mathrm{Re}_1 \geq 10^4 \end{cases}$$

$$(15)$$

For the flow in the inner annulus, the heat transfer coefficient h_2 has been calculated from the following correlations [1, 5, 12, 14, 15]:

$$Nu_2 = \begin{cases} Hausen : 3.657 + \frac{0.0668(\mathrm{Re}_2 \, \mathrm{Pr}_2 \, d_{e1}/L_1)^{1/3}}{0.04 + (\mathrm{Re}_2 \, \mathrm{Pr}_2 \, d_{e1}/L_1)^{-2/3}}, & \mathrm{Re}_2 < 2300 \\ Gnielinski : \frac{(f/8)(\mathrm{Re}_2 - 1000)\,\mathrm{Pr}_2}{1 + 12.7\sqrt{(f/8)}\left(\mathrm{Pr}_2^{2/3} - 1\right)}\left(1 + \left(\frac{d_{1e}}{L_1}\right)^{2/3}\right), & 2300 \leq \mathrm{Re}_1 < 10^4 \\ Monrad \; and \; Pelton : 0.020\mathrm{Re}_2^{0.8}\,\mathrm{Pr}_2^{2/3}(d_{2i}/d_{1e})^{0.53}, & \mathrm{Re}_2 \geq 10^4 \end{cases}$$

$$(16)$$

where factor f of the Eq. (16) is the Darcy friction factor, detailed by Colebrook equation:

$$f = (0.782 \ln \mathrm{Re}_2 - 1.51)^{-2}. \tag{17}$$

Heat transfer coefficient for the outside surface of the intermediate tube, h_3, has been calculated based on the same correlations (16).

5.4 The calculus of the effective overall heat transfer coefficient

The following expression for the effective overall heat transfer coefficient related to the total effect of convective thermal resistances has been proposed:

$$U = \frac{1}{\frac{1}{h_2} + \left(A_{1,o} + A_{2,i}\right)\left(\frac{1}{h_1 A_{1,i}} + \frac{1}{h_3 A_{2,o}}\right)} \tag{18}$$

The Eq. (18) has been obtained considering, for simplicity, the h_2 and an average heat transfer area in the inner annulus. Also, the fouling resistance has not been consid-

Table 3 Relations used for the calculation of the heat transfer coefficients

Fluid	1	2	3
Variable			
Flow space	Inner tube	Inner annulus	Outer annulus
Characteristic length [m]	$d_{1,i}$	$d_{e,1} = d_{2,i} - d_{1,o}$	$d_{e,2} = d_{3,i} - d_{2,o}$
Linear velocity [m/s]	$w_1 = 4\frac{m_1}{\rho_1 d_{1,i}^2}$	$w_2 = 4\frac{m_2}{\rho_2 \left(d_{2,i}^2 - d_{1,o}^2\right)}$	$w_3 = 4\frac{m_3}{\rho_3 \left(d_{3,i}^2 - d_{2,o}^2\right)}$
Re	$Re_1 = \frac{w_1 \rho_1 d_{1,i}}{\mu_1}$	$Re_2 = \frac{w_2 \rho_2 d_{e,1}}{\mu_2}$	$Re_3 = \frac{w_3 \rho_3 d_{e,2}}{\mu_3}$
Pr	$Pr_1 = \frac{c_{p,1} \mu_1}{\lambda_1}$	$Pr_2 = \frac{c_{p,2} \mu_2}{\lambda_2}$	$Pr_3 = \frac{c_{p,3} \mu_3}{\lambda_3}$
h [W/m² °C]	$h_1 = \frac{Nu_1 \lambda_1}{d_{1,i}}$	$h_2 = \frac{Nu_2 \lambda_2}{d_{e,1}}$	$h_3 = \frac{Nu_3 \lambda_3}{d_{e,2}}$

ered because the tubes of the heat exchanger have been clean during experimental tests.

6 The experimental study

6.1 The experimental setup and the experimental determinations

The experimental setup used for the study of oil–water heat transfer in the triple concentric-tube heat exchanger is similar to the experimental set-up used by Radulescu et al. [8] when studying water–water heat transfer in annuli in laminar and transitional flow regimes.

The schematic diagram of the experimental setup for conducting oil–water heat transfer in a triple concentric-tube heat exchanger is shown in Fig. 3. It includes a test section (an insulated triple concentric-tube heat exchanger), a thermostatic bath, a pump and instrumentations for measurement (flow meters and digital probe thermometers).

The triple concentric-tube heat exchanger being tested consists of three concentric cooper tubes and it has been thermally insulated during the test by closed cell foam insulation (thickness of 25.4 mm and thermal conductivity of 0.046 W/m K). Oil flows through the inner annulus and cold water from the supply network flows through the inner tube and outer annulus.

The temperatures of the hot fluid inlet to the test section has been adjusted to 60 and 80 by using electric heaters controlled by temperature controller (Model MGW Lauda R400, range −60…300 °C, resolution 0.1 °C, 3.2 kW). After the temperatures of the oil have been adjusted in order to achieve the desired levels, the oil has been pumped (pump Model Jürgens, 2.2 kW) from the thermostatic bath (Model MGW Lauda U12) and passed through a flow meter and the test section and then returned to the thermostatic bath.

Table 5 Comparison between the experimental and predicted values of the outlet temperatures

No.	Fluid 1		Fluid 2		Fluid 3	
	Exp	Model	Exp	Model	Exp	Model
1	13.3	12.3	54.4	56.5	13.4	13.0
2	12.4	11.9	53.8	55.6	12.6	12.4
3	11.9	11.7	53.2	55.2	12.1	11.9
4	13.1	12.2	52.2	55.2	13.3	12.9
5	12.5	11.9	51.9	54.2	12.6	12.3
6	11.8	11.6	51.4	53.6	12.0	11.9
7	17.5	15.3	74.4	74.6	19.1	16.3
8	15.0	14.8	76.6	73.6	16.4	15.4
δ (%)	5.06		3.70		4.15	

The oil flow rates have been changed from 120 to 180 l/h and cold water flow rates have been changed from 100 to 400 l/h. The cold water inlet temperatures during the test ranged between 11.4 and 14.2.

The fluids flow rates have been controlled by adjustment and measured by the flow meters with magnetic float (Models HFL-2-05 and HFB-2-05, accuracy ±4 % over entire range and ±2.5 % over centre third of the measuring range, repeatability ±1 % of full scale, range 1–19 lpm) Cold and hot fluids temperatures at inlet and outlet of the test section have been measured by digital probe thermometers (Model Kangarro 4,430 Control Company with accuracy ±1 %, range −50 to 300 °C, resolution 0.1 °C). The uncertainty of the temperature measurements is ±0.1 °C.

Before any data were read, the system has been allowed to approach the steady state. In Table 4 there are presented the experimental values associated to the inlet values of the test section.

6.2 The mathematical model simulation

For this study we have developed a simulation program of a triple concentric-tube heat exchanger, based on the

Table 4 The numerical values of the input variables

No.	Fluid 1		Fluid 2		Fluid 3	
	Flow rate (l/h)	Inlet temperature (°C)	Flow rate (l/h)	Inlet temperature (°C)	Flow rate (l/h)	Inlet temperature (°C)
1	100	11.4	180	60	100	11.4
2	200	11.4	180	60	200	11.4
3	400	11.4	180	60	400	11.4
4	100	11.4	120	60	100	11.4
5	200	11.4	120	60	200	11.4
6	400	11.4	120	60	400	11.4
7	100	14.2	150	80	100	14.2
8	200	14.2	150	80	200	14.2

Table 6 Values of heat transfer coefficients and effective overall heat transfer coefficient (W/m² °C)

No.	Fluid 1		Fluid 2		Fluid 3		U
	Flow regime	h_1	Flow regime	h_2	Flow regime	h_3	
1	Transition	630.1	Laminar	78.5	Laminar	261.8	42.6
2	Transition	1,886.0	Laminar	78.7	Laminar	319.8	52.8
3	Transition	3,880.0	Laminar	78.8	Laminar	406.2	58.6
4	Transition	628.5	Laminar	67.5	Laminar	261.7	39.1
5	Transition	1,884.9	Laminar	67.7	Laminar	319.8	47.6
6	Transition	3,879.3	Laminar	67.8	Laminar	406.2	52.3
7	Laminar	736.6	Laminar	65.9	Laminar	264.1	39.8
8	Transition	2,035.2	Laminar	66.1	Laminar	322.3	47.2

Table 7 Values of range and the uncertainties

Property	Range (W/m² °C)	Uncertainty (%)
h_1	630.1–3,880.0	0.43–0.98
h_2	65.9–78.8	2.49–3.23
h_3	261.7–406.2	0.89–2.31
U	39.1–58.6	1.50–2.50

mathematical model presented in paragraph 3 and adapted for the experimental heat exchanger. The simulation program has been run for the eight sets of the experimental data presented in Table 4. Table 5 lists the comparative values of the outlet temperatures, both experimental and predicted, and the average deviation. Average deviation is defined as

$$\delta = \frac{1}{N} \sum_1^N \frac{T^{predicted} - T^{experimental}}{T^{experimental}} \tag{19}$$

There can be observed that the average deviations from the predicted and experimental outlet temperatures are lower than 5.1 %. The difference between the experimental and the predicted values can have three main causes:

- Errors generated by the measuring systems;
- Human errors;
- Errors generated by the mathematical models used for the calculation of the heat transfer coefficients.

The errors for the cold fluids are higher than those for the hot fluid. One of the causes is represented by the low reporting mean temperature (13.4 °C) used to calculate the average deviation for the outlet temperature of the cold fluid. For the hot fluid, the reporting mean temperature is 58 °C, fact that generates lower values of the average deviations. For the experimental data presented in Table 4, the calculation program has generated values of the heat transfer coefficients, as well as the value of the effective overall heat transfer coefficient, as presented in Table 6.

The uncertainty in calculation the local heat transfer coefficients and effective overall heat transfer coefficient was performed on the system using the method suggested by Kline and McClintock [16]. The range and the uncertainties of the local heat transfer coefficients and effective overall heat transfer coefficient are presented in Table 7.

The values of the local heat transfer coefficients have been compared with the data for liquid–liquid heat transfer. The values of the overall heat transfer coefficient for this type of heat exchanger to the oil–water heat transfer have not been found in the technical literature. Therefore, the values of the effective overall heat transfer coefficients have been compared with the data for tubular heat exchangers [11, 12].

7 Conclusions

In this paper, there has been elaborated a mathematical model of the triple concentric-tube heat exchanger for the estimation of the fluids outlet temperatures, in a steady state regime. The mathematical model is based on several assumptions, the most important being the heat exchanger treating with a lumped parameters system. Based on the mathematical model, there have been predicted outlet temperatures of the fluids for an experimental triple concentric-tube heat exchanger, tested for cooling a mineral oil with water in laminar and transition flow regimes. The validation of the mathematical model has been performed using the following criteria:

- The predicted values of the outlet temperatures have been compared with the experimental temperatures, the average deviation ranging in the domain 3.5…4.8 %.
- The calculated values for the heat transfer coefficients have been similar with the literature data. EXAMPLE
- The calculated values for the effective overall heat transfer coefficient have been close to those of tubular heat exchangers, presented in literature.

Therefore, it may be considered that the mathematical model presented in this paper has been validated and can be used for the simulation of this type of heat exchangers. Future work will be devoted for testing the elaborated mathematical model for turbulent flow regime in an industrial triple concentric-tube heat exchanger.

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