

# Thermal conductivity of sands

Shan Xiong Chen

Received: 10 April 2007 / Accepted: 16 October 2007 / Published online: 8 January 2008  
© Springer-Verlag 2007

**Abstract** The effect of porosity and saturation degree on thermal conductivity of sands is investigated through laboratory test. The thermal conductivity of four sands is measured using laboratory thermal probe. It is found that thermal conductivity increases with decrease of porosity and increase saturation degree. An empirical equation of thermal conductivity expressed as the function of porosity and saturation degree is developed, its reliability is verified by comparing the predicted results with measured ones as well as results available in the literature.

## 1 Introduction

The thermal properties of soils are of importance in many engineering applications, ranging from geotechnical engineering problems such as ground freezing or soil shrinkage to nuclear waste disposal, underground energy conservation and buried electric cables. Thermal properties of soils also affect the microclimate around plants, and hence affect the plants themselves. The most important thing is the exact evaluation of the thermal conductivity of soils. Unfortunately, the demand for this property is not balanced by experimental data. To partially meet the demand, some well-known predictive models have been extended. De Vries [10] published experimental results for quartz sand. Johansen [6] found that the thermal conductivity of an unsaturated soil should be estimated by interpolation

between dry and saturated values. Donazzi [2] used the exponential function to express the relationship of thermal resistivity with porosity and saturation degree. Tarnawski and Leong [9] developed a model based on de Vries' approach, enabling easy estimation of thermal conductivity of soils with approximately log-normal particle-size distribution. Nevertheless, many existing models have limitations.

The thermal conductivity of soils depends upon various factors, i.e., type of soil, particle size distribution, soil structure, porosity, saturation degree and temperature, etc. Amongst all those factors, porosity and saturation degree have remarkable effects on the thermal conductivity. This study focuses on the effects of porosity and saturation degree on the thermal conductivity of sands.

## 2 Principle of measurements

Several methods have been used to measure the thermal conductivity of soils, and details of these methods have been presented [7]. The methods can be divided into two major categories: steady heat flow method and transient heat flow method. The steady heat flow method generally requests a long testing time and the moisture within soil may migrate during the test under the action of temperature gradient, leading to the variation of the uniform water content during the measurement. Consequently, steady-heat-flow method in rigorous sense is not suitable for unsaturated soils.

On the contrary, the transient heat flow method is rapid and only a small temperature increment occurs in the process of test. A short time is required for one measurement cycle. As a result, the moisture content remains constant during the measurement.

---

S. X. Chen (✉)  
Institute of Rock and Soil Mechanics,  
Chinese Academy of Sciences,  
Wuhan 430071, People's Republic of China  
e-mail: sxchen@whrsm.ac.cn

The transient thermal probe method is a transient heat flow method. It determines the thermal conductivity of a medium through measuring the rate of temperature rise caused by an infinite line heat source within an infinite homogeneous medium.

The temperature response at any time  $t$  was given by Carslaw and Jaeger [1]:

$$T = \frac{Q}{4\pi K} \int_x^\infty \frac{e^{-u}}{u} du \quad (1)$$

where  $T$  is the temperature at any radius  $r$ ;  $Q$  is the power input per unit length of probe;  $K$  is the thermal conductivity;  $x = \frac{r^2}{4at}$ ;  $a$  is the thermal diffusivity;  $r$  is radial distance from heat source;  $t$  is the time.

For large value of  $t$ ,  $t \gg \frac{r^2}{4a}$ , Eq. (1) results in

$$T = \frac{Q}{4\pi K} \left( \ln \frac{4at}{r^2} - \gamma \right) \quad (2)$$

where  $\gamma$  is Euler's constant with a value of 0.5772.

Then, within an increment of time, Eq. (2) can be written as:

$$\Delta T = \frac{Q}{4\pi K} (\Delta \ln t) \quad (3)$$

If  $Q$  is kept constant during the test, the measured temperature  $T$  at the line heat source has a linear relationship with the logarithm of time  $t$ . The thermal conductivity  $K$  of the measured medium can be determined from the slope of this relationship. In practice, an infinite soil mass can be replaced with a large enough soil cylinder and the line heat source can be replaced by a probe equipped with a resistance wire and a thermocouple for measuring temperature.

### 3 Sample preparation and test procedures

Four kinds of quartz sands with different gradations were tested. Their quartz contents all exceed 99% and the air-dried moisture contents are lower than 0.1%. The identification of the four sands are A for uniform sand of medium size, B for uniform coarse sands, C for uniform silty sands and D for medium sized well graded sands. The grain size distributions of the sands are given in Fig. 1. The maximum porosity of sample A, B, C, and D are 0.490, 0.547, 0.547, and 0.472 respectively, accordingly, the minimum are 0.396, 0.434, 0.434, and 0.354, respectively.

For interpretation of test data, both porosity and saturation degree of the samples were artificially controlled. According to the presumed saturation degree and porosity, soil samples with certain moisture content were prepared and cured for at least 12 h under wrap in air tight

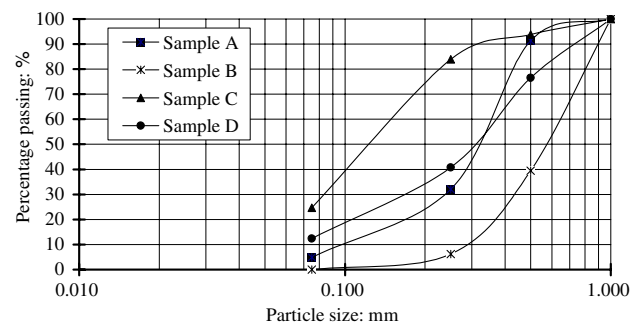


Fig. 1 Particle size distribution curve for different soils

container. The moisture contents of the samples were measured by the drying method, ensuring the accurate moisture content. Then the samples were compacted to the required dry density using a conventional compaction procedures. Upon completion of measurements, the samples were weighed to check the accuracy of the dry density, if the dry density was far from the requirement, the test must be conducted again.

Once a sample was compacted, the thermal probe was inserted into the center of the soil sample directly, or using a drill press. For a loose sample, the thermal probe can be inserted directly to have a tight contact between the two, whereas for a dense soil specimen, a borehole was drilled in advance with the same diameter of the probe, and then the probe was inserted. After the appropriate electrical connections between the Thermal Property Analyzer and the probe, the test was performed and the data were obtained. Moisture content and density determinations were made upon completion of the thermal test. Moisture content samples were obtained from a 2.5-cm inner core as well as a 5-cm diameter outer core.

The measurement of the thermal conductivity of saturated sandy soil samples has a little bit difference from the measurement of other soil samples. Due to the poor retention ability of sandy soils, the moistures in the soils is liable to loss, leading to inaccuracy of saturation degree. In order to solve this problem, the marinating method was adopted, i.e., a porous stone and a filter-paper were placed on the bottom of the prepared samples with certain dry density, and then the samples is marinated in distilled water whose level was slightly lower than the soil sample top surface.

### 4 Experimental results and discussions

The thermal conductivity of the above mentioned four kinds of sandy soils have been measured. Figures 2, 3, 4, and 5 plot the relationship curve of thermal conductivity with porosity, Figs. 6, 7, 8, and 9 show the relationship between the thermal conductivity with saturation degree.

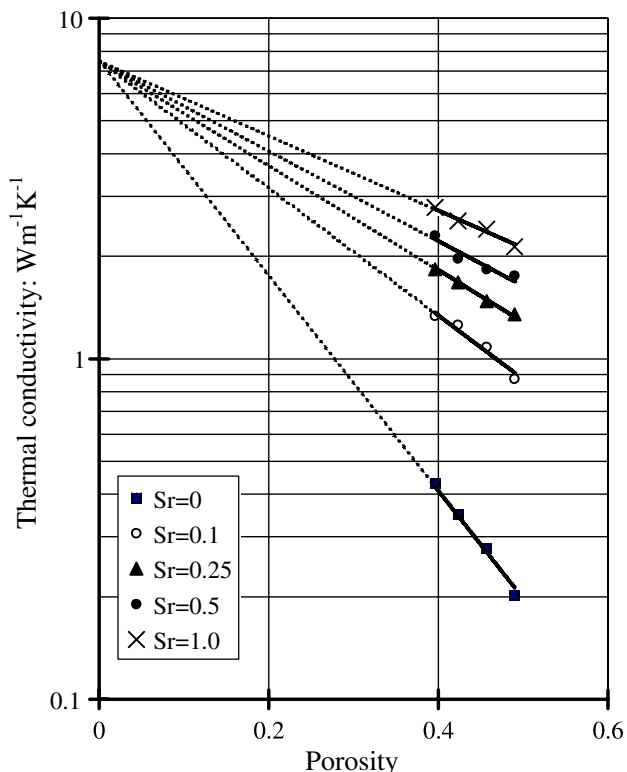


Fig. 2 Variation of thermal conductivity with porosity for sample A

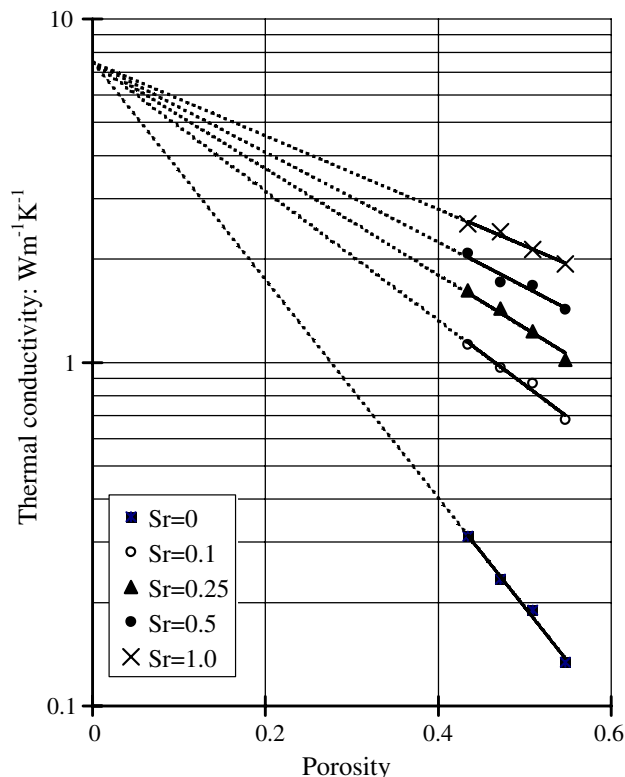


Fig. 4 Variation of thermal conductivity with porosity for sample C

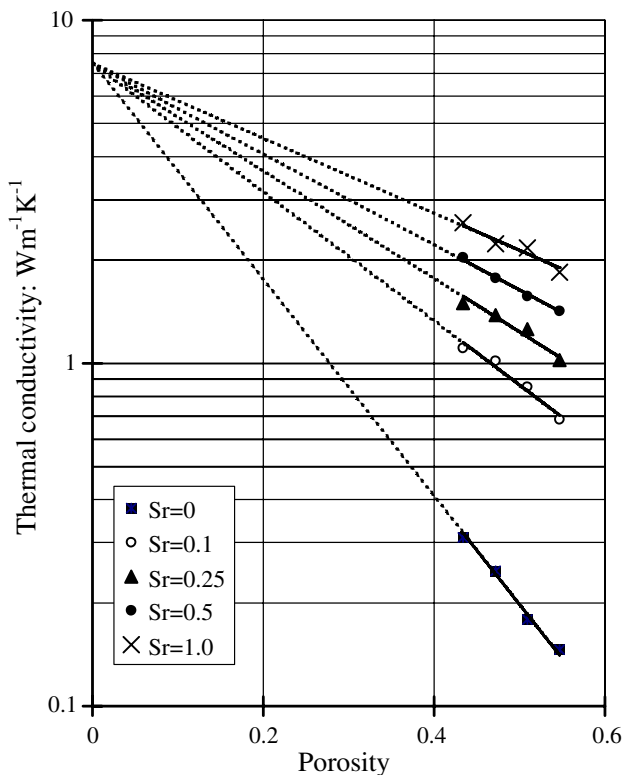


Fig. 3 Variation of thermal conductivity with porosity for sample B

From the relationship curves of thermal conductivity with saturation degree, it can be observed that the thermal conductivity increases as the saturation degree increases for a given dry density. As water is added to the soil, it forms a thin film on the soil particles, which eases the flow of heat [9].

Figures 2, 3, 4, and 5 exhibit an increment in the thermal conductivity of soils with decrease of porosity. This is due to the improvement in contact between soil particles, which leads to a better conduction of heat. It is obvious that for a given moisture content, an increase in density means an increase in its saturation degree, indicating that there is less air resisting the heat flow.

### 5 The proposed equation for estimating thermal conductivity of sands

It can be easily seen from Figs. 2, 3, 4, and 5 that the thermal conductivity  $K$  of the above four sandy soils has a logarithm relationship with its porosity  $n$ , i.e., there exists a linear relation between  $\ln K$  and  $n$ , and the slope of the line is a function of the saturation degree  $S_r$ . Therefore, the following equation can be used to describe the thermal conductivity of sandy soils.

$$\ln K = ng(S_r) + \ln K_0 \tag{4}$$

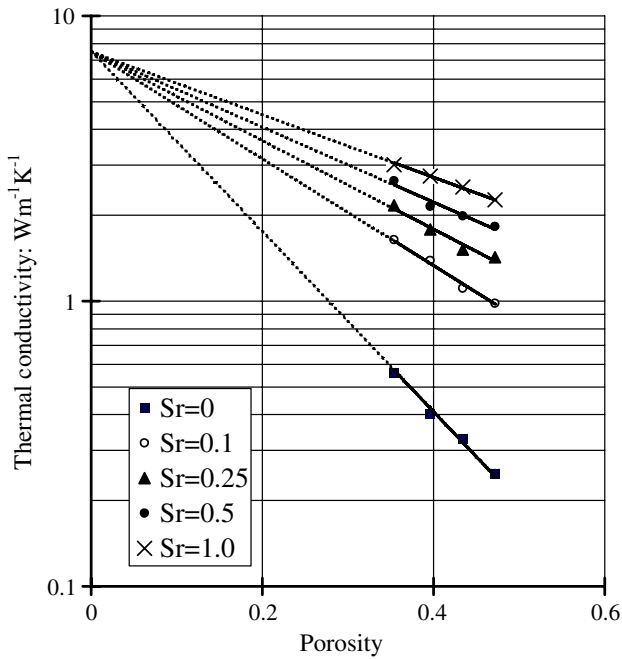


Fig. 5 Variation of thermal conductivity with porosity for sample D

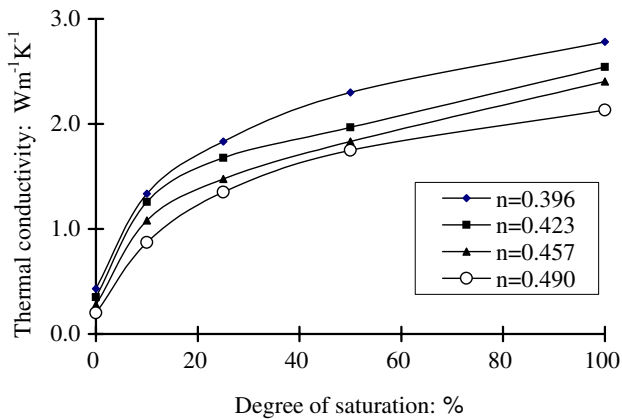


Fig. 6 Variation of thermal conductivity with saturation degree for sample A

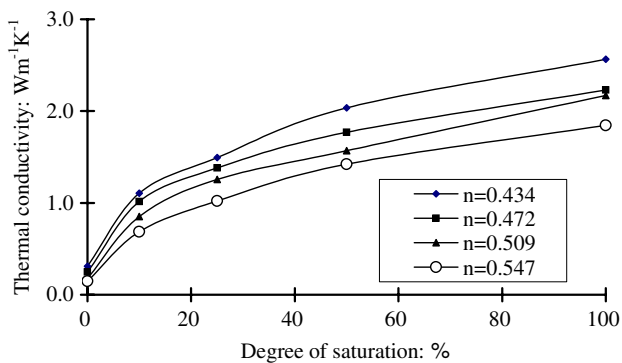


Fig. 7 Variation of thermal conductivity with saturation degree for sample B

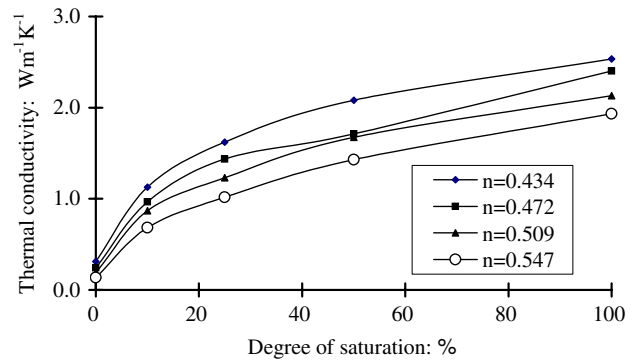


Fig. 8 Variation of thermal conductivity with saturation degree for sample C

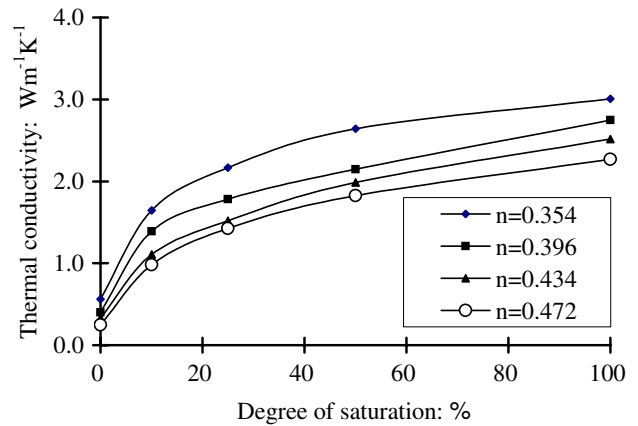


Fig. 9 Variation of thermal conductivity with saturation degree for sample D

where  $K_0$  is the thermal conductivity of the solid grain itself.

From Figs. 6, 7, 8, and 9, it can be known that the thermal conductivity of sands can be described as a power function of saturation degree. Consequently, the term of  $g(S_r)$  in Eq. (4) can be written as

$$g(S_r) = c \ln[(1 - b)S_r + b] + \ln d \tag{5}$$

where  $b$ ,  $c$ , and  $d$  are empirical coefficients.

Upon substitution of Eq. (5) into Eq. (4) and through interpretation, the following relation between soil thermal conductivity and porosity/saturation degree can be obtained

$$K(n, S_r) = K_0 d^m [(1 - b)S_r + b]^{cn} \tag{6}$$

Let  $S_r = 1.0$  in Eq. (6), the thermal conductivity of saturated sandy soils is

$$K(n, 1.0) = K_0 d^m \tag{7}$$

Referring to Johansen's model [6] for thermal conductivity of a saturated coarse soil, let  $d = \frac{K_w}{K_0}$ , Eq. (7) can be rewritten as

$$K_{\text{sat}} = K_0^{1-n} K_w^n \tag{8}$$

where  $K_w$  is thermal conductivity of water with a value of 0.613 at temperature of 300 K [4].

Upon substitution of Eq. (8) into Eq. (6), an empirical equation of thermal conductivity expressed as the function of porosity and saturation degree can be obtained

$$K(n, S_r) = K_0^{1-n} K_w^n [(1 - b)S_r + b]^{cn} \tag{9}$$

Fitting the measured data of the thermal conductivity of the above four sandy soils using DPS code [8], the regression equation can be obtained

$$K(n, S_r) = 7.5^{1-n} 0.61^n [(1 - 0.0022)S_r + 0.0022]^{0.78n} \tag{10}$$

$(R^2 = 0.996)$

where the unit of thermal conductivity is  $\text{W m}^{-1} \text{K}^{-1}$ .

### 6 Comparison with existing empirical equations

Previous researchers have conducted a vast amount of studies on the thermal conductivity of soils, accumulated a wealth of data and many empirical equations. Owing to the differences in relevant material characteristics such as type of the soil, particle size distribution, composition and structure, grain shape, etc., it is not easy to make comparison. In this section, Eq. (10) will be compared with relevant results and variation law reflected by the empirical equation of thermal conductivity will be evaluated.

### 7 Comparison with Donazzi’s empirical equation

Donazzi [2] used the following exponential function to express the relationship of thermal resistivity  $\rho$  (the inverse function of thermal conductivity, i.e.,  $\rho = 1/K$ ) with porosity  $n$  and saturation degree  $S_r$

$$\rho = \rho_w^n \rho_0^{1-n} \exp(3.08(1 - S_r)n) \tag{11}$$

where  $\rho_w$  is the thermal resistivity of water with a value of  $1.70 \text{ m K W}^{-1}$ ,  $\rho_0$  is the thermal resistivity of soil grains ( $\rho_0 = 0.25 \text{ m K W}^{-1}$  for sandy soils).

Equations (10) and (11) both reflect the following two aspects:

- (i) In the case of  $n = 0$ , the thermal conductivity has the value of soil mass in the solid form  $K_0$ , as we know,  $K_0$  is related to the composition of soils. Johansen’s results [5] found that  $K_0$  of sandy soils depends upon the quartz content of soils.  $K_0$  values in Eqs. (10) and (11) are 7.5 and  $4.0 \text{ W m}^{-1} \text{K}^{-1}$ , respectively;

- (ii) For  $S_r = 1$  and  $n = 1$ , the thermal conductivity has the value of pure water  $K_w$ ,  $K_w$  values in Eqs. (10) and (11) are  $0.61$  and  $0.60 \text{ W m}^{-1} \text{K}^{-1}$ , respectively.

The variation laws of  $K - n$  described by Eqs. (10) and (11) are coincident with each other, i.e.,  $\ln K$  and  $n$  has a linear relationship, the slope is a function of  $S_r$ . Whereas the variation laws of  $K - S_r$  described by Eqs. (10) and (11) are different. In the interest of comparison,  $\rho_0 = 1/7.5 \text{ m K W}^{-1}$  and  $n = 0.354$  are given in Eq. (11), the  $K - S_r$  curve is shown in Fig. 10. For other values of  $n$ , similar conclusion can also be obtained.

According to experiences, the thermal conductivity  $K$  of sandy soils change rapidly, provided that saturation degree  $S_r$  is low (for instance  $S_r = 0-0.3$ ) and the variation of thermal conductivity  $K$  slows down with the increase of saturation degree. It can be seen from Fig. 10 that Eq. (10) reflects these feature and Eq. (11) does not reflect these feature.

### 8 Comparison with Johansen’s empirical equation

The empirical equation for soils thermal conductivity proposed by Johansen [6] is

$$K = (K_{\text{sat}} - K_{\text{dry}})K_e + K_{\text{dry}} \tag{12}$$

where  $K_{\text{sat}}$  is the thermal conductivity of saturated soils;  $K_{\text{dry}}$  is the thermal conductivity of dry soils and  $K_e$  is the Kersten number. The values of  $K_{\text{sat}}$ ,  $K_{\text{dry}}$  and  $K_e$  are calculated using the following formulae

$$K_{\text{dry}} = \frac{0.137\gamma_d + 64.7}{2700 - 0.947\gamma_d} \tag{13}$$

in which  $\gamma_d$  is the dry density of soils with an unit of  $\text{kg m}^{-3}$

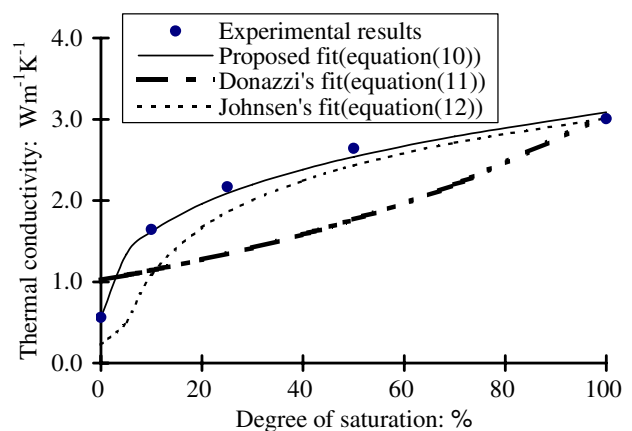


Fig. 10 Thermal conductivity vs. saturation degree relationship for sample D ( $n = 0.354$ )

$$K_e = 0.71g S_r + 1 \quad (S_r > 0.05) \quad (14)$$

$$K_{\text{sat}} = 0.57^n K_s^{1-n} \quad (15)$$

in which  $K_s = 7.7^\alpha 2.0^{1-\alpha}$ ,  $\alpha$  is the quartz content.

In the interest of comparison,  $\gamma_d = 1,712 \text{ kg m}^{-3}$  and  $K_s = 7.5 \text{ W m}^{-1} \text{ K}^{-1}$  are given in Eqs. (10), (12), respectively, the relation curve of  $K - S_r$  is shown in Fig. 10. It is known that Johansen's equation underestimates the thermal conductivity of soils at low moisture contents (Ewen and Thomas [3]). This is reconfirmed here in Fig. 10.

It can be known from the comparisons that the empirical equation proposed in this paper can rationally describe the changing law of sandy soils in thermal conductivity, but the equation becomes simpler and clearer with only two empirical coefficients.

## 9 Conclusions

For specific soil, the thermal conductivity depends upon porosity and saturation degree, etc. In terms of test and interpretation, we propose an empirical equation to characterize the relationship between thermal conductivity and porosity/ saturation degree. Comparison show that the empirical equation can reflect the mechanism of the variation in thermal conductivity of sandy soils.

1. With the increase of dry density, decrease of porosity and improvement of contacts between grains, the thermal conductivity of soils increases correspondingly.
2. The main paths for thermal conduction between soil grains in a dry state are confined to grains contact points. Small increment of water contents around the contact points will lead to rapid increment of the

thermal conductivity. However, further increment of water contents will not result in obvious increment of effective contact faces, and the increment of thermal conductivity slows down.

The comparison of the proposed empirical equation with models of Donazzi and Johansen shows that the proposed equation can rationally describe the thermal conductivity of sands and can be used for analysis of practical problems.

## References

1. Carslaw HS, Jaeger JC (1959) Conduction of heat in solids. Oxford University Press, Oxford
2. Donazzi F (1977) Soil thermal and hydrological characteristics in designing underground cables. Proc IEE 123:506–516
3. Ewen J, Thomas HR (1987) The thermal probe—a new method and its use on a unsaturated sand. Géotechnique 37(1):91–105
4. Frank PI, David PD (1981) Fundamentals of heat transfer. Wiley, New York
5. Farouki O (1982) Evaluation of methods for calculating soil thermal conductivity. Cold Regions Research and Engineering Laboratory Report 82–8, US Army Corps of Engineering, Hanover, New Hampshire
6. Johansen O (1975) Thermal conductivity of soils. Ph.D. thesis, University of Trondheim, Trondheim, Norway. (English Translation 637, US Army Corps of Engineers, Cold Regions Research and Engineering Laboratory, Hanover, New Hampshire)
7. Mitchell JK, Kao TC (1978) Measurement of soil thermal resistivity. J Geotech Eng ASCE 101(GT10):1307–1320
8. Qiyi T, Mingguang F (2002) DPS data processing system for practical statistics. Science, Beijing
9. Tarnawski VR, Leong WH (2000) Thermal conductivity of soils at very low moisture content and moderate temperatures. Transport Porous Media 41:137–147
10. De Vries DA (1963) Thermal properties of soils. In: van Wijk WR (ed) Physics of plant environment. North-Holland, Amsterdam, pp 210–235