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Transient heat conduction in functionally graded thick hollow cylinders by analytical method

S. M. Hosseini · M. Akhlaghi · M. Shakeri

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Abstract In this article, transient heat conduction in a cylindrical shell of functionally graded material is studied by using analytical method. The shell is assumed to be in axisymmetry conditions. The material properties are considered to be nonlinear with a power law distribution through the thickness. The temperature distribution is derived analytically by using the Bessel functions. To verify the proposed method the obtained numerical results are compared with the published results. The comparisons of temperature distribution between various time and material properties are presented.

List of symbols

a	inner radius
b	outer radius
B_1, B_2	constant coefficients
B_{1i}, B_{2i}	constant coefficients
C (kJ/kg K)	specific heat
h (W/m^2 K)	heat convection coefficients
k (W/m K)	heat conductivity
$\bar{k}$	nondimentional heat conduction coefficient
k_0	material constant
m_1, m_2	material constant
N	number of eigenvalues
r (m)	radius
$\bar{r}$	nondimensional radius
t	time
$\bar{t}$	nondimensional time
T (°K)	temperature distribution
$\bar{t}$	nondimensional temperature distribution
T_0 (°K)	constant temperature
ρc_c (Kj/m^3 K)	the product of density and specific heat of ceramic
ρc_0	material constant
$\rho \bar{c}$	nondimensional
ρ (kg/m^3)	density
θ (°C)	temperature of shell body
θ_1 (°C)	temperature of fluid
γ, γ_i	eigenvalues
λ	constant value

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1 Introduction

Functionally graded materials (FGMs) are new kind of materials. These materials are spatial composite within which the mechanical properties vary continuously in the macroscopic sense from one surface to the other. These materials are expected to be used for thermal applications and high rate temperature loading. An important and useful aspect in this case is the determination of transient temperature distribution. Analytical and computational studies of appointing stresses and displacements in cylindrical shell made of FGM have been carried out by some of researchers as following.

Temperature and stress distributions were determined in a stress-relief-type plate of FGMs with steady state and transient temperature distributions by Awaji [1]. A multi-layered material model was employed to solve the transient temperature field in an FGM strip with continuous and piecewise differentiable material properties by Jin [2]. He obtained a closed form asymptotic solution of the temperature field for short times, by using an asymptotic analysis and an integration technique and the Laplace transform. A general analysis of one dimensional steady state thermal stresses in a thick hollow cylinder under axisymmetry and nonaxisymmetry loads was developed by Jabbari et al. [3, 4].

A local boundary integral equation method with the moving least squares approximation of physical fields was applied to transient heat conduction analysis in functionally graded materials by Sladek et al. [5]. They solved the initial boundary value problem in the Laplace transform domain with a subsequent numerical Laplace inversion to obtain time-dependent solutions. Tarn et al. [6] have studied the end effects of steady state heat conduction in a hollow or solid circular cylinder of FGM under 2D thermal loads with arbitrary end conditions. They evaluated the decay length that characterizes the end effects on thermal field by using matrix algebra and eigen function expansion. The sensivity analysis of heat conduction for functionally graded materials and the steady state, transient problem treated with the direct method and the adjoint method were presented by Chen et al. [7]. The precise time integration method is employed to solve the transient problem by them. Transient temperature field and associated thermal stresses in functionally graded materials have been determined by using a finite element-finite difference method (FEM/FDM) by Wang et al. [8]. Thermal shock fracture of a FGM plate and the thermal shock resistance of FGMs were analyzed by them. A finite element/finite difference method (FEM/FDM) was developed also to solve the time-dependent temperature field in non-homogeneous materials such as functionally graded materials by Wang et al. [9].

This paper presents an analytical solution for transient temperature distribution in functionally graded thick hollow cylinders.

2 Temperature field

To determine the temperature distribution in functionally graded thick hollow cylinder with inner and outer radii a and b, the following boundary and initial conditions for the temperature field are considered. The inner surface of shell is considered to be made of a ceramic material. The boundary conditions are:

$$k\frac{\partial T}{\partial r} + hT = 0 \quad \text{at } r = a \tag{1}$$

$$\frac{\partial T}{\partial r} = 0 \quad \text{at } r = b \tag{2}$$

where k is the thermal conductivity and h is the heat transfer coefficient and T is defined by:

$$T(r,t) = \theta(r,t) - \theta_1 \tag{3}$$

where $\theta(r,t)$ is the temperature of shell body and θ_1 is the temperature of fluid that flows in the cylinder. The initial condition for temperature field is:

$$T(r,0) = T_0 \tag{4}$$

where T_0 is a constant temperature. The heat transfer equation in functionally graded hollow cylinder for plane strain and axisymmetry case is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k \cdot r \cdot \frac{\partial T}{\partial r}\right) = \rho c \frac{\partial T}{\partial t} \tag{5}$$

where ρ is the density and c is the specific heat. We used the following nondimensional variables for temperature field.

$$\bar{T} = \frac{T}{T_0}, \quad \bar{r} = \frac{r}{a}, \quad \bar{t} = \frac{th}{\rho c_c \cdot a}, \quad \bar{\rho}\bar{c} = \frac{\rho c}{\rho c_c}, \quad \bar{k} = \frac{k}{ah}$$

Where $\rho\ c_c$ is the standard value (the density and specific heat of inner surface ceramic material). Using these terms, the heat transfer and boundary conditions equations can be written as follows:

$$\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{k} \cdot \bar{r} \cdot \frac{\partial \bar{T}}{\partial \bar{r}}\right) = \bar{\rho}\bar{c}\frac{\partial \bar{T}}{\partial \bar{t}} \tag{6}$$

$$\bar{k}\frac{\partial \bar{T}}{\partial \bar{r}} + \bar{T} = 0 \quad at\ \bar{r} = 1 \tag{7}$$

$$\frac{\partial \bar{T}}{\partial \bar{r}} = 0 \quad at\ \bar{r} = b/a \tag{8}$$

$$\bar{T}(r,0) = 1 \tag{9}$$

It is assumed that the thermal conductivity $\bar{k}$ and $\bar{\rho}\bar{c}$ are the power functions of r as:

$$\bar{k} = k_0 \bar{r}^{m_1} \tag{10}$$

$$\bar{\rho}\bar{c} = \rho c_0 \bar{r}^{m_2} \tag{11}$$

In the above equations k_0, $\rho\, c_0$, m_1, m_2 are the material constants and $\rho\, c_0 = 1$ for the ceramic material. With introducing Eqs. 10 and 11 in Eq. 6 the following equation is obtained:

$$k_0\bar{r}^m \frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + k_0(m_1+1)\bar{r}^{m-1}\frac{\partial \bar{T}}{\partial \bar{r}} = \rho c_0 \frac{\partial \bar{T}}{\partial \bar{t}} \tag{12}$$

where:

$$m = m_1 - m_2 \tag{13}$$

To solve the partial differential Equation (12), by using the separation of variables method, the following solution is assumed:

$$\bar{T}(\bar{r},\bar{t}) = R(\bar{r})\cdot g(\bar{t}) \tag{14}$$

Substituting Eq. 14 into Eq. 12 yields:

$$k_0\bar{r}^m \frac{\partial^2 R}{\partial r^2}\cdot g + k_0(m_1+1)\bar{r}^{m-1}\frac{\partial R}{\partial r}\cdot g = \rho c_0 \frac{\partial g}{\partial t}\cdot R \tag{15}$$

or,

$$\frac{k_0\bar{r}^m \frac{\partial^2 R}{\partial \bar{r}^2} + k_0(m_1+1)\bar{r}^{m-1}\frac{\partial R}{\partial \bar{r}}}{R} = \frac{\rho c_0 \frac{\partial g}{\partial \bar{t}}}{g} = -\lambda^2 \tag{16}$$

In the above equations, λ is a constant value. From Eq. 16, two ordinary differential equations are obtained as follows:

$$\rho c_0 \frac{\partial g}{\partial \bar{t}} + \lambda^2 g = 0 \tag{17}$$

$$k_0\bar{r}^m \frac{\partial^2 R}{\partial \bar{r}^2} + k_0(m_1+1)\bar{r}^{m-1}\frac{\partial R}{\partial \bar{r}} + \lambda^2 R = 0 \tag{18}$$

Solutions of Eqs. 17 and 18 are:

$$g(\bar{t}) = A_1 e^{-\frac{\lambda^2}{\rho c_0}\bar{t}} \tag{19}$$

$$R(\bar{r}) = \bar{r}^{-\frac{m_1}{2}}\left\{B_1 J_n\left(\gamma \bar{r}^{\frac{2-m}{2}}\right) + B_2 Y_n\left(\gamma \bar{r}^{\frac{2-m}{2}}\right)\right\} \tag{20}$$

where:

$$n = \frac{m_1}{m-2} \tag{21}$$

$$\gamma = \frac{2\lambda}{\sqrt{k_0}(m-2)} \tag{22}$$

Thus the temperature distribution is:

$$\bar{T}(\bar{r},\bar{t}) = e^{-\frac{\lambda^2}{\rho c_0}\bar{t}}\cdot \bar{r}^{-\frac{m_1}{2}}\left\{B_1 J_n\left(\gamma \bar{r}^{\frac{2-m}{2}}\right) + B_2 Y_n\left(\gamma \bar{r}^{\frac{2-m}{2}}\right)\right\} \tag{23}$$

By using Eqs. 23 and 10, the boundary condition (7) is rewritten as the follows:

$$\begin{aligned} &k_0\bar{r}^{m_1}\left[e^{-\frac{\lambda^2}{\rho c_0}\bar{t}}\left(-\frac{m_1}{2}\right)\bar{r}^{-\frac{m_1}{2}-1}\left\{B_1 J_n\left(\gamma \bar{r}^{\frac{2-m}{2}}\right)+B_2 Y_n\left(\gamma \bar{r}^{\frac{2-m}{2}}\right)\right\}\right] \\ &+k_0\bar{r}^{m_1}\left[e^{-\frac{\lambda^2}{\rho c_0}\bar{t}}\cdot \bar{r}^{-\frac{m_1}{2}}\gamma\left(\frac{2-m}{2}\right)\bar{r}^{-\frac{m}{2}} \right.\\ &\left.\left\{B_1 J_n'\left(\gamma \bar{r}^{\frac{2-m}{2}}\right)+B_2 Y_n'\left(\gamma \bar{r}^{\frac{2-m}{2}}\right)\right\}\right] \\ &+e^{-\frac{\lambda^2}{\rho c_0}\bar{t}}\cdot \bar{r}^{-\frac{m_1}{2}}\left\{B_1 J_n\left(\gamma \bar{r}^{\frac{2-m}{2}}\right)+B_2 Y_n\left(\gamma \bar{r}^{\frac{2-m}{2}}\right)\right\}=0 \end{aligned} \tag{24}$$

where $\bar{r}=1$, thus:

$$\frac{B_2}{B_1} = \frac{f_2}{f_1} \tag{25}$$

where

$$\begin{aligned} f_1 = &\left((1)^{-\frac{m_1}{2}} - \frac{m_1}{2}k_0(1)^{\frac{m_1}{2}-1}\right)Y_n\left(\gamma(1)^{\frac{2-m}{2}}\right) \\ &+\left(\frac{2-m}{2}\right)\gamma k_0(1)^{\frac{m_2}{2}}\cdot Y_n'\left(\gamma(1)^{\frac{2-m}{2}}\right) \end{aligned} \tag{26}$$

$$\begin{aligned} f_2 = &\left((1)^{-\frac{m_1}{2}} - \frac{m_1}{2}k_0(1)^{\frac{m_1}{2}-1}\right)J_n\left(\gamma(1)^{\frac{2-m}{2}}\right) \\ &+\left(\frac{2-m}{2}\right)\gamma k_0(1)^{\frac{m_2}{2}}\cdot J_n'\left(\gamma(1)^{\frac{2-m}{2}}\right) \end{aligned} \tag{27}$$

and the boundary condition (8) is written as:

$$\begin{aligned} \frac{\partial \bar{T}}{\partial \bar{r}} = &e^{-\frac{\lambda^2}{\rho c_0}\bar{t}}\left(-\frac{m_1}{2}\right)\bar{r}^{-\frac{m_1}{2}-1}\left\{B_1 J_n\left(\gamma \bar{r}^{\frac{2-m}{2}}\right) + B_2 Y_n\left(\gamma \bar{r}^{\frac{2-m}{2}}\right)\right\} \\ &+ e^{-\frac{\lambda^2}{\rho c_0}\bar{t}}\cdot \bar{r}^{-\frac{m_1}{2}}\gamma\left(\frac{2-m}{2}\right)\bar{r}^{-\frac{m}{2}} \\ &\left\{B_1 J_n'\left(\gamma \bar{r}^{\frac{2-m}{2}}\right) + B_2 Y_n'\left(\gamma \bar{r}^{\frac{2-m}{2}}\right)\right\} = 0 \end{aligned} \tag{28}$$

where $\bar{r} = b/a$, thus:

$$\frac{B_2}{B_1} = \frac{f_4}{f_3} \tag{29}$$

where

$$f_3 = \left(\frac{2-m}{2}\right)\gamma(b/a)^{\frac{m_2-2m_1}{2}} \cdot Y'_n\left(\gamma(b/a)^{\frac{2-m}{2}}\right) - \frac{m_1}{2}(b/a)^{-\frac{m_1}{2}-1} \cdot Y_n\left(\gamma(b/a)^{\frac{2-m}{2}}\right) \tag{30}$$

$$f_4 = \left(\frac{2-m}{2}\right)\gamma(b/a)^{\frac{m_2-2m_1}{2}} \cdot J'_n\left(\gamma(b/a)^{\frac{2-m}{2}}\right) - \frac{m_1}{2}(b/a)^{-\frac{m_1}{2}-1} \cdot J_n\left(\gamma(b/a)^{\frac{2-m}{2}}\right) \tag{31}$$

and

$$J'_n\left(\gamma\bar{r}^{\frac{2-m}{2}}\right) = \frac{d}{d\eta}\left[J_n\left(\gamma\bar{r}^{\frac{2-m}{2}}\right)\right] \tag{32}$$

$$Y'_n\left(\gamma\bar{r}^{\frac{2-m}{2}}\right) = \frac{d}{d\eta}\left[Y_n\left(\gamma\bar{r}^{\frac{2-m}{2}}\right)\right] \tag{33}$$

where $\eta = \gamma\bar{r}^{\frac{2-m}{2}}$. The right-hand side of both Eqs. 25 and 29 shall be equal.

$$\frac{f_2}{f_1} = \frac{f_4}{f_3} \tag{34}$$

The eigenvalue γ can be obtained from Eq. 35 as the follows:

$$f_1 \cdot f_4 - f_3 \cdot f_2 = 0 \tag{35}$$

The Eq. 35 can be solved by using numerical methods, such as Newton–Rophson method and using MATLAB software. The boundary condition (7) can be derived by using Eqs. 26 and 27 as follows:

$$e^{-\frac{\lambda^2}{\rho c_0}\bar{t}}(B_2 \cdot f_1 - B_1 \cdot f_2) = 0 \tag{36}$$

The coefficient B_1 replaces by Eq. 29 and then:

$$e^{-\frac{\lambda^2}{\rho c_0}\bar{t}}\left(B_2 \cdot f_1 - B_2\frac{f_3}{f_4} \cdot f_2\right) = e^{-\frac{\lambda^2}{\rho c_0}t}\frac{B_2}{f_4}(f_1 \cdot f_4 - f_3 \cdot f_2) = 0 \tag{37}$$

It means that the boundary condition (7) is satisfied for each eigenvalues. By using similar method, the boundary condition (8) can be satisfied. There are infinite numbers of roots for Eq. 35 thus Eq. 23 can be written:

$$\bar{T}(\bar{r},\bar{t}) = \sum_{i=1}^{\infty} e^{-\frac{\lambda_i^2}{\rho c_0}\bar{t}} \cdot \bar{r}^{-\frac{m_1}{2}}\left\{B_{1i}J_n\left(\gamma_i\bar{r}^{\frac{2-m}{2}}\right) + B_{2i}Y_n\left(\gamma_i\bar{r}^{\frac{2-m}{2}}\right)\right\} \tag{38}$$

where γ_i (eigen values) are the roots of Eq. 35 and $i = 1,..., \infty$. To determine of the value of B_{1i} and B_{2i}, we consider N numbers of eigenvalues. By using the initial conditions at $t = 0$, Eq. 38 can be satisfied.

$$1 = \sum_{i=1}^{\infty} \bar{r}^{-\frac{m_1}{2}}\left\{B_{1i}J_n\left(\gamma_i\bar{r}^{\frac{2-m}{2}}\right) + B_{2i}Y_n\left(\gamma_i\bar{r}^{\frac{2-m}{2}}\right)\right\} \tag{39}$$

For $2N$ values of r between 1 and b/a, the Eq. 39 can be calculated and $2N$ equations will derive. These derived equations can be written in matrix form as follows:

$$\begin{bmatrix} k(1,1) & . & . & . & k(1,2N) \\ . & & . & & . \\ . & & & . & . \\ . & & & . & . \\ k(2N,1) & . & . & . & k(2N,2N) \end{bmatrix} \begin{Bmatrix} B_{11} \\ B_{21} \\ . \\ B_{1N} \\ B_{2N} \end{Bmatrix} = \begin{Bmatrix} 1 \\ . \\ . \\ . \\ 1 \end{Bmatrix} \tag{40}$$

The components of k matrix are:

$$k(i, 2j-1) = \bar{r}_i^{-\frac{m_1}{2}} \cdot J_n\left(\gamma_j \cdot \bar{r}_i^{\frac{2-m}{2}}\right) \tag{41}$$

$$k(i, 2j) = \bar{r}_i^{-\frac{m_1}{2}} \cdot Y_n\left(\gamma_j \cdot \bar{r}_i^{\frac{2-m}{2}}\right) \quad j = 1,\ldots,N \text{ and } i = 1,\ldots,2N \tag{42}$$

The B_{1i} and B_{2i} coefficients are determined from Eq. 40 as:

$$\begin{Bmatrix} B_{11} \\ B_{21} \\ . \\ B_{1N} \\ B_{2N} \end{Bmatrix} = [k]^{-1} \begin{Bmatrix} 1 \\ . \\ . \\ . \\ 1 \end{Bmatrix} \tag{43}$$

To achieve high precision results, the number of eigenvalues in Eq. 38 should be increased.

3 Numerical results and discussion

To verify the proposed method, the functionally graded hollow cylinder with $b/a = 1.1$ where a and b are inner and outer radii, is considered. Suppose that the inner surface is made of graphite/epoxy with thermal conductivity of $k = 0.72$ W/m K. This shell is a thin hollow cylinder.

For t=0 and $m_1 = 0$ (homogeneous material), the first six eigenvalues are obtained by using the boundary conditions (flux-prescribed at inner and outer surfaces) in Ref. [6]. For simplicity of analysis the power law coefficients for m_1 and m_2 are considered to be the same. These eigenvalues are compared with the results presented in Ref. [6]. This comparison is shown in Table 1.The results for various m_1 and $b/a = 1.1$ are in good agreement with those obtained according to Ref. [6]. Consider the functionally graded thick hollow cylinder with inner radius a and outer radius b. The boundary and initial conditions are defined in Eqs. 7, 8 and 9. Suppose that the inner surface is made of alumina (ceramic). The alumina specifications are $k_c = 46$ W/m K, $c_c = 0.76$ kJ/kg K, $\rho_c = 3{,}800$ kg/m^3, and inner radius a is 0.25 m.

The convection coefficient and temperature of the fluid flowing within hollow cylinder are $h = 4{,}600$ W/m^2 K and $\theta_1 = 200°$ C. The first five eigenvalues of the functionally graded thick hollow cylinder for various values of power index m_1 and b/a and infinite length are given in Tables 2 and 3. It can be seen that the eigenvalues for various power law exponents are increased as the value of b/a is decreased. By using the proposed method, the behavior of a shell subjected to a transient thermal loading can be shown. Figure 1 shows the radial distribution of temperature for $\bar{t} = 0.5$ and $b/a = 1.5$ and various values of power law exponents. It is evident that the errors decrease as the numbers of eigenvalues are increased. The effects of eigenvalue numbers on the accuracy of the solution are shown in Fig. 2. The accuracy of the results can be improved, by increasing the number of applied eigenvalues. For large values of power exponents, one can see higher rates of heat transfer. Histories of the temperature distribution across the thickness of the shell are illustrated in Fig. 3. Figures 4 and 5 show the radial distribution of temperature and histories of temperature distribution across thickness of shell for $b/a = 2$. It can be seen from the Figs. 3 and 5 that for small values of b/a, the shell reaches the steady-state temperature profile faster than the case with larger values of b/a. By decreasing the value of b/a, the behavior of the shell approaches to the behavior of a shell made of homogeneous material. The comparison of the present method and a numerical method (using MATLAB software) for $m_1 = m_2 = 0.5$ and $b/a = 2$, for different times, are illustrated in Fig. 6. The results are in good agreement with obtained results from numerical method (MATLAB software). For m_1 and m_2 differing from each other, assume that the outer surface of shell is aluminum with specifications as: $k_m = 204$ W/m K, $c_m = 0.896$ kJ/kg K, $\rho_m = 2{,}707$ kg/m^3

Using Eqs. 10 and 11, the power function exponents m_1 and m_2 can be calculated as, $m_1 = 4.018$, $m_2 = -0.432$.The history of temperature distribution across the thickness is shown in Fig. 7.

Table 2 The first eight eigenvalues for FGM hollow cylinder ($b/a = 1.5$)

	$m_2 = m_1$				
	0.1	0.5	1	2	5
γ_1	3.147	3.0541	2.9392	2.7171	2.114
γ_2	10.1309	10.1106	10.0887	10.0569	10.0605
γ_3	16.8679	16.8604	16.8533	16.8468	16.8888
γ_4	23.4873	23.4848	23.4834	23.4862	23.5392
γ_5	30.022	30.0217	30.0228	30.0292	30.0835
γ_6	36.4963	36.4972	36.4993	36.507	36.559
γ_7	42.9279	42.9292	42.9317	42.9399	42.9885
γ_8	49.3287	49.3302	49.3329	49.3409	49.3859

Table 3 The first eight eigenvalues for FGM hollow cylinder ($b/a = 2$)

	$m_2 = m_1$				
	0.1	0.5	1	2	5
γ_1	1.4107	1.3328	1.2387	1.0621	0.6303
γ_2	4.8398	4.8226	4.8064	4.7923	4.9018
γ_3	8.1344	8.1259	8.1187	8.1153	8.1958
γ_4	11.4028	11.3983	11.395	11.3966	11.4668
γ_5	14.6544	14.6521	14.6511	14.6556	14.7206
γ_6	17.8919	17.891	17.8914	17.8977	17.9589
γ_7	21.1169	21.1169	21.1183	21.1256	21.1836
γ_8	24.3312	24.3318	24.3337	24.3415	24.3965

Table 1 The first six eigenvalues for thin hollow cylinder ($b/a = 1.1$)

m_1	γ_1		γ_2		γ_3		γ_4		γ_5		γ_6	
	Present	Ref. [6]	Present	Ref. [6]	Present	Ref. [6]	Present	Ref. [6]	Present	Ref. [6]	Present	Ref. [6]
0	0	0	31.4268	31.4292	62.8373	62.8385	94.2514	94.2522	125.6664	125.667	157.0818	157.0823
0.5	0	0	31.4226	31.4391	62.8352	62.8434	94.25	94.2555	125.6654	125.6695	157.081	157.0843
1	0	0	31.4308	31.4512	62.8392	62.8495	94.2527	94.2596	125.6674	125.6726	157.0825	157.0867
2	0	0	31.4699	31.4821	62.8589	62.865	94.2659	94.2699	125.6773	125.6803	157.0905	157.0928
5	0	0	31.5498	31.6271	62.8902	62.9378	94.2849	94.3185	125.6909	125.7167	157.1011	157.1221

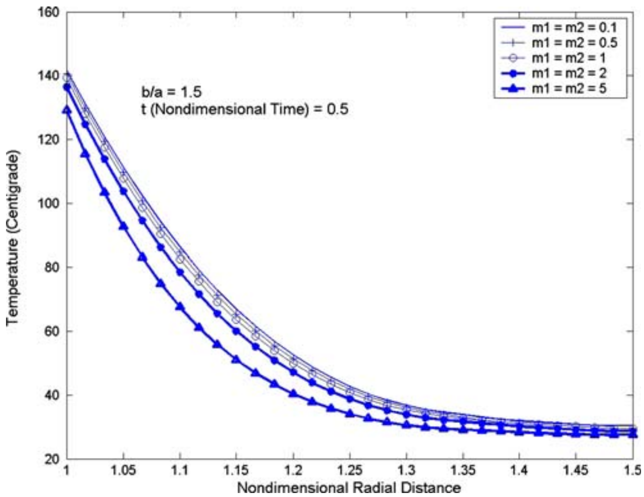

Fig. 1 Radial distribution of temperature for = 0.5 and b/a = 1.5

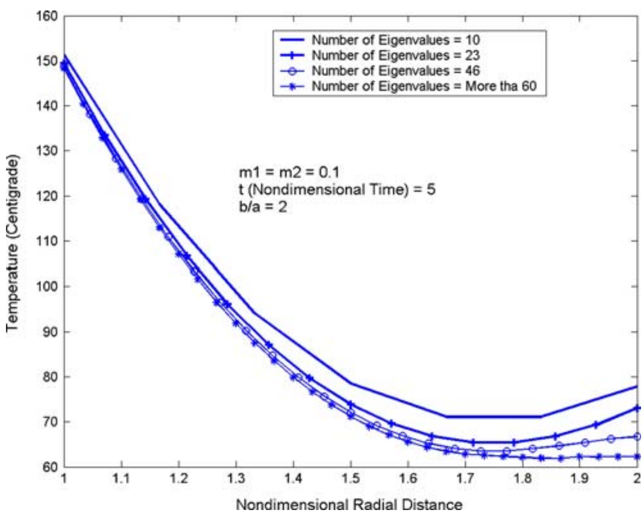

Fig. 2 The effects of eigenvalue numbers on results

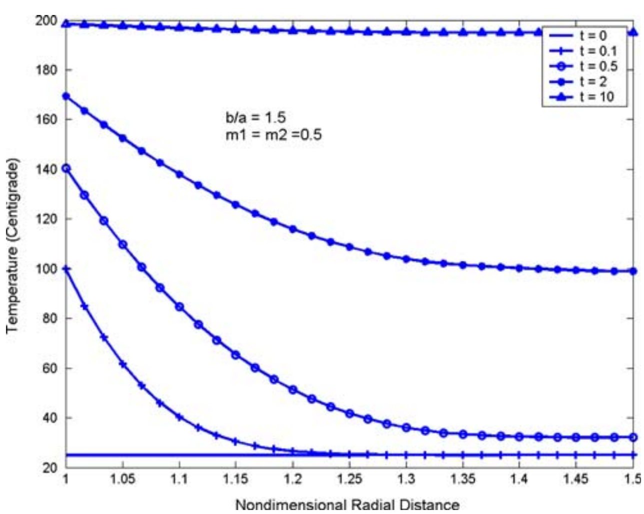

Fig. 3 History of temperature radial distribution for $m_1 = m_2 = 0.5$ and $b/a = 1.5$

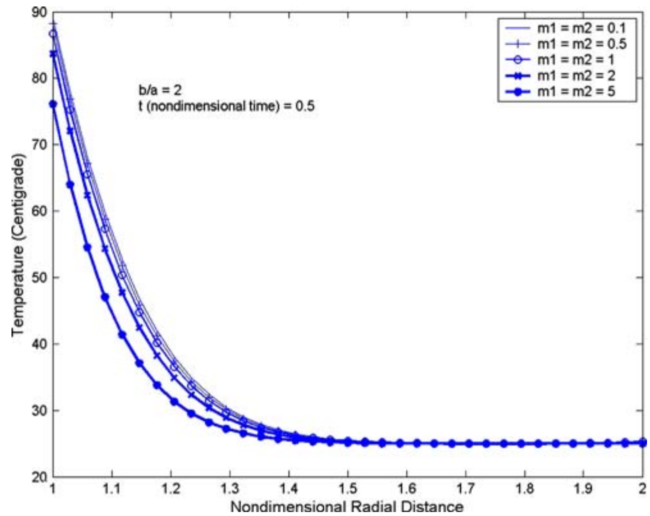

Fig. 4 Radial distribution of temperature for t = 0.5 and b/a = 2

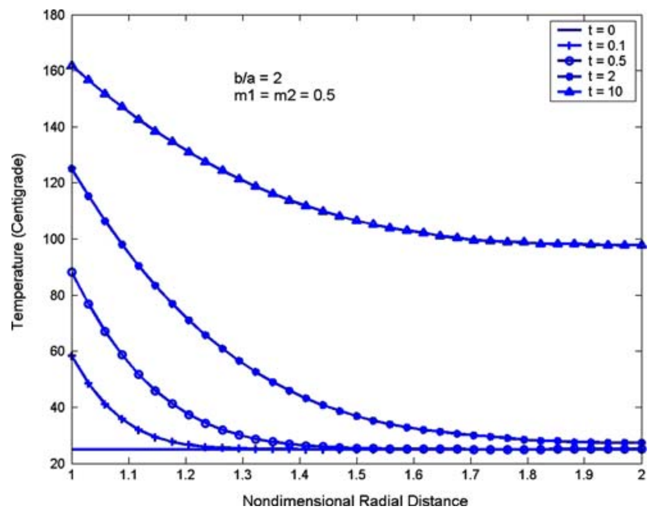

Fig. 5 History of temperature radial distribution for $m_1 = m_2 = 0.5$ and $b/a = 2$

4 Conclusion

In this paper, an analytical solution for transient heat conduction of functionally graded thick hollow cylinder is presented in axisymmetry conditions. The material properties through the thickness of the shell are assumed to be nonlinear with a power law distribution. The temperature distribution of functionally graded cylindrical shells are investigated for various power function exponent m_1 and m_2. The results of this procedure can be outlined as:

1. The trend of the variation of the first eight eigenvalues of functionally graded thick hollow cylinder for various power law exponents and infinite length

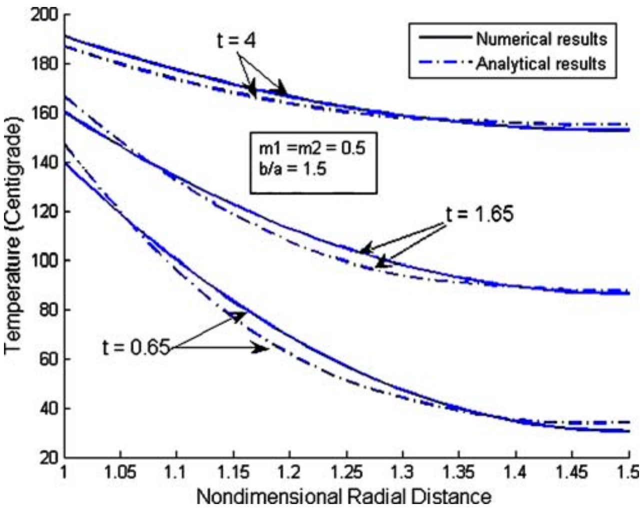

Fig. 6 The comparison of present results and numerical method

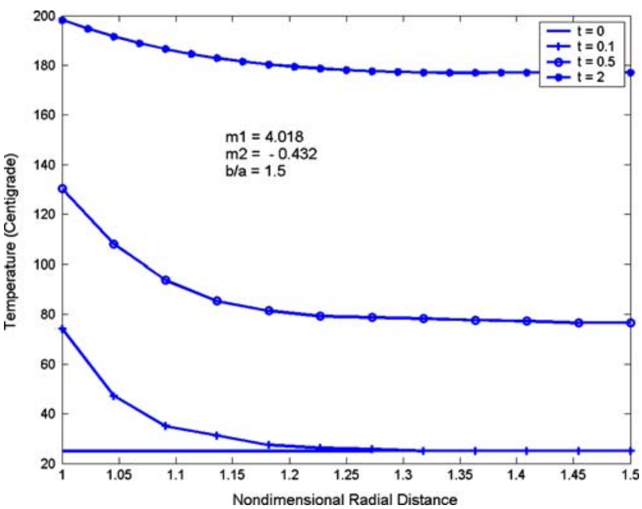

Fig. 7 History of temperature radial distribution for $m_1 = 4.018$, $m_2 = -0.432$ and $b/a = 1.5$

show that the accuracy of the results can be increased by using more number of eigenvalues.

2. The transient temperature distribution in functionally graded thick hollow cylinder is obtained analytically in closed form. This distribution can be useful in determining the thermal stress fields. The optimization of heat conduction can be carried out by using this form of solution.

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