Heat Mass Transfer (2006) 42: 759–766
DOI 10.1007/s00231-005-0027-0

ORIGINAL

Chun Wang

Analytic solutions for a liquid film on an unsteady stretching surface

Received: 14 January 2005 / Accepted: 22 July 2005 / Published online: 12 April 2006

Abstract The momentum and heat transfer in a laminar liquid film on a horizontal stretching sheet is analyzed by the Homotopy analysis method (HAM). Analytic series solutions are given and compared with numerical results given by other authors. The good agreement between them shows the effectiveness of HAM to the problem of liquid film on an unsteady stretching surface.

Keywords Homotopy analysis method · Liquid film · Unsteady stretching surface

1 Introduction

Flow and heat transfer within a thin liquid film is important for the understanding and design of various heat exchangers and chemical processing equipment. Typical applications include wire and fiber coating, reactor fluidization, polymer processing, food stuff processing, transpiration cooling, and so on. The problem of laminar-film condensation on a vertical plate was first considered by Sparrow and Gregg [1] based on the boundary layer theory and similarity transformation. They also considered the heat and mass transfer in a liquid film on a rotating disk [2]. The melting from a horizontal rotating disk was investigated by Wang [3], where the nonlinear governing equations were solved by perturbation method and numerical integration. Dandapat and Ray [4, 5] studied the cooling and thermocapillarity effect of a thin liquid film above a rotating disk.

A class of flow problems with obvious relevance to polymer extrusion is the flow induced by the stretching motion of a flat elastic sheet. For example, in a melt-spinning process, the extrudate from the die is generally drawn and simultaneously stretched into a filament or sheet, which is thereafter solidified through rapid quenching or gradual cooling by direct contact with water or chilled metal rolls. In fact, stretching imports a unidirectional orientation to the extrudate, thereby improving its mechanical properties and the quality of the final product greatly depends on the rate of cooling.

The hydrodynamics of a thin liquid film on a stretching sheet was first considered by Wang [6] who reduced the unsteady Navier–Stokes equations to a nonlinear ordinary differential equations by means of similarity transformation. Usha and Sridharan [7] considered a similar problem of axisymmetric flow in a liquid film. Recently, several authors extended Wang's work by including the non-Newtonian effect of fluid, heat transfer and the thermocapillarity effect (see Andersson et al. [8, 9], Dandapat et al. [10] and Chen [11]). More recently, Kumari and Nath [12] studied the unsteady MHD film over a rotating infinite disk.

Since the quality of the final products in many manufacturing industries depends considerably on the flow and heat transfer within a thin liquid film above a stretching sheet, the analysis and fundamental understanding of the momentum and thermal transports for such processes are very important. Most of the previous studies were investigated with the aid of perturbation or numerical method. However, to the best of our knowledge, no one had reported uniformly valid, totally analytical solutions to these kind of problems.

This paper presents a kind of analytic method to deal with the problem of liquid film on an unsteady stretching sheet. The method employed is based on the Homotopy analysis method (HAM [13]) which is developed recently for nonlinear problems. HAM has been successfully applied to many boundary layer problems, such as the Darcy or non-Dary free convection heat and mass transfer in porous medium (see Wang et al. [14–16]). The problem discussed in present paper is the heat transfer in a liquid film on an unsteady stretching surface. This problem was first considered by Andersson et al. [9], however, using a kind of multiple shooting numerical

C. Wang
School of Naval Architecture, Ocean and Civil Engineering,
Shanghai Jiao Tong University, Shanghai 200030, China
E-mail: chunwang@sjtu.edu.cn

method. We hope the analytical solutions given by HAM is helpful to understand the flow and heat transfer mechanisms of the liquid film and would find applications in technological and manufacturing industries, such as polymer extrusion.

2 Governing equation

Consider the Newtonian fluid flow in a thin liquid film of uniform thickness $h(t)$ on a horizontal elastic sheet, which emerges from a narrow slot at the origin of a Cartesian coordinate system, as shown in Fig. 1. The fluid motion within the film arises due to the stretching of the elastic sheet. The velocity and temperature fields in the thin liquid film are governed by the two-dimensional boundary layer equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\kappa\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where (x,y) are the Cartesian coordinates along and normal to the flat surface, respectively, u and v are the velocity components in x-and y-directions, respectively, t is the time, T temperature, ν kinematic viscosity of fluid, κ thermal diffusivity.

Assuming that the surface of the planar liquid film is smooth and free of surface waves, and the viscous shear stress as well as the heat flux vanishing at the adiabatic free surface, the boundary conditions considered are

$$y=0: \quad u=U, \quad v=0, \quad T=T_s, \tag{4}$$

$$y=h: \quad \frac{\partial u}{\partial y}=\frac{\partial T}{\partial y}=0, \quad v=\frac{\mathrm{d}h}{\mathrm{d}t}, \tag{5}$$

where U and T_s are the surface velocity and the temperature of the stretching sheet, respectively. It is noted that condition (5) imposes a kinematic constraint of the fluid motion.

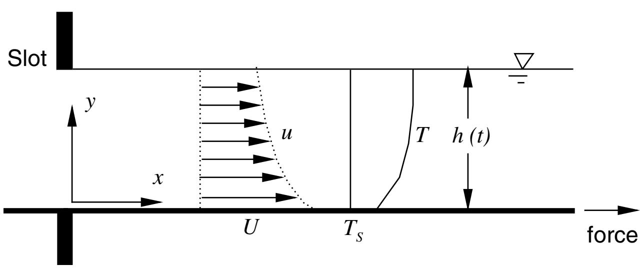


Fig. 1 Schematic of a liquid film on an unsteady stretching sheet

In this paper, the flow is caused by the stretching of the sheet which moves in its own plane with velocity

$$U=\frac{bx}{(1-\alpha t)}, \tag{6}$$

where b and α are both positive constants with dimension time^{-1}. It is seen from (6) that the effective stretching rate $b/(1-\alpha\ t)$ increases with time since $\alpha > 0$. The surface temperature T_s of the sheet varies with the distance x from the slot and time t in the form

$$T_s=T_o-T_{\mathrm{ref}}\left[\frac{bx^2}{2\nu}\right](1-\alpha t)^{-3/2}, \tag{7}$$

where T_o denotes the temperature at the slit, and T_{ref} is the reference temperature which can be taken either as a constant reference temperature or a constant temperature difference. Expression (7) represents a situation in which the sheet temperature decreases from T_o at the slot in proportion to x^2 and the amount of temperature reduction along the sheet increases with time. It should be noticed that expressions (6) and (7), on which the following analysis is based, are valid only for time $t < \alpha^{-1}$.

The special form of (6) and (7) allows similarity transformation which converts the governing partial differential equations into a set of ordinary differential equations. We introduce the following dimensionless variable $f(\eta)$ and $\theta(\eta)$ as well as the similarity variable η

$$\psi=[\nu b(1-\alpha t)^{-1}]^{1/2}x\beta^{-1}f(\eta), \tag{8}$$

$$T=T_o-T_{\mathrm{ref}}\left[\frac{bx^2}{2\nu}\right](1-\alpha t)^{-3/2}\theta(\eta), \tag{9}$$

$$\eta=\left(\frac{b}{\nu}\right)^{1/2}(1-\alpha t)^{-1/2}\beta^{-1}y, \tag{10}$$

where β is a yet unknown constant denoting the dimensionless film thickness, and $\psi(x,y,t)$ is the stream function defined by

$$u=\frac{\partial\psi}{\partial y}=bx(1-\alpha t)^{-1}f'(\eta), \tag{11}$$

$$v=-\frac{\partial\psi}{\partial x}=-[\nu b(1-\alpha t)^{-1}]^{1/2}\beta f(\eta), \tag{12}$$

which automatically satisfies the continuity equation (1). Let $\eta=1$ at the free surface, we have from (10)

$$\beta=\left(\frac{b}{\nu}\right)^{1/2}(1-\alpha t)^{-1/2}h(t), \tag{13}$$

which gives

$$\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{\alpha\beta}{2}\left(\frac{\nu}{b}\right)^{1/2}(1-\alpha t)^{-1/2}. \tag{14}$$

In terms of new variables $f(\eta)$, $\theta(\eta)$ and η, Eqs. (2) and (3) with boundary conditions (4) and (5) can be rewritten as

$$f''' + \gamma\left(ff'' - S\eta f''/2 - f'^2 - Sf'\right) = 0, \tag{15}$$

$$\mathrm{Pr}^{-1}\theta'' + \gamma(f\theta' - 2\theta f' - S\eta\theta'/2 - 3S\theta/2) = 0, \tag{16}$$

$$f(0) = 0, \quad f'(0) = 1, \quad f(1) = \frac{S}{2}, \quad f''(1) = 0, \tag{17}$$

$$\theta(0) = 1, \quad \theta'(1) = 0, \tag{18}$$

where primes denote differentiation with respect to η, $\gamma = \beta^2$ is an unknown constant which must be determined as a part of the present problem, $S = \alpha/b$ is a dimensionless measure of the unsteadiness, $\mathrm{Pr} = \nu/\kappa$ is the Prandtl number. It is noteworthy that the momentum boundary layer problem defined by the ODE (15) subject to the relevant boundary conditions (17) is decoupled from the thermal boundary layer problem, while the temperature field $\theta(\eta)$ is on the other hand coupled to the velocity field.

3 Homotopy analysis method

We assume that the solutions $f(\eta)$ and $\theta(\eta)$ could be expressed by

$$\{\eta^m | m = 0, 1, 2, \cdots\}. \tag{19}$$

From (17) and (18), it is straightforward to choose

$$f_0(\eta) = \eta + \frac{3S-6}{4}\eta^2 + \frac{2-S}{4}\eta^3, \tag{20}$$

$$\theta_0(\eta) = 1, \tag{21}$$

as the initial guess of $f(\eta)$ and $g(\eta)$. We choose

$$\mathcal{L}_f = \frac{\partial^3}{\partial\eta^3}, \quad \mathcal{L}_\theta = \frac{\partial^2}{\partial\eta^2}, \tag{22}$$

as the linear operator, which have properties

$$\mathcal{L}_f[C_1 + C_2\eta + C_3\eta^2] = 0, \tag{23}$$

$$\mathcal{L}_\theta[C_1 + C_2\eta] = 0, \tag{24}$$

where C_1, C_2 and C_3 are constants.

We construct the so-called zero-th order deformation equation

$$\begin{aligned}(1-q)\mathcal{L}_f[F(\eta,q) - f_0(\eta)] \\ = q\hbar_f H_f(\eta)\mathcal{N}_f[F(\eta,q), \Gamma(q)],\end{aligned} \tag{25}$$

$$\begin{aligned}(1-q)\mathcal{L}_\theta[\Theta(\eta,q) - \theta_0(\eta)] \\ = q\hbar_\theta H_\theta(\eta)\mathcal{N}_\theta[F(\eta,q), \Theta(\eta,q), \Gamma(q)],\end{aligned} \tag{26}$$

subject to boundary conditions

$$F(0,q) = 0, \quad F'(0,q) = 1, \quad F(1,q) = \frac{S}{2}, \tag{27}$$

$$F''(1,q) = 0, \quad \Theta(0,q) = 1, \quad \Theta'(1,q) = 0, \tag{28}$$

where $q \in [0,1]$ is an embedding parameter, $H_f(\eta)$ and $H_\theta(\eta)$ are non-zero auxiliary functions, and

$$\begin{aligned}&\mathcal{N}_f[F(\eta,q), \Gamma(q)] \\ &\quad = F''' + \Gamma\left(FF'' - \frac{1}{2}S\eta F'' - F'^2 - SF'\right),\end{aligned} \tag{29}$$

$$\begin{aligned}&\mathcal{N}_\theta[F(\eta,q), \Theta(\eta,q), \Gamma(q)] \\ &\quad = \mathrm{Pr}^{-1}\Theta'' + \Gamma\left(F\Theta' - 2\Theta F' - \frac{1}{2}S\eta\Theta' - \frac{3}{2}S\Theta\right).\end{aligned} \tag{30}$$

Here prime denotes differentiation with respect to η. Obviously, we have

$$F(\eta,0) = f_0(\eta), \quad \Theta(\eta,0) = \theta_0(\eta), \tag{31}$$

when $q = 0$, and

$$F(\eta,1) = f(\eta), \quad \Theta(\eta,1) = \theta(\eta), \quad \Gamma(1) = \gamma, \tag{32}$$

when $q = 1$. Thus, as q increases from 0 to 1, $F(\eta,q)$ and $\Theta(\eta,q)$ vary from the initial guess $f_0(\eta)$ and $\theta_0(\eta)$ to the solutions $f(\eta)$ and $\theta(\eta)$ of Eqs. (15, 16, 17, 18), respectively. So does $\Gamma(q)$ from initial guess γ_0 to γ. Assuming that the auxiliary parameters $\hbar_f$ and $\hbar_\theta$ are so properly chosen that the Maclaurin series of $F(\eta,q)$, $\Theta(\eta,q)$ and $\Gamma(q)$ expanded with respect to q are convergent at $q = 1$, we then have due to (31) and (32)

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{+\infty} f_m(\eta), \tag{33}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{+\infty} \theta_m(\eta), \tag{34}$$

$$\gamma = \gamma_0 + \sum_{m=1}^{+\infty} \gamma_m, \tag{35}$$

where

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m F(\eta,q)}{\partial q^m}\bigg|_{q=0}, \tag{36}$$

$$\theta_m(\eta) = \frac{1}{m!}\frac{\partial^m \Theta(\eta,q)}{\partial q^m}\bigg|_{q=0}, \tag{37}$$

$$\gamma_m = \frac{1}{m!}\frac{\partial^m \Gamma(q)}{\partial q^m}\bigg|_{q=0}, \tag{38}$$

Differentiating the zero-th order deformation equations (25) and (26) m times with respect to q, then dividing by $m!$, and finally setting $q = 0$, we have the mth-order deformation equations

$$\mathscr{L}_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_f H_f(\eta)\mathscr{R}_m(\eta), \tag{39}$$

$$\mathscr{L}_\theta[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar_\theta H_\theta(\eta)\mathscr{S}_m(\eta), \tag{40}$$

subject to the boundary conditions

$$f_m(0) = 0, \ f'_m(0) = 0, \ f_m(1) = 0, \ f''_m(1) = 0, \tag{41}$$

$$\theta_m(0) = 0, \quad \theta'_m(1) = 0, \tag{42}$$

for $m \geq 1$, where

$$\begin{aligned}\mathscr{R}_m(\eta) = & f'''_{m-1} + \sum_{n=0}^{m-1} \gamma_{m-1-n} \sum_{i=0}^{n} (f_i f''_{n-i} - f'_i f'_{n-i}) \\ & - \frac{\eta S}{2} \sum_{n=0}^{m-1} \gamma_n f''_{m-1-n} - S \sum_{n=0}^{m-1} \gamma_n f'_{m-1-n},\end{aligned} \tag{43}$$

$$\begin{aligned}\mathscr{S}_m(\eta) = & \mathrm{Pr}^{-1} \theta''_{m-1} + \sum_{n=0}^{m-1} \gamma_{m-1-n} \sum_{i=0}^{n} (f_i \theta'_{n-i} - 2\theta_i f'_{n-i}) \\ & - \frac{\eta S}{2} \sum_{n=0}^{m-1} \gamma_n \theta'_{m-1-n} - \frac{3}{2} S \sum_{n=0}^{m-1} \gamma_n \theta_{m-1-n},\end{aligned} \tag{44}$$

and

$$\chi_m = \begin{cases} 1, & m > 1, \\ 0, & m = 1. \end{cases} \tag{45}$$

The solutions of (39) and (40) can be expressed as

$$f_m(\eta) = f_m^*(\eta) + \chi_m f_{m-1}(\eta) + C_1 + C_2\eta + C_3\eta^2, \tag{46}$$

$$\theta_m(\eta) = \theta_m^*(\eta) + \chi_m \theta_{m-1}(\eta) + C_1 + C_2\eta, \tag{47}$$

where C_1, C_2 and C_3 are integral constants and $f_m^*(\eta)$ and $\theta_m^*(\eta)$ are the special solutions of (39) and (40), respectively. It should be pointed out that $f_m^*(\eta)$ and $\theta_m^*(\eta)$ contain the unknown γ_{m-1} which should be determined along with C_1, C_2 and C_3 by the boundary conditions (41) and (42).

We solve Eqs. (39) and (40) for $m = 1,2,3,\ldots$ successively, and at the Mth-order approximation we have the analytical solutions of $f(\eta)$, $\theta(\eta)$ and γ

$$f(\eta) \approx \sum_{m=0}^{M} f_m(\eta), \tag{48}$$

$$\theta(\eta) \approx \sum_{m=0}^{M} \theta_m(\eta), \tag{49}$$

$$\gamma \approx \sum_{m=0}^{M} \gamma_m, \tag{50}$$

4 Results analysis

The basic idea of HAM described above can be accomplished easily with the aid of symbol computing software, such as MATHEMATICA. It is found that the solutions of (39) and (40) can be expressed by

$$f_m(\eta) = \sum_{k=2}^{4m+3} a_{m,k}\eta^k, \quad \theta_m(\eta) = \sum_{k=1}^{4m} c_{m,k}\eta^k, \tag{51}$$

for $m \geq 1$, where $a_{m,k}$ and $c_{m,k}$ are coefficients, which can be obtained recursively for $m = 1,2,3,\ldots$ using

$$a_{0,1} = 1, \ a_{0,2} = \frac{3S-6}{4}, \ a_{0,3} = \frac{2-S}{4}, \ b_{0,0} = 1, \tag{52}$$

given by (20) and (21). When $m = 1$, we have

$$f_1(\eta) = \sum_{k=2}^{7} a_{1,k}\eta^k, \quad \theta_1(\eta) = \sum_{k=1}^{4} c_{1,k}\eta^k, \quad \gamma_0 = \frac{105(2-S)}{2R} \tag{53}$$

where

$$A_{1,2} = \frac{h_f(72 + 228S - 282S^2 + 75S^3)}{16R}, \tag{54}$$

$$A_{1,3} = \frac{h_f(2R - 70 - SR - 35S + 35S^2)}{4R}, \tag{55}$$

$$A_{1,4} = \frac{105h_f(8 + 4S - 10S^2 + 3S^3)}{64R}, \tag{56}$$

$$A_{1,5} = \frac{21h_f(-24 + 20S - 2S^2 - S^3)}{64R}, \tag{57}$$

$$A_{1,6} = \frac{21h_f(8 - 12S + 6S^2 - S^3)}{64R}, \tag{58}$$

$$A_{1,7} = \frac{3h_f(-8 + 12S - 6S^2 + S^3)}{64R}, \tag{59}$$

and

$$C_{1,1} = \frac{525h_g\mathrm{Pr}(2S - S^2)}{4R}, \tag{60}$$

$$C_{1,2} = \frac{105h_g\mathrm{Pr}(-8 - 2S + 3S^2)}{8R}, \tag{61}$$

$$C_{1,3} = \frac{105h_g\mathrm{Pr}(4 - 4S + S^2)}{4R}, \tag{62}$$

$$C_{1,4} = \frac{105h_g\mathrm{Pr}(-4 + 4S - S^2)}{16R}, \tag{63}$$

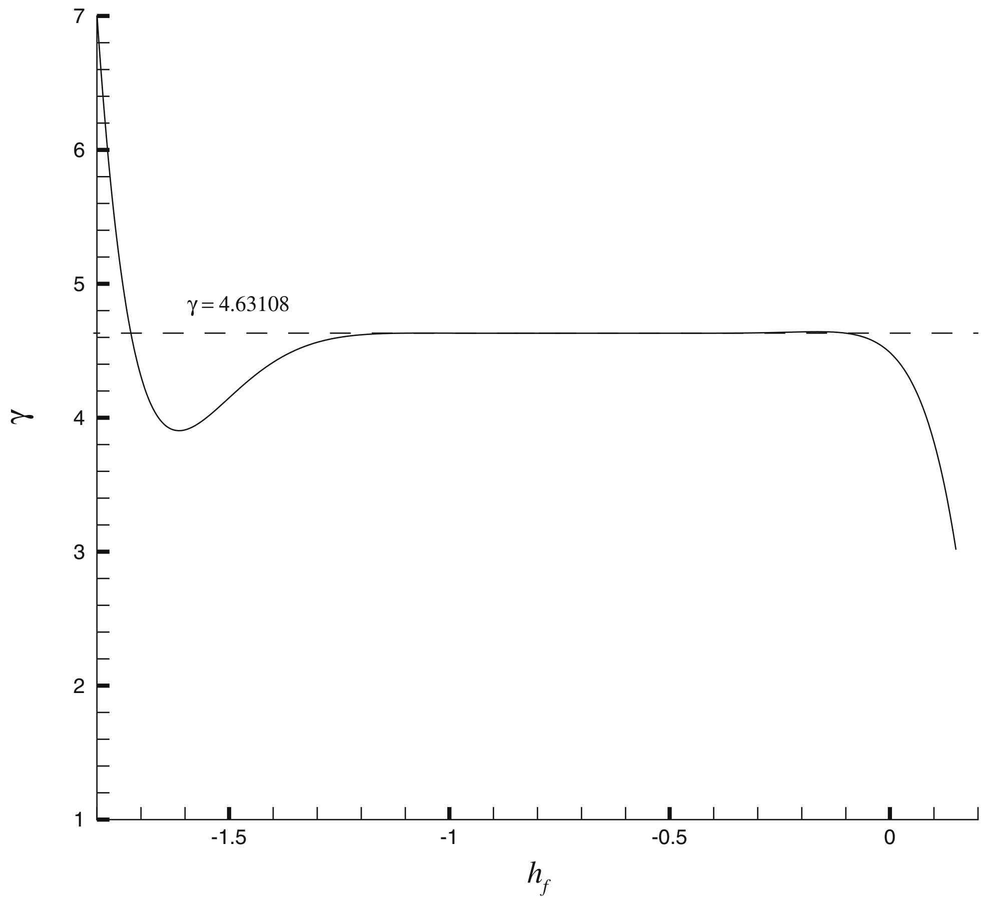


Fig. 2 Variation of $\gamma(=\beta^2)$ with $\hbar_f$ under tenth-order of approximation, in case of $S=0.8$. *Solid line* analytical solution given by HAM; *dashed line* numerical solution given by Andersson et al. [9]

with definition

$$R = 11 - 25S + 36S^2. \tag{64}$$

It is noted that the right-hand-sides of (39) and (40) contain two non-zero auxiliary functions $H_f(\eta)$ and $H_\theta(\eta)$. For simplicity, we choose $H_f(\eta)=H_\theta(\eta)=1$. It is also noted that our analytical series solutions contain two non-zero auxiliary parameters $\hbar_f$ and $\hbar_\theta$ which could adjust and control the convergence of the solutions. In general, by means of the so-called $\hbar$-curve [13], it is straightforward to choose appropriate ranges for $\hbar_f$ and $\hbar_\theta$ to ensure the convergence of the solutions. Figure 2 shows the variation of $\gamma(=\beta^2)$ with $\hbar_f$ in case of $S=0.8$. It is seen that convergent result can be obtained when $-1.1 \le \hbar_f \le -0.2$ under tenth-order of approximation. Thus, we can choose an appropriate value for $\hbar_f$ in this range to get convergent solution for γ. For example, when $\hbar_f = -0.6$, the value of γ is 4.63108, which is the same as the numerical result given by Andersson et al. [9]. When $S=1.2$ and $\hbar_f = -1.0$, the solution of γ is 1.27189 which is also the same as that given by Andersson et al. [9].

Let us first consider the hydrodynamic part of the problem. For positive values of the unsteadiness parameter S, Wang [6] found that solutions exist only for $0 \le S \le 2$. The limiting case of $S \to 0$ stands for the case of an infinitely thick layer of fluid, i.e. $\beta \to \infty$, whereas the limiting case of $S \to 2$ represents a liquid film of infinitesimal thickness, i.e. $\beta \to 0$. Variations of the dimensionless film thickness β and the surface gradient $f''(0)$ of the velocity component u parallel to the sheet with respect to the unsteadiness parameter S are shown in Table 1. The order of approximation M and the values of $\hbar_f$ used in the calculation are also listed in Table 1. It should be pointed out that it is difficult to perform the calculations for the limiting cases when $S \to 0$. Table 1 reveals that both the dimensionless film thickness β and the magnitude of $f''(0)$ decrease with the increasing of S.

Similarity solutions for the dimensionless temperature $\theta(\eta)$ were obtained for two representative values of the unsteadiness parameter S and for Prandtl numbers in the range from 0.01 to 3. Variation of free surface temperature $\theta(1)$ with Pr is shown in Table 2 for $S=0.8$ and $S=1.2$. It is shown that a larger unsteadiness parameter gives rise to a higher dimensionless free-surface temperature. In addition, it is interesting to note that $\theta(1)$ vanishes at sufficiently large Prandtl numbers, which means that the free-surface temperature becomes the temperature T_0 at the slot. In the limiting case $\mathrm{Pr} \to 0$, the dimensionless free-surface temperature approaches to unity; namely, the temperature becomes uniform within the thin film in the vertical direction and equals the sheet temperature T_s. Table 2 also shows the variation of surface heat flux $-\theta'(0)$ from the liquid film

Table 1 Variations of the dimensionless film thickness β and the surface velocity gradient $f''(0)$ with respect to the unsteadiness parameter S

S	$\hbar_f$	M	$\beta(=\gamma^{1/2})$	$f''(0)$	S	$\hbar_f$	M	$\beta(=\gamma^{1/2})$	$f''(0)$
0.4	−0.2	20	5.12249	−6.69912	1.2	−1.0	10	1.127780	−1.442631
0.5	−0.5	15	3.90950	−4.55108	1.3	−1.0	10	0.964219	−1.218322
0.6	−0.5	15	3.13125	−3.74233	1.4	−1.0	10	0.821032	−1.012784
0.7	−0.6	10	2.57701	−3.14965	1.5	−1.0	10	0.693144	−0.821842
0.8	−0.6	10	2.15199	−2.68094	1.6	−1.0	10	0.576173	−0.642397
0.9	−0.8	10	1.81599	−2.29683	1.7	−1.0	10	0.465770	−0.472094
1.0	−0.8	10	1.54362	−1.97238	1.8	−1.0	10	0.356389	−0.309137
1.1	−1.0	10	1.31810	−1.69136	1.9	−1.0	10	0.237000	−0.152134

Table 2 Variations of free surface temperature $\theta(1)$ and heat flux from the liquid film to the stretching sheet $-\theta'(0)$ with Pr and S under 10-th order of approximation

S = 0.8				S = 1.2			
Pr	$\hbar_\theta$	$\theta(1)$	$-\theta'(0)$	Pr	$\hbar_\theta$	$\theta(1)$	$-\theta'(0)$
0.01	−1	0.960480	0.090474	0.01	−1	0.982331	0.037734
0.1	−0.8	0.692533	0.756162	0.1	−1	0.843622	0.343931
1	−0.4	0.097884	3.595970	1	−0.5	0.286717	1.999590
2	−0.2	0.024941	5.244150	2	−0.3	0.128124	2.975450
3	−0.2	0.008785	6.514440	3	−0.25	0.067658	3.698830

to the stretching sheet with Pr and S. It is found that an increase in unsteadiness parameter S produces an decrease in the surface heat flux for a given Prandtl number Pr. The influence of the unsteadiness parameter S on the heat flux from the liquid film to the stretching sheet is more pronounced at low Prandtl numbers than for

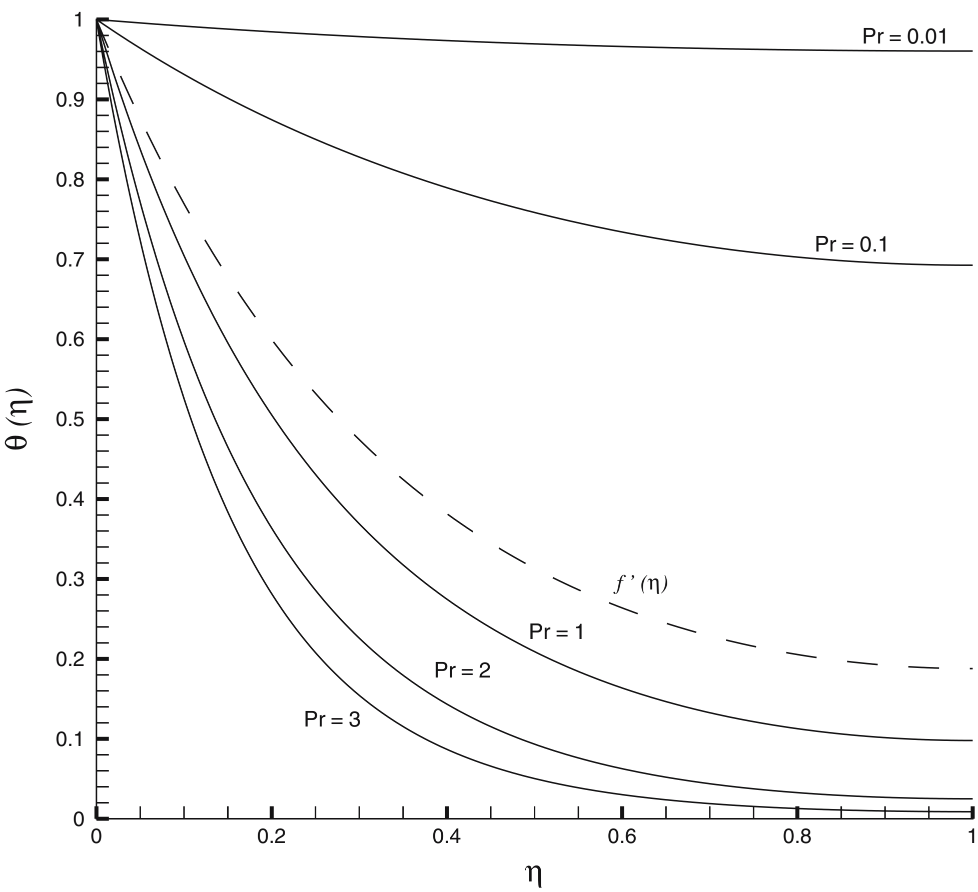
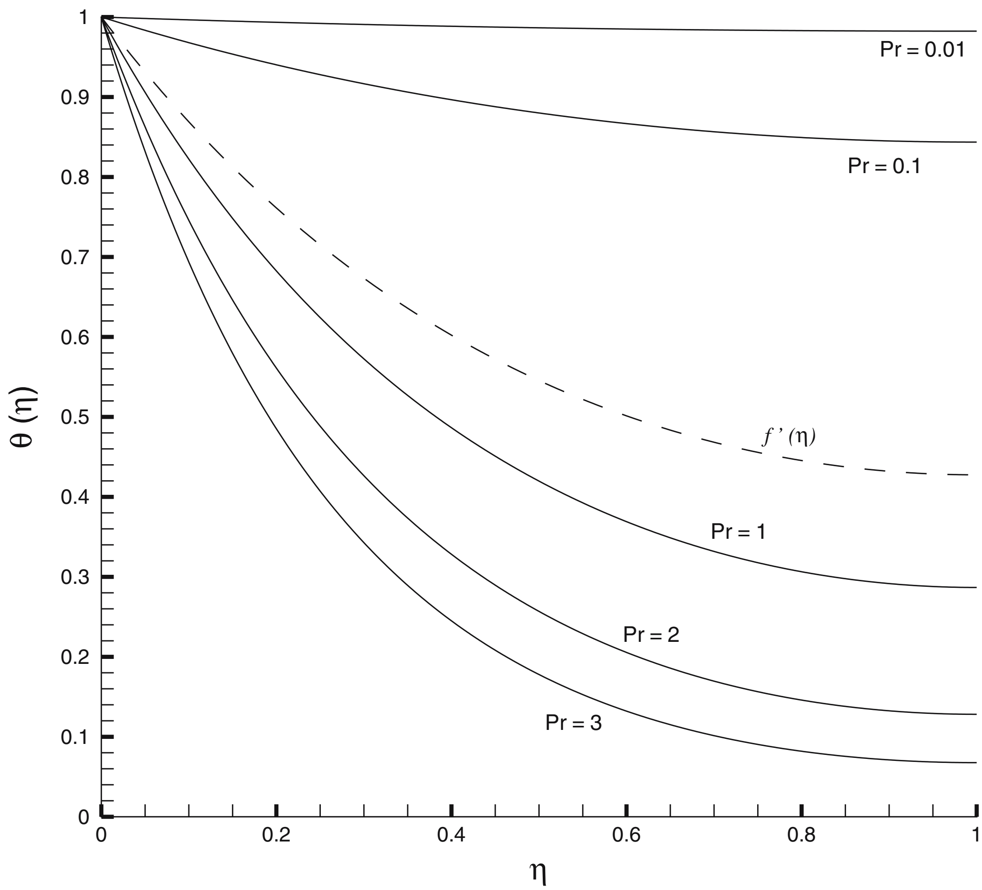


Fig. 3 Solutions of $f'(\eta)$ (*dashed line*) and $\theta(\eta)$ (*solid lines*) under tenth-order of approximation, in case of $S=0.8$. In this case, $\beta=2.15199$ and $f''(0)/\beta=-1.2458$, which is the same as that given by Andersson et al. [9]. Note that, $f''(0)/\beta$ in this paper is the same as $f''(0)$ appeared in [9], due to the difference of the similarity transformation employed

Fig. 4 Solution of $f'(\eta)$ (*dashed line*) and $\theta(\eta)$ (*solid line*) under tenth-order of approximation, in case of $S=1.2$. In this case, $\beta=1.12778$ and $f''(0)/\beta=-1.27918$, which is also the same as that given by Andersson et al. [9]



Pr > 1. The values of $\hbar_\theta$ used for the calculation are also listed in Table 2. It should be pointed out that due to the capacity of our computer, we can't get accurate enough analytical solutions for Prandtl numbers higher than 10 at this moment.

Temperature distributions $\theta(\eta)$ (solid lines) and velocity distributions $f'(\eta)$ (broken lines) for $S=0.8$ and $S=1.2$ are shown in Figs. 3 and 4, respectively. It is shown that the temperature decreases with the distance from the stretching sheet for all Prandtl numbers. In addition, a nearly uniform temperature distribution, $\theta(1)\approx 1$ or $T\approx T_s$, is observed to prevail within the liquid film at a very low Prandtl number (e.g. $\mathrm{Pr}=0.01$). For a particular value of S, the temperature is reduced by increasing the Prandtl number.

5 Conclusion

The purpose of this paper is to present exact analytical solutions for the momentum and heat transfer within an unsteady liquid film whose motion is caused solely by the linear stretching of a horizontal elastic sheet. Comparisons with previously published work were performed and found to be in excellent agreement. We hope the analytical solutions given by HAM is helpful to understand the flow and heat transfer mechanisms of the liquid film and would find applications in technological and manufacturing industries, such as polymer extrusion.

References

1. Sparrow EM, Gregg JL (1959) A boundary-layer treatment of laminar film condensation. ASME J Heat Transfer 81:13–18
2. Sparrow EM, Gregg JL (1960) Mass transfer, flow and heat transfer about a rotating disk. ASME J Heat Transfer 82:294–302
3. Wang CY (1989) Melting from a horizontal rotating disk. Trans ASME J Appl Mech 56:47–50
4. Dandapat BS, Ray PC (1990) Film cooling on a rotating disk. Int J Non-Linear Mech 25:569–582
5. Dandapat BS, Ray PC (1994) The effect of thermacapilarity on the flow of a thin liquid film on a rotating disc. J Phys D Appl Phys 27:2041–2045
6. Wang CY (1990) Liquid film on an unsteady stretching surface. Quart Appl Math XLVIII:601–610
7. Usha R, Sridharan R (1993) On the motion of a liquid film on an unsteady stretching surface. ASME Fluids Eng 150:43–48
8. Andersson HI, Aarseth JB, Braud N, Dandapat BS (1996) Flow of a power-law fluid film on an unsteady stretching surface. J Non-Newtonian Fluid Mech 62:1–8
9. Andersson HI, Aarseth JB, Dandapat BS (2000) Heat transfer in a liquid film on an unsteady stretching surface. Int J Heat Mass Transfer 43:69–74
10. Dandapat BS, Santra BS, Andersson HI (2003) Thermocapillarity in a liquid film on an unsteady stretching surface. Int J Heat Mass Transfer 46:3009–3015

11. Chen CH (2003) Heat transfer in a power-law fluid film over a unsteady stretching sheet. Heat Mass Transfer 39:791–796
12. Kumari M, Nath G (2004) Unsteady MHD film flow over a rotating infinite disk. Int J Eng Sci 42:1099–1117
13. Liao SJ (2003) Beyond perturbation:introduction to the homotopy analysis method. Chapman Hall/CRC, Boca Raton
14. Wang C, Zhu JM, Liao SJ, Pop I (2003) On the explicit analytic solution of Cheng-Chang equation. Int J Heat Mass Transfer 46:1855–1860
15. Wang C, Liao SJ, Zhu JM (2003) An explicit analytic solution for the combined heat and mass transfer by natural convection from a vertical wall in a non-Darcy porous medium. Int J Heat Mass Transfer 46:4813–4822
16. Wang C, Liao SJ, Zhu JM (2003) An explicit analytic solution for non-Darcy natural convection over horizontal plate with surface mass flux and thermal dispersion effects. Acta Mechanica 165:139–150