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Md. Mamun Molla · Md. Anwar Hossain
Rama Subba Reddy Gorla

Natural convection flow from an isothermal horizontal circular cylinder with temperature dependent viscosity

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Abstract Free convection flow from an isothermal horizontal circular cylinder immersed in a fluid with viscosity proportional to an inverse linear function of temperature is studied. The governing boundary layer equations are transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations is reduced to local non-similarity equations which are solved numerically by a very efficient implicit finite difference method together with Keller box scheme. Numerical results are presented by velocity and viscosity profiles of the fluid as well as heat transfer characteristics, namely the local heat transfer rate and the local skin-friction coefficients for a wide range of viscosity parameter ϵ (= 0.0, 0.5, 1.0, 2.0, 3.0, 4.0) and the Prandtl number Pr (=1.0, 7.0, 10.0, 15.0, 20.0, 30.0).

Nomenclature

a	Radius of the circular cylinder
C_p	Specific heat at constant pressure
C_f	Local skinfriction
f	Dimensionless stream function
g	Acceleration due to gravity
Gr	Grashof number
k	Thermal conductivity
Nu	Local Nusselt number
Pr	Prandtl number
q_w	Heat flux at the surface
T	Temperature of the fluid in the boundary layer
T_w	Temperature at the surface
u, v	The dimensionless x and y component of the velocity
$\hat{u}, \hat{v}$	The dimensional $\hat{x}$ and $\hat{y}$ component of the velocity
x, y	Axis in the direction along and normal to the surface

Greek symbols

β	Volumetric coefficient of thermal expansion
ψ	Stream function
τ_w	Shearing stress
ϵ	Viscosityvariation parameter
γ	Constant
ρ	Density of the fluid
v_∞	Reference kinematic viscosity
$\mu\ (T)$	Viscosity of the fluid
μ_∞	Dynamic viscosity of the ambient fluid
θ	Dimensionless temperature function

Subscript

w	Wall conditions
∞	Ambient temperature
x	Differentiation with respect to x

Superscript

$'$	Differentiation with respect to y

M. M. Molla
Department of Mathematics,
Bangladesh University of Engineering & Technology,
Dhaka, 1000, Bangladesh

M. A. Hossain
Department of Mathematics,
University of Dhaka, Dhaka, 1000, Bangladesh

R. S. R. Gorla (✉)
Department of Mechanical Engineering,
Cleveland State University, Cleveland, OH, USA
E-mail: r.gorla@csuohio.edu

1 Introduction

Natural convection flow of a viscous incompressible fluid from a horizontal circular cylinder represents an important problem, which is related to numerous engineering applications. Sparrow and Lee [17] looked at the problem of a vertical stream over a heated horizontal circular cylinder. They obtained a solution by expanding velocity and temperature profiles in powers of x, the co-ordinate measuring distance from the front stagnation point on the cylinder. The exact solution is still out of reach due to the non-linearity in the Navier–Stokes equations. It appears that Merkin [13, 14], was the first who presented a complete solution of this problem using Blasius and Gortler series expansion methods along with an integral method and a finite-difference scheme. Also the problem of free convection boundary layer flow on a cylinder of elliptic cross-section was studied by Merkin [15]. Ingham [7] investigated the boundary layer flow on an isothermal horizontal cylinder. Hossain and Alim [3] have investigated natural convection–radiation interaction on boundary layer flow along a vertical thin cylinder. Hossain et al. [4], have studied radiation–conduction interaction on mixed convection from a horizontal circular cylinder. Recently, Nazar et al. [16], have considered the problem of natural convection flow from the lower stagnation point to the upper stagnation point of a horizontal circular cylinder immersed in a micropolar fluid.

All the above studies were confined to a fluid with constant viscosity. However, it is known that this physical property may change significantly with temperature. For instance, the viscosity of water decreases by about 240% when the temperature increases from 10°C ($\mu = 0.00131$ kg m^{-1} s^{-1}) to 50°C ($\mu = 0.000548$ kg m^{-1} s^{-1}). To predict accurately the flow behavior, it is necessary to take into account this variation of viscosity. Gray et al. [2] and Mehta and Sood [12], have shown that when this effect is considered, the flow characteristics may substantially be changed compared to constant viscosity. Kafoussius and Williams [9] and Kafoussius and Rees [8], have investigated the effect of temperature dependent viscosity on the mixed convection flow from a vertical flat plate. Recently Hossain et al. [5, 6] have investigated the natural convection flow from a vertical wavy cone and a vertical wavy surface, respectively, with variable viscosity proportional to an inverse linear function of temperature. Also Lings and Dybbs [11] studied the natural convection with variable viscosity proportional to an inverse linear function of temperature.

The present study was undertaken in order to investigate the natural convection flow of a viscous incompressible fluid over an isothermal horizontal circular cylinder. The viscosity $\mu(T)$ of the fluid is assumed to be temperature dependent. The surface temperature T_w of the cylinder is higher than that of the ambient fluid temperature T_∞. It has been assumed that the viscosity of the fluid is inversely proportional to a linear function of temperature. A semi-empirical formula for the viscosity versus temperature had been used by Lings and Dybbs [11]. The governing partial differential equations are reduced to locally non-similar partial differential forms by adopting appropriate transformations. The transformed boundary layer equations were solved numerically using very efficient finite-difference scheme known as Keller box technique [10]. Effect of viscosity-variation parameter ϵ, on the velocity and viscosity distribution of the fluid as well as on the local rate of heat transfer in terms of the Nusselt number Nu and the local skin-friction are shown graphically for fluids having Prandtl number, Pr ranging from 1.0 to 30.0.

2 Formulation of problem

Consideration is given to a steady two-dimensional laminar free convective flow of a viscous and incompressible fluid over a uniformly heated horizontal circular cylinder of radius a. We assume that the fluid viscosity is temperature dependent. It is assumed that the surface temperature of the cylinder is T_w, where $T_w > T_\infty$. Here, T_∞ is the ambient temperature of the fluid, the configuration considered is as shown in Fig. 1.

Under the usual Bousinesq approximation, the equations governing the flow are

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0, \tag{1}$$

$$\rho\left(\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}}\right) = \frac{\partial}{\partial \hat{y}}\left(\mu\frac{\partial \hat{u}}{\partial \hat{y}}\right) + \rho g\beta(T - T_\infty)\sin\left(\frac{\hat{x}}{a}\right) \tag{2}$$

$$\hat{u}\frac{\partial T}{\partial \hat{x}} + \hat{v}\frac{\partial T}{\partial \hat{y}} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

The boundary conditions of Eqs. 1, 2, 3 are

$$\hat{u} = \hat{v} = 0, \quad T = T_w, \text{ at } \hat{y} = 0, \tag{4a}$$

$$\hat{u} \to 0, \quad T \to T_\infty \quad \text{as } \hat{y} \to \infty, \tag{4b}$$

where $(\hat{u}, \hat{v})$ are velocity components along the $(\hat{x}, \hat{y})$ axes, g is the acceleration due to gravity, ρ is the density, k is the thermal conductivity, β is the coefficient of thermal expansion, $\mu(T)$ is the viscosity of the fluid depending on the fluid temperature T.

Out of the many forms of viscosity variation, which are available in the literature, we will consider the following form proposed by Lings and Dybbs [11]:

$$\mu = \frac{\mu_\infty}{1 + \gamma(T - T_\infty)}, \tag{5}$$

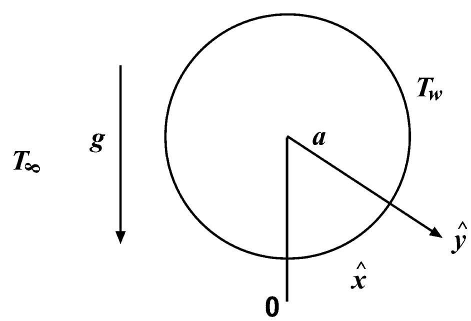

Fig. 1 The flow model and coordinate system

where γ is a constant and μ_∞ is the viscosity of the ambient fluid.

We now introduce the following non-dimensional variables:

$$x = \frac{\hat{x}}{a}, \quad y = \mathrm{Gr}^{\frac{1}{4}}\left(\frac{\hat{y}}{a}\right), \quad u = \frac{\rho a}{\mu_\infty}\mathrm{Gr}^{\frac{1}{2}}\hat{u},$$

$$\upsilon = \frac{\rho a}{\mu_\infty}\mathrm{Gr}^{-\frac{1}{2}}\hat{\upsilon}, \quad \theta = \frac{T - T_\infty}{T_\mathrm{w} - T_\infty}, \quad \mathrm{Gr} = \frac{g\beta(T_\mathrm{w} - T)a^3}{\nu_\infty^2}, \tag{6}$$

where $\nu_\infty(=\mu_\infty/\rho)$ is the reference kinematic viscosity and Gr is the Grashof number and θ is the non-dimensional temperature.

Substituting variables (Eq. 6) into Eqs. 1, 2, 3 leads to the following non-dimensional equations

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0, \tag{7}$$

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \frac{\varepsilon}{(1+\varepsilon\theta)^2}\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y} + \frac{1}{1+\varepsilon\theta}\frac{\partial^2 u}{\partial y^2} + \theta\sin x, \tag{8}$$

$$u\frac{\partial \theta}{\partial x} + \upsilon\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2}. \tag{9}$$

With the boundary conditions (Eq. 4) become

$$u = \upsilon = 0, \theta = 0 \text{ at } \quad x = 1, \quad \text{for any } y, \tag{10a}$$

$$u = \upsilon = 0, \quad \theta = 1 \quad \text{at}\, y = 0, \quad x > 0, \tag{10b}$$

$$u \to 0, \quad \theta \to 1 \quad \text{as}\, y \to \infty, \quad x > 0. \tag{10c}$$

In Eq. 8 the viscosity variable parameter ϵ is defined as

$$\varepsilon = \gamma(T_\mathrm{w} - T_\infty). \tag{11}$$

To solve Eqs. 7, 8, 9, subject to the boundary conditions (Eq. 10), we assume the following variables

$$\psi = xf(x,y), \quad \theta = \theta(x,y), \tag{12}$$

where ψ is the non-dimensional stream function defined in the usual way as

$$\psi = xf(x,y), \quad \theta = \theta(x,y), \quad u = \frac{\partial \psi}{\partial y}, \quad \upsilon = -\frac{\partial \psi}{\partial x}. \tag{13}$$

Substituting (Eq. 12) into Eqs. 8 and 9 we get the following transformed equations:

$$\frac{1}{1+\varepsilon\theta}\frac{\partial^3 f}{\partial y^3} + f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \frac{\varepsilon}{(1+\varepsilon\theta)^2}\frac{\partial \theta}{\partial y}\frac{\partial^2 f}{\partial y^2} + \frac{\theta\sin x}{x} = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial x}\right), \tag{14}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} + f\frac{\partial \theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \theta}{\partial y}\frac{\partial f}{\partial x}\right). \tag{15}$$

The transformed boundary conditions may be written as:

$$f = \frac{\partial f}{\partial y} = 0, \quad \theta = 1 \quad \text{at}\, x = 0, \quad \text{any}\, y, \tag{16a}$$

$$f = \frac{\partial f}{\partial y} = 0, \quad \theta = 1 \quad \text{at}\, y = 0, \quad x > 0, \tag{16b}$$

$$\frac{\partial f}{\partial y} \to 0, \quad \theta \to 0 \quad \text{as}\, y \to \infty, \quad x > 0. \tag{16c}$$

It can be seen that near the front stagnation point of the cylinder i.e. $x \approx 0$, Eqs. 14 and 15 reduce to the following ordinary differential equations:

$$\frac{1}{1+\varepsilon\theta}\frac{\partial^3 f}{\partial y^3} + f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 - \frac{\varepsilon}{(1+\varepsilon\theta)^2}\frac{\partial \theta}{\partial y}\frac{\partial^2 f}{\partial y^2} + \theta = 0, \tag{17}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} + f\frac{\partial \theta}{\partial y} = 0. \tag{18}$$

Subject to the boundary conditions

$$f(0) = \frac{\partial f}{\partial y}(0) = 0, \quad \theta(0) = 1, \tag{19a}$$

$$\frac{\partial f}{\partial y} \to 0, \quad \theta \to 0 \quad \text{as}\, y \to \infty. \tag{19b}$$

In practical applications, the physical qualities of principal interest are the rate heat transfer and the skin-friction coefficients, which can be written, in non-dimensional form as

$$\mathrm{Nu} = \frac{a\mathrm{Gr}^{-1/4}}{k(T_\mathrm{w} - T_\infty)}q_\mathrm{w}, \quad C_\mathrm{f} = \frac{\mathrm{Gr}^{-3/4}a^2}{\mu_\infty \nu_\infty}\tau_\mathrm{w}, \tag{20}$$

where

$$\text{where}\, q_\mathrm{w} = -k\left(\frac{\partial T}{\partial \hat{y}}\right)_{\hat{y}=0}, \quad \tau_\mathrm{w} = \left(\mu\frac{\partial \hat{u}}{\partial \hat{y}}\right)_{\hat{y}=0}. \tag{21}$$

Using the variables (Eq. 6), (Eq. 12) and the boundary condition (Eq. 16b), we get the following expressions for the local Nusselt number and friction factor:

$$\mathrm{Nu} = -\frac{\partial \theta}{\partial y}(x,\, 0), \tag{22}$$

$$C_\mathrm{f} = \frac{x}{1+\varepsilon}\frac{\partial^2 f}{\partial y^2}(x, 0). \tag{23}$$

We also discuss the effect of the viscosity-variation parameter ϵ and the Prandtl number, Pr, on the velocity and viscosity distribution. The values of the velocity and

Table 1 Numerical values of $\partial\theta/\partial y$ (x, 0) for different values of curvature parameter x while Pr = 1.0 and ϵ = 0.0

$-\frac{\partial\theta}{\partial y}$ (x, 0)			
x	Merkin [13]	Nazar et al. [15]	Present
0.0	0.4214	0.4214	0.4241
$\pi/6$	0.4161	0.4161	0.4161
$\pi/3$	0.4007	0.4005	0.4005
$\pi/2$	0.3745	0.3741	0.3740
$2\pi/3$	0.3364	0.3355	0.3355
$5\pi/6$	0.2825	0.2811	0.2812
π	0.1945	0.1916	0.1917

Table 2 Numerical values of $\partial^2 f/\partial y^2$ (x, 0) for different values of curvature parameter x while Pr = 1.0 and ϵ = 0.0

$\frac{\partial^2 f}{\partial y^2}$ (x, 0)			
x	Merkin [13]	Nazar et al. [15]	Present
$\pi/6$	0.4151	0.4148	0.4145
$\pi/3$	0.7558	0.7542	0.7539
$\pi/2$	0.9579	0.9545	0.9541
$2\pi/3$	0.9756	0.9698	0.9696
$5\pi/6$	0.7822	0.7740	0.7739
π	0.3391	0.3265	0.3264

viscosity distribution are calculated from the following relations:

$$u = \frac{\partial f}{\partial y}(x, y), \quad \frac{\mu}{\mu_\infty} = \frac{1}{1 + \varepsilon\theta}. \tag{24}$$

3 Results and discussion

Equations 14 and 15 subject to the boundary conditions (Eq. 16) were solved numerically using a very efficient implicit finite-difference together with Keller box, which is described by Cebeci and Bradshaw [1]. The numerical solutions start at the lower stagnation point of the cylinder, $x = 0$, with initial profiles as given by Eqs. 17 and 18 along with the boundary conditions (Eq. 19) and proceed around the cylinder up to the rear stagnation point, $x = \pi$. Solutions are obtained for fluids having Pr = 1.0, 7.0, 10.0, 15.0, 20.0, 30.0 and for a wide range of values of the variable viscosity parameter ϵ = 0.0, 0.5, 2.0, 2.0, 3.0 and 4.0.

Since values of $\partial\theta/\partial y(x, 0)$ and $\partial^2 f/\partial y^2\ (x, 0)$ are known from the solutions of the coupled Eqs. 14 and 15, numerical values of the local heat transfer rate, Nu from Eq. 22 and the local skin-friction coefficients C_f from Eq. 23 are calculated. Numerical values of $\partial\theta/\partial y\ (x, 0)$ and $\partial^2 f/\partial y^2\ (x, 0)$ are depicted in Tables 1 and 2, Figs. 2 and 3.

Numerical values of $\partial\theta/\partial y(x, 0)$ and $\partial^2 f/\partial y^2\ (x, 0)$ are depicted in Tables 1 and 2, respectively, for Pr = 1.0, and the results of Merkin [13] and Nazar et al. [16] show excellent agreement among these three solutions.

Figure 2a and b deals with the effect of viscosity-variation parameter ϵ (=0.0, 0.5, 1.0, 2.0, 4.0) for Pr = 7.0 on the rate of heat transfer and the local skin-friction coefficient respectively. From Fig. 2a it is seen that the rate of heat transfer, $\partial\theta/\partial y\ (x, 0)$ increases monotonically with the increase of the viscosity-variation parameter ϵ. We also observe that the value of $\partial\theta/\partial y(x, 0)$ reaches to some minimum values at $x = \pi$, the rear stagnation point of the cylinder for every values of ϵ. For ϵ = 0.0, 0.5, 1.0, 2.0 and 4.0 the minimum values attained by the rate of heat transfer $\partial\theta/\partial y$ (x, 0) are 0.2681, 0.2852, 0.2955, 0.3092 and 0.3312, respectively. From these we may conclude that these minimum values increase with the increase of the viscosity-variation parameter ϵ. Figure 2b shows that for increasing values of ϵ, the local skin-friction coefficients decrease. The decreasing values of $\partial^2 f/\partial y^2\ (x, 0)$ converge to individual finite values for every values of ϵ. For ϵ = 0.0, 0.5, 1.0, 2.0 and 4.0 the limiting values of $\partial^2 f/\partial y^2\ (x, 0)$ are 0.1368, 0.1220, 0.1103, 0.0949 and 0.0813 respectively.

The effects of Pr, on the rate of heat transfer, $\partial\theta/\partial y\ (x, 0)$ and the skin-friction, $\partial^2 f/\partial y^2\ (x, 0)$ are illustrated in Fig. 3a and b respectively while ϵ = 3.0. Figure 3a reveals that, increase in the value of Pr leads

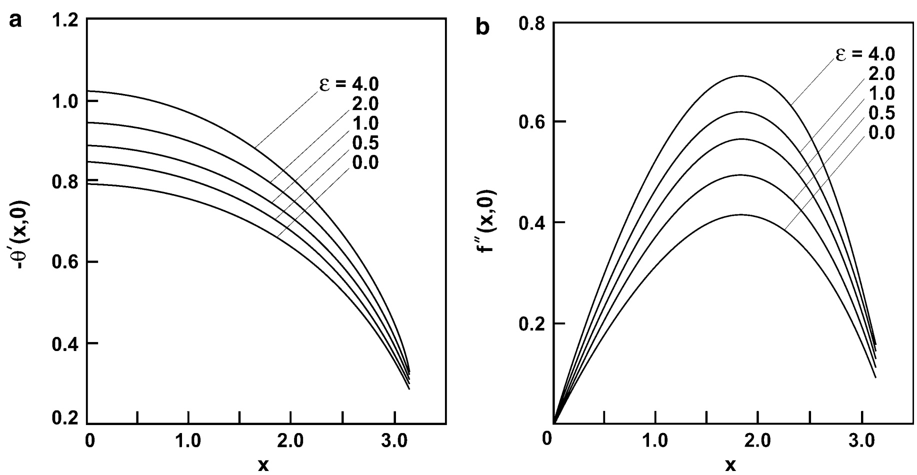

Fig. 2 **a** Heat transfer rate distribution. **b** Friction factor distribution

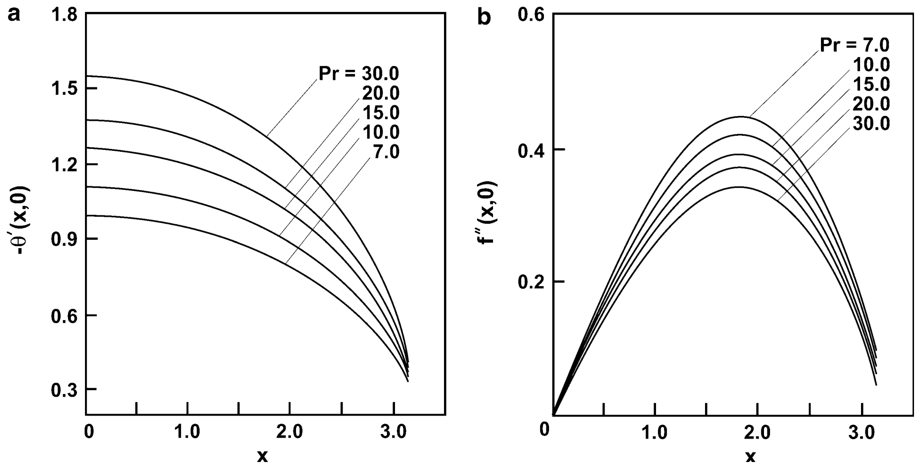

Fig. 3 **a** Heat transfer rate distribution. **b** Friction factor distribution

to increase the values of the rate of heat transfer. Opposite effects on the local skin-friction is observed due to increase of the value of Prandtl number. We may also observe that the absolute maxima of the local skin-friction shifts towards the middle of the surface.

4 Conclusions

The effect of temperature-dependent viscosity on the natural convection boundary layer flow from an isothermal horizontal circular cylinder has been investigated theoretically. Numerical solutions of the equations governing the flow are obtained by using a very efficient implicit finite difference method together with Keller box scheme. From the present investigation the following conclusions may be drawn.

1. The velocity distribution increases and the viscosity of the fluid decrease at the middle of the surface for increasing value of viscosity-variation parameter ϵ.
2. Increasing the value of the viscosity-variation parameter ϵ leads to an increase in the local heat transfer rate and to a decrease the local skin-friction.
3. It has been observed that the velocity distribution and skin-friction decrease as well as the viscosity distribution and the rate of heat transfer increase with an increase of Pr.
4. The results have demonstrated that the assumption of constant fluid properties may introduce severe errors in the prediction of surface friction factor and heat transfer rate.

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