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Heat transfer over an unsteady stretching surface

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Abstract Similarity solution of the laminar boundary layer equations corresponding to an unsteady stretching surface have been studied. The governing time-dependent boundary layer are transformed to ordinary differential equations containg Prandtl number and unsteadiness parameter. The effect of various governing parameters such as Prandtl number and unsteadiness parameter which determine the velocity and temperature profiles and heat transfer coefficient are studied.

1 Introduction

The continuous moving surface heat transfer problem has many practical applications in industrial manufacturing processes.

Since the pioneering work of Sakiadis [1, 2], various aspects of the problem have been investigated by many authors. Most studies have been concerned with constant surface velocity and temperature (see, Tsou et al. [3]), but for many practical applications the surface undergoes stretching and cooling or heating that cause surface velocity and temperature variations. Crane [4], Vleggaar [5] and Gupta and Gupta [6] have analysed the stretching problem with a constant surface temperature, while Soundalgekar and Ramana [7] investigated the constant surface velocity case with a power- law temperature variation. Grubka and Bobba [8] have analysed the stretching problem for a surface moving with a linear velocity and with a variable surface temperature.

Ali [9] has reported flow and heat characteristics on a stretched surface subject to power-law velocity and temperature distributions. The flow field of a stretching wall with a power-law velocity variation was discussed by Banks [10]. Recently, Ali [11] and Elbashbeshy [12] extended Banks's work for a porous stretched surface for different values of the injection parameter. Even more recently, Elbashbeshy [13] have analysed the stretching problem which was discussed by Elbashbeshy [12] to include an uniform porous medium.

The present work is to study the heat transfer over an unsteady stretching surface. This has not been studied in the literature. It may be remarked that the present analysis is an extension of and a complement to the earlier paper [12]

2 Formulation of the problem

Consider an unsteady, two dimensional laminar flow on a continuous stretching surface with surface temperature T_ω and velocity $U_\omega = bx(1-\gamma t)^{-1}$ (see, Anderson et. [14]) where b (is the stretching rate) and γ are positive constant.

The x- axis runs along the continuous surface in the direction of the motion and y- axis is perpendicular to it.

The conservation equations of the laminar boundary layer are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

with the associated boundary conditions

$$\left.\begin{array}{c} y = 0, u = U_\omega(x,t), v = 0,\ T = T_\omega(x,t) \\ y \to \infty, \quad u = 0, \quad T = T_\infty \end{array}\right\} \tag{4}$$

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where u and v are the velocity components in the x and y directions, T is the temperature inside the boundary layer, v is the kinematic viscosity, α is the thermal diffusivity and t is the time and T_∞ is the free stream temperature.

The equation of continuity is satisfied if we choose a stream function $\psi(x, y)$ such that

$$u = \frac{\partial\psi}{\partial y}, \quad v = -\frac{\partial\psi}{\partial x}$$

The mathematical analysis of the problem is simplified by introducing the following dimensionless coordinates:

$$\eta = \sqrt{\frac{b}{v(1-\gamma t)}}y \tag{5}$$

$$\psi(x,y) = \sqrt{\frac{vb}{(1-\gamma t)}}xf(\eta) \tag{6}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_\omega - T_\infty}, T_\omega - T_\infty = \frac{b}{2vx^2}(1-\gamma t)^{\frac{-3}{2}} \tag{7}$$

Substituting (5) – (7) into Eqs (2) and (3) we obtain

$$f''' + ff'' - f'^2 - A\left(f' + \frac{1}{2}\eta f''\right) = 0 \tag{8}$$

$$P_r^{-1}\theta'' + 2f'\theta + f\theta' - \frac{A}{2}(3\theta + \eta\theta') = 0 \tag{9}$$

where $P_r = v/\alpha$ is the Prandtl number and A is dimensionless measure of the unsteadiness. The primes denote differentiation with respect to η.

The boundary conditions (4) now becomes

$$\left.\begin{array}{c} \eta = 0, f = 0, f' = 1, \theta = 1 \\ \eta \to \infty,\ f' = 0, \theta = 0 \end{array}\right\} \tag{10}$$

3 Numerical method

The transformed momentum Eq. (8) and the energy Eq. (9) subject to the boundary condition (10) were integrated numerically by the well-known fourth-order Runge-Kutta-Merson method. The half interval method was used to search for $f'(0)$ until $f'(\eta)$ decayed exponentially to zero full stop. The Gaussian elimination method was used to solve the energy equation, and the algorithm was modified following Chow [15]. The solution provided f'', and θ and the forward finite difference equation was employed to obtain $\theta'(0)$ for uniform temperature boundary conditions. The numerical results were found to depend upon η_∞ and step size. A step size of $\Delta\eta = 0.1$ gave sufficient accuracy for a Prandtl number of 1.0. The value of η_∞ was chosen as large as possible between 4 to 14 depending upon the Prandtl number and unsteadiness parameter without causing numerical oscillation in the values of f', f'' and θ. The computation was carried out on an IBM compatible 586 PC.

4 Result and Discussion

Velocity profiles $f'(\eta)$ and temperature profiles $\theta(\eta)$ for A = 0.8, 1.2 and 2.0 are shown in Figs. 1, 2 and 3 respectively. The temperature gradient $\theta'(0)$ at stretching surface is displayed in Fig. 4. The latter quantity is of particular importance since the heat transfer between the stretching surface and the fluid is conventionally expressed in dimensionless form as a local Nusselt number.

$$Nu = \frac{-\lambda\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_\omega - T_\infty}$$

$$\frac{Nu}{\sqrt{R_e}} = -\theta'(0)$$

where λ is the thermal conductivity and

$$R_e = \frac{xU\omega}{v}$$

is a local Reynolds number based on the surface velocity U_ω.

Since the fluid motion is driven solely by the stretching surface, the surface gradient $f'(0)$ of the velocity component u parallel to the surface stretching is negative and decreases with unsteadiness parameter. From Table 1, we note that, the surface gradient decreases with increasing the unsteadiness parameter while the rate of heat transfer increases with unsteadiness parameter and Prandtl number.

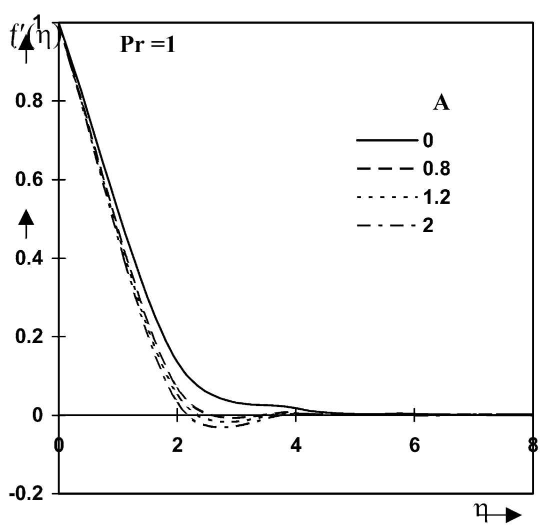

Fig. 1 Velocity distribution as a function of η for various values of A

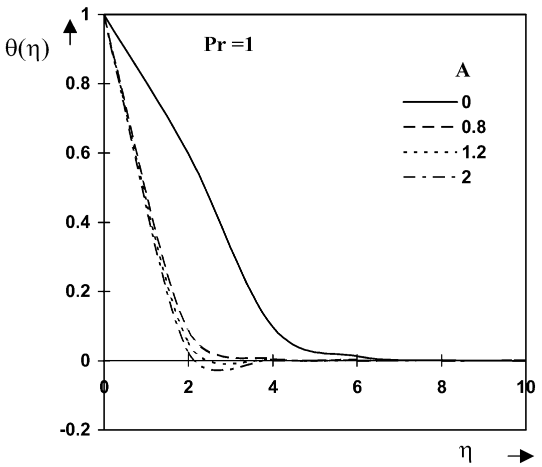

Fig. 2 Temperature distribution as a function of η for various values of A

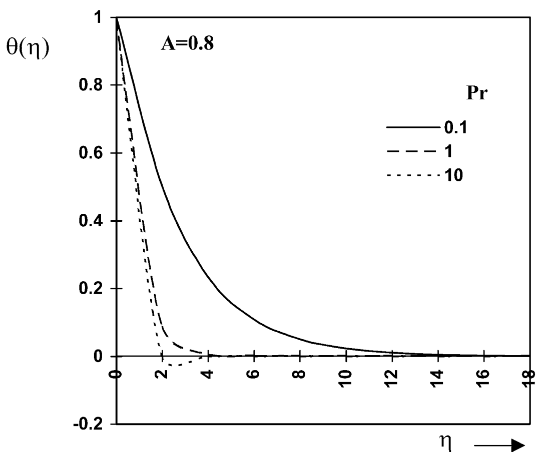

Fig. 3 Temperature distribution as a function of η for various values of Pr

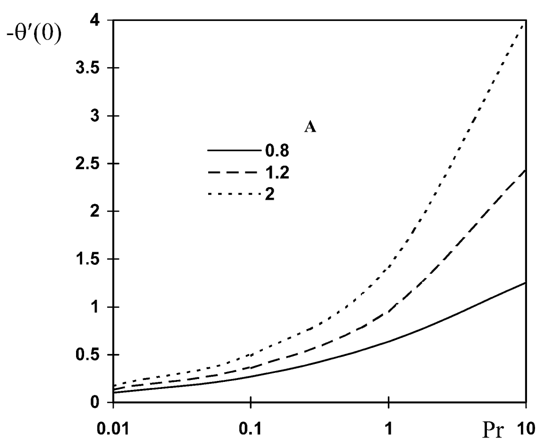

Fig. 4 Variation of the heat transfer coefficient

Table 1 Comparison of $\frac{Nu}{\sqrt{R_e}}$ for A = 0.0 and Pr = 1.0 to previously published data

Grubka and Bobba [16]	Ali [9]	present results
1.00000	1.0054	0.99999

Table 2 $\frac{Nu}{\sqrt{R_e}}$ and surface gradient $-f''(0)$ as a function of Prandtl number and unsteadiness parameter

A	0.8		1.2		2	
Pr	$-\theta'(0)$	$-f''(0)$	$-\theta'(0$	$-f''(0)$	$-\theta'(0)$	$-f''(0)$
0.01	0.1016	1.3321	0.1319	1.4691	0.1723	1.7087
0.1	0.2707	1.3321	0.3576	1.4691	0.4916	1.7087
1.0	0.6348	1.3321	0.9491	1.4691	1.4086	1.7087
10	1.2552	1.3321	2.4177	1.4691	3.9814	1.7087

The similarity solutions for the dimensionless velocity in Fig. 1 show that the boundary layer thickness decreases with unsteadiness parameter.

The similarity solutions for the dimensionless temperature in Fig. 2 show that θ (η) decreases monotonically with η, i.e. with the distance from the stretching surface, for all Prandtl numbers. This implies that the temperature T gradually decreases with η from T_ω at $\eta=0$. It is noteworthy that θ (η) vanishes at the free stream η = 4 for sufficiently large Prandtl number.

The thermal boundary layer thickness decreases with Prandtl number, for all unsteadiness parameter. Figure 4 reveals that uniformity of lower Prandtl number for A = 0.8, 1.2 and 2 and the temperature gradients vanishes at free stream. Moreover, when A tends to zero the solution approached the numerical solution by Ali [9] and Grubka et al. [8], we note that, from Table 2, it is clear that the numerical solution gives good results in comparison with refs. [8] and [9].

5 Conclusion

The purpose of this paper was to present an exact similarity solution for momentum and heat transfer in an unsteady flow whose motion is caused solely by the linear stretching of a horizontal stretching surface. The following results are obtained

1. A new similarity solution for the temperature field has been obtained, which transforms the time-dependent thermal energy equation to an ordinary differential equation.
2. Thermal boundary layer thickness decreases with unsteadiness parameter and Prandtl number while the momentum boundary layer thickness decreases with unsteadiness parameter.
3. The surface gradient is negative and decreases with unsteadiness parameter.
4. The rate of heat transfer increases with unsteadiness parameter and Prandtl number.

References

1. Sakiadis BC (1961) Boundary layer behavior on continuous solid surfaces: I. Boundary-layer equations for two- dimensional and axisymmetric flow. AIChE J 7(1): 26–28
2. Sakiadis BC (1961) Boundary layer behavior on continuous solid surface : II. Boundary layer behavior on continuous flat surface. AIChE J 7(1): 221–225
3. Tsou FK, Sparrow EM, Goldstein RJ (1967) Flow and heat transfer in the boundary layer on a continuous moving surface, Int J Heat Mass Transfer 10: 219–235
4. Crane LJ (1970) Flow past a stretching plane. Z Amgew Math Phys 21: 645–647
5. Vleggaar J (1977) Laminar boundary layer behavior on continuous, accelerating surfaces. Chem Eng Sci 32: 1517–1525
6. Gupta PS, Gupta AS (1977) Heat and mass transfer on a stretching sheet with suction or blowing. Canadian J Chem Eng 55(6): 744–746
7. Soundalgekar VM, Ramana TV (1980) Heat transfer past a continuous moving plate with variable temperature Warme-Und Stoffuber tragung.14: 91–93
8. Grubka LJ, Bobba KM (1985) Heat transfer characteristics of a continuous stretching surface with variable temperature. J Heat Transfer 107: 248–250
9. Ali ME (1994) Heat transfer characteristics of a continuos stretching surface. Warme-Und Stoffubertragung 29: 227–234
10. Banks WHH (1983) Similarity solutions of the Boundary layer equation for a stretching wall. J Mec Theor Appl 2: 375–392
11. Ali ME (1995) On thermal Boundary layer on a power law stretched surface with suction or injection. Int J Heat Mass Flow 16: 280–290
12. Elbashbeshy EMA (1998) Heat transfer over a stretching surface with variable heat flux. J Physics D: Appl Physics 31: 1951–1955
13. Elbashbeshy EMA, Bazid MAA (2000) Heat transfer over a continuously moving plate embedded in non-Darcian porous medium. Int J Heat and Mass Transfer 43: 3087–3092
14. Andersson HT, Aarseth JB, Dandapat BS (2000) Heat transfer in a liquid film on an unsteady stretching surface. Int J Heat Transfer 43: 69–74
15. Chow CY (1983) An introduction to computational fluid mechanics. Seminole, Boulder, Co
16. Grubka LT, Bobba KM (1985) Heat transfer Characteristics of a continuous stretching surface with variable temperature. ASME J Heat Transfer 107: 248–250