# Onset of convective instabilities in a binary liquid mixtures with fast chemical reactions in a porous medium

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Abstract In this paper, we study the instabilities, which can occur when a horizontal sparsely filled porous layer saturated with a binary mixture with fast chemical reaction is heated from below or from above. Analytic expressions are found for the criteria for the onset of stationary and oscillatory convection in binary mixtures with chemical reaction. The reaction is assumed to be fast compared with the diffusion rate. We find that both a stationary instability and an oscillatory instability can occur as the first bifurcation, depending on the sign and the value of the heat of reaction. The effect of the Prandtl number, the separation parameter and the porous parameter are shown graphically. Some of the known results are derived as special cases.

#### List of symbols

- A average specific heat
- *a* wave number

 $a_x, a_y$ 

- wave numbers in x and y directions respectively
- *C* concentration field
- $C_{eq}$  concentration at the equilibrium state
- $C_p$  specific heat at constant pressure
- d layer thickness
- F porous parameter,  $\left(\frac{d^2}{k}\right)$
- $\vec{g}$  acceleration due to gravity
- k permeability of the porous medium
- Pr Prandtl number,  $\left(\frac{v}{\kappa}\right)$
- *p* pressure
- $p_c$  chemical potential
- $\vec{q}$  velocity vector, (u, v, w)
- *R* thermal Rayleigh number,  $\left(\frac{\alpha_T g d^4 T_z}{\nu_K}\right)$
- *T* Temperature field
- $\bar{T}_z$  applied temperature gradient
- t time
- $t_r$  relaxation time of chemical reaction

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M.S. Malashetty (⊠), S.N. Gaikwad Department of Mathematics Gulbarga University Gulbarga-585 106 Karnataka, India E-mail: malashettyms@yahoo.com X dimensionless parameter,  $\left(\frac{d^2}{\kappa \tau_1}\right)$ space coordinates

#### Greek symbols

- $\alpha_C$  coefficient of solute expansion
- $\alpha_T$  coefficient of thermal expansion
- $\epsilon$  porosity of the medium
- $\kappa$  thermal diffusivity
- μ fluid viscosity
- $\mu_e$  effective Brinkman viscosity
- $\lambda$  average heat conductivity
- *v* kinematics viscosity of fluid
- $\rho$  fluid density
- $\phi$  heat of reaction
- $\sigma$  growth rate
- $\eta$  dimensionless parameter,  $(\phi \tau_2 / \tau_1)$
- $\psi$  chemical reaction parameter,  $(\phi \alpha_C / \alpha_T)$
- $\omega$  frequency

## Other symbols

$$\begin{array}{ccc} \nabla^2 & \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \\ \nabla_1^2 & \nabla^2 - \frac{\partial^2}{\partial z^2} \end{array}$$

#### Subscripts

- o reference value
- f fluid
- s solid

#### Superscripts

- \* dimensionless quantity
- deviation from the conduction state

#### 1 Introd

# Introduction

Convective instability in a porous media has been studied with great interest for more than half a century. Convection, driven by buoyancy, has found increased applications in underground coal gasification, solar energy conversion, oil reservoir simulation, ground water contaminant transport, geothermal energy extraction and many other areas. In the beginning, interest was mainly focussed on problems in which the driving force was due to the applied temperature/concentration gradients at the boundaries of the system. Recently, the focus is also on the study of convective instabilities in reactive fluids.

In recent years there has been considerable interest, from various branches of fluid mechanics and condensed matter physics, in systems that show an oscillatory instability as the first bifurcation. The reasons for considering the instability in a porous medium are twofold. First, a porous medium is often used as a catalyst enhancing the rate of reaction. On the other hand, use of a porous medium significantly simplifies the descriptions of an average hydrodynamic flow and allows us to consider realistic boundary conditions for the velocity. Therefore, the considered system is expected to be a convenient object for studying the nonlinear behavior above the critical level.

The stability of a binary mixture without chemical reaction is studied extensively in case of viscous fluid [1–3] as well as in porous medium [4–7]. The effect of fast chemical reaction on the stability of a binary liquid mixtures with and without porous medium is studied by Steinberg and Brand [8] and Gitterman and Steinberg [9].

The instability in binary mixtures can lead to stationary convection as well as to oscillatory convection. The stationary convection occurs when heating is done either from below or from above depending on the sign of the thermal diffusion coefficient, whereas the oscillatory one occurs just when heating is from below. A qualitatively similar behavior was predicted for binary mixtures with chemical reactions. The reasons for the similarity is based on the fact that in a system subject to a temperature gradient the chemical reaction produces a concentration gradient like the thermal diffusion effect in a nonreactive binary mixture. The direction of the concentration gradient with respect to the temperature gradient depends on the properties of the reaction under consideration [10]. Correspondingly, the stability of the system also depends on the reaction under consideration. For example, in the case of dissociating fluids, [11], the behavior of the system is similar to one of a nonreactive binary mixture with normal thermal diffusion effect ( $\kappa$ <0). But there are also reactions, as, for example, polymerization that produces the same concentration distribution as the abnormal thermal diffusion effect ( $\kappa$ >0). In the latter case one should expect stationary convection when heated from above and an oscillatory one when heated from below. This mechanism is called chemically driven instability since only the effect of a chemical reaction on the stability of the system is considered. The thermal diffusion usually has much smaller influence on the stability than a chemical reaction and can be neglected.

The purpose of this paper is to present the results of a linear stability analysis of the chemically driven instability of a reactive binary mixture in a sparsely packed porous medium as this study is not carried out so far. In order to study the effect of chemical reaction on the hydrodynamic stability and to obtain an analytical solution, we make two approximations. First, we neglect the effect of thermal diffusions since, its influence on the stability is much smaller than that of a chemical reaction. Thus, we consider only chemically driven instability. Second, we assume that rate of chemical reaction in a porous medium subjected to external temperature gradient is fast, compared with the diffusion rate. In this approximation, the linear stability analysis is simplified considerably and it becomes possible to carry out the analysis analytically for realistic boundary conditions.

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## Mathematical formulation

We consider a horizontal porous layer saturated with a reactive binary mixture, of depth d and of infinite extent in the horizontal (x, y) plane, subject to a vertical gradient of temperature in the gravitational field  $\vec{g}$ . In the Boussinesq approximation, the hydrodynamic equations for the flow through a sparsely packed porous medium are given by ([9, 12]).

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

$$\frac{\partial \vec{q}}{\partial t} + \vec{q} \cdot \nabla \vec{q} = -\frac{\nabla p}{\rho_0} + \frac{\rho}{\rho_0} \vec{g} - \frac{\mu}{\rho_0 k} \vec{q} + \frac{\mu_e}{\rho_0} \nabla^2 \vec{q}, \qquad (2)$$

$$A\frac{\partial T}{\partial t} + \vec{q} \cdot \nabla T = \kappa \nabla^2 T + \frac{C - C_{eq}}{\tau_1}, \qquad (3)$$

$$\frac{\partial C}{\partial t} + \vec{q} \cdot \nabla C = -\frac{C - C_{eq}}{\tau_2},\tag{4}$$

$$\rho = \rho_0 [1 - \alpha_T (T - T_0) + \alpha_C (C - C_0)],$$
(5)

where

$$\tau_1 = t_r \rho C_p \left[ T \left( \frac{\partial p_c}{\partial T} \right)_{P,C} \right]^{-1}$$
 and  $\tau_2 = \rho t_r$ .

Here  $A = \rho C_p / (\rho C_p)_f$ ,  $\kappa = \lambda / (\rho C_p)$  where  $\rho C_p = (1 - \varepsilon) (\rho C_p)_s$ +  $\varepsilon(\rho C_p)_f$  and  $\lambda = (1 - \varepsilon)\lambda_s + \varepsilon\lambda_f$  are the average specific heat and heat conductivity respectively; the indices s and f corresponds to solid and fluid respectively and  $\varepsilon$  is the porosity of the medium,  $t_r$  is the relaxation time of the chemical reaction,  $C_p$  is the specific heat at constant pressure,  $C_{eq}$  is the concentration in equilibrium state,  $p_c$  is the chemical potential,  $\kappa$  is the renormalized thermal diffusivity containing contribution from the porous medium and the fluid, k is the permeability of the porous medium,  $\mu$  is the fluid viscosity and  $\mu_e$  is the effective viscosity. In general the effective Brinkman viscosity will be different from the fluid viscosity. However for simplicity we assume that both are equal  $(\mu_e = \mu)$ . The equation (4) is written down in the approximation of a chemical reaction fast compared with the diffusion rate. We also set A equal to 1 for simplicity in our further discussion.

It is well known that in the convection free steady state, in contrast to nonreactive fluids, the temperature and concentration gradients depend in the general case of reactive fluids on the vertical coordinate z. This makes the stability analysis considerably more difficult for an analytical approach. However, in the case of a fast chemical reaction considered here, equations (1)–(4) admit the following stationary solution.

$$\vec{q} = 0, \quad \nabla T_0 = -\bar{T}_z \hat{g}, \quad C = C_{eq}(T),$$
(6)

where  $\overline{T}_z$  denotes the applied temperature gradient which can have either sign and  $\hat{g} = \vec{g}/|\vec{g}|$ . For a small temperature difference across the porous layer saturated with liquid, we expand the law of mass action  $C_{eq} = C_{eq}(T)$  into a power series around the appropriate value of concentration  $C_0$  and temperature  $T_0$  on the lower boundary as

$$C_{eq} = C_0 + \phi(T - T_0),$$
 (7)

where  $\phi$  is the parameter which indicates the nature of heat of reaction. The equation (7) yields

$$\nabla C_0 = \phi \nabla T_0, \tag{8}$$

and for the density gradient we obtain

$$\frac{d\rho}{dz} = -\rho \alpha_T \left( 1 + \phi \frac{\alpha_C}{\alpha_T} \right) \frac{dT}{dz}.$$
(9)

The parameter  $(\phi \alpha_C / \alpha_T)$  determines the contribution to the density distribution made by the chemical reaction. It is similar to the separation parameter  $-(\kappa/T)(\alpha_C / \alpha_T)$  in a binary mixture without chemical reaction. Here the parameter  $\phi$  can be either positive or negative depending on the reaction under consideration.

We now discuss the boundary conditions to equations (1)–(4). There are two types of boundary conditions for the concentrations. For theoretical calculations, it is convenient to require an equilibrium concentration at the boundaries, but this is hardly realizable experimentally. A more realistic condition is that of impermeable boundaries through which there exists zero mass flux. For convenience, we therefore assume that the concentrations on the boundaries are constant. The velocity boundary conditions depend on whether the boundaries are both rigid or both free, and whether one is rigid and the other is free. The qualitative results are the same in all cases, although the critical Rayleigh numbers are different. Hence, we consider the simplest case, that of free boundaries.

For the deviations from the basic heat conduction state, we then find the following nonlinear equations

$$\frac{\partial \vec{q}}{\partial t} + \vec{q} \cdot \nabla \vec{q} = -\frac{\nabla p}{\rho_0} + (\alpha_T T' + \alpha_C C') \hat{g} - \frac{v}{k} \vec{q} + v \nabla^2 \vec{q},$$
(10)

$$\frac{\partial T'}{\partial t} + \vec{q} \cdot \nabla T' - \bar{T}_z(\vec{q} \cdot \hat{g}) = \kappa \nabla^2 T + \frac{C - C_{eq}}{\tau_1}, \quad (11)$$

$$\frac{\partial C'}{\partial t} + \vec{q} \cdot \nabla C' - \phi \bar{T}_z(\vec{q} \cdot \hat{g}) = -\frac{C - C_{eq}}{\tau_2}, \qquad (12)$$

The boundary conditions for these equations as discussed above are

$$w = 0, \quad \frac{\partial^2 w}{\partial z^2} = 0, \quad T' = 0, \quad C' = 0 \quad \text{at } z = 0, d,$$
(13)

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where prime denotes the deviation from the steady state solution equation (6).

# Linear stability analysis

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Linearizing the hydrodynamic equations (10)-(12) for the deviation from the heat conduction state, we obtain the following linear equations:

$$\frac{\partial \vec{q}'}{\partial t} = -\frac{\nabla p'}{\rho_0} + (\alpha_T T' + \alpha_C C') \vec{g} - \frac{\nu}{k} \vec{q}' + \nu \nabla^2 \vec{q}', \qquad (14)$$

$$\frac{\partial T'}{\partial t} - \bar{T}_z(\vec{q} \cdot \hat{g}) = \kappa \nabla^2 T' + \frac{C - C_{eq}}{\tau_1}, \qquad (15)$$

$$\frac{\partial C'}{\partial t} - \phi \bar{T}_z(\vec{q} \cdot \hat{g}) = -\frac{C - C_{eq}}{\tau_2}, \qquad (16)$$

We eliminate p' form equation (14) and render the resulting equation and equations (15) and (16) dimensionless by setting

$$\begin{split} w^* &= \frac{d}{\kappa} w', \quad t^* = \frac{\kappa}{d^2} t, \quad (x^*, y^*, z^*) = \frac{1}{d} (x, y, z), \\ T^* &= \frac{T'}{|\nabla T_0| d}, \quad C^* = \frac{C'}{|\nabla C_0| d}, \\ \Pr &= \frac{v}{\kappa}, \quad R = \frac{g \alpha_T \bar{T}_z d^4}{\kappa v}, \quad F = \frac{d^2}{k}, \end{split}$$

to obtain (on dropping asterisks)

$$\left(\frac{\partial}{\partial t} + F + \nabla^2\right) \nabla^2 w = R \Pr^{-1} \left[ \nabla_1^2 T + (\phi \alpha_C / \alpha_T) \nabla_1^2 C \right],$$
(17)

$$\left(\frac{\partial}{\partial t} - \Pr^{-1}\nabla^2 + \frac{\phi d^2}{\kappa \tau_1}\right)T = w + \left(\frac{\phi d^2}{\kappa \tau_1}\right)C,$$
(18)

$$\left(\frac{\partial}{\partial t} + \frac{d^2}{\kappa \tau_2}\right)C = w + \left(\frac{d^2}{\kappa \tau_2}\right)T,\tag{19}$$

where  $\nabla_1^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$ .

The dimensionless groups that appear are Pr, the Prandtl number, R, the thermal Rayleigh number and F, the porous parameter. The porous parameter F is the reciprocal of the Darcy number Da.

### Method of solution

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We now assume the solutions for the equations (17)–(19) in the form

$$\begin{bmatrix} w(x,y,z,t) \\ T(x,y,z,t) \\ C(x,y,z,t) \end{bmatrix} = \begin{bmatrix} W \\ T \\ C \end{bmatrix} e^{\sigma t} \sin \pi z \cos a_x x \cos a_y y.$$
(20)

Substituting (20) into (17)–(19) we get the following equations

$$(\sigma + \pi^2 + a^2 + F)(\pi^2 + a^2)W = R\Pr^{-1}a^2(T + \psi C),$$
(21)

$$\left(\sigma + \Pr^{-1}(\pi^2 + a^2) + \frac{\phi d^2}{\kappa \tau_1}\right)T = W + \frac{\phi d^2}{\kappa \tau_1}C,$$
(22)

$$\left(\sigma + \frac{d^2}{\kappa \tau_2}\right)C = W + \frac{d^2}{\kappa \tau_2}T,$$
(23)

where  $a^2 = a_x^2 + a_y^2$  and  $\psi = \phi \alpha_C / \alpha_T$  is the analog of the separation parameter. We eliminate *W*, Tand *C* from the equations (21)–(23), to obtain an expression for the thermal Rayleigh number, *R* 

 $R = \frac{(\pi^2 + a^2)(\sigma + \pi^2 + a^2 + F) \left[ \left( \sigma + \frac{d^2}{\kappa \tau_2} \right) \left( \sigma + \Pr^{-1}(\pi^2 + a^2) + \frac{\phi d^2}{\kappa \tau_1} \right) - \frac{\phi d^4}{\kappa^2 \tau_1 \tau_2} \right]}{a^2 \Pr^{-1} \left[ \sigma (1 + \psi) + \frac{d^2}{\kappa \tau_2} (1 + \phi \tau_2 / \tau_1) (1 + \psi) + \psi \Pr^{-1}(\pi^2 + a^2) \right]}$ 

Standard analysis used in the study of convective instability gives the criteria for the stationary instability (i.e., when  $\sigma = 0$ ) as

$$R = \frac{X(\pi^2 + a^2)^2(\pi^2 + a^2 + F)}{a^2[X(1+\psi)(1+\eta) + \psi \Pr^{-1}(\pi^2 + a^2)]},$$
 (25)

where

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 $X = rac{d^2}{\kappa au_2}, \quad \eta = rac{\phi au_2}{ au_1}.$ 

In the absence of chemical reaction and single component fluid equation (25) reduces to

$$R = \frac{(\pi^2 + a^2)^2(\pi^2 + a^2 + F)}{a^2}.$$
 (26)

The minimum value occurs at  $a = a_c$  where  $a_c$  is given by equation

$$a_{c} = \frac{1}{4} \left\{ -\left(\pi^{2} + F\right) + \left[9\pi^{4} + 10\pi^{2}F + F^{2}\right]^{1/2} \right\}.$$
 (27)

These are the values given by Rudraiah [13].

Further in the absence of porous media i.e. for a clear viscous fluid layer heated from below equation (26) reduces to

$$R = \frac{(\pi^2 + a^2)^3}{a^2}.$$
 (28)

In this case the critical value of R equal to  $\frac{27\pi^4}{4}$  occurs at  $a_c = \pi/\sqrt{2}$ . These are the results of classical Rayleigh Benard problem for clear viscous fluid with free-free boundaries.

Next we obtain the oscillatory Rayleigh number  $R^{Osc}$ and the oscillatory frequency from equation (24) respectively (i.e., when  $\sigma = i\omega$ ,  $\omega$  is real and positive) as where  $R^{Osc}$  represents the thermal Rayleigh number for oscillatory case and

$$P = \Pr^{-1}(\pi^2 + a^2) + (d^2/\kappa\tau_2)(1 + \phi\tau_2/\tau_1) + (F + \pi^2 + a^2),$$
  

$$Q = \Pr^{-1}(\pi^2 + a^2)^2 (d^2/\kappa\tau_2)(F + \pi^2 + a^2).$$

The expressions for the oscillatory instability and the oscillatory frequency are simplified considerably if we assume  $X \ll 1$ . With this assumption, equations (29) and (30) reduces to

(24)

$$R^{Osc} = \frac{\left\{ (\pi^2 + a^2)^3 (F + \pi^2 + a^2) \Pr^{-1} + (\pi^2 + a^2)^2 (F + \pi^2 + a^2)^2 \right\}}{a^2 [(\pi^2 + a^2) (\Pr^{-1} + 1 + \psi) + (1 + \psi)F]}$$
(31)

$$\omega^{2} = \omega_{c}^{2} = \frac{-\left[ (R^{Osc} a_{c}^{2} (\mathbf{Pr}^{-1})^{2} \psi) \right]}{\left[ (\mathbf{Pr}^{-1} + 1)(\pi^{2} + a_{c}^{2}) + F \right]}.$$
(32)

Minimizing equation (25) with respect to  $a^2$  yields the cubic equation in  $a^2$  as

$$\psi \operatorname{Pr}^{-1}(a^{2})^{3} + \left[\psi \operatorname{Pr}^{-1}\pi^{2} + 2X(1+\psi)(1+\eta)\right](a^{2})^{2} + (\pi^{2}+F) \\ \times \left[(1+\psi)(1+\eta)X - \psi \operatorname{Pr}^{-1}\pi^{2}\right](a^{2}) \\ - (\pi^{2}+F)\pi^{2}\left[(1+\psi)(1+\eta)X + \psi \operatorname{Pr}^{-1}\pi^{2}\right] = 0.$$
(22)

Solving the equation (33) for  $a^2$  gives the critical wave number  $a_c^2$ . Further substituting this critical wave number  $a_c^2$  into the equation (25) yields the critical Rayleigh number  $R_c$  for stationary instability.

Similarly on minimizing equation (31) with respective to  $a^2$  yields the fourth order equation in  $a^2$  as

$$a_0(a^2)^4 + a_1(a^2)^3 + a_2(a^2)^2 + a_3(a^2) + a_4 = 0,$$
 (34)

where

$$\begin{split} a_0 &= 2(\Pr^{-1} + 1)(\Pr^{-1} + 1 + \psi), \\ a_1 &= (\Pr^{-1} + 1) \big[ 5\pi^2 (\Pr^{-1} + 1 + \psi) + 3F(1 + \psi) \big] \\ &+ F(\Pr^{-1} + 2)(\Pr^{-1} + 1 + \psi), \\ a_2 &= (\Pr^{-1} + 1) \big[ 3\pi^4 (\Pr^{-1} + 1 + \psi) + 5\pi^2 F(1 + \psi) \big] \\ &+ (\Pr^{-1} + 2)F \big[ \pi^2 (\Pr^{-1} + 1 + \psi) + 2F(1 + \psi) \big], \end{split}$$

$$R^{Osc} = \frac{(\pi^2 + a^2) \left( \Pr^{-1}(\pi^2 + a^2) \left( \frac{d^2}{\kappa \tau_2} \right) \left( 1 + \frac{\phi \tau_2}{\tau_1} \right) \right) \left[ \Pr^{-1}(\pi^2 + a^2) d^2 / \kappa \tau_2 + (\pi^2 + a^2 + F) P \right]}{a^2 \Pr^{-1}[(1 + \psi)(\pi^2 + a^2 + F) + \Pr^{-1}(\pi^2 + a^2)]},$$
(29)

and

$$\omega^{2} = \frac{\left\{-R^{0sc}a^{2}\Pr^{-1}\left[\frac{d^{2}}{\kappa\tau_{2}}(1+\psi)\left(1+\frac{\phi\tau_{2}}{\tau_{1}}\right)+\psi\Pr^{-1}(\pi^{2}+a^{2})\right]+\frac{d^{2}}{\kappa\tau_{2}}Q\right\}}{P(\pi^{2}+a^{2})}, \quad a_{3} = (\Pr^{-1}+1)\left[\pi^{4}(1+\psi)F - \pi^{6}(\Pr^{-1}+1+\psi)\right] + (\Pr^{-1}+2)F\left[\pi^{2}(1+\psi)F - \pi^{4}(\Pr^{-1}+1+\psi)\right] + (Pr^{-1}+2)F\left[\pi^{2}(1+\psi)F - \pi^{4}(\Pr^{-1}+1+\psi)\right] + (F)^{2}\left[(1+\psi)F - \pi^{2}(\Pr^{-1}+1+\psi)\right],$$

$$\begin{split} a_4 &= - \big\{ (\Pr^{-1} + 1) \big[ \pi^8 (\Pr^{-1} + 1 + \psi) + \pi^6 (1 + \psi) F \big] \\ &+ (\Pr^{-1} + 2) F \big[ \pi^6 (\Pr^{-1} + 1 + \psi) + \pi^4 (1 + \psi) F \big] \\ &+ (F)^2 \big[ \pi^4 (\Pr^{-1} + 1 + \psi) + \pi^4 (1 + \psi) F \big] \big\}. \end{split}$$

We solve equation (34) for  $a^2$  which gives the critical wave number  $a_c^2$  and substituting this critical wave number into the equation (31) yields the critical Rayleigh number  $(R^{Osc})_c$  for the oscillatory instability. Substituting this critical wave number and critical Rayleigh number of oscillatory instability into the equation (32) gives the critical frequency for oscillatory case.

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# Conclusions

In the present study we have employed a linear stability analysis for chemically driven instabilities in a binary mixture with a chemical reaction that is fast compared with the diffusion rate. Analytical expressions have found for the onset of stationary, oscillatory instabilities and oscillatory frequency, which depend on the rate and the heat of reaction. The critical wave number also depends strongly on these quantities.

The region of stationary instability for different values of the porous parameter F is shown in Figure 1. We observe from this figure that an increase in the value of the porous parameter increases the critical Rayleigh number of the stationary mode.

Figure 2 illustrates the variation of critical Rayleigh number with chemical reaction parameter  $\psi$  for different



**Fig. 1.** Variation of critical Rayleigh number  $R_c$  with chemical reaction parameter  $\psi$  for different values of porous parameter F



Fig. 2. Variation of critical Rayleigh number  $R_c$  with chemical reaction parameter  $\psi$  for different values of Pr



**Fig. 3.** Variation of critical Rayleigh number  $R_c$  with chemical reaction parameter  $\psi$  for different values of  $\eta$ 



**Fig. 4.** Variation of critical Rayleigh number  $(R^{osc})_c$  with chemical reaction parameter  $\psi$  for differnt values of porous parameter F



**Fig. 5.** Variation of critical Rayleigh number  $(R^{osc})_c$  with chemical reaction parameter  $\psi$  for different values of Prandtl number Pr

values of Prandtl number Pr for stationary case. We find from this figure that an increase in the value of the Prandtl number Pr increases the critical Rayleigh number hence increases the region of marginal stability. The effect of the parameter  $\eta$  on the stationary instability is shown in Fig. 3. We find that an increase in the value of  $\eta$  decreases the critical Rayleigh number of the stationary state. Hence its effect is destabilizing.

Figures 4 and 5 show the variation of critical Rayleigh number with chemical reaction parameter  $\psi$  for different values of the porous parameter *F* and the Prandtl number Pr for oscillatory case. We observe from Fig. 4 that for fixed value of the Prandtl number Pr an increase in the value of the porous parameter *F* increases the critical Rayleigh number for oscillatory state. It can be seen from Fig. 5 that for fixed value of the porous parameter *F* an increase in the value of Prandtl number Pr decreases the region of stable state for oscillatory mode. We can also observe from these figures that the effect of increasing the value of chemical reaction parameter  $\psi$  is to decrease the critical Rayleigh number for both stationary and oscillatory state. Thus the effect of  $\psi$  is to destabilize the system.

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