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An analytic model for air drying of impermeable wood

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Abstract An analytic model for the process of air drying of boards of impermeable wood was developed in the light of some new experimental results. The work includes a new and informative way of plotting the data, a formula for the drying time and a derivation for the diffusion coefficient function within the wood. It was found necessary to take into account the aspect ratio of the board, and that the correction term for evaporative cooling was also significant. The notion of impermeability implies that there is no movement of cell lumen water under the influence of capillary pressure.

Symbols

a, b	half thickness and half width of board respectively, m
D(r)	diffusion coefficient function for wood as a function of equilibrium relative humidity, kg/m s Pa
d	film coefficient for mass transfer, kg/m^2 s Pa
F	mass flux, kg/m^2 s
p	pressure of water vapour, Pa
r	relative humidity
S	surface area of board, m^2
T	temperature, K
t	time, s (unless otherwise specified)
V	volume of board, m^3
x, z	distance parameters
y	average moisture content parameter
y_0	y axis intercept, Fig. 1
α	shape factor, (15)
β	6800 K
γ	parameter in diffusion coefficient function

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Δp	wet-bulb depression vapour pressure, Pa
ΔT	wet-bulb depression temperature, K
v	distance parameter
ρ_g	basic density, kg/m^3
ω	moisture content of wood
ϖ	average moisture content

Subscripts

d	drying
e	empty lumen
i	initial
s	surface
w	wet-bulb
x	corresponding to the position parameter, x
o	corresponding to $y = 0$; except for y_0
l	corresponding to $y = 1$
∞	ambient conditions

Introduction

Traditionally drying models for wood have relied on an analogy with Fick's law for the diffusion of a solute in a liquid solution. This gives rise to the "diffusion equation" with moisture content as the potential, and enables standard solutions of this equation to be applied to wood drying. Such solutions usually assume one-dimensional flow. The present model takes the pressure of water vapour to be the potential for water movement, and flow towards the short sides of the board is taken into account.

King (1945) and then Joy (1951) used a graphical method of differentiation to evaluate the diffusion coefficient as a function of moisture concentration and equilibrium relative humidity respectively. Skaar (1954) then formalised the process by differentiating under the integral sign to make the determination explicit. Such method is employed here and extended to enable the diffusion coefficient function to be determined from the drying curve.

Experiment

The experimental work under consideration concerned a section of *Eucalyptus regnans* (basic density, 485.6 kg/m^3), 200 mm long $\times$ 90 $\times$ 40 mm^2 cross-section. It was dried under nominally constant conditions of 43 °C, 0.47 rh and about 0.5 m/s air flow velocity. For the purpose of the simulation, the drying was deemed to commence when the average moisture content returned to its initial value after some increase took place through condensation. The initial average moisture content was 1.127 and the empty lumen moisture content (fibre saturation) was taken as 0.28. The term "empty lumen" has been used because it may be that the cell wall is saturated over the entire hygroscopic range (see Hunter 1996).

A useful descriptor for the average moisture content is defined

$$y = \frac{\omega_i - \varpi}{\omega_i - \omega_e} \tag{1}$$

ω_i is the initial, ϖ the average and ω_e the empty lumen moisture content.

In terms of y we have for the mass flux

$$F = \frac{\rho_g V}{S}(\omega_i - \omega_e)\frac{dy}{dt} \tag{2}$$

where ρ_g is the basic density, S is the surface area where drying takes place, V the volume of the board and t is time.

At the beginning of the drying, the flux F is also given by

$$F = d\Delta p \tag{3}$$

where d is the film coefficient for mass transfer, and Δp is the wet-bulb depression vapour pressure. From (2) and (3) we have for the film coefficient

$$d = \frac{\rho_g V(\omega_i - \omega_e)}{S\Delta p}\left.\frac{dy}{dt}\right|_0 \tag{4}$$

Now also at the start of drying, both t and y approach zero and so by L'Hospitals rule (Taylor 1952)

$$\lim_{y\to 0}\frac{t}{y} = \left.\frac{dt}{dy}\right|_0 = 1\bigg/\left.\frac{dy}{dt}\right|_0 \tag{5}$$

It is useful therefore to graph the drying data in the form y versus t/y and from the intercept with the t/y axis, d can be calculated immediately from (4).

Referring to Fig. 1 for the above experiment we find

$(t/y)_0 = 48.3$ h

and $d = 4.64 \times 10^{-8}$.

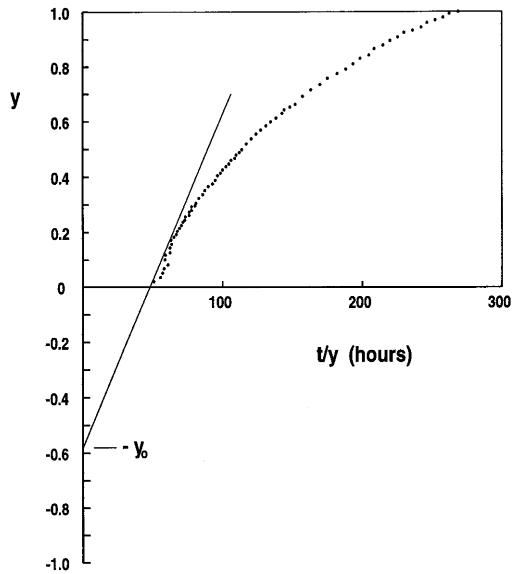

Fig. 1. Drying data. y is the moisture content parameter and t is time

Clearly the data in Fig. 1 present the shape of a logarithm and so in Fig. 2, ln t/y is plotted against y.
The functional relationship is

$$y = y_0 \ln\left[(t/y)/(t/y)_0\right] \tag{6}$$

If y = 1 we have

$$y_0 = \frac{1}{\ln\left[(t/y)_1/(t/y)_0\right]} \tag{7}$$

y_0 is also the negative of the y intercept of the tangent to the data at y = 0.
If the drying time t_d corresponds to when the moisture content parameter y = 1, then from (6)

$$t_d = (t/y)_0 e^{1/y_0} \tag{8}$$

Psychrometry and the mass flux

A vapour pressure – temperature diagram for the process is shown in Fig. 3. The wood surface initially achieves the wet-bulb temperature of the drying air. The surface then rises in temperature (assumed to prevail throughout the board), and follows the wet-bulb line, ultimately to the state of the drying air.

The mass flux

Differentiating (6) and using (2) and (3) we find for the mass flux at the surface

$$F_s = \frac{d\Delta p\ y_0 e^{-y/y_0}}{y + y_0} \tag{9}$$

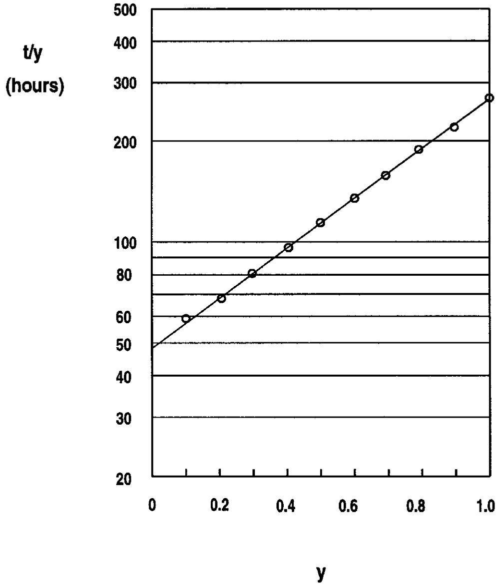

Fig. 2. Logarithmic plot of drying data

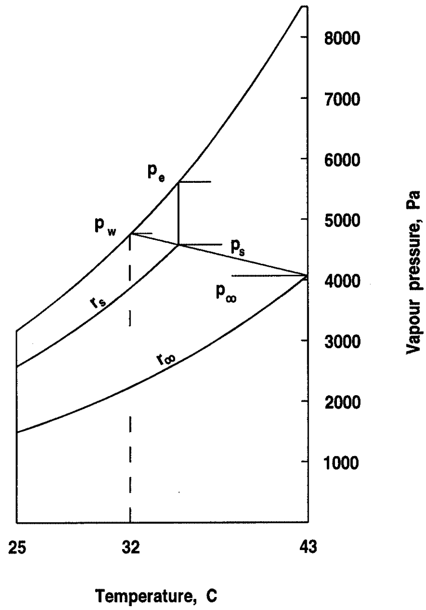

Fig. 3. Thermodynamic states on a psychrometric chart

Again at the surface we have

$$F_s = d(p_s - p_\infty) \tag{10}$$

so with (9) we have

$$p_s = p_\infty + \frac{\Delta p\ y_0 e^{-y/y_0}}{y + y_0} \tag{11}$$

With reference to Fig. 3 therefore, a reasonable approximation for the relative humidity at the surface is

$$r_s = r_\infty + \frac{(1 - r_\infty) y_0 e^{-y/y_0}}{y + y_0} \tag{12}$$

and also

$$T_e = T_\infty - \frac{\Delta T (r_s - r_\infty)}{1 - r_\infty} \tag{13}$$

The moisture plateau

If the wood is considered to be impermeable (no movement of cell lumen water under the influence of capillary pressure), then we may assume a plateau of moisture content of fixed, uniform height whose plan area is diminishing (Fig. 4). Most of the water resides within this plateau and so it is reasonable to neglect accumulation in the hygroscopic region.

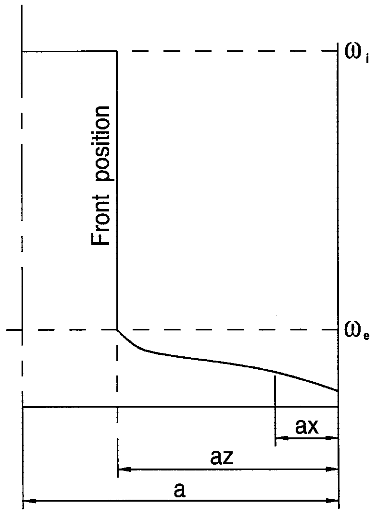

Fig. 4. Moisture content diagram showing square front

The shape factor, α

With reference to Fig. 5, it is clear that at any time the mass flux is at a minimum at the external surface (F_s), and the variation of flux is described approximately by

$$F = \frac{F_s}{1 - z/\alpha} \tag{14}$$

where a is the half thickness of the board, za is the distance in from the surface and

$$\alpha = \frac{a + b}{2a} \tag{15}$$

If the drying front progresses the same distance za in from each surface, it will be found that the parameter y above, is related to z by

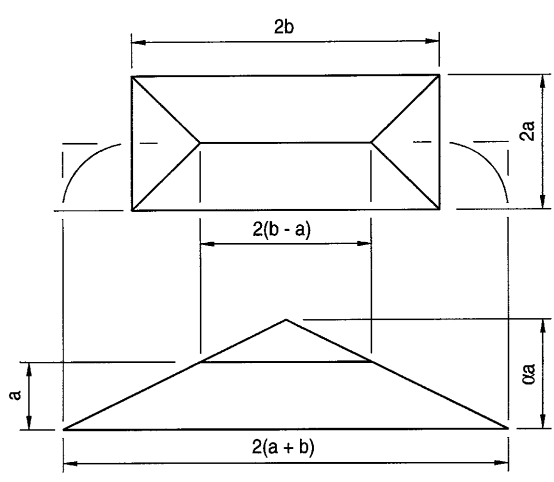

Fig. 5. Defining the shape factor, α

$$y = \frac{z(2\alpha - z)}{2\alpha - 1} \tag{16}$$

Hence

$$1 - z/\alpha = \sqrt{1 - (2\alpha - 1)y/\alpha^2} \tag{17}$$

and

$$\frac{dz}{dy} = \frac{2\alpha - 1}{2\alpha(1 - z/\alpha)} \tag{18}$$

Note that for large α, the flux becomes one-dimensional and $y = z$. With reference to (9), for small y, z

$$\frac{y_0 e^{-y/y_0}}{y + y_0} \cong 1 - 2y/y_0 \tag{19}$$

and with (16) above

$$\frac{y_0 e^{-y/y_0}}{y + y_0} \cong 1 - \frac{4\alpha z}{(2\alpha - 1)y_0} \tag{20}$$

Substituting into (12)

$$(1 - r_s) \cong (1 - r_\infty)\frac{4\alpha z}{(2\alpha - 1)y_0} \tag{21}$$

The diffusion coefficient function D(r)

The diffusion coefficient function D(r), where vapour pressure is the potential, is defined by

$$F = \frac{D(r)}{a}\frac{dp}{dx} \tag{22}$$

where ax is the distance from the surface. The value of D(r) at $r = 1$, D_e (saturation or empty lumen) is therefore the value of the diffusion coefficient at the wood surface at the commencement of drying. At such time we can write

$$F_s = d\Delta p \cong \frac{D_e(p_e - p_s)}{za} \tag{23}$$

and therefore

$$d\Delta p \cong \frac{D_e p_e(1 - r_s)}{za} \tag{24}$$

Substituting for $1 - r_s$ from (21)

$$d\Delta p \cong \frac{D_e p_e (1 - r_\infty)}{a y_0} \frac{4\alpha}{2\alpha - 1} \tag{25}$$

In the limit as $y \to 0$, p_e becomes p_w and we have

$$D_e = \frac{2\alpha - 1}{4\alpha} \frac{d\, a\, \Delta p\, y_0}{(1 - r_\infty) p_w} \tag{26}$$

From our experiment, $y_0 = 0.585$ and inserting the remaining values we find for D_e the value 5.23×10^{-11}. Equation (26) gives a means for calculating the value of the diffusion coefficient D_e at saturation from the intercept y_0; the other parameters in the equation are known. Substituting from (4), (5) and (26) into (8), we have for the drying time

$$t_d = \frac{\rho_g V(\omega_i - \omega_e)}{S\, d\, \Delta p} \exp\left[\frac{2\alpha - 1}{4\alpha} \frac{d\, a \Delta p}{D_e (1 - r_\infty) p_w}\right] \tag{27}$$

or by eliminating α using (15)

$$t_d = \frac{\rho_g V(\omega_i - \omega_e)}{S\, d\, \Delta p} \exp\left[\frac{1}{2} \frac{Vd}{SD_e} \frac{\Delta p}{(1 - r_\infty) p_w}\right] \tag{28}$$

For times for different stages of drying, the y value corresponding to particular average moisture content should multiply the coefficient and the exponent in (28).

Determining the diffusion coefficient function D(r) from the drying curve

For the flux within the hygroscopic region we have

$$F = \frac{p_e}{a} D(r) \frac{dr}{d\nu} \tag{29}$$

where νa is measured in from the wood surface. Applying (14) and integrating in from the surface

$$F_s \int_0^z \frac{d\nu}{1 - \nu/\alpha} = \frac{p_e}{a} \int_{r_s}^1 D(r) dr \tag{30}$$

$$\text{or} \quad -\alpha F_s \ln(1 - z/\alpha) = \frac{p_e}{a} \int_{r_s}^1 D(r) dr \tag{31}$$

Taking the flux F_s from (9) and differentiating (31) with respect to r_s we have

$$D(r_s) = \alpha d\, a\, \Delta p \frac{d}{dr_s} \left[\frac{e^{-y/y_0}}{1 + y/y_0} \frac{1}{p_e} \ln(1 - z/\alpha)\right] \tag{32}$$

Performing the differentiation

$$\frac{D(r_s)}{\alpha d\ a\ \Delta p} = \frac{e^{-y/y_0}}{1+y/y_0}\frac{1}{p_e}\frac{d}{dz}\ln(1-z/\alpha)\frac{dz}{dy}\frac{dy}{dr_s} - \frac{e^{-y/y_0}}{1+y/y_0}\frac{1}{p_e}\ln(1-z/\alpha)\frac{dp_e}{dT_e}\frac{dT_e}{dr_s} + \frac{d}{dy}\frac{e^{-y/y_0}}{1+y/y_0}\frac{1}{p_e}\ln(1-z/\alpha)\frac{dy}{dr_s} \quad (33)$$

$\frac{dz}{dy}$ is obtained from (18), $\frac{dy}{dr_s}$ from (12), $\frac{dT_e}{dr_s}$ from (13) and

$$\frac{dp_e}{dT_e} = \frac{p_e}{T_e}\left[\frac{\beta}{T_e} - 5\right] \quad (34)$$

which is a form of the Clapeyron equation (Hunter 1991).

With these substitutions

$$\frac{D(r_s)(1-r_\infty)p_e}{\alpha d\ a\ \Delta p\ y_0} = \frac{1}{y_0}\ln\sqrt{1-(2\alpha-1)y/\alpha^2} + \frac{1}{y_0}\frac{e^{-y/y_0}}{1+y/y_0}\frac{\Delta T}{T_e}\left[\frac{\beta}{T_e}-5\right]\ln\sqrt{1-(2\alpha-1)y/\alpha^2} + \frac{2\alpha-1}{\alpha^2}\frac{1+y/y_0}{(1-(2\alpha-1)y/\alpha^2)(2+y/y_0)} \quad (35)$$

With a substitution for y_0 from (26) into the left hand side of (35) we have

$$\frac{D(r_s)}{D_e} = \frac{p_w}{p_e}\left\{\begin{array}{l} \dfrac{2(1+y/y_0)}{(1-(2\alpha-1)y/\alpha^2)(2+y/y_0)} \\ + \dfrac{4\alpha^2}{(2\alpha-1)y_0}\left[\ln\sqrt{(1-(2\alpha-1))y/\alpha^2}\right. \\ \left. + \dfrac{e^{-y/y_0}}{1+y/y_0}\dfrac{\Delta T}{T_e}\left[\dfrac{\beta}{T_e}-5\right]\ln\sqrt{1-(2\alpha-1)y/\alpha^2}\right] \end{array}\right\} \quad (36)$$

A substitution of zero for y in terms (i), (ii) and (iii) in (36) gives zero for terms (ii) and (iii). If $p_e \to p_w$, term (i) gives $D(r_s) \to D_e$ as required. In order to graph $D(r_s)$, (r_s) is given in terms of y in (12).

The function $D(r_s)$ corresponding to the above experiment is shown in Fig. 6 together with a previous determination for Scots pine (Hunter 1993). Refer also to Table 1.

Relative humidity in the hygroscopic region

The equilibrium relative humidity in the hygroscopic region enables the moisture content profile to be determined.

If the distance into an arbitrary position is xa where the relative humidity is r_x, the lower limit in (30) can be changed to give

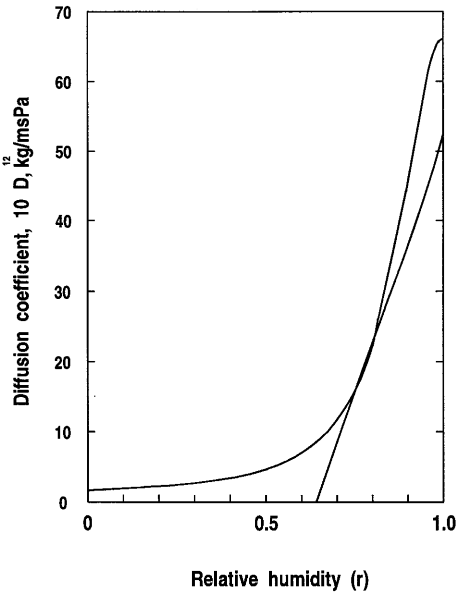

Fig. 6. The diffusion coefficient function. The curve intersecting the humidity axis is from the present work

Table 1. The diffusion coefficient function

Y	z	$\frac{e^{-y/y_0}}{1+y/y_0}$	r_s	T_e °C	p_e	p_s	$D(r)/D_e$
0	0	1	1	32.05	4775	4775	1
0.05	0.0350	0.8458	0.9183	33.74	5251	4667	0.748
0.10	0.0708	0.7198	0.8515	35.12	5670	4579	0.561
0.20	0.1449	0.5294	0.7506	37.20	6356	4443	0.297
0.30	0.2230	0.3958	0.6798	38.67	6883	4349	0.108
0.40	0.3057	0.2998	0.6289	39.72	7282	4282	−0.041
0.50	0.3939	0.2294	0.5916	40.49	7587	4232	−0.164
0.60	0.4889	0.1770	0.5638	41.06	7820	4195	−0.261
0.70	0.5927	0.1376	0.5429	41.49	8000	4167	−0.313
0.80	0.7081	0.1076	0.5270	41.89	8171	4146	−0.269
0.90	0.8404	0.0846	0.5148	42.07	8249	4130	0.016
1.0	1.0	0.0668	0.5054	42.27	8336	4117	1.096

$$-\alpha F_s \ln\frac{1 - z/\alpha}{1 - x/\alpha} = \frac{p_e}{a}\int_{r_x}^{1} D(r)dr \tag{37}$$

Eliminating F_s using (31)

$$\frac{\ln\frac{1-X/\alpha}{1-Z/\alpha}}{\ln(1 - z/\alpha)} = \frac{\int_{r_x}^{1} D(r)dr}{\int_{r_s}^{1} D(r)dr} \tag{38}$$

which is the required result.

If, for example, we take

$$D(r) \propto \frac{1}{1 + \gamma(1 - r)^2} \qquad (39)$$

which is quite realistic (Hunter 1993) we find

$$\frac{\ln \frac{1-X/\alpha}{1-Z/\alpha}}{\ln(1 - z/\alpha)} = \frac{\arctan\left[(1 - r_x)\sqrt{\gamma}\right]}{\arctan\left[(1 - r_s)\sqrt{\gamma}\right]} \qquad (40)$$

For the above experiment the equilibrium relative humidity curves are shown in Fig. 7.

Discussion

The model described arose out of the observation of the logarithmic relationship presented by the data when graphed in the form of Figs. 1 and 2. A one dimensional, constant temperature model, when examined in the light of the above data proved to be inadequate and so the geometric factor and the wet-bulb depression terms were incorporated. The resulting model shows how, for example, the drying time may be affected by changes in the humidity of the drying air. Also, graphing of the data in the forms suggested by Figs. 1 and 2 gives a direct determination of the internal and external diffusion coefficients. Reference is made to Eqs. (4), (5) and (26).

The diffusion coefficient function derived from the experiment is very similar to such a function derived directly from diffusion cup experiments although from a different wood species.

The diffusion coefficient function derived in this way is limited to the range of relative humidity greater than that of the drying air. It will be noticed also that in the vicinity of r_∞, the diffusion coefficient function exhibits some negative values. Such values are, of course, impossible and must be attributed to the sensitivity of such values to the approximations involved in the model.

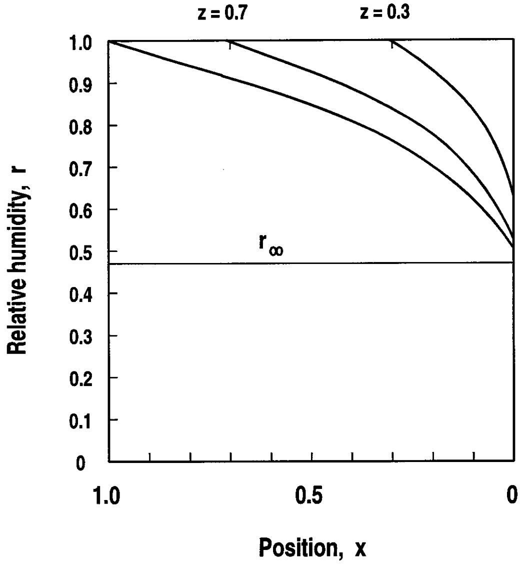

Fig. 7. Relative humidities prevailing across the hygroscopic range

It is interesting to note that (28) suggests that for given values of the other parameters, the product $d\Delta p$ could be selected to minimise the drying time. This is achieved when the exponent in (28) is equal to unity.

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