ORIGINAL

The biomechanics of bamboo: investigating the role of the nodes

**David Taylor · Billy Kinane · Ciara Sweeney ·
Darragh Sweetnam · Peter O'Reilly · Kai Duan**

Abstract The stem of the bamboo plant consists of a hollow, tubular culm with periodic nodes, characterised by an internal diaphragm and an external ridge. Bamboo is a highly anisotropic material having a strong fibre orientation, but in the vicinity of the nodes, the fibres diverge from their longitudinal orientations. Previous researchers have claimed that the nodes have a biomechanical function, preventing failure by making the tube stiffer and stronger. To investigate this claim, tensile and bending tests were carried out on material samples and intact culms, both with and without nodes, to investigate culm stiffness and strength. Crack propagation tests were also carried out to determine the effect of nodes on fracture toughness. The results suggest that far from being a point of strength, the node may be a point of weakness when loaded in tension. Material in the node has a significantly lower tensile strength; in bending tests on intact culm lengths, failure occurs when the stress on the tensile side is exactly equal to the node's tensile strength. Failure occurs by longitudinal splitting: it is proposed that this may be initiated by cracks forming in the nodes. The spacing of nodes is too large to affect the stiffness and strength of the tube as a whole and also greater than the critical crack length for brittle fracture. Thus, the diaphragm and ridge structure of the node can be explained as an attempt to reinforce a biologically essential feature which would otherwise be a point of weakness.

D. Taylor (✉) · B. Kinane · C. Sweeney · D. Sweetnam · P. O'Reilly
Trinity Centre for Bioengineering, Trinity College Dublin, Dublin, Ireland
e-mail: dtaylor@tcd.ie

K. Duan
Central Queensland University, Rockhampton, Australia

Introduction

The biomechanics of bamboo is important not only for our understanding of the living plant, but also because it is used extensively as a structural, engineering material. The bamboo plant grows as a series of hollow tubes known as culms. Mechanically, this tubular structure needs to have sufficient stiffness to keep the culm upright and sufficient strength to prevent failure. The principal type of deformation is cantilever bending, in which the lower end is fixed whilst the rest of the tube bends sideways and downwards as a result of self-weight and, especially, wind loading. This loading creates a region on one side of the tube which experiences axial tensile loading, and a corresponding region on the other side which experiences axial compression. An integral part of the structure of the culm is a series of regularly spaced nodes, characterised by an external ridge and an internal diaphragm (see Fig. 1). Previous researchers have argued that these nodes have a mechanical function, improving the stiffness and strength of the culm as a whole. For example, Shao et al. (2010) stated: "Bamboo node is crucial to improve stiffness and stability of the slender bamboo culm during growth"; Kappel et al. (2003) claimed "nodes support the culm to prevent failure due to local buckling". In conversations with botanists, we found that it appears to be an accepted fact that these nodes have evolved to improve the biomechanics of the plant. But the literature on this matter is very sparse, consisting only of these two papers (Kappel et al. 2003; Shao et al. 2010) plus a recent paper on the toughness of nodes (Wang
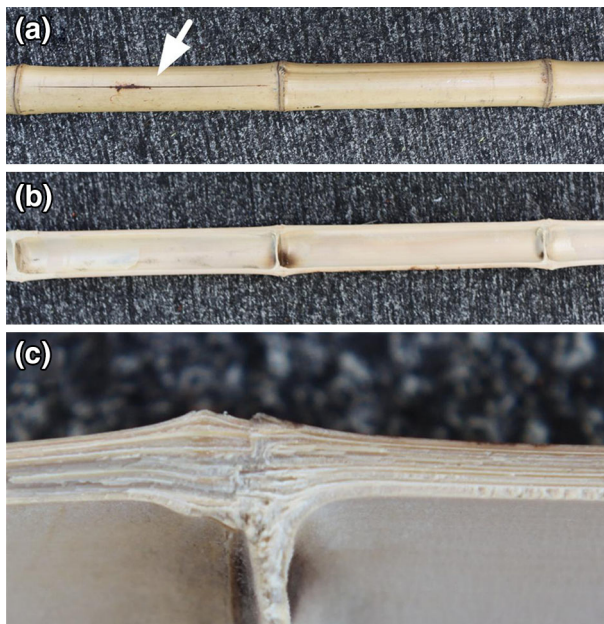
Fig. 1 **a** Bamboo culm with nodes and a longitudinal crack (*arrowed*). **b** Longitudinal section through the culm showing the internal diaphragms at the nodes. **c** Magnified view showing the external ridge and the fibrous structure near the node

et al. 2014). Further work is required to understand the roles of two different factors: (1) the material itself and (2) the geometric structure of the culm and nodes.

The aim of the present study was to investigate the mechanical role of these nodes, using experimental and theoretical methods which are already well established in the field of engineering design. Engineers frequently make use of tubular structures and so have developed considerable understanding of the ways in which tubes deform and fail. Failure can occur by one of several different modes, including fracture (in tension or compression), buckling and splitting (Wegst and Ashby 2007). We formulated the following: stiffness, strength and toughness:

a. Stiffness: do nodes reduce the amount of elastic, reversible deformation which occurs when bending forces are applied to the culm?
b. Strength: do nodes increase the force and bending moment at which the culm breaks?
c. Toughness: do nodes prevent the culm from splitting as a result of cracks which propagate along its length?

Materials and methods

Culms of Moso bamboo (*Phyllostachys pubescens*) were obtained from a local supplier, in an air-dried condition in which it is normally used for construction purposes. The mechanical properties are affected by moisture content (Chung and Yu 2002), so air-dried material will differ from fresh-living material (see below). Most researchers use material in the air-dried state, and this gives consistent results. For all the tests described below, lengths of culm were chosen which had a diameter of approximately 30 mm. To carry out tests to simulate the natural loading of culms, which approximates to a state of pure bending, 3 m lengths were cut and each was divided into two 1.5 m lengths, thus creating a pair of samples with approximately identical size and nodal spacing. One sample of each pair was tested in its original state; for the second sample, the nodes were removed before testing by mechanically planing away the external ridges and machining out the internal diaphragms. Each sample was loaded in four-point bending with an outer span of 1,000 mm and an inner span which varied from 380 to 600 mm (see Fig. 2). The inner span was adjusted so as to always have at least two nodes in the central portion of the sample, over which the applied bending moment is constant. Each sample was loaded to failure, recording the force and displacement continuously. The bending stiffness was defined as the ratio of applied force (in Newtons) to deflection (in millimetres). The stress to failure (σ_f, in units of MPa, equivalent to Newtons per square millimetre) was calculated using the following standard equation:

$$\sigma_f = \frac{Mr_o}{I} \tag{1}$$

Fig. 2 **a** Test set-up for four-point bending of lengths of culm. **b** Geometry of tensile test specimens taken from nodal and internodal regions. All dimensions in millimetres
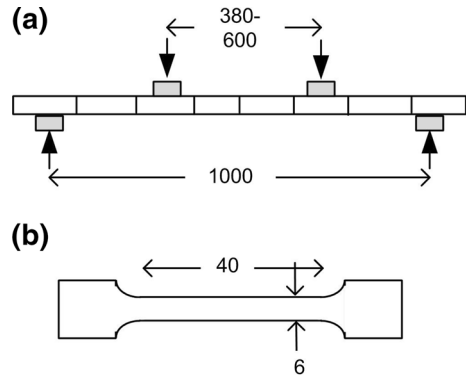
In this equation, M is the applied bending moment, r_o is the outer radius of the tube of the culm, and I is the moment of inertia of the cross section, given by:

$$I = \left(\frac{\pi}{4}\right)\left(r_o^4 - r_i^4\right) \tag{2}$$

Here, r_o and r_i are the outer and inner radii of the tube, respectively. Care was taken to avoid failure occurring at the loading points by loading through nodes and using rubber pads to spread the contact force. As a result, all samples failed within the central span. Ten pairs of samples were tested in this way.

In a second series of tests, small samples were cut from the culm walls and machined into standard specimens for tensile testing (see Fig. 2). The samples were approximately rectangular in cross section, but slightly curved owing to culm's tubular shape, and were provided with larger ends to facilitate gripping in the testing machine. In the central test section, the samples had a width of 6 mm and a thickness equal to the original culm wall thickness: approximately 4 mm. Each sample was loaded in axial tension until failure, recording the force to failure and dividing by the area of the cross section to obtain the tensile strength (i.e. the stress to failure).

A final series of tests was conducted to measure the fracture toughness of bamboo in nodal and internodal regions. Fracture toughness, K_c, is a parameter which measures the resistance of the material to crack propagation. It was clear from all these tests and from those of previous researchers (e.g. Chung and Yu 2002; Lo et al. 2004; Shao et al. 2010) that when bamboo is loaded to failure, it almost always fails by splitting along its length, whatever type of loading is applied. This is due to the fibrous structure, which makes the material much stronger in the longitudinal direction than in the transverse direction.

Bamboo culms frequently show small cracks/splits which often stop at or near nodes (see Fig. 1). We reasoned that a possible role for the nodes could be to prevent these cracks from propagating along the entire length of the culm. Figure 3 shows the test set-up and a typical result. Tensile forces were applied at two points across the diameter of a culm sample (length 100 mm) into which two notches had been machined (length 30 mm). The applied force was gradually increased until cracks propagated from these notches. Six samples were made using internode material, i.e. lengths of culm containing no nodes; in a further five samples, nodes

(a)

(b)

(c)
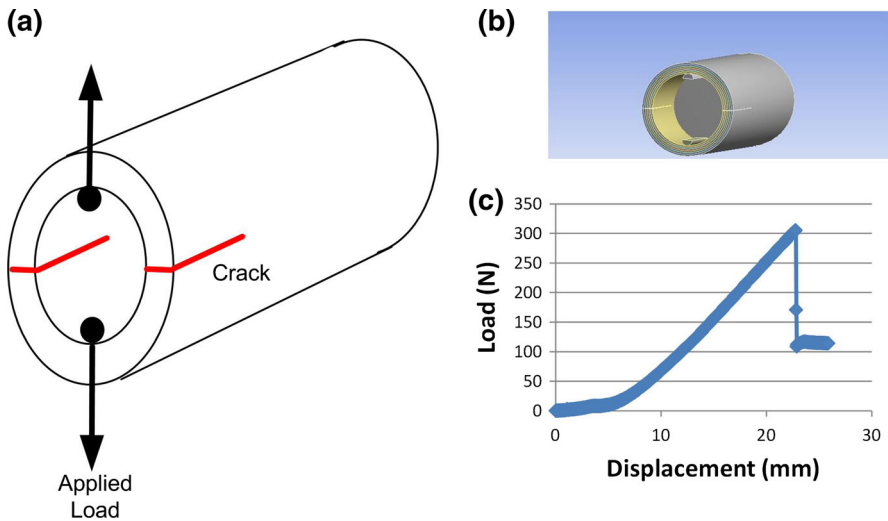
Crack

Applied
Load

Fig. 3 **a** Test set-up used to measure fracture toughness: tensile load is applied at one end of a length of culm containing two cracks, which propagate longitudinally. **b** Finite element model created to simulate this test. **c** Typical test results; the sudden drop in load occurs when the cracks propagate. From this load drop, the energy released during crack propagation can be calculated

were included midway along the sample so that the ends of the notches were located at or near the nodes.

The fracture toughness was estimated in two different ways: firstly, by measuring the energy released from the sample when the cracks propagated. This can be found from the load drop which occurs, as seen in Fig. 3. Dividing this energy by the area of new surface created by the crack gives a parameter G_c known as the critical strain energy release rate. The fracture toughness K_c can then be found from G_c and the material's Young's modulus (E) using the following equation:

$$K_c = \sqrt{G_c E} \tag{3}$$

Secondly, K_c can be found from the force needed to cause crack extension, which is the maximum force on the graph shown in Fig. 3. For this approach, it was necessary to make a computer model of the test specimen, using ANSYS finite element software (see Fig. 3). From this model, information about the stresses close to the tip of the crack as a function of the applied force was obtained. Using a standard result from fracture mechanics theory (Janssen et al. 2002), K_c was calculated using the following equation:

$$K_c = \sigma \sqrt{2\pi x} \tag{4}$$

Here, σ is the tensile stress at a distance x from the crack tip, at the time when crack propagation occurs. A complication in making this model which does not normally arise with engineering materials is that the material properties of bamboo vary within the culm wall: the density of fibres decreases from outside to inside, causing a drop in the elastic modulus (Low et al. 2006). This was taken into account in this

Fig. 4 Results from the four-
point bending tests of culms in
which the nodes were left intact,
or extracted. There was no
significant difference in the
bending stiffness (*upper graph*
p = 0.48); however, there was a
significant difference in the
bending strength (*lower graph*
p = 0.03)

Bending Stiffness (N/mm)

■ p=0.48

Bending Strength (MPa)

■ p=0.03
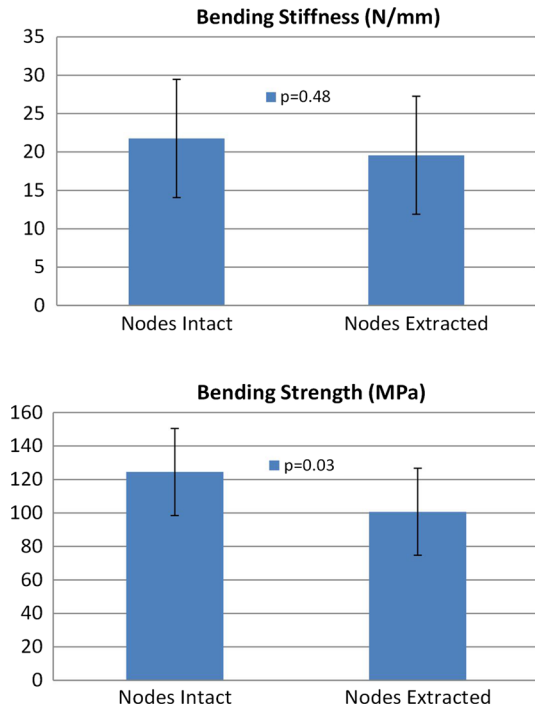
computer model by dividing the thickness into six concentric layers, each with a different Young's modulus. The modulus values were estimated using data from Nogata and Takahashi, who showed that a standard composite material model could be applied to predict the Young's modulus of bamboo (Nogata and Takahashi 1995). They successfully predicted the longitudinal stiffness of the material; their approach was adapted to predict the transverse stiffness using test data from another study (Torres et al. 2007). We validated our model by comparing its predictions of sample deflection with those measured experimentally, making slight changes to the Young's modulus values until a perfect prediction was achieved. The average Young's modulus—1.85 GPa—was used in Eq. 3 above.

Results

Figure 4 shows the results of the bending tests conducted on lengths of intact culm and paired samples from which the nodal ridges and diaphragms had been removed. Paired Student's *t* tests were used to determine statistical significance. Removing the nodes caused a small decrease in stiffness from 21.8 N/mm (SD = 7.5) to 19.6 N/mm (SD = 7.9). This 10 % drop in the mean stiffness was not statistically significant ($p = 0.48$). On the other hand, removing nodes caused a 19 % decrease in mean strength (from 124.4 MPa (SD = 23.1) to 100.7 MPa (SD = 29.0) and this difference was statistically significant ($p = 0.033$).

Figure 5 shows the results of the tensile tests conducted on small specimens cut from the culm wall. For comparison, the results of a similar study (Shao et al. 2010) are also included. The average tensile strength of material at locations remote from nodes was 158 MPa (SD = 17.1), very similar to the value of 154 MPa (SD = 13.9) measured by Shao et al. Test samples which were cut in such a way as to include a node always failed at the node, with a reduced strength of 101 MPa (SD = 18.2), which agrees with a value of 103 MPa (SD = 15.8) found by Shao et al. In these tests, the specimens were planed into regular, flat, rectangular shapes. Interestingly, Shao et al. also carried out tests in which the outer surfaces of the samples were not planed, thus retaining the ridge feature at the node. As Fig. 5 shows, these unplaned samples had a strength of 126 MPa (SD = 20.0), higher than the planed nodal samples but lower than the internodal material.

Fig. 5 Results from tensile tests and fracture toughness tests, measuring the tensile strength using specimens containing no nodes (internode material *upper graph*) and specimens containing nodes (*middle graph*). The *lower graph* shows the fracture toughness measured for cracks in internode material and cracks placed close to nodes. Also shown are tensile test data from Shao et al. (2010)
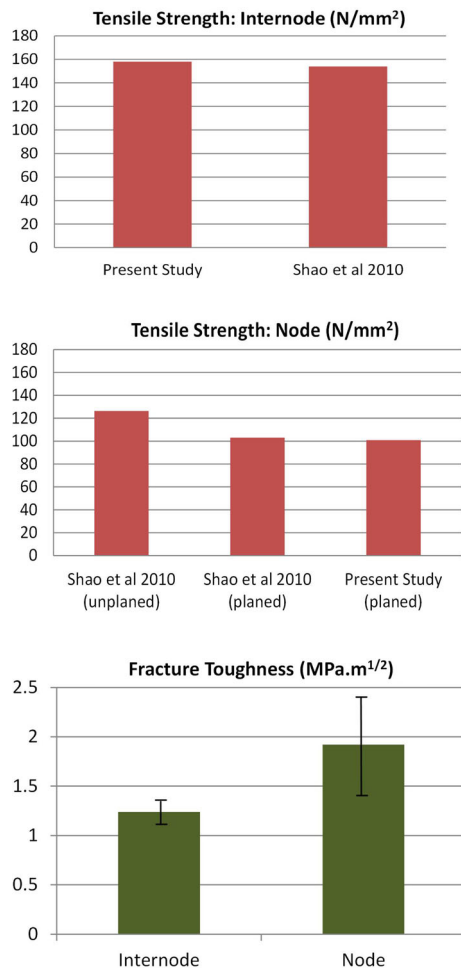
Figure 5 also shows the results of the fracture toughness tests. As noted above, two different methods were used to estimate K_c. The results shown in the figure are from the strain energy method (Eq. 3); the presence of the node increased the mean toughness considerably, by a factor of 1.55, from 1.24 MPa.m$^{1/2}$ (SD = 0.13) to 1.92 MPa.m$^{1/2}$ (SD = 0.53). This difference was statistically significant ($p = 0.013$). It was found that the second method for estimating K_c, using the computer model, returned slightly higher values of 1.62 and 2.79, respectively, giving a toughness increase at the node by a factor of 1.72. These results are in broad agreement with those in a recent paper (Wang et al. 2014) where an increase by a factor of 1.87 was recorded using a different technique.

Discussion

The work described above is, to the best of the authors' knowledge, the most comprehensive study yet undertaken on the role of nodes in the bamboo culm. In what follows the significance of the results is discussed in terms of the effect on stiffness, strength and toughness. One limitation of the study should be mentioned: culms were used which had been air-dried. Living bamboo has a significantly higher water content which can be expected to alter the mechanical properties of the material itself. Though many previous studies have been carried out on the mechanical properties of bamboo, almost all of these used dry bamboo, being focussed on its use as a structural engineering material. One previous study was found which tested freshly cut bamboo of the same species: Chung and Yu (2002) found that drying increased the strength slightly, by about 14 %, and also increased the stiffness, by about 26 %. We had the opportunity to carry out toughness tests on a few freshly cut samples of tropical bamboo obtained from Dublin's botanic gardens. This bamboo was of a different species and so cannot be directly compared with the authors' other results; the toughness was found to be 0.73 MPa.m$^{1/2}$, which is lower than the values given above for internodal material (1.24 MPa.m$^{1/2}$). These results suggest that whilst all measured properties are somewhat lower in fresh material, the changes are not too great and the relative differences between properties remain more or less the same, implying that our conclusions should also be valid for living bamboo.
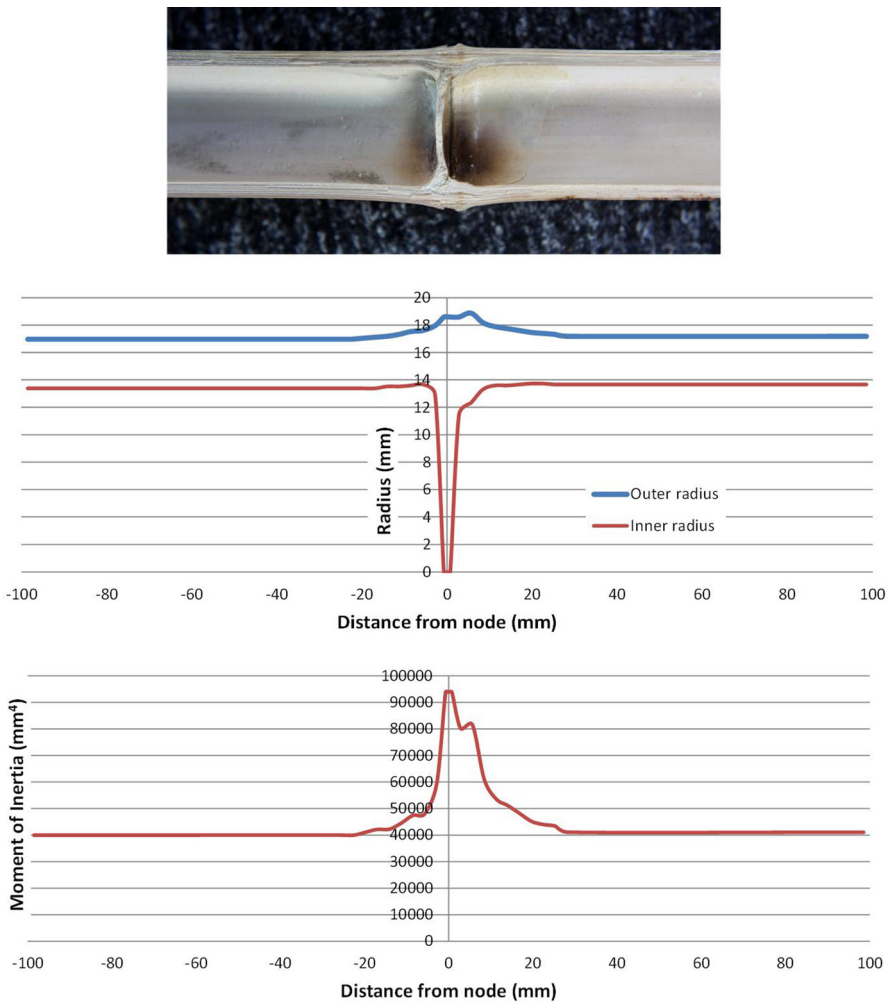
Stiffness

The ability to resist deformation under applied forces is important for any plant with an upright growth habit. As the stem becomes longer, forces due to its own weight may cause it to become permanently bent, reducing access to light in competition with surrounding plants. Larger forces potentially arise during wind loading, but being temporary in nature, the deformation induced is perhaps not so significant, provided this deformation is entirely elastic and therefore recoverable. Indeed some flexibility may even prevent stem failure.

In this work, it was found that nodes had little if any effect on the bending stiffness; the average stiffness increased by only 10 %, and this was not statistically

significant. However, some scatter would have arisen in these data due to variation in shape and size from sample to sample, even though it was attempted to minimise this by using paired samples from the same culm. In any case, this experimental result can be compared with a simple theoretical analysis, as follows. The bending stiffness of a tube is determined by the moment of inertia, I, which was defined above (Eq. 2). Figure 6 shows a longitudinal section through a culm, including a node, along with measurements of the inner and outer radii and the moment of inertia at different points along its length. As expected, the moment of inertia increases near the node, but actually the increase is quite modest because much of the nodal material lies close to the middle of the tube and so does not contribute

Fig. 6 Measurements taken from a longitudinal section of culm including a node. The *upper graph* shows the outer and inner radii, from which can be calculated the moment of inertia, *I*, shown in the *lower graph*

very much to bending stiffness. When bending forces are applied, the higher moment of inertia of the node will mean that it will bend less, but this will not affect the amount of bending in the rest of the tube, because the deformation in each part of the tube only depends on the local moment of inertia. To find the bending stiffness of a length of culm containing several nodes, the average moment of inertia was used. The result in this case is that removing the nodes only reduces the bending stiffness by 6.2 %. This prediction is comparable with experimental results, confirming that the nodes have only a small effect on the stiffness of the culm as a whole. In order to have a significant effect, the nodes would have to be much more bulky or would have to be spaced much more closely together.

Strength

The amount of force or stress needed to cause failure is a more difficult property to assess, because failure can occur in several different ways. Wegst and Ashby ([2007]) discussed this matter in considerable detail in an excellent paper which develops the theory necessary to predict the strength of tubes made from anisotropic materials, including bamboo. Essentially, there are three possible modes of failure: transverse fracture, buckling and longitudinal splitting.

Transverse fracture of a bending tube occurs when the maximum stress reaches the tensile strength of the material, or its compressive strength, whichever is the lower, since during bending both tensile and compressive stresses occur. A node would be able to improve strength by reducing the stress for a given loading: the moment of inertia parameter I is again relevant here. However, a node can only affect the stress in its immediate vicinity, so the material remote from the nodes would still experience the same stresses. So, it can be concluded that a node cannot improve strength if failure occurs by transverse fracture.

Tubes with relatively thin walls fail by a process known as "ovalisation buckling"; bending causes the initially circular cross section to become oval in shape, reducing its moment of inertia and increasing local stresses. This leads to a run-away situation in which the deformation becomes unstable and the tube suddenly buckles. A node can be very effective in resisting this type of failure, largely due to its internal diaphragm which prevents ovalisation. However, this will only work if the nodes are sufficiently close together to prevent significant ovalisation occurring in the gaps between them. The relevant parameter here is Ω, which is given by:

$$\Omega = \left(\frac{E_{\text{long}}}{E_{\text{trans}}}\right)^{1/4} \left(\frac{(r_o - r_i)L^2}{r_o^3}\right)^{1/2} \tag{5}$$

Here, L is the distance between nodes. In order to prevent ovalisation, the value of Ω must be less than 0.5 (Wegst and Ashby [2007]). For the bamboo culms tested in this study, an average value of 8.1 was found. This indicates that the nodal spacing is too large to affect failure by buckling.

Nevertheless, these bending test results showed a significantly higher strength (by 19 %) for culms with nodes compared with those in which the nodes had been

removed. Some insight into this apparent anomaly can be obtained by comparing the bending test results (Fig. 4) with those from small material samples (Fig. 5). These latter tests showed that when the geometrical features (the ridge and diaphragm) were removed, material in the node had a strength of only 100.1 MPa, lower than that of internode material. When the ridge was left in place, the nodal material had a strength of 126 MPa (Shao et al. 2010). These two values are almost identical to the failure strengths of the culms which were tested in bending (Fig. 4): 124.4 MPa (with nodes intact) and 100.7 MPa (with nodes planed flat). All of these values are less than the measured strength of internode material (158 MPa). What these results imply is that in the bending tests, failure always begins in the region of the node, whether or not the ridge and diaphragm are planed off. If the nodes were truly protecting the culm, then failure would occur elsewhere, in the internodal region, at a higher stress of the order of 158 MPa.

In these bending tests, the culms tended to fail suddenly, almost always by longitudinal splitting, and it was difficult to determine whether the failures initiated at nodes or not. However, evidence that failure does start at the nodes was provided by Shao et al. (2010). Figure 7 is a reproduction of one of their figures, showing a splitting failure starting from a node, in a specimen in which the nodal ridge had been retained.

Thus, it appears that the node, rather than being a source of strength, is a potential source of weakness when loaded in tension. This probably occurs as a result of changes in the orientation of fibres in the vicinity of the node, which deviate from their longitudinal orientations (see Fig. 1). Under bending loads, failure occurs first in the node, in the form of local splitting, creating small cracks which subsequently run along the length of the culm from node to node.

The above-proposed mechanism of failure also helps to explain an anomaly in the work of Wegst and Ashby (2007). Their theoretical model predicted that failure would always occur by transverse fracture and not by splitting or ovalisation buckling. The bending moment required for splitting was predicted to be much too large, but this is contradicted by the experimental results which always show longitudinal splitting. Our proposal to resolve this anomaly is to note that the Wegst and Ashby theory did not allow for the presence of nodes. It is proposed that tensile stress on one side of the node (due to bending in the culm as a whole) causes local failure of material inside the node, initiating the whole failure process.

Fig. 7 Photograph (taken from Shao et al. 2010: figure 4) showing part of a tensile test specimen containing a node. After application of tensile force, cracking has started in the nodal region and is spreading longitudinally. Reproduced with permission from the Forest Research Institute, Malaysia

Toughness

Several previous workers have measured the fracture toughness of bamboo, obtaining a wide range of different values. These differences are partly due to the anisotropy of the material: for example, Amada and Untao (2001) attempted to measure the toughness for cracks propagating in the transverse direction and obtained very high values in the range 20–100 MPa.m$^{1/2}$, whilst Tan et al. (2011), who propagated cracks in the longitudinal direction, deduced a value for G_c of 1700 J/m^2 which leads to a fracture toughness value of 1.7 MPa.m$^{1/2}$ (using Eq. 3), slightly higher than the result of the current study for internode material. Shao et al. (2009) obtained a slightly lower value of 0.78 MPa.m$^{1/2}$. Mitch et al. (2010) obtained a much lower value of 0.18 MPa.m$^{1/2}$; however, this result is invalid owing to an incorrect stress analysis, which underestimated the applied stress intensity by about a factor of 10.

It was hypothesised that nodes may act to make crack propagation more difficult. It was found that this is indeed the case: the fracture toughness parameter K_c was measured in two different ways, and it was found that nodes increased K_c by factors of 1.55 and 1.72, respectively; Wang et al. (2014) recently obtained comparable results. As a result, splits may tend to stop at nodes rather than running the whole length of the culm. This seems promising but when we attempted to quantify the effect, a problem arose. The relevant equation linking the stress needed to propagate the crack (σ_p) with the length of the crack (c, measured from tip to tip) is:

$$\sigma_p = \frac{K_c}{\sqrt{\pi c/2}} \tag{6}$$

The transverse strength of the material is approximately 25 MPa. This is the stress needed to cause a split to occur. Using this stress value and the measured value of K_c at the node in the above equation gave a crack length of 3.8 mm; this is much less than the typical distance between nodes, which is 100–200 mm. This means that by the time the crack has run the full distance from one node to the next, it will be so long that it will easily pass through the node. Provided there is enough stress to cause a crack to form, the crack will be able to propagate without being arrested at the node. This analysis assumes that the necessary stress is being provided by some form of loading on the entire culm, e.g. self-weight or wind loading. Nodes may be effective in limiting crack growth caused by local loading, such as an impact event or damage by an animal.

Nodes are biologically necessary: they act as branch points for the thin stems which carry the leaves. At these branch points, a potential weakness occurs because the fibres which reinforce the material are required to deviate from their longitudinal orientations. Thus, the features of the node—its external ridge and internal diaphragm—can be seen as attempts to strengthen a potentially weak region in the culm, attempts which are only partially successful.

Conclusion

This work has investigated the mechanical role of the nodes in the bamboo culm. Conducting three separate analyses and using both experimental methods and theoretical modelling, it was attempted to determine the effect of the nodes on culm stiffness, strength and toughness. It was concluded that rather than being a point of strength, the node may be a point of weakness. Nodes are certainly necessary features because they provide the branch points for the leaves of the plant. The results of the present work suggest that the morphological features of the node—the internal diaphragm and external thickening of the culm—have evolved as an attempt to avoid failure in the vicinity of the branch. Evidence was provided to show that failure may occur preferentially from nodes, and analysis to demonstrate that nodes would have to be much more closely spaced to have a significant effect on the mechanical performance of the culm as a whole.

References

Amada S, Untao S (2001) Fracture properties of bamboo. Compos Part B Eng 32:451–459

Chung KF, Yu WK (2002) Mechanical properties of structural bamboo for bamboo scaffoldings. Eng Struct 24:429–442

Janssen M, Zuidema J, Wanhill R (2002) Fracture mechanics. Spon Press, London

Kappel R, Mattheck C, Bethge K, Tesari I (2003) Bamboo as a composite structure and its mechanical failure behaviour. In: Collins M, Brebbia CA (eds) Design and nature II: comparing design in nature with science and engineering. WIT Press, Southampton, pp 285–293

Lo TY, Cui HZ, Leung HC (2004) The effect of fiber density on strength capacity of bamboo. Mater Lett 58:2595–2598

Low IM, Che ZY, Latella BA (2006) Mapping the structure, composition and mechanical properties of bamboo. J Mater Res 21:1969–1976

Mitch D, Harries KA, Sharma B (2010) Characterization of splitting behavior of bamboo culms. J Mater Civ Eng 22:1195–1199

Nogata F, Takahashi H (1995) Intelligent functionally graded material: bamboo. Compos Eng 5:743–751

Shao ZP, Fang CH, Tian GL (2009) Mode I interlaminar fracture property of moso bamboo (*Phyllostachys pubescens*). Wood Sci Technol 43:527–536

Shao ZP, Zhou L, Liu YM, Wu ZM, Arnaud C (2010) Differences in structure and strength between internode and node sections of moso bamboo. J Trop For Sci 22:133–138

Tan T, Rahbar N, Allameh SM, Kwofie S, Dissmore D, Ghavami K, Soboyejo WO (2011) Mechanical properties of functionally graded hierarchical bamboo structures. Acta Biomater 7:3796–3803

Torres LA, Ghavami K, Garcia JJ (2007) A transversely isotropic law for the determination of the circumferential Young's modulus of bamboo with diametric compression tests. Lat Am Appl Res 37:255–260

Wang F, Shao Z, Wu Y, Wu D (2014) The toughness contribution of bamboo node to the mode I interlaminar fracture toughness of bamboo. Wood Sci Technol 48(6):1257–1268

Wegst UGK, Ashby MF (2007) The structural efficiency of orthotropic stalks, stems and tubes. J Mater Sci 42:9005–9014