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Originals

Comparison between modulus of elasticity values calculated using 3 and 4 point bending tests on wooden samples

L. Brancheriau, H. Bailleres, D. Guitard

Abstract Several data banks on wooden properties of different species contain mechanical characteristics of which the bending modulus of elasticity. This modulus can be calculated using different test methods, the more ordinary used are the 3 point and 4 point bending tests. The values obtained by one method cannot be directly compared with those of other methods. So the bending properties read in a data bank have to be converted before using them and correctly compared with other data from different references. The aim of this study is to make an analytic formula of a crossing coefficient between 3 point and 4 point bending concerning the longitudinal modulus of elasticity measured following the French standards (NF 1942; NF 1987). This formula includes a study of the shear force influence, and a study of supports and loading head indentation effect, in a 3 point bending test. The analytical study and the experiences have shown that the supports and loading head indentation effect are not negligible but have the same influence as the shear effect. The indentation is the result of the competition between two physical phenomena which are the wood stiffness and the load level applied on the piece of wood during a bending test. The practical result of this study is the development of a crossing analytic formula from a 3 point bending modulus of elasticity to a 4 point bending one, verified by the experimentation.

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L. Brancheriau (✉)
CIRAD-forêt – programme Bois 73,
rue J. F. Breton, 34032 Montpellier Cedex 01, France
e-mail: brancheriau@cirad.fr

H. Bailleres
e-mail: henri.bailleres@cirad.fr

D. Guitard
LEPT-ENSAM UMR 8508,
Esplanade des arts et métiers,
33405 Talence Cedex, France

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Notations

Physical properties

e	width
h	height
l	length between the two supports
L	span
d	density at a moisture content of 12%

Bending tests

P	applied load
f	deflection at midspan
F	deflection measurement
a	distance between the load point
m	central gauge length
R	radius

Mechanical properties

E_{L3}	3 point bending modulus of elasticity
E_{L3A}	apparent modulus of elasticity in 3 point bending
E_{L4}	4 point bending modulus of elasticity
E_{L4C}	3 point bending modulus of elasticity converted in a 4 point bending one
E_T	tangential modulus of elasticity
σ_{TL}	tangential shear strain
σ_{RL}	radial shear strain
G_{TL}	tangential shear modulus
ν_{LT}	tangential Poisson's ratio
ν_{LR}	radial Poisson's ratio
t	indentation deflection
K	indentation coefficient
K'	shear coefficient

Introduction

Since the beginning of their activity, first the wood study laboratories of the Centre Technique Forestier Tropical (C.T.F.T.), then the Centre de Coopération Internationale en Recherche Agronomique pour le Développement (C.I.R.A.D.), have determined the technological properties of tropical species by doing hundreds of thousands of tests. More than 40,000 physical and mechanical tests have been done to this date, enabling the characterisation of about 1,100 tropical wood species. Nowadays theses results are set up in a data bank. It constitutes both a collective memory and an "information tank", but one of its most important uses is to be the study base of the links between wood properties and forest products use.

Other data banks on wooden properties of different species exist and are sold as computer software, atlas or technical guidebooks. They propose in particular values concerning mechanical characteristics of which the bending modulus of elasticity. This modulus can be obtained by different test methods, the more commonly used are the 3 point and 4 point bending tests. For all data banks, a problem appears when the modulus of elasticity values have to be used. Indeed, the values calculated using a method cannot be directly compared with those obtained by the other method. So the bending properties read in a data bank have to be converted before using and correctly comparing them with the other data from different references.

For example, the calculation of the CIRAD-Forêt wood data bank modulus of elasticity, with small specimens of clear wood, is made using 3 point bending tests in agreement with the old French standard (NF 1942). Knowing the French present-day standards (NF 1987; EN 1995) it is necessary to be able to convert these old values into 4 point bending values, in order to be correctly compared to data based on the new standards. Contrary to 3 point bending, a 4 point loading does not induce a shear effect, moreover the indentation of the supports and the loading head doesn't influence the deflection measurement. In this article, we propose to study a crossing analytic formula from 3 point bending modulus of elasticity to the one obtained in 4 point bending according to the French standards (NF 1942; NF 1987).

Theoretical approach

Calculation methods

In 3 point bending, as in 4 point bending, the modulus of elasticity is determined using the classical equations of strength of materials applied to straight beams (Figs. 1, 2, 3 and 4).

(a) For 3 point bending, the modulus of elasticity is:

$$E_{L3} = \frac{l^3}{4eh^3}k \quad \text{with } k = \frac{|\Delta P|}{|\Delta f|} \tag{1}$$

The deflection (f) is measured by the machine cross-head moving which sets the load (Fig. 2).

(b) For 4 point bending, the modulus of elasticity is:

$$E_{L4} = \frac{3(l-a)\mathrm{m}^2}{8eh^3}k \quad \text{with } k = \frac{|\Delta P|}{|\Delta f|} \tag{2}$$

The deflection (f) is measured independently of the applied load system (Fig. 4).

The 3 point bending test systematically underestimates the calculated value (Sales 1977; Perstorper 1994). This phenomenon is due to the fact that the shear effect and the indentation effect of the loading head and the supports are

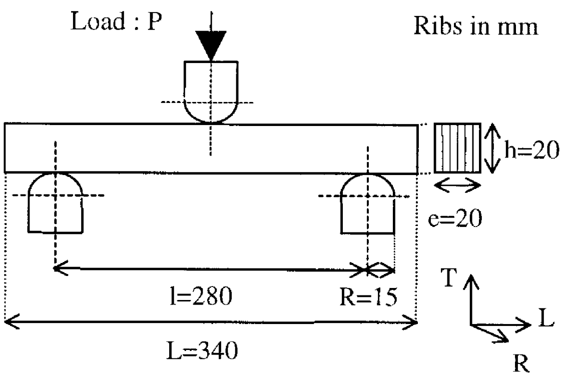

Fig. 1. Three points static bending test, French standard NF B 51-008 (1942)

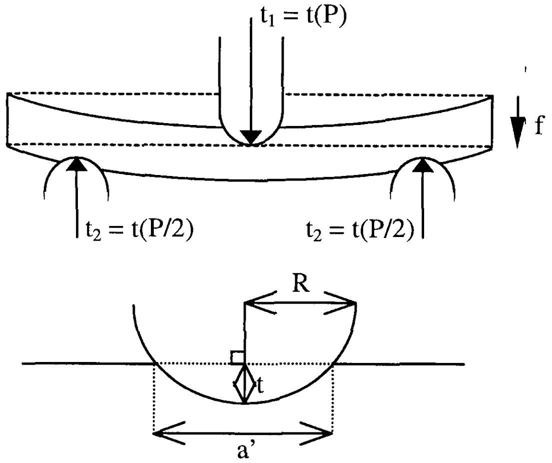

Fig. 2. Geometrical description of the indentation effect

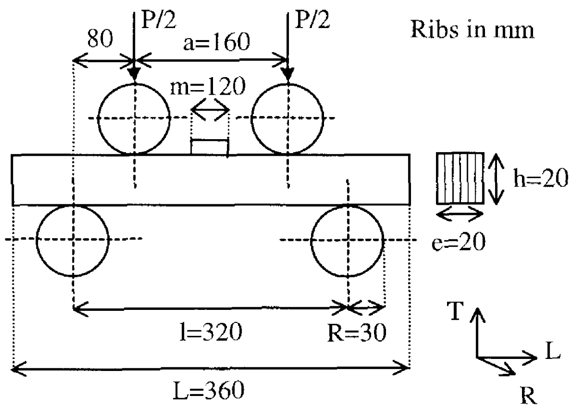

Fig. 3. Four points static bending test, French standards NF B 51-008 and NF B 51-016 (1987)

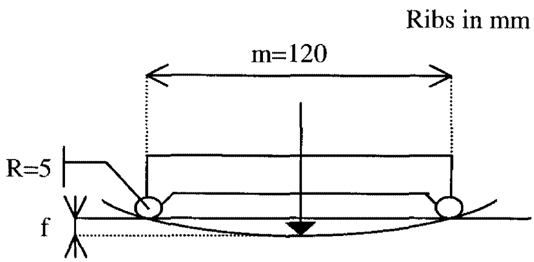

Fig. 4. Curvature measurement

neglected. The 4 point bending test gives the best evaluation of the elastic modulus because the beams theory hypothesis are valid at the place of measurement.

(c) The wood building American standards (ASTM 1995) take into account the shear effect by using the following equation:

$$\frac{1}{EL_{3A}} = \frac{1}{E_{L3}} + \frac{1}{K'G_{TL}} \frac{h^2}{l^2} \qquad (3)$$

By referring to the Eq. 1, the apparent modulus of elasticity E_{L3A} is:

$$EL_{3A} = \frac{Pl^3}{4eh^3 f}$$

K′ is the shear coefficient. It is defined as the ratio of average shear strain on a section to shear strain at the centroid. For a rectangular section, the value of K' is given according to Poisson's ratio (isotropic material) by:

$$K' = \frac{10(1+\nu)}{12+11\nu} \quad (4)$$

$\nu \in [0.05; 0.5] \Rightarrow K' \in [0.84; 0.86]$ and $1/K' \in [1.17; 1.20]$

The modulus E_{L3} is a function of the specimen ratios l/h and E_L/G_{TL}. The anisotropy of wood is taken into account by the shear modulus. The standardisation sets l/h = 0.071 (NF 1942). The average value of E_L/G_{TL} is evaluated at 17 (Kollman and Côté 1968; Guitard 1987). Under these conditions, the underestimation of the elastic modulus, induced by the shear stress in a 3 point bending test, is about 9.5% (Kollman and Côté 1968). This is due to the ratio E_L/G_{TL} particularly high for standardised wooden specimens, in comparison with an isotropic elastic material.

(d) Guerrin (1990) took into account the shear effect in a mechanics of solids calculation using results of mechanics of materials. The analytic solution proposed on the neutral axis (Eq. 5) can be written as the following formula:

$$E_{L3} = E_{L3A}\left[1 + \frac{E_{L3}}{G_{TL}}\frac{h^2}{l^2}\left(\frac{1+G_{TL}}{E_{L3}}\left(\frac{\nu_{LT}}{2} - \nu_{LR}\frac{e^2}{h^2}\right)\right)\right] \quad (5)$$

By referring to the Eq. 3, K′ is equal to:

$$K' = \left(1 - \frac{\nu}{2}\frac{G_{TL}}{E_{L3}}\right)^{-1} \quad (6)$$

$\nu \in [0.05; 0.5] \Rightarrow K' \in [1.00; 1.01]$ and $1/K' \in [0.99; 1.00]$

The Eq. 5 leads to an underestimation of about 8%, Which is comparable to 9.5% found with the American standards, taking into account the uncertainty of the parameters value into the calculation and the uncertainty of the elastic modulus determination with a 3 point loading.

(e) Lekhnitskii (1963) and Laroze (1988) have developed theoretical models with tridimensional states of stress ($\sigma_{RL} \neq 0$). It's showed that σ_{TL} and σ_{RL} can take the following forms:

$$\sigma_{TL} = \frac{\partial\varphi}{\partial z} + \frac{Ty}{2I_z}\left(az^2 - y^2\right) \text{ et } \sigma_{RL} = -\frac{\partial\varphi}{\partial y} \quad (7)$$

with 'a' constant.

φ(y,z) is a harmonic function. It's written in a Fourier's series form (Lekhnitskii 1963; Laroze 1988). This approach, although mathematically correct, is relatively complex to use in a practical way.

(f) Phang (1979) has taken into account the shear effect by several analysis methods. In particular he has developed an exact solution of a linear elasticity problem, in a Fourier's series form, which agrees with the models of Lekhnitskii (1963) and Laroze (1988). Phang also used the mixed variational principles of the linear elasticity introduced by Verchery in 1973, making the hypothesis of a pure bending mixed field with 3 and 7 terms. The difference between the 7 terms mixed analysis and the 3 terms mixed analysis is relatively low (about 1.6% for an orthotropic beam glass-resin). For a 3 terms analysis, we get the following relation:

$$E_{L3} = E_{L3A}\left[1 + \frac{E_{L3}}{G_{TL}}\frac{h^2}{l^2}\left(1.2 - 0.9\nu_{LT}\frac{G_{TL}}{E_{L3}}\right)\right] \tag{8}$$

$$\frac{1}{K'} = 1.2 - 0.9\nu\frac{G_{TL}}{E_{L3}} \tag{9}$$

$\nu \in [0.05;\ 0.5] \Rightarrow K' \in [0.84;\ 0.85]$ and $1/K' \in [1.17;\ 1.20]$
The shear coefficient values are similar to those calculated with Eq. 4. Phang's model is more accurate than the Guitard one. However the latter reaches a satisfactory result with a simple calculation of linear elasticity. The relative error of the elastic modulus value caused by the Eq. 5 is about 1.5% referring to the American standardised model, Which remains very low in relation to the error made neglecting the shear effect in 3 point bending.

By including the shear effect and the indentation effect in the calculation of the elastic modulus for a 3 point bending test, it is possible to evaluate the 4 point bending modulus of elasticity in an analytic way. In the following of this article, the study of the shear effect is based on an approach realised by Guitard in 1996. An application of the normal contact Hertz's theory allows us to take into account the indentation effect (François, Pineau and Zaoui 1993).

Analytical study

On the neutral axis, the analytic solution chosen (Eq. 5) can be written as:

$$E_{L3} = \frac{\nu_{LT}\frac{h^2}{2} - \nu_{LR}e^2 + l^2}{\frac{4eh^3f}{lP} - \frac{h^2}{G_{TL}}} \tag{10}$$

Neglect of Poisson's terms in Eq. 10 induces an absolute theoretical relative error on the modulus of elasticity calculation lower than 0.2%. The interval of relative errors is indeed the following:

$$\nu \in [0.05; 0.5] \Rightarrow \frac{|\Delta E_L|}{E_L} \in [0.013\%; 0.13\%]$$

So the Poisson's terms are neglected in the following of this article. The supports and the loading head used in a 3 point bending test cause three local deformations (Fig. 2). The link between the absolute deflection measurement (F) and the absolute deflection of the theoretical calculation (f) is given by the equation:

$$F = f + t_1 + t_2 \tag{11}$$

To value the deflection induced by the supports and the loading head we used the Hertz's theory of normal contact (1881), and also results of the mechanic of contact applied to a normal concentrated linear load (François, Pineau and Zaoui 1993). For a support or a loading head with a cylindrical shape, the indentation effect is characterised by the following relation between the indentation deflection t (mm) and the applied load P (N), keeping into the elastic behaviour of the material.

$$t = KP^{\frac{2}{3}} \tag{12}$$

K is called indentation coefficient. Its inverse 1/K is the stiffness of wood, making an analogy with the stiffness of a spring. The coefficient K can be evaluated analytically by the formula:

$$K = 0.515\left(\frac{h}{e\sqrt{RE_T}}\right)^{2/3}\left(mm.N^{3/2}\right) \tag{13}$$

Using Eqs. 10, 11 and 12 we get the following analytic expression of the crossing coefficient between 3 point and 4 point bending:

$$E_{L4} = \frac{1}{1 - 1.906K\left(E_{L3}\frac{eh^3}{l^3}\right)^{2/3} - \frac{E_{L3}}{G_{TL}}\frac{h^2}{l^2}}E_{L3} \tag{14}$$

The value of the load (P), used to determinate the indentation deflection (t) in Eq. 12, is deduced from the 3 point modulus of elasticity by setting the absolute deflection (f) in Eq. 1 at 10 mm. Eq. 14 allows to value the under-estimation at about 12% when the indentation effect is neglected in a 3 point bending test. The average values of different parameters are estimated at:

$$\left.\begin{array}{l} E_L = 18108\,\mathrm{N/mm^2} \\ G_{TL} = 1065\,\mathrm{N/mm^2} \\ E_L/G_{TL} = 17 \\ E_T = 890\,\mathrm{N/mm^2}(\mathrm{Eq.13}) \Rightarrow K = 0.0023\,\mathrm{mm/N^{3/2}} \end{array}\right\} \frac{|\Delta E_L|}{E_L} = 12\%$$

This effect, not negligible for wooden specimens, is caused by the anisotropy of the study material, and especially by the high suppleness to the compression in the transversal way. The ratio E_L/E_T is indeed particularly high for wood, but it's unitary for an isotropic material. Eq. 14, obtained by the theoretical approach, has to be verified by experiences to check the rates of the shear effect and also the indentation effect in a 3 point bending test.

Materials and methods

Specimens

Six species of wood have been chosen within a wide range of density (Table 1). For each 6 species, 5 groups of 2 match test specimens have been made, except the Ferreol. Only 2 groups of this species were made because of a lack of raw material. So the total number of clear wood specimens was 54. The specimens have been taken in such way that the growth rings got a negligible bend and were parallel to the wood tangential direction.

Table 1. List of the chosen species, load threshold 'LT' and load-unload cycle upper limit value 'ULV'

Common name	Scientific name	Density d (g/cm^3)	Breaking load (N)	LT (N)	ULV (N)
Obeche	*Triplochiton scleroxylon K.S.*	0.35	990	100	50
Okoumé	*Aucoumea klaineana Pierre*	0.50	1619	160	80
Kotibé	*Nesogordonia papaverifera R.C.*	0.70	2267	230	115
Amarante	*Peltogyne venosa Benth.s.venosa*	0.90	3409	340	170
Jatoba	*Hymenaea parvifolia Huber*	1.13	3848	385	193
Ferréol	*Swartzia panacoco Cowan*	1.26	4781	480	240

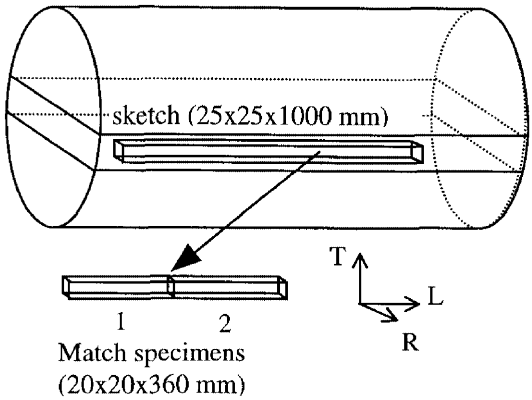

Fig. 5. Sawing plan of the twin samples

The twin specimens were taken on the same wood grain, as a sketch of 25 × 25 × 1000 mm, to get the same mechanical properties (Fig. 5). After the conditioning stage, at 65% ± 5% of air relative humidity and 20 °C ± 2 °C, the specimens were put to test size of 20 × 20 × 360 mm.

Test procedure

Tests were made to compare the modulus of elasticity values calculated using 3 point and 4 point bending. Each specimen is tested in 3 point bending, then in 4 point bending. The tests are done at a low level of load so the elastic behaviour of wooden specimens is not modified. The difference between the 3 point and 4 point modulus of elasticity is only due to the difference between the two test methods. A specific indentation test is realised then to value the indentation effect. The specimens moisture content is finally determined to check if these are well conditioned at the standardised moisture content of 12% ± 2%.

Static bending tests

An electromechanical machine, Adamel Lhomargy DY36, of 100 kN capacity in traction-compression has been used. The loading equipment was able to measure the load with a maximum of 2 kN for the low level of load tests. The load threshold, and the load and unload cycles limit values for a low level of load test are linked. The load threshold was set to 10% of the breaking load, which was evaluated using the CIRAD-Forêt wood data bank. The load and unload cycle upper limit value was set to 5% of the breaking load. The lower limit value was constant at 20 N. All the values for each species are shown in the Table 1.

The test specimens were loaded using the 3 point bending test method as shown in Fig. 1, in agreement with the old French standard of 1942 (NF 1942). The load increased by the machine cross-head moving was applied at a continuous rate of 0.02 mm/s to the upper limit value of the load and unload cycles (Table 1). A load (P)-absolute deflection (f) record permitted to calculate the modulus of elasticity value. The absolute deflection was equal to the machine cross-head moving. The Eq. 1 was used to do the calculation. The absolute theoretical relative errors of these test results, caused by measurement uncertainties, have been evaluated (without taking care about humidity corrections) at:

$$\frac{|\Delta E_L|}{E_L} = 9\% \text{ for the modulus of elasticity.}$$

The 4 point bending test, shown in Fig. 3, is relative to current French standard of 1987 (NF 1987). This loading method is also recommended by the European standardisation of 1995 for timber in structural sizes in wood building (EN 1995). The determination of the elastic modulus was realised with 3 successive cycles of load and unload, between 200 N and 600 N, checking the linearity of the load-relative deflection record (NF 1987). The load increased at a continuous rate of 0.02 mm/s. The relative deflection measurement was done at the midspan of the specimen, in the volume submitted to pure bending, by using a deflection measuring apparatus described at the Fig. 4 (Guitard 1987).

The deflection measuring apparatus is an instrument which measures the central point vertical moving of the specimen upper or bottom face. Concerning a 4 point bending test, the results were obtained by using the Eq. 2. The absolute theoretical relative errors of these test results, caused by measurement uncertainties, have been evaluated (without taking care about humidity corrections) at:

$$\frac{|\Delta E_L|}{E_L} = 6\% \text{ for the modulus of elasticity.}$$

Indentation test

In order to study the indentation effect of the loading head and the supports during a 3 point bending test, it was necessary to make a very specific test. This

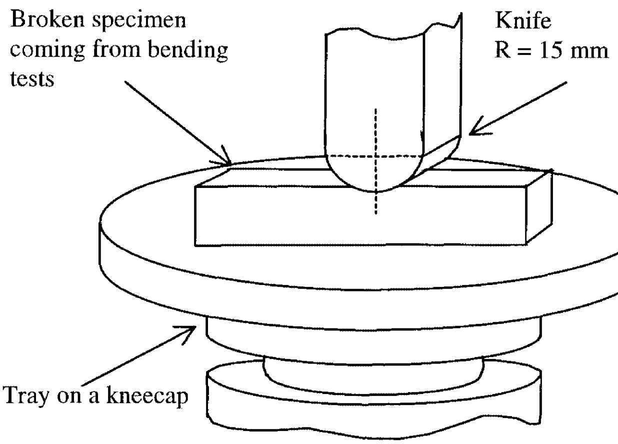

Fig. 6. Principle of the indentation test

one was based on the determination of the Monnin hardness (Fig. 6), in agreement with the NF B 51-013 standard (NF 1985). A direct measurement of the indentation deflection (t) with the increasing of the applied load (P) constitutes the difference between the specific indentation test and the standard one. The indentation deflection was equal to the machine cross-head moving. The load increased at a continuous rate of 0.02 mm/s up to reach the elastic limit.

The absolute theoretical relative error about the measurement of the indentation deflection has been evaluated at 2%.

Results

(a) The change from the modulus of elasticity evaluated in 3 point bending to the 4 point bending modulus has been studied experimentally (Fig. 7). An empirical crossing coefficient, getting an absolute theoretical relative error of 12%, has been determined (Eq. 15).

$$E_{L4} = 1.24 \times E_{L3} \quad R^2 = 0.99 \tag{15}$$

Level of significance of the adjustment: 1%

A 3 point bending test under-estimates about 19% the modulus of elasticity value in relation to a 4 point loading. However, the relative difference between these two bending tests is not continuous according to the density (Fig. 8). This difference can be caused directly by the density or by other wood anatomical differences between the species tested.

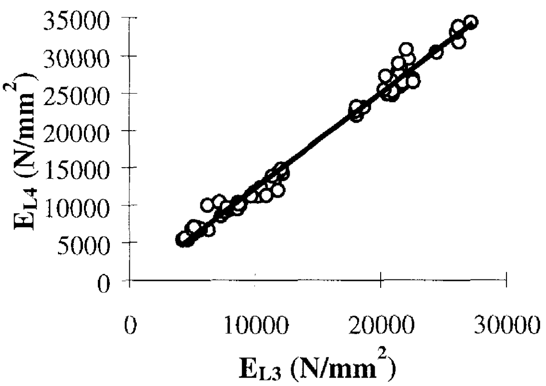

Fig. 7. Linear regression plot between the MOE obtained by three and four points bending tests

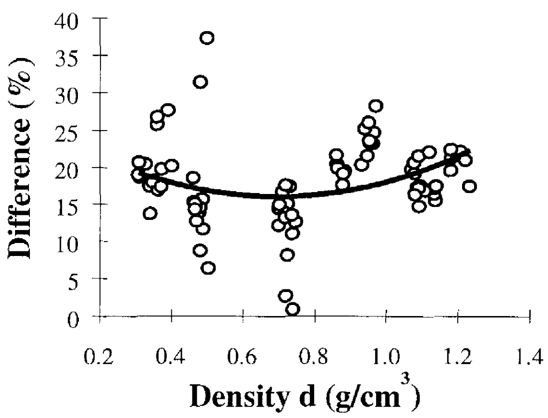

Fig. 8. Density versus relative difference between MOE measurements obtained by three and four points bending tests with respect to four points bending measurements

We notice a parabolic type of evolution, reaching a minimum of about 12% at the average density of 0.7 g/cm^3 (Fig. 8). The relative difference is maximal (21%) at the extreme densities of 0.3 g/cm^3 and 1.2 g/cm^3. This phenomenon is due to the influence of the supports and loading head indentation in 3 point bending.

(b) The determination of the indentation coefficient K has been done by making a linear adjustment on the set of pairs ($P^{2/3}$; t) for each tested specimen ($\bar{R}^2 = 0.90$ with a level of significance of 1%). The experimental adjustment of the indentation coefficient K chosen (Fig. 9), for a cylindrical shape with a radius of 15 mm, is according to the density:

$$K = 0.001|d - 0.7| + \frac{0.0018}{d} \quad R^2 = 0.72 \tag{16}$$

(c) The Fig. 10 compares the experimental values of the elastic modulus E_{L4} with those calculated E_{L4C} using the Eq. 14. The parameters G_{TL} and K of the crossing analytic formula are calculated according to the density using Eqs. 16 and 17.

$$G_{TL} = 976\left(\frac{d}{0.65}\right)^{1.18} \quad R^2 = 0.86 \tag{17}$$

This last equation is obtained by making an adjustment on the shear modulus values listed by Guitard (1987) and measurements realised during this study on specimens of Table 1.

$$E_{L4C} = 0.96 \times E_{L4} \quad R^2 = 0.99 \tag{18}$$

Level of significance of the adjustment: 1%
The Eq. 18, obtained in this way, allows to make an experimental correction of the analytic formula (Eq. 14) which becomes:

$$E_{L4} = \frac{1.042}{1 - 1.906K\left(E_{L3}\frac{eh^3}{l^3}\right)^{2/3} - \frac{E_{L3}}{G_{TL}}\frac{h^2}{l^2}} E_{L3} \tag{19}$$

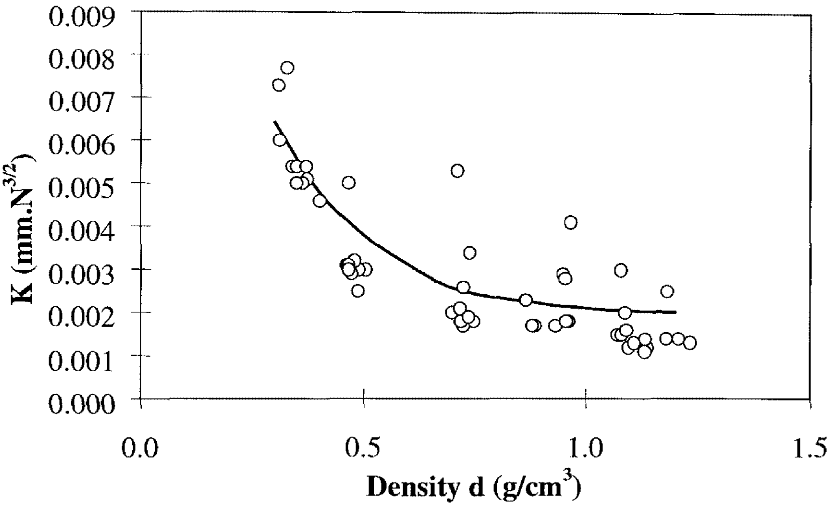

Fig. 9. Experimental adjustment of the indentation coefficient K according to the density

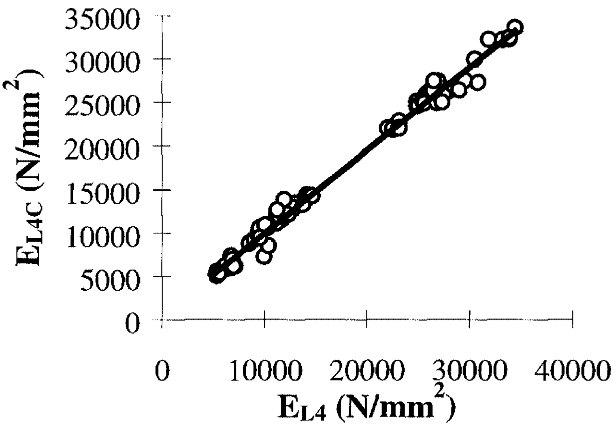

Fig. 10. Linear regression plot between the MOE obtained by four points bending tests and analytic formula

The absolute theoretical relative error, evaluated on the analytic crossing coefficient only, is 12%. Using a modulus of elasticity stemmed from the CIRAD-Forêt wood data bank, the relative error of the elastic modulus evaluated with the Eq. 19 is about 21%.

The expression of the analytic crossing coefficient (Eq. 19) allows to find the same evolution type than the one shown at Fig. 8 concerning the relative difference between 3 point and 4 point bending tests (Fig. 11). This last remark would lead us to confirm the major influence of the density in the difference between modulus of elasticity measured by 3 and 4 point bending tests.

Eq. 19 also allows to value respectively at 8% and 11% the under-estimations which led in practice to the 3 point bending test when the shear and indentation effects were neglected. However, the influence of the indentation on

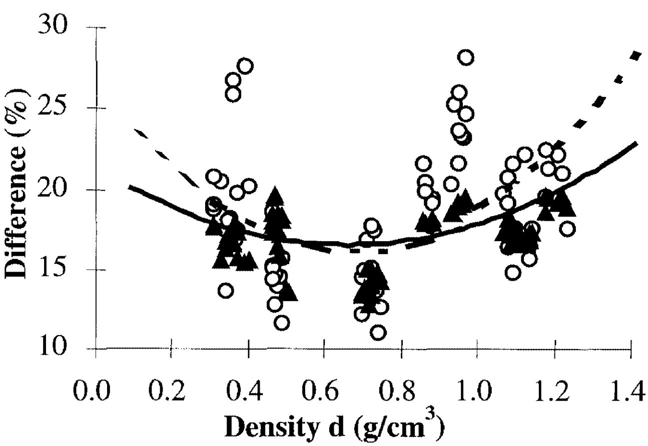

Fig. 11. Density versus relative difference between MOE measurements obtained by three and four point bending tests with respect to four points bending measurements. (o) Experimental data in four points bending test. (ζ) Calculated data using the analytic crossing coefficient

the evaluation of the elastic modulus in 3 point bending isn't continuous according to the density (Fig. 12).

The trend, put in a prominent position by Fig. 12, gets a parabolic type of evolution, varying from 9% for the average density of 0.7 g/cm^3 to 12% for the extreme densities of 0.3 g/cm^3 and 1.2 g/cm^3. This phenomenon can be explained by the fact that the indentation is the result of the competition between two factors, a physical one characterised by the stiffness of wood 1/K, and the other methodological characterised by the load P applied on the specimen (Fig. 13).

The indentation is significant for density values of about 0.3 g/cm^3 because the stiffness of wood is low. A high level of load during a 3 point bending test for species of density values close to 1.2 g/cm^3 also implies a significant indentation effect. For a density close to the average value of 0.7 g/cm^3 the opposite effects of the wood stiffness and the applied load balance each other, implying a low level of indentation.

Discussion

Specimens moisture content

The standardised moisture content is different from one bending test to the other. It is set to 15% for a 3 point bending test and to 12%±2% for a 4 point loading (NF 1942; NF 1987). However, the CIRAD-Forêt experimental procedure has always recommended to make the tests at 12% of moisture content. So, making a correction of the reference moisture content concerning the mechanical properties was not necessary.

The moisture content distribution of the 54 specimens, conditioned at 20°C ± 2°C and 65% ± 5% of relative air humidity, is situated between 11% and 15%. The moisture content correction formulas, recommended by the current standards, were not used in the calculations (NF 1987). Indeed, the influence of the variations in the moisture content was neglected within the interval [11%; 15%].

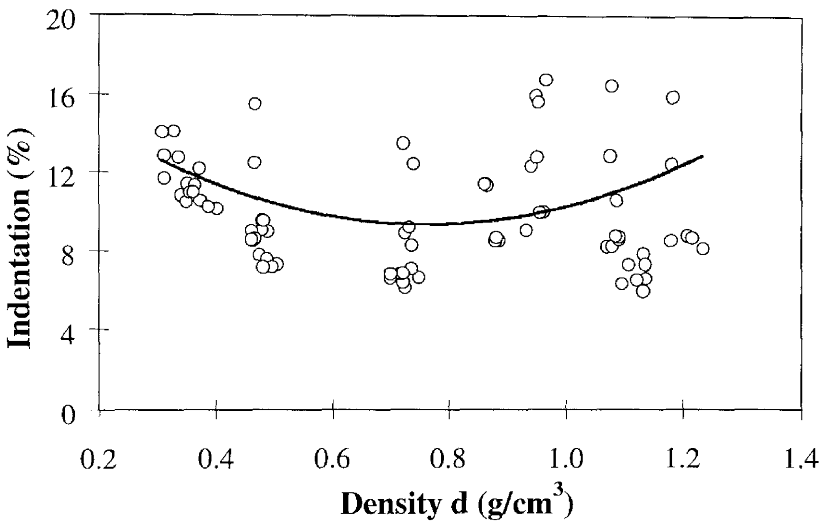

Fig. 12. Density versus indentation effect

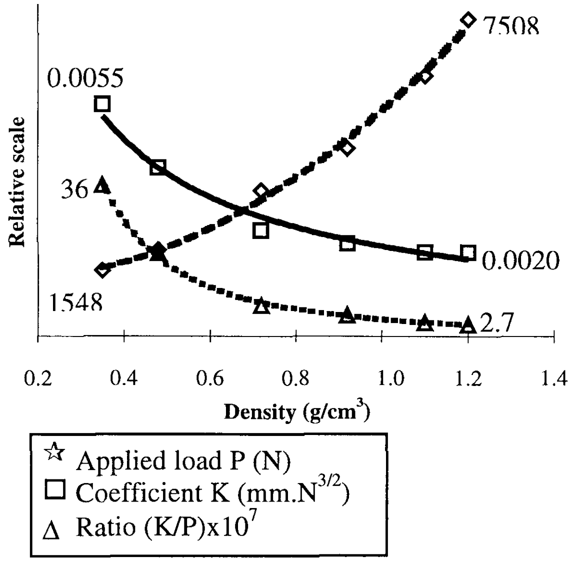

Fig. 13. Density versus indentation coefficient and applied load

We must keep in mind that mechanical behaviour within a set surrounding is the standardised reference which regulates the common use. Each species of wood reaches its own hygrocopic balance which depends on its sorption/desorption isotherm. This balance varied according to the chemical composition and the density of wood, but also to the history of each specimen. Going deeper into the analysis brings rheological considerations which overstep the study context.

Modulus of elasticity

(a) The Guitard's hypothesis, of a bidimensional state of stress, causes the non-similarity between Eq. 5 and the law of behaviour recommended by the American standards (Eqs. 3 and 4). Eq. 5 is obtained supposing σ_{RL} negligible, which implies a tangential shear state:

$$\sigma_{TL} = aG_{TL}\left[\frac{-E_{L3}}{2\,G_{TL}}\left(\frac{h^2}{4}-y^2\right)+\nu_{LR}\left(\frac{e^2}{12}-z^2\right)\right] \tag{20}$$

with 'a' constant.
The shear stress have a parabolic type distribution and cancels out for $z = \pm\, e/(2\sqrt{3})$ as it is showed at Fig. 14. The tangential stress σ_{TL} is set to an average of zero on the rectangular section S, Which allows to determine the mechanical state in a better way. Indeed the static and kinematics conditions are satisfied.

We deduce that the hypothesis, σ_{RL} negligible, distorts the Poisson's ratio term of Eq. 20. However this simplification does not cause an appreciable error concerning the estimation of the shear effect influence on the elastic modulus value.

(b) The distribution of indentation coefficient values, shown at Fig. 9, leads to chose an hyperbolic type adjustment. A lack of matter implies an infinitely high indentation coefficient, or a zero stiffness 1/K. On the contrary, for

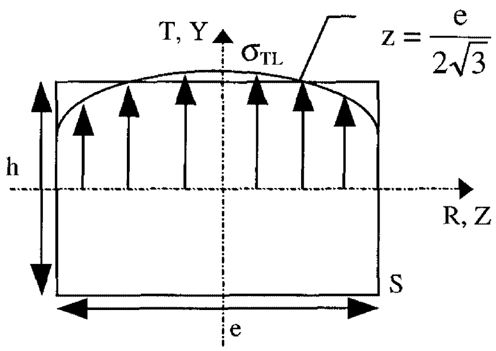

Fig. 14. Shear stress evolution σ_{TL} on sample face (y = h/2)

infinitely dense wood, K is nil so the stiffness is infinite. However it is necessary to correct the extreme trends of the chosen adjustment, because experimentally the trends of coefficient K converge faster towards infinity and slower towards zero (Fig. 9) than the hyperbolic model (Eq. 16).
The coefficient K is evaluated analytically by Eq. 13. Using this last equation with Eq. 16, the distribution of tangential elastic modulus values according to the density can be determined (Fig. 15). These values were added to the data listed by Guitard (1987). The following relation is obtained by making an adjustment on the two sets of values.

$$E_T = 795\left(\frac{d}{0.65}\right)^{1.53} \quad R^2 = 0.80 \tag{21}$$

Indeed the evolution of modulus E_T is well described by a power law in accordance with the Ashby and Gibson models (Ashby and Gibson 1988).

Conclusion

The crossing analytic formula from 3 point bending modulus of elasticity to the one obtained using a 4 point bending test gives results verified by the experience. The use of known theories to problem of changing datas from 3 point bending to 4 point bending allows to make this analytic formula. A 3 point bending test under-estimates about 19% the modulus of elasticity value in relation to a 4 point

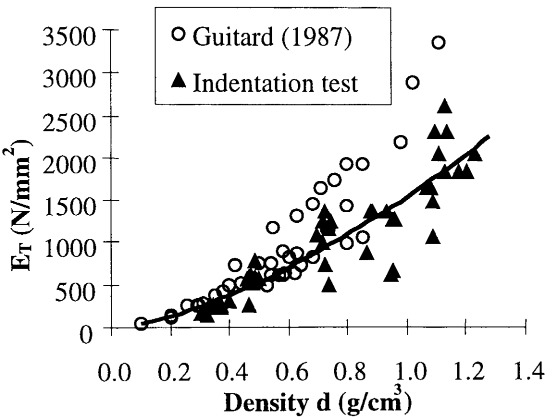

Fig. 15. Density versus tangential elastic modulus

loading. The under-estimations, which lead to a 3 point bending test when the shear effect and the indentation effect are neglected, are valued respectively to 8% and 11%. However the relative difference between these two bending tests is not continuous according to the density. The following crossing formula takes into account this phenomenon by the estimation of mechanical properties according to the density.

$$E_{L4} = \frac{1.042}{1 - 1.906K\left(E_{L3}\frac{eh^3}{l^3}\right)^{2/3} - \frac{E_{L3}}{G_{TL}}\frac{h^2}{l^2}} E_{L3}$$

with

$$G_{TL} = 976\left(\frac{d}{0.65}\right)^{1.18} \quad \text{and,}$$

for a support and a loading head with a cylindrical shape of 15 mm radius

$$K = 0.001|d - 0.7| + \frac{0.0018}{d} \quad \text{or,}$$

for a support and a loading head with a cylindrical shape of any size radius

$$K = 0.515\left(\frac{h}{e\sqrt{R}E_T}\right)^{2/3}, \text{ using } \quad E_T = 795\left(\frac{d}{0.65}\right)^{1.53}$$

The use of these formulas can be very useful when data from different references has to be compared. Their correct use is given by the experimental conditions and following the written mechanical hypothesis. These formulas only concern homogeneous clear wood, with straight grain following the principal specimen axis.

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