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## Somatosensory attunement to the rigid body laws

Received: 24 August 2000 / Accepted: 5 October 2000 / Published online: 16 November 2000  
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**Abstract** In the most general case, haptic perception of an object's heaviness is most likely the perception of the object's resistance to movement, determined jointly by the object's mass and mass distribution. In two experiments with occluded objects wielded freely in three dimensions, we showed additive effects on perceived heaviness of mass and the inertia tensor. Our manipulations of the inertia tensor were directed specifically at the volume and symmetry of the inertia ellipsoid, quantities that can be understood as important to controlling the level and patterning of muscular forces, respectively. Ellipsoid volume and symmetry were found to have separate effects on perceptual reports of heaviness that were invariant over different tensors. Independent sensitivities to translational inertia and particular characterizations of rotational inertia suggest specialized somatosensory attunement to the rigid body laws.

**Keywords** Heaviness perception · Somatosensory · Mechanical variables · Movement

### Introduction

Everyday human interactions with manually grasped objects (e.g., wielding, hefting, or transporting them) typically consist of translations and rotations. These interactions combine forces proportional to an object's resistance to translation (its mass) and torques scaled to an object's resistance to rotation (its inertia tensor). They involve the rigid body laws of translation and rotation. A somatosensory ability to register the parameters (mass, inertia tensor) of the rigid body laws in ways relevant to controlling muscular forces is suggested by experiments directed at perceiving the heaviness of freely wielded,

nonvisible objects. In the research in question, different inertias for translation (measured in kilograms) were combined factorially with different inertias for rotation (measured in  $\text{kg}\times\text{m}^2$ ) (Turvey et al. 1999). The experiments found that mass and the tensor of inertia had separate effects on haptically perceived heaviness (see also Amazeen 1997, 1999)<sup>1</sup>.

Achieving the requisite factorial combinations required the use of "tensor objects" (Amazeen and Turvey 1996), an example of which is shown in Fig. 1A. Tensor objects are composed of five rods of fixed mass and linear dimensions and a variable number of attached metal rings. One rod is the stem. The other rods are branches attached to the stem so as to form two cross-pieces perpendicular to the stem and to each other. The attachment of the branches is through a hub that can be positioned freely along the length of the stem. The total mass of a tensor object can be prescribed by simply selecting particular magnitudes of the masses of the attached metal rings. Specific tensors of inertia relative to  $O$  (the rotation point in the wrist when the stem of a tensor object is grasped at one end, see Fig. 1A) can be prescribed by selecting specific positions of the hub and/or specific positions and mass magnitudes of the metal rings attached to the branches and the stem.

The somatosensory components most engaged by wielding are the mechanoreceptors embedded in muscles and in the attachments of muscles to tendons. Collectively, they define a somatosensory subsystem that

<sup>1</sup> Because the translation and rotation laws are additive, we might expect separate, additive perceptual effects of mass and the inertia tensor. The additivity of the laws is made particularly transparent through the *geometric algebra* in which quantities of different grade or kind can be added (Gull et al. 1993; Hestenes 1986). Thus, linear momentum,  $\mathbf{P}$ , and angular momentum,  $\mathbf{I}$  can be combined into a single quantity  $P$  (complex momentum) defined by  $P=\mathbf{P}+i\mathbf{I}$ . Similarly, force,  $\mathbf{F}$ , and torque,  $\mathbf{T}$ , can be combined into a single quantity  $W$  (complex force or wrench) defined by  $W=\mathbf{F}+i\mathbf{T}$  (Hestenes 1986). In both cases, a vector is combined with a bivector (an oriented plane) capturing, respectively, translation and rotation. When combined, the two laws yield a single equation,  $dP/dt=W$ , the complex law of motion for rigid bodies (Hestenes 1986).

has been referred to as *dynamic touch* (Gibson 1966). The latter subsystem seems to be broadly responsive to manipulations of the eigenvalues and eigenvectors of the tensor (for reviews, see Carello and Turvey 2000; Pagano and Turvey 1998; Turvey 1996; Turvey and Carello 1995).

The relative values of the eigenvalues  $I_k$  ( $k=1, 2, 3$ ) or principal moments of a rigid object determine three dynamically distinct classes. A rigid object is *centrosymmetric* when all three moments are equal, *axially symmetric* when two moments are equal, and *asymmetric* when all three moments are distinct (Hestenes 1986). An object's rotational dynamics about  $O$ , that is, its response to applied torques, depends on the symmetry of the object's rotational inertia with respect to  $O$ . In contrast to axially symmetric and asymmetric objects, a centrosymmetric object resists rotation to an equal degree about any arbitrarily chosen axis. Experiments have shown that haptically perceived heaviness of a freely wielded tensor object decreases as the object's principal moments become more nearly identical (Amazeen and Turvey 1996). That is, a non-visible object of fixed mass feels lighter the more closely it approximates centrosymmetry (Turvey et al. 1999).

A useful quantification of dynamical symmetry ( $S$ ) is the ratio

$$S = 2 \cdot I_3 / (I_1 + I_2)$$

(given  $I_1 \geq I_2 \geq I_3$ ) (Turvey et al. 1999).  $S$  increases as  $I_1$ ,  $I_2$ , and  $I_3$  become more nearly identical, attaining its highest value of one when the object is centrosymmetric. When the mass and  $S$  of tensor objects are combined factorially, their effects on perceived heaviness are additive, with perceived heaviness increasing with mass and decreasing with  $S$  (Turvey et al. 1999). The effect of  $S$  in linear combination with mass suggests that the somatosensory registration of the heaviness of a hand-held object is more properly the somatosensory registration of an object's disposition to move in response to the patterning and levels of muscular forces that bring about translation and rotation. The restriction on the patterning of forces is imposed by  $S$ . The restriction on the level of forces is imposed by mass and by the absolute values of  $I_k$  quantified through the *determinant* ( $I_1 \times I_2 \times I_3$ ) of the inertia ten-

sor or, equivalently, through the reciprocal of the *volume* ( $V$ ) of the ellipsoid of inertia:

$$V = 4\pi/3 (\text{Determinant } I_{ij})^{-1/2}$$

Turvey et al. (1999) suggested that, for a fixed inertia of translation and fixed  $S$ , judgments of heaviness may increase with a rotational inertia's determinant or, synonymously, with the inverse of  $V$ .

In summary, when a person remarks on the heaviness of a nonvisible wielded or hefted object, it seems that the remark is in reference to the object's disposition for being moved. In the two experiments reported in the present article, we evaluated the hypothesis implied by the preceding review. Namely, that so-called heaviness perception reflects the attunement of the somatosensory system to the rigid body laws with the particular invariant forms of the rotational-inertia parameter defined relative to the muscles and their neural control. According to this hypothesis: (1) mass and  $I_k$  contribute independently to perceptual reports of heaviness, and (2)  $S$  and  $V$  are the action-relevant forms of  $I_k$  that similarly contribute independently to perceptual reports of heaviness.

## Experiment 1

In experiment 1, we evaluated the hypothesis in four ways through a single factorial design. We asked whether the effect on heaviness perception was invariant for: (1) a given mass over different magnitudes of  $I_1$ ,  $I_2$ , and  $I_3$ ; (2) a given  $S$  over different magnitudes of mass and  $I_1$ ,  $I_2$ , and  $I_3$ ; (3) a given  $V$  over different magnitudes of mass and  $I_1$ ,  $I_2$ , and  $I_3$ ; and (4) a given  $S$  over different magnitudes of  $V$ , and vice versa.

### Materials and methods

#### Participants

Ten undergraduates at the University of Connecticut participated in partial fulfillment of a course requirement. All participants were informed as to the nature of the study and provided consent to participate.

#### Apparatus

The details of the eight tensor objects used in experiment 1 are given in Table 1. As Table 1 reveals, the tensor objects were con-

**Table 1** Object mass ( $M$ ), symmetry ( $S$ ), volume ( $V$ ), eigenvalues ( $I_1$ ,  $I_2$ ,  $I_3$ ), and mean and standard deviation (SD) of perceived heaviness in experiment 1

| Object | $M$ (g) | $S$  | $V$  | $I_1$ (kg·m <sup>2</sup> ) | $I_2$ (kg·m <sup>2</sup> ) | $I_3$ (kg·m <sup>2</sup> ) | Mean (SD) <sup>a</sup> |
|--------|---------|------|------|----------------------------|----------------------------|----------------------------|------------------------|
| 1      | 374.3   | 0.60 | 1.43 | 2.49                       | 2.39                       | 1.45                       | 88.1 (17.1)            |
| 2      |         | 0.35 | 1.43 | 2.91                       | 2.89                       | 1.02                       | 99.8 (8.5)             |
| 3      |         | 0.60 | 1.11 | 2.92                       | 2.82                       | 1.72                       | 95.4 (13.6)            |
| 4      |         | 0.35 | 1.11 | 3.46                       | 3.42                       | 1.20                       | 112.8 (11.5)           |
| 5      | 454.3   | 0.60 | 1.43 | 2.49                       | 2.36                       | 1.46                       | 98.3 (12.9)            |
| 6      |         | 0.35 | 1.43 | 2.93                       | 2.89                       | 1.02                       | 113.2 (15.3)           |
| 7      |         | 0.60 | 1.11 | 2.93                       | 2.81                       | 1.72                       | 107.8 (7.4)            |
| 8      |         | 0.35 | 1.11 | 3.47                       | 3.44                       | 1.20                       | 120.9 (22.9)           |

<sup>a</sup> Standard deviations were calculated over participants

structured such that two levels of  $S$  were crossed with two levels of  $V$  which, in turn, were crossed with two levels of mass. Achieving the preceding factorization meant that the two levels of  $S$  and, likewise, the two levels of  $V$  had to be crossed with two different configurations of  $I_1$ ,  $I_2$ , and  $I_3$ <sup>2</sup>.

### Procedure

The tensor objects were never seen in the experiment (participants were blindfolded), and no information was given about their design or variety. The objects were firmly grasped in the right hand with the proximal end of the stem flush with the bottom of the fist and with the stem always parallel to the fist (see Amazeen and Turvey 1996). Wielding was by rotations of the hand in three dimensions with the forearm supported. Participants were free to wield for as long as needed and to elect whatever pattern and vigor of wielding they wished. Experiments have shown that haptic sensitivity to the inertial parameters is independent of the forcefulness of wielding. For example, the perceptual effects of rotational inertia are constant over variations in mean torque levels brought about by experimenter-imposed restrictions on angular acceleration (Amazeen and Turvey 1996; Solomon and Turvey 1988).

Perceived heaviness was reported by magnitude estimation relative to a standard (object 5 of Table 1) assigned a value of 100. The standard was wielded on every trial prior to wielding the test object for that trial. Object presentations were randomized with five trials per object. This procedure was approved by the institutional review board of the University of Connecticut.

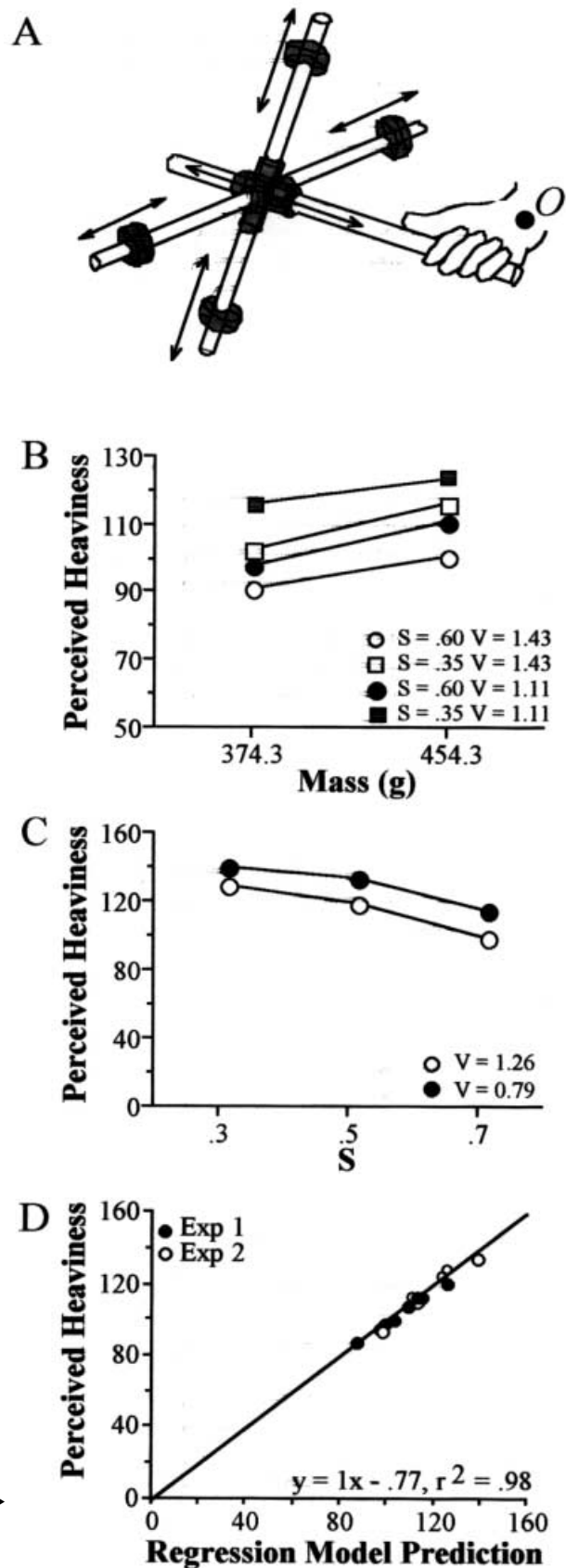
### Results and discussion

The mean judgments of heaviness as a function of  $S$ ,  $V$ , and mass are reported in Fig. 1B and Table 1. An analysis of variance (ANOVA) found main effects of mass (heaviness increased with mass) [ $F(1,80)=14.68$ ,  $P<0.0005$ ],  $S$  (heaviness decreased with  $S$ ) [ $F(1,80)=24.436$ ,  $P<0.0001$ ], and  $V$  (heaviness decreased with  $V$ ) [ $F(1,80)=10.51$ ,  $P<0.005$ ], with no interactions (all  $F_s<1$ ).

The expectations that mass and  $I_k$  affect heaviness perception, and do so non-interactively, were confirmed. Further, the expectations that  $S$  and  $V$  (scalars derived from  $I_k$ ) influence heaviness perception, and do so non-interactively, were confirmed. These confirmations are an important advance over Turvey et al. (1999). In their experiments, although  $S$  and  $V$  were fixed for different magnitudes of mass, these fixed magnitudes were produced in each case by different inertia tensors (e.g., a specific  $I_k$  was used to produce a specific  $S$ ). Further, in Turvey et al., although  $S$  and  $V$  were partially decorrelated, they were not combined factorially. In the present experiment,  $S$  and  $V$  were combined factorially and fixed in magnitude for different magnitudes of mass and  $I_k$ . If  $S$  and  $V$  are meaningful quantities for the neuromuscular control of movements, as hypothesized, then their effects should be invariant over the amounts of mass and the particulars of the mass distribu-

<sup>2</sup> Computations of  $I_k$ ,  $S$ , and  $V$  were by means of the Inertia Tensor Calculation Graphical User Interface (K. Shockley, University of Connecticut) in the Matlab (Mathworks, Mass., USA) programming environment. This software permits the user to manipulate a tensor-object's mass distribution graphically, while providing on-line calculations and schematic representations of its diagonalized tensor and inertia ellipsoid.

**Fig. 1** **A** A tensor object. The cross-bars are moveable as a unit (as indicated by the *arrows*), and the metal rings attached to them can be varied in mass and location (as indicated by the *arrows*). These variations permit the construction of particular inertia tensors relative to the origin of rotation axes at  $O$ . **B** The results of experiment 1. **C** The results of experiment 2. **D** A single linear regression captures the results of experiments 1 and 2



tions (variations in  $I_1$ ,  $I_2$ , and  $I_3$ ) that give rise to them. The results of experiment 1 showed that the latter invariances were indeed the case. The results were also consistent with the hypothesis that the effects of  $S$  and  $V$  are independent. The implication is that  $S$  and  $V$  are important degrees of freedom for understanding how rotational inertia affects perceptual reports of an object's heaviness.

### Experiment 2

In experiment 2, we conducted a further evaluation of the hypothesized orthogonal effects of  $S$  and  $V$  and, in so doing, sought to verify the invariance of their effects over variations in the coordinate-independent tensors. We did so using the widest ranges of  $S$  and  $V$  jointly possible, with tensor objects constrained to the inertial range of objects typically wielded and hefted unimanually. To achieve these ranges, mass was not manipulated. It was fixed at 454.3 g.

### Materials and methods

#### Participants

Nine undergraduates and one graduate student at the University of Connecticut participated in partial fulfillment of a course requirement or on a voluntary basis. None had participated in experiment 1. All participants were informed as to the nature of the study and provided consent to participate.

#### Apparatus

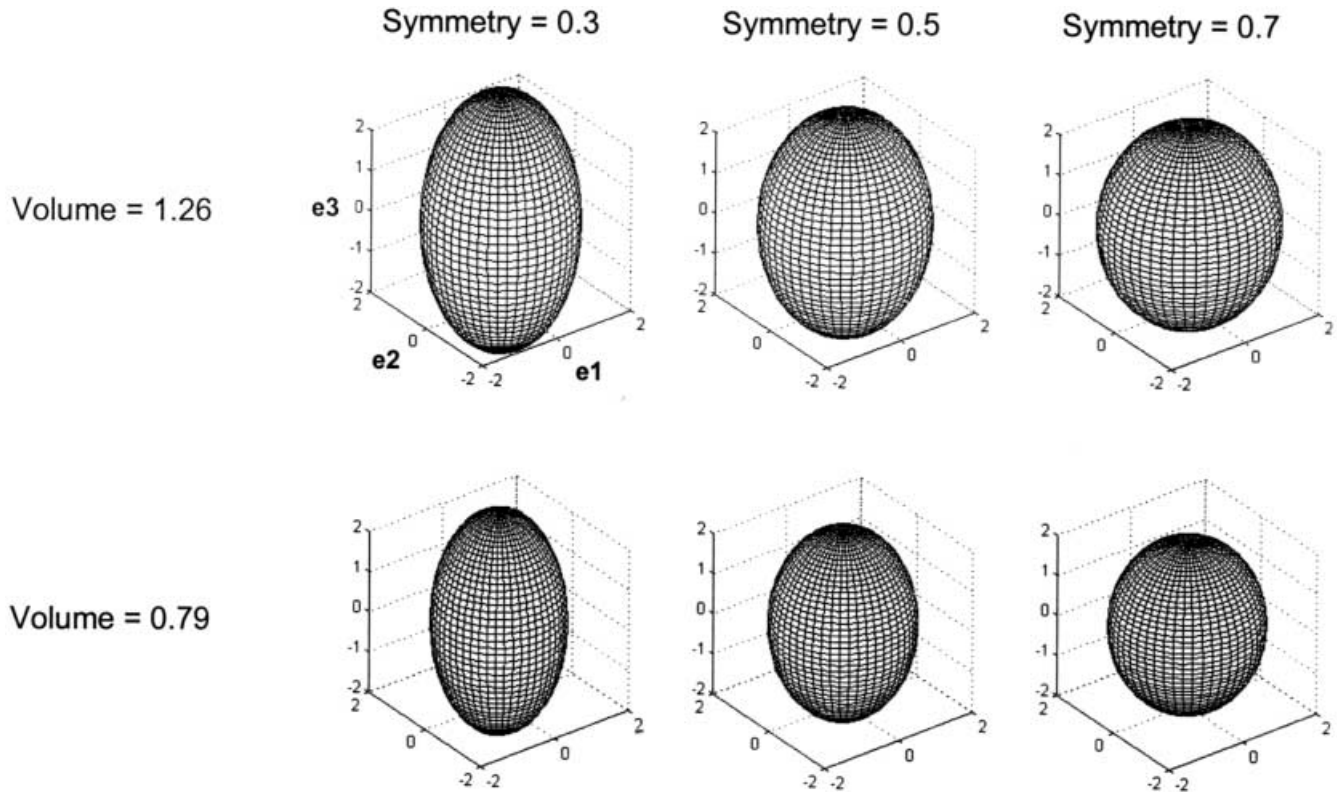
There were seven tensor objects of identical mass (see Table 2). Six of the objects constituted two levels of  $V$  crossed with three levels of  $S$ . Their ellipsoids of inertia are shown in Fig. 2. The remaining tensor object (object 7 in Table 2) was the standard. It matched object 5 of experiment 1 in order to ensure a common basis for heaviness estimates in the two experiments.

**Table 2** Symmetry ( $S$ ), volume ( $V$ ), eigenvalues ( $I_1$ ,  $I_2$ ,  $I_3$ ), and mean and standard deviation (SD) of perceived heaviness in experiment 2

| Object         | $S$  | $V$  | $I_1$ (kg·m <sup>2</sup> ) | $I_2$ (kg·m <sup>2</sup> ) | $I_3$ (kg·m <sup>2</sup> ) | Mean (SD) <sup>a</sup> |
|----------------|------|------|----------------------------|----------------------------|----------------------------|------------------------|
| 1              | 0.30 | 1.26 | 3.32                       | 3.29                       | 1.00                       | 124.0 (14.2)           |
| 2              | 0.50 | 1.26 | 2.84                       | 2.77                       | 1.40                       | 112.6 (13.1)           |
| 3              | 0.70 | 1.26 | 2.62                       | 2.40                       | 1.75                       | 94.4 (11.9)            |
| 4              | 0.30 | 0.79 | 4.55                       | 4.52                       | 1.36                       | 134.8 (15.1)           |
| 5              | 0.50 | 0.79 | 3.87                       | 3.79                       | 1.91                       | 127.6 (14.2)           |
| 6              | 0.70 | 0.79 | 3.52                       | 3.30                       | 2.40                       | 110.2 (14.0)           |
| 7 <sup>b</sup> | 0.60 | 1.43 | 2.49                       | 2.36                       | 1.46                       |                        |

<sup>a</sup> Standard deviations were calculated over participants

<sup>b</sup> Object 7 served as the standard, with an assigned value of 100, for magnitude estimations. This object was not used as a test object



**Fig. 2** The ellipsoids of inertia of the six tensor objects used in experiment 2. The axes and radii of inertia ellipsoids are given by the eigenvectors  $e_k$  and the inverse square roots of the eigenvalues  $I_k$ , respectively, of the objects' inertia tensors (where  $k=1, 2, 3$ )

## Procedure

The same procedure was used as in experiment 1, with the exception that the standard was not used as a test object. Object presentations were randomized with five trials per object.

## Results and discussion

Mean perceived heaviness as a function of  $S$  and  $V$  is reported in Table 2 and plotted in Fig. 1C. Inspection of Fig. 1C suggests that heaviness was an additive function of  $S$  and  $V$ . In confirmation, an ANOVA found main effects of  $S$  [ $F(2,48)=19.77$ ,  $P<0.0001$ ] and  $V$  [ $F(1, 48)=13.21$ ,  $P<0.001$ ] with no interaction [ $F<1$ ].

The results of experiment 2 were combined with those of experiment 1 (given the common standard object). The 14 mean perceived-heaviness values from Tables 1 and 2 were subjected to a multiple linear regression on mass,  $S$ , and  $V$ . The regression accounted for 98% of the variance and yielded the regression equation: perceived heaviness =  $113.78 + 0.15 \cdot (\text{mass}) - 63.76 (S) - 31.17 (V)$ . Figure 1D plots mean perceived heaviness against the model obtained from the regression analysis with lower and upper 95% confidence intervals of [0.1, 0.9], [-74.08, -53.65], [-37.86, -24.47], respectively. The uniformity of the results across experiments 1 and 2 is clear from inspection of Fig. 1D.

## General discussion

We have assumed that, when charged with answering the question of “how heavy?”, the somatosensory system answers the question of “how moveable?” The latter question is more general and reflects the somatosensory system’s essential role in controlling the movements of limbs and hand-held objects. To perform this role in a manner consistent with the laws of rigid body motion requires that the somatosensory system has the ability to detect independently the inertia for translation and the inertia for rotation. Evidence for such an ability was provided by experiment 1: the perceptual effects of mass and  $I_k$  were separate and additive.

In satisfying the aforementioned role in controlling movement, the somatosensory system must also be responsive to the requirements of the motor system. That is, it must be responsive to the motion-laws’ parameters in ways that bear on the control of muscular forces. Both experiments 1 and 2 showed that, in perceiving how moveable (or how heavy) an object is, somatosensory re-

sponsiveness to  $I_k$  assumed two orthogonal forms. These were detecting  $V$ , which would be of relevance to constraining the levels of rotational force, and detecting  $S$ , which would be of relevance to constraining the directions in which the rotational force can be most readily applied. Both experiments also showed that a given  $V$  and a given  $S$  had the same perceptual effects when defined by different eigenvalues. This latter fact indicates a responsiveness of the somatosensory system to mechanical invariants of higher order and underscores how special its attunement to the rigid body laws may be.

**Acknowledgement** This research was supported by NSF grant no. SBR 97-09678.

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