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The use of motion cues in the haptic sense of circularity

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Abstract We tested the hypothesis that speed cues are used to haptically identify changes in the curvature of the hand's trajectory. Subjects grasped the handle of a robotically-controlled manipulandum that was moved in the horizontal plane along various elliptical arcs following one of three different speed profiles. In one profile, a circular arc was traced at a constant speed whereas in the other two speed was constant for ellipses whose aspect ratios differed from unity. A two-alternative forced choice procedure was used to identify the ellipse that was sensed to be circular in each of the three experimental conditions. In unconstrained movements, speed varies with the radius of curvature. If speed cues are used to identify curvature during passive movements, one would expect that subjects' responses should be biased towards the ellipse traced at a constant speed. The results did not support this hypothesis, indicating that speed cues are not a major contributor in the haptic sensing of shape.

Keywords Haptic sensing · Motion cues · Curvature · Arm

Introduction

Haptic sensing epitomizes sensorimotor integration. It involves the integration of sensory information arising

from proprioceptors and cutaneous afferents with efferent signals and cognitive factors to deduce an object's properties such as its shape and texture (Goodwin and Wheat 2004). The most exquisite example of this is provided by the hand and finger movements in object manipulation. However, even movements that are largely restricted to the proximal arm are adequate to derive information about an object's contours with a high degree of accuracy and acuity (Hogan et al 1990; Fasse et al 2000). For example, in detecting deviations from a straight edge, subjects typically exhibit a bias of no more than 1 mm over an arc length of 15 cm, and a detection threshold of about 3 mm (Henriques and Soechting 2003). Similarly, subjects are able to detect deviations from a circular shape when the aspect ratio (AR) (the ratio of the minor to the major axis of an ellipse) deviates from unity by as little as 15%.

During unconstrained movements, such as drawing, it is well known that variations in speed are related to the curvature of the hand's path (Lacquaniti et al 1983). Specifically, straight portions of a trajectory are executed faster than are portions that are curved. This relationship between speed and curvature has been formalized as the "two-thirds power law", speed being (approximately) proportional to the radius of curvature raised to the 1/3 power (Viviani and Cenzato 1985). While various explanations have been proposed to account for this observation (Richardson and Flash 2002; Schwartz 1994; Soechting and Terzuolo 1986; Viviani and Flash 1995), the phenomenon is robust, holding even for ocular tracking (de'Sperati and Viviani 1997).

The possibility arises that cues related to speed could be used to sense an object's curvature during exploratory movements or when the arm is passively displaced. If so, a constant speed would be equated to a constant curvature (a circle) and a decrease in speed would be equated to an increase in curvature. In fact Viviani et al (1997) have reported a haptic illusion that follows from this hypothesis. They passively displaced a subject's hand along elliptic arcs, with speed profiles that corresponded either to a circle, a tall narrow ellipse, or one

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that was short and wide. A circle traced with a speed profile that normally corresponded to a tall narrow ellipse was reported by the subjects to be a tall narrow ellipse. Conversely, a circle traced with a speed profile that corresponded to a short, wide ellipse was reported to be distorted in that direction.

We reinvestigated this phenomenon, using a modification of the experimental design of Viviani et al (1997), with the aim of more precisely defining the relative importance of position and motion (speed) cues in the haptic sensing of the shape of contours. However, we found that under our experimental conditions, subjects did not manifest this illusion.

Materials and methods

Experimental overview

Subjects grasped the handle of a manipulandum that consisted of a two-jointed robotically-controlled arm as it was moved in the horizontal plane along elliptical trajectories. The aspect ratio (AR) of the ellipse was varied from trial to trial, the major axis being oriented either medio-laterally (x -axis) or along the anterior-posterior direction (y -axis). With their eyes closed, subjects were asked to report whether the ellipse was wide (major axis in the x -direction) or narrow (major axis in the y -direction). Each subject was presented with three blocks of 60 trials.

The blocks differed in the speed profiles of the trajectories experienced by the subjects. In one block (Normal, N), the motion conformed to the 2/3 power law (Fig. 1A). The x and y components of the velocity (V_x and V_y) were sinusoidal. Consequently, a circle was traversed at a constant speed (middle panel, Fig. 1A), whereas it was maximal at the flatter portions of the ellipses (top and bottom panels, Fig. 1A). In the other two blocks of trials, the speed profiles were altered such that ellipses with AR differing from unity were produced at a constant speed. Figure 1B shows examples from one block (*Faster on Sides*, FS) in which a wide ellipse with an AR of 3/2 resulted in a constant speed. The situation was reversed in the third block of trials (*Faster on Top*, FT) in which a narrow ellipse was traced at a constant speed.

Our reasoning was the following: if subjects relied exclusively on positional cues, a circular trace (middle panels, Fig. 1) should result in an equal probability of responding “wide” or “narrow”. Conversely, if they relied exclusively on speed cues, the point of equi-probability should be shifted to the ellipse with an AR of 3/2 in Fig. 1B. Finally, if they used a mixture of the two cues, the point of equi-probability should be somewhere in between the two extremes and one should be able to deduce the relative weighting of the two cues.

We performed two experiments, each comprising six subjects. The experimental design was similar in the two, except for the average speed and the amount of distortion in the velocity profiles. In the second experiment,

these parameters were chosen to more closely approximate the experimental conditions used by Viviani et al (1997). A total of eight subjects participated, four of whom took part in both experiments. They gave their informed consent to experimental procedures approved by the Institutional Review Board of the University of Minnesota.’

Details of experimental design

Subjects grasped the handle (10×4 cm diameter, free to rotate about the vertical axis) of a lightweight robot arm (Interactive Motion Technologies) that was programmed to passively displace the subject’s dominant arm along a prescribed trajectory. All but two of the subjects were right handed. Motions were generated by means of an elastic force field

$$\begin{aligned} f_x &= K(x_d - x_a) - C \frac{dx_a}{dt} \\ f_y &= K(y_d - y_a) - C \frac{dy_a}{dt} \end{aligned} \quad (1)$$

where f_x and f_y are the forces along the x -axis and y -axis generated by the torque motors, x_a and y_a are the actual x and y components of the handle’s position, and x_d and y_d correspond to the desired positions. The stiffness K was chosen to be 10 N/cm and damping C (0.5 N/cm/s) was added to insure stability. The desired trajectories were given by

$$x_d = A \sin \theta \quad \text{and} \quad y_d = B \cos \theta \quad (2)$$

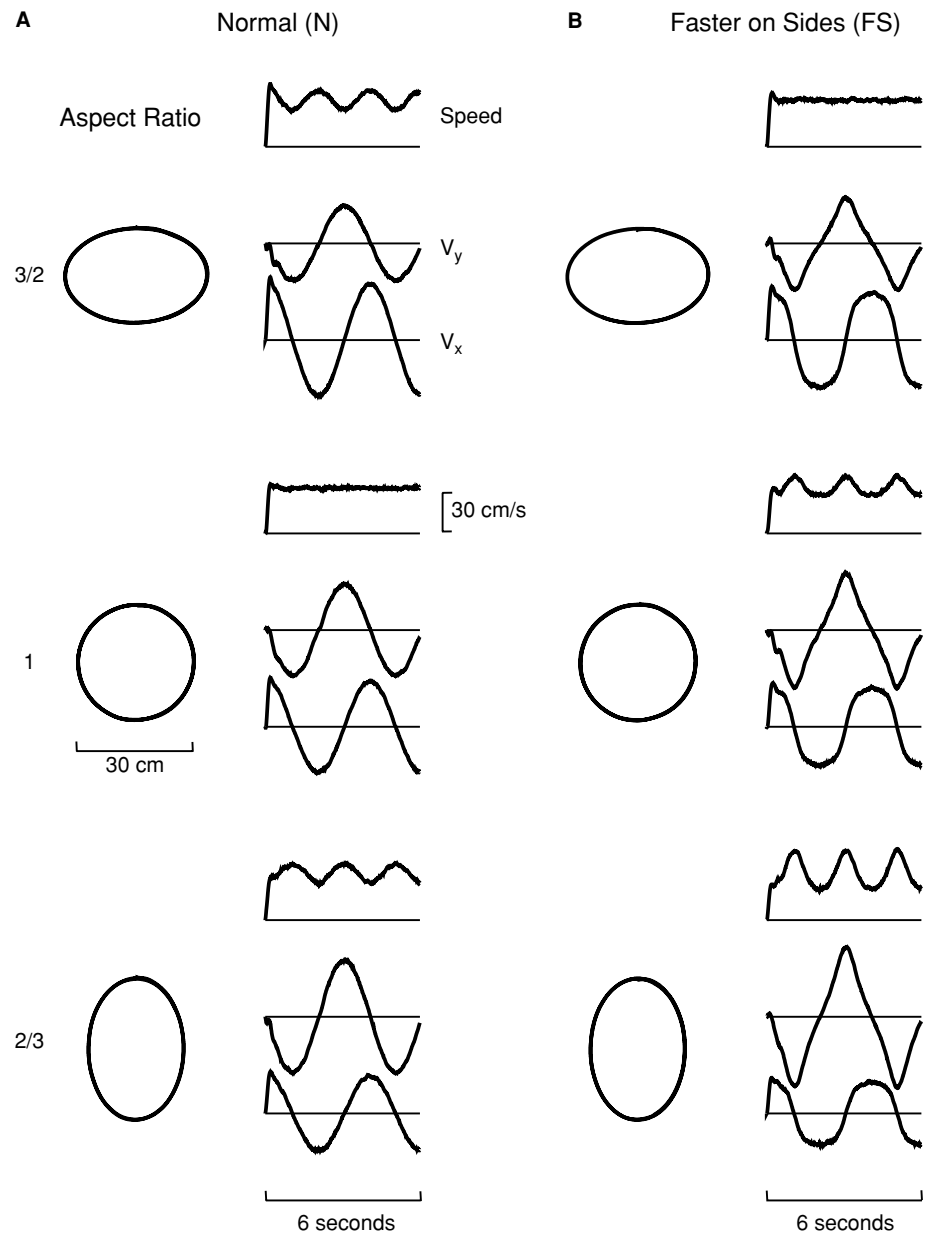
The aspect ratio (AR) A/B was varied from trial to trial, keeping the area of the ellipse constant. For a circle, the radius A was equal to 15 cm.

In the normal condition (N), in which the motion satisfied the 2/3 power law, the angular velocity $\dot{\theta}(\omega)$ was constant, resulting in a period of 4 s in experiment 1 and a period of 2.5 s in experiment 2. In the other two conditions, the period was the same but the angular velocity varied, such that an ellipse with an AR different from unity was followed at a constant speed. In experiment 1, these AR were 3/2 and 2/3 (see Fig. 1B). In experiment 2, the motion was more distorted, such that ellipses with AR of 2/1 and 1/2 were followed at a constant speed. The angular velocity profile was the same from trial to trial in these two conditions, and it was derived from a Fourier series expansion to best approximate a constant speed condition at the desired eccentricity:

$$\theta = \omega t + c_1 \sin 2\omega t + c_2 \sin 4\omega t \quad (3)$$

In experiment 1, $c_1 = \pm 0.0991$ and $c_2 = 0.0118$. In experiment 2, these values were ± 0.1558 and 0.0321. (The sign of c_1 determined whether a wide or narrow ellipse was followed at a constant speed.) A non-constant angular velocity resulted in x and y velocities that were distorted from the normal sinusoidal waveform, more closely approximating a triangular (v_y) or a square (v_x) waveform for condition FS (Fig. 1B). (For condi-

Fig. 1A–B Experimental design. Subjects grasped the handle of a manipulandum that moved along elliptical trajectories. The AR of the ellipse was varied from trial to trial. In the condition that corresponded to biological motion (**A**), the x and y components of velocity (v_x and v_y) were modulated sinusoidally, and a circle was traced at a constant speed. In another experimental condition (**B**), the velocity components were distorted from sinusoidal such that an ellipse with an AR of 3/2 (*top panel*) was traced at a constant speed. In each panel, the traces depict measured values during one trial



tion FT, the two waveforms were reversed.) When the motion was distorted, the ratio of the minimum to the maximum speed for circular motion was 0.68 in experiment 1 and 0.54 in experiment 2.

Each experimental condition consisted of 60 trials. We used a two-alternative forced choice (2AFC) adaptive staircase design (Henriques and Soechting 2003; Kesten 1958; Treutwein 1995); on each trial subjects had to report whether the ellipse was wide or narrow. Ascending and descending staircases were interleaved randomly (see Fig. 2), beginning with AR equal to 2 and to 1/2. We used the shape parameter α , defined as

$$\alpha = \text{AR} - 1 \text{ for } \text{AR} > 1 \quad \alpha = 1 - \frac{1}{\text{AR}} \text{ for } \text{AR} < 1 \quad (4)$$

to vary the AR from trial to trial. Using this definition, positive α 's correspond to wide ellipses, negative α 's

correspond to narrow ellipses and a value of zero corresponds to a circle. Initially, from one trial to the next, we moved α closer to zero in increments of 1/8. This was continued as long as the subject's response (wide or narrow) was consistent with his or her response on the previous trial in that staircase. If the response differed, the direction was reversed and the step size was decreased. We took the average value of α on the last 30 trials to represent the subject's bias, that is, the AR that was sensed to be circular.

On each trial, the robot arm moved the subject's hand through the prescribed trajectory for several cycles. In experiment 1, the motion persisted for three cycles (12 s) and it comprised five cycles (12.5 s) in experiment 2. At the start of each trial, the arm was slowly moved (over 5 s) to the starting position ($x=0$, $y=B$), a warning tone was sounded and then the arm was moved along

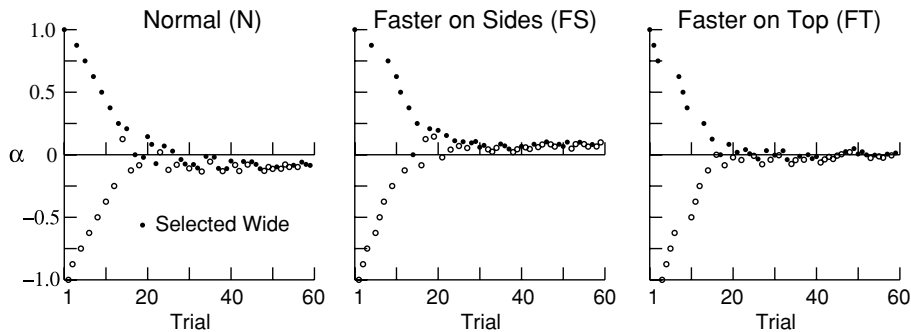


Fig. 2 Estimation of circularity. On each trial, the subjects reported whether the ellipse was wide or narrow, the AR of the ellipse being varied in a double staircase procedure from trial to trial. A positive shape parameter (α) indicates a wide ellipse while a negative value denotes a narrow ellipse. The panels depict the results from one subject for the three conditions of motion in experiment 2. Note that the responses converged to about the same value in all three conditions, indicating that the pattern of motion did not affect the estimation of circularity

the ellipse using the forces given by Eq. 1. After the required number of cycles, or after the subject had responded, the forces were turned off. On each trial, we recorded the subject's response and the time, as well as the arm's actual displacement (x_a and y_a), velocity (v_x and v_y) and the forces (f_x and f_y) at a sampling rate of 200 Hz.

Data analysis

Since the actual displacement of the hand (x_a and y_a) need not correspond precisely to the desired one (x_d and y_d) we also computed the AR of the actual motion. We first found the extrema in the x and y positions and computed the AR from the ratio $\Delta x_a / \Delta y_a$. As an alternative measure, we fitted an ellipse to the actual motion by means of linear regression

$$y_a^2 = a_1 x_a^2 + a_2 x_a y_a + a_3 + a_4 x_a + a_5 y_a \quad (5)$$

equating AR to $a_1^{1/2}$ as well as computing it from the ratio of the major and minor axes derived from the full set of coefficients.

To obtain a measure of the uncertainty in subjects' responses (their difference threshold), we first combined the responses from the six subjects for each experimental condition, subtracting that particular subject's bias in that experimental condition. From these pooled data we computed a response probability (p_{wide}) as a function of the shape parameter α of the ellipses. We fitted these data with the logistic distribution function

$$p_{\text{wide}} = \frac{1}{1 + e^{-\frac{-(\alpha - \alpha_0)}{\beta}}} \quad (6)$$

In this representation, p_{wide} is equal to 0.27 and 0.73 when $\alpha - \alpha_0 = \pm \beta$ and we defined the difference threshold to be 2β .

Finally, we used a bootstrap method to determine whether the difference threshold depended on the experimental condition. We generated 200 sets of simulated responses by drawing, with replacement, from the pooled data of 360 responses for each experimental condition. We then fitted each set with Eq. 6 to obtain the 95% confidence limits on β .

Results

In these experiments, the hand was moved along a prescribed trajectory by a robot manipulandum using a visco-elastic force field (Eq. 1). The actual hand trajectories corresponded very closely to the desired trajectories, as may be ascertained from the trials illustrated in Fig. 1 for one subject and two of the three experimental conditions in experiment 1. The traces depict the measured displacements and velocities of the handle for the first 1.5 cycles of the motion. From data such as these we computed the actual shape parameter α in several different ways (see "Methods") and compared it with the desired one. The actual and the desired shape parameters showed a high degree of correlation, with r^2 values that exceeded 0.99 (and usually 0.999) in all but one of the 12 sessions (six subjects and two experiments). The slope of the regression between the actual and desired shape parameters was close to 1.0, with values ranging from 0.995 (± 0.019 SD) to 0.997 (± 0.018). Similarly, the intercepts were close to 0, with values ranging from -0.001 (± 0.008) to -0.005 (± 0.012).

In each experiment, we asked subjects to judge whether an ellipse was wide (major axis along the medio-lateral direction) or whether it was narrow (antero-posterior direction). This was done in three blocks of trials, which differed in the relation between speed and curvature. In one block of trials (Normal, Fig. 1A), the relation between speed and curvature was governed by the $2/3$ power law, such that a circle was traversed at a constant speed and the x and y components of the velocity were modulated sinusoidally. In a second block of 60 trials, motion was faster on the sides of the ellipse (FS, Fig. 1B), such that a wide ellipse (AR of $3/2$, $\alpha = 0.5$ in experiment 1) was traversed at a constant speed. In this instance, the angular velocity was not constant (see Eq. 3) and the x and y components of the velocity were distorted. In the third condition (FT), the x and y

velocities were exchanged, such that a narrow ellipse (AR of 2/3, $\alpha = -0.5$) was traced at a constant speed. In the first experiment, each cycle lasted 4 s.

Since the experimental conditions in experiment 1 did not produce any consistent trends in what subjects judged to be a circular shape (see below), in experiment 2 we increased the speed at which the shapes were traced and increased the sizes of the distortions in conditions FS and FT. Specifically, in experiment 2, each cycle lasted only 2.5 s and ellipses whose shape parameters α were ± 1 (in conditions FT and FS) were produced at speeds that were close to constant. In this second experiment, the cycle duration was identical to the one used by Viviani et al (1997). In their experimental design the variations in speed were the same on every trial, irrespectively of the ellipse's eccentricity. Accordingly, only ellipses with a shape parameter $\alpha = \pm 1.29$ were traced in accordance with the 2/3 power law. In their experimental design, the ratio of minimum to maximum speed was 0.44 on every trial. In the present experiments, this value depended on the ellipse's eccentricity; for a circle this ratio equalled 0.54 when the velocity profiles differed from normal.

Figure 2 shows representative results from one subject in experiment 2 and illustrates that the three different motion conditions (Normal, FS and FT) had no appreciable influence on the shape of the ellipse that was judged to be circular. In this instance, the shape parameter averaged over the last 30 trials (where the probability of reporting wide or narrow was about equal) was $-0.087 (\pm 0.031)$ for the normal trajectories (left panel). This value was slightly more positive (0.069 ± 0.022 in FS and -0.010 ± 0.028 in FT) in the other two conditions. If speed-related cues influenced the judgment of wide versus narrow, one would expect the effects of FS and FT to be oppositely directed compared to normal (in the positive direction for one and in the negative direction for the other). The data in Fig. 2 are not in accord with this expectation.

The results shown in Fig. 2 were representative of the results from all subjects in both experiments (Fig. 3). Averaged over both experiments, α was equal to 0.001

(N), 0.028 (FS) and 0.012 (FT). Using values only from the second experiment, α was equal to -0.019 (N), 0.030 (FS) and -0.002 (FT). In no case did these trends approach significance (ANOVA, $F_{(2,15)} = 0.159$, $P = 0.85$ for experiment 2). This conclusion did not change when we computed α from the actual motion using three different ways to estimate this parameter.

Distorting the motion profile did appear to have a small effect on the consistency with which subjects discriminated wide from narrow ellipses, at least in the second experiment. This is illustrated in Fig. 4, which shows the subjects' response probability as a function of the shape parameter α . To generate these plots, we combined the data from all six subjects for each experiment, first subtracting their biases for each experimental condition. Accordingly, the probability of choosing wide or narrow is equal at the origin. The binned data were fitted with the logistic distribution function (Eq. 6), and the parameter β can be equated to one half the width of the detection threshold. It is evident that β did not differ appreciably in the three conditions in experiment 1; in fact β was largest in the normal condition (N , top panel). Statistically, the detection threshold did not depend on the experimental condition in experiment 1, the 95% confidence limits for the three estimates overlapping. The detection threshold did increase in experiment 2 when the motion deviated from the 2/3 power law. The detection threshold for condition FS ($\beta = 0.113$, 95% confidence limits 0.076, 0.160) was larger than it was in the normal condition ($\beta = 0.040$, 0.028, 0.062). The difference threshold for the third condition (FS) did not differ statistically from either of the other two. Finally, these values are comparable to the ones we found previously when subjects actively generated arm movements to discriminate the eccentricity of ellipses (Henriques and Soechting 2003).

In summary, motion-related cues had little or no effect on subjects' ability to discriminate wide from narrow ellipses in our experimental conditions. The bias did not differ in any of the three motion profiles (Fig. 3) and the detection thresholds increased modestly, if at all, when the motion was distorted from normal (Fig. 4). However, based on interviews conducted with the subjects at the conclusion of the two experiments, it does appear that the motion profiles led to illusions that were not measured by our experimental paradigm. Most subjects did notice that speed was not constant in conditions FS and FT. However, most reported that the ellipses appeared to be tilted away from the cardinal directions. When asked to provide a sketch, they drew an ellipse whose major axes were tilted about 30° from the medio-lateral direction. Our previous studies indicated that subjects are not as reliable in estimating tilt as they were in estimating curvature, with an average bias of about 5° (Henriques and Soechting 2003). Finally, some of the subjects reported that the ellipses appeared to be distorted and when asked to provide a sketch, they drew a figure that no longer possessed two axes of symmetry.

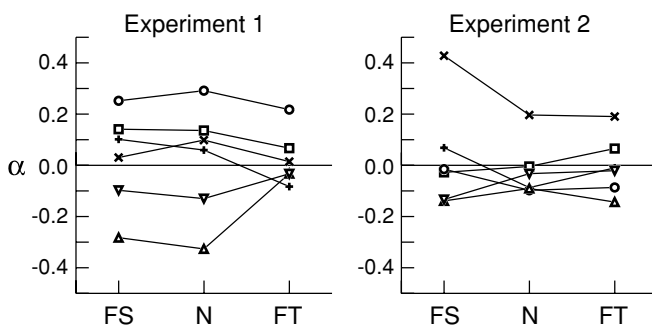
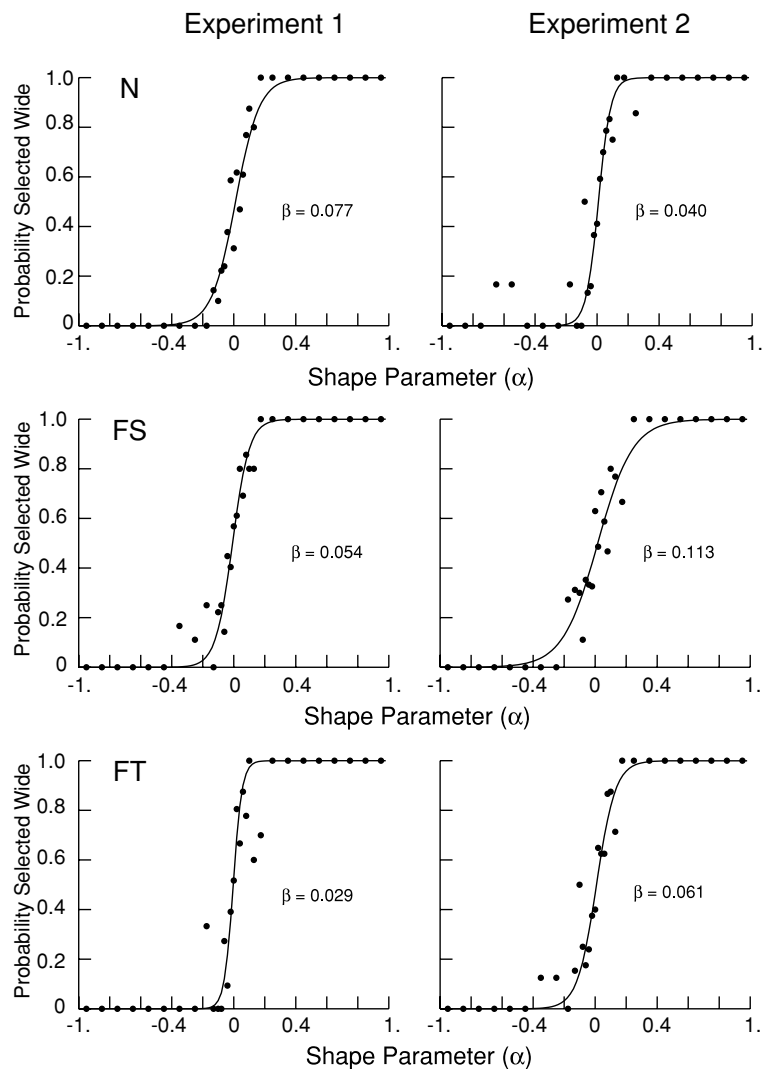


Fig. 3 Summary of results. The plots depict the average shape parameter α on the last 30 trials in each of the three experimental conditions (N normal, FS faster on sides, FT faster on top) from each of the six subjects

Fig. 4 Probability of responding wide as a function of shape parameter. Each of the panels shows the response probability, computed from the pooled data from six subjects. The bias (shown in Fig.3) for each subject was subtracted before combining the data. They were then binned (in intervals of 0.02 for $|\alpha| < 0.11$ and intervals of 0.1 for $|\alpha| > 0.2$) and fitted with a psychometric function. The parameter β , estimated from the fit, represents the half-width of the difference threshold for discrimination



These reports are clearly anecdotal, but when they are combined with the results shown in Fig. 5, they suggest that force-related cues affected the haptically sensed motion. The plots in this figure show averages of hand displacement, velocity and the forces generated by the torque motor for the three motion profiles in experiment 2. The data were obtained by averaging over the last 24 trials, segregating the trials according to the response (wide or narrow). It is clear that this subject exhibited little bias in any of the three motion conditions; the traces in the left column are close to circular. The velocity traces in the middle column of Fig. 5 show the distortion introduced in conditions FS and FT. However, it is noteworthy that the velocity profiles in these two conditions were approximately elliptical, with the major axis oriented either medio-laterally or antero-posteriorly. Furthermore, these profiles were symmetric about the cardinal axes, in accordance with the experimental design.

In the normal motion condition (*N*), forces generated by the torque motors were small. They were much larger

when the motion profile deviated from normal (left column, top and bottom panels in Fig. 5). Furthermore, in those conditions, the force profiles were clearly asymmetrical and tilted away from the cardinal axes. For example, in FT, the forces appear to be oriented along an axis at about 45° up and to the right, whereas in FS there was an axis of symmetry oriented up and to the left. Furthermore, in both conditions, the overall force pattern exhibits two lobes. The results from this subject were typical of the results from all six subjects in the second experiment. In conditions FS and FT, the modulation in the forces was tilted to varying degrees from the cardinal axes, exhibiting a symmetrical shape that consisted of two major lobes.

The pattern of forces shown in Fig. 5 can be accounted for by the viscoelastic properties of the arm. We demonstrated this by regressing the forces f_x and f_y on the hand's position (x_a, y_a) and velocity (v_x, v_y)

$$\begin{aligned} f_x &= k_{xx}x_a + k_{xy}y_a + C_{xx}v_x + C_{xy}v_y \\ f_y &= k_{yx}x_a + k_{yy}y_a + C_{yx}v_x + C_{yy}v_y \end{aligned}$$

Fig. 5 Force and movement patterns. The *traces* depict x - y plots of the measured position (*left column*), velocity (*middle column*) and force (*right column*) in the three experimental conditions. Data are averages over the last 24 trials from one subject in experiment 2, segregated according to the subject's response (*solid lines* narrow, *dashed lines* wide)

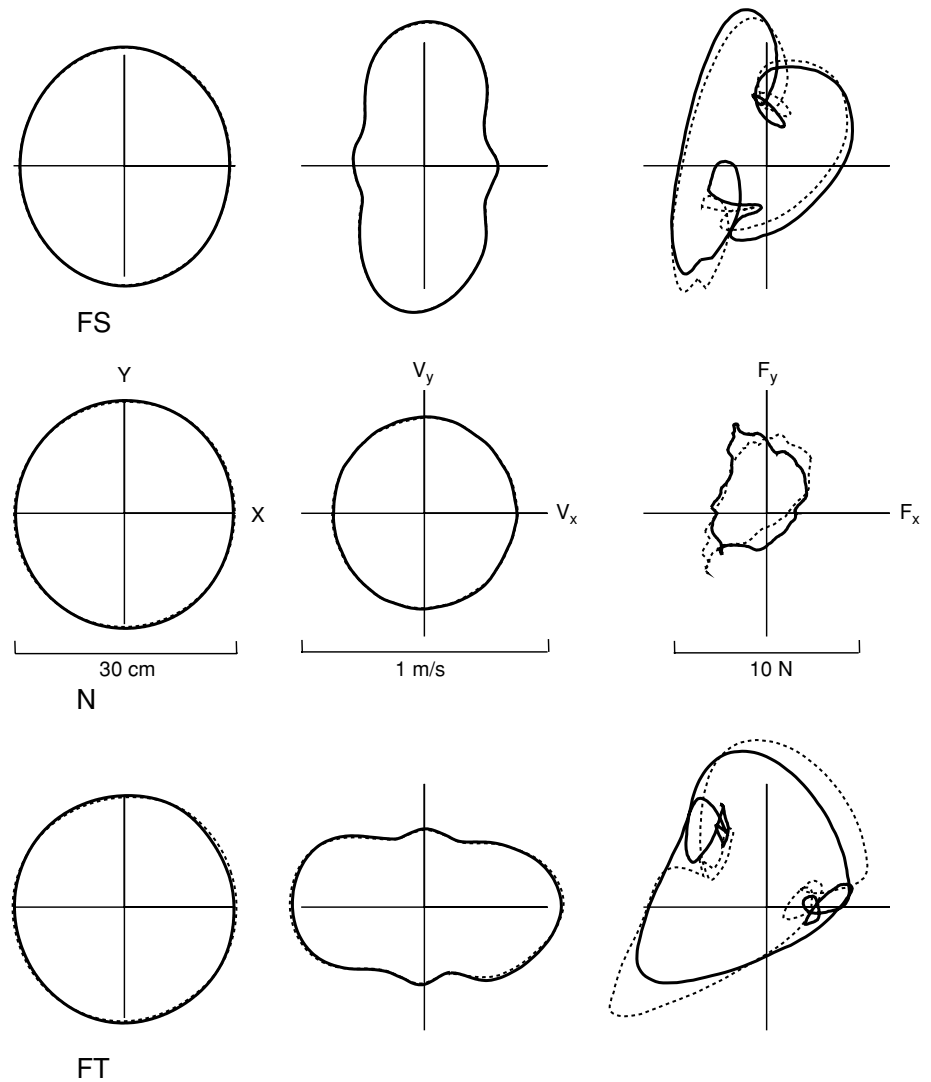


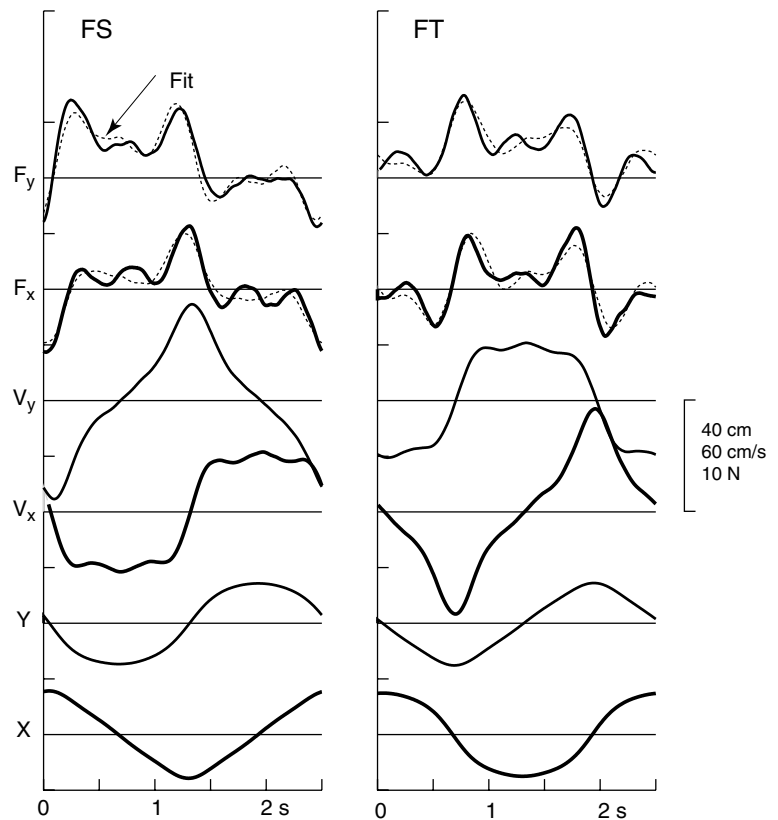
Figure 6 shows the results of this analysis for one subject for FS (left) and FT (right). In both instances, the model (dashed lines) gave an excellent fit to the force data, the variance not accounted for ranging from 8 to 18%. These results are representative of the results from all six subjects; on average the variance not accounted for was 9%. With one exception, the eigenvalues of the stiffness matrix (k_{xx} , k_{xy} , k_{yx} , k_{yy}) were all positive, but unequal (82.9 ± 23.4 and 27.3 ± 10.1 N/m). These values are about 20% as large as the stiffness values estimated from small perturbations applied when subjects maintained a static posture (Mussa-Ivaldi et al 1985) or during arm movements (Bennett et al 1992; Gomi and Kawato 1996). The damping coefficients are more difficult to evaluate because we added damping to the system to ensure stability (see “Methods”). Furthermore, it should be noted that these estimates are subject to uncertainty because of the similarity of the time courses of the x -component of position and the y -component of velocity, and vice versa (see Fig. 6). Nevertheless, this analysis supports the premise that the arm motion in the

present experiments was primarily passive, that the forces applied by the torque motors counteracted limb viscoelastic forces and that inertial forces played a lesser role.

Discussion

We set out to test the hypothesis that subjects make use of variations in speed, in addition to other parameters, to identify the curvature of objects whose shape is explored haptically. This hypothesis was based on a recent report by Viviani et al (1997) that suggested that this was indeed the case. The hypothesis seemed plausible for other reasons as well. As mentioned in the “Introduction”, during normal unconstrained movements speed and curvature are related by the two-thirds power law. For example, a circle is drawn at a constant speed. Speed decreases at the extremes of the major axis of an ellipse, and it is largest at the extremes of the minor axis. It is certainly plausible that when a subject's arm is passively

Fig. 6 Fit of a viscoelastic model to force data. The *traces* show average position (x, y), velocity (V_x, V_y) and force (F_x, F_y) data for the two conditions of distorted motion (*FS* left and *FT* right). Results are shown for one cycle of motion, and the averages were obtained from the last 20 trials in each condition for one subject. The *dashed lines* show the fit of a viscoelastic model to the force data



displaced, as in our experiments, he or she matches variations in speed to a template derived from prior experience.

Speed variations could also provide a useful cue when subjects actively explore an object's shape (Henriques and Soechting 2003) by moving the hand along rigid contours. Consider the case when a straight contour is explored. The force generated by the subjects can be decomposed into components perpendicular and tangential to the contour—the force in the parallel direction moving the arm along at a desired speed. Now, if the contour begins to curve towards the subject, some of this force will be directed against the contour, and speed will decrease. Thus it is conceivable that variations in speed could also be useful during active movements to determine changes in curvature.

These arguments presuppose that subjects can actually sense variations in speed at the hand. The extent to which speed is encoded in cortical structures presumably involved in haptic sensing (namely parietal areas) is not known, although it is clear that this parameter constitutes a prominent component of the signal in motor structures (Coltz et al 2000; Johnson and Ebner 2000; Reina et al 2001).

The results of our experiments did not support the hypothesis. We moved the subjects' hands along elliptical trajectories with three different speed profiles, one in which a circle was traced at a constant speed and the other two in which ellipses with AR of 2:1 (or 1.5:1) were traced at constant speed. We found that the subjects'

bias (the eccentricity that they sensed to correspond to circular motion) was unaffected by the variations in speed, although we did find a small effect on the detection threshold. Thus, we were unable to replicate the observation reported by Viviani et al (1997), even though in most aspects our experimental conditions in experiment 2 were comparable to theirs. Specifically, the size of the ellipses was comparable, as was the speed of hand movement and the distortion of the motion with respect to the $2/3$ power law.

Our experimental design did differ in two respects. In the previous experiments, the modulation in speed was the same, in a particular experimental condition, irrespective of the ellipse's eccentricity. In our experiments, angular velocity was the same from trial to trial (Eq. 3), but the speed profile depended on the eccentricity. It is doubtful that this detail had an effect on the results. The previous experiments tested hand motion in the frontal plane, whereas in our experiments the motion was confined to the horizontal plane. It is possible that this could account for the difference. Motion in the frontal plane involved primarily the shoulder whereas horizontal plane motion involved the elbow as well as the shoulder.

Moreover, it should be noted that the effect reported by Viviani et al (1997) was modest. On average, their subjects identified an ellipse whose shape parameter $\alpha = -0.01$ to be circular in the constant speed condition. When the speed was fastest along the sides (FS), α increased to 0.01, and it decreased to -0.091 when speed

was fastest along the top and bottom (FT) of the ellipse. (They reported their results in terms of eccentricity, rather than AR.)

Even though changing the speed profile did not introduce a bias into what our subjects sensed to be circular, this manipulation did appear to introduce distortions in the haptically sensed shape. Subjects reported that the major axis appeared to be tilted away from the medio-lateral direction and some also reported that they felt that the curved trajectories were asymmetrical. Our experimental design did not permit us to investigate these reports in any quantitative fashion. However, these qualitative reports are consistent with the changes in the forces on the hand responsible for the motion. We found that the force trajectories were also distorted and oriented away from the cardinal axes when the angular velocity was not constant (FS and FT, Fig. 5). Thus our present results are compatible with the notion that haptic sense involves the integration of proprioceptive and tactile cues, proprioceptive cues being related to the posture of the arm (the shoulder, elbow and wrist) and tactile cues in the hand arising from forces exerted through the handle. This interpretation is consistent with a recent report by Robles-de-la-Torre and Hayward (2001). Although their experimental design differed from ours in important aspects (active, exploratory finger movements rather than primarily passive arm movements), they found that force cues could induce illusory changes in an object's contour.

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