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Dimers based on the $\alpha + \alpha$ potential and chain states of carbon isotopes

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Abstract. Using the well established binding energies of one and two valence neutrons in the two-center $\alpha+\alpha$ system (forming the states in ^{9}Be and ^{10}Be*) the structure of these nuclear dimers and their rotational bands including those with more than 2 nucleons are discussed using published transfer reaction data for Be and Boron isotopes. Based on the 0_2^+ state in ^{12}C which is supposed to be an 3α particle chain at an excitation energy of 7.65 MeV and using the binding energy of these valence neutrons in ^{9}Be and ^{10}Be*, chain states in the system ^{12}C* + x neutrons are constructed. The energy position of the lowest chain states are estimated and ways for their population in reactions on ^{9}Be and using radioactive beams are proposed. It is expected that these states are metastable and could have appreciable branches for γ-decay. Further extrapolations to longer chain states (polymers) in neutron rich light isotopes are made.

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I Introduction

There has been a renewed interest in chain states consisting of α-particles in 4N nuclei both experimentally and theoretically [1, 2]. After the well established structure of ^{8}Be, which shows a rotational band based on a two-α-particle structure, the second 0_2^+ at 7.65 Mev in ^{12}C has been proposed [3, 4] to be a chain of three α-particles. Various approaches based on a cranked cluster model have confirmed that such structures exist [5] not only in ^{12}C but also in other nuclei. Experimental evidence exists for example for 16O [6] and recently also for ^{24}Mg [7]. The origin of the existence of such states is the structure of the α-α potential, which (in its local form) shows an attractive part and a repulsive core, which describes well the 0^+ and 2^+ states in ^{8}Be at 0.092 MeV and 3.04 MeV excitation energy. This particular property of the α-α potential gives rise to the unique two-center structure of the ^{9}Be states (one valence particle in the two α-system with a binding energy of 1.6 MeV) and a group of four states in ^{10}Be* at about 6 MeV excitation energy which correspond to two valence neutrons in the two α-particle system [8]. The experimental information on the binding energy of the valence neutrons in ^{9}Be (^{8}Be + n, 1.66 MeV) and for the 4 states in ^{10}Be in the energy range of 5.96 MeV to 6.26 MeV (these are ca 2.5 MeV below the threshold of ^{8}Be + 2n at 8.47 MeV in ^{10}Be), which has been discussed by the author in [8], are the building blocks for the present work.

In the following I will use as in physical chemistry the notion dimer for a two center structure and polymers for multicenter (chain) states. We also use in Sect. II the molecular orbital classification for valence particles (see [8] for further references).

In the present work I have compiled the information on the existence of dimer structure in ^{9}Be and ^{10}Be as well as the possible existence of further dimers (see Figs. 1 and 6) and their corresponding rotational band structure in the isotopes of Beryllium (^{11}Be - ^{12}Be) and Boron (^{10}B - ^{13}B). There exists a considerable amount of experimental data for these nuclei [9], the information is, however, often incomplete and a variety of new experiments have to be performed to establish the properties of these states. The available information is used to construct rotational bands in the neutron rich isotopes; these rotational bands give immediately values for the moments of inertia, which can be used for the interpretation of the strongly deformed states (Sect. II.3.). The dimers at lower excitation energies will often be mixed with states with normal deformation or even with shell model states. A study of the decay schemes and transition probabilities is thus needed.

Once the properties of the dimers are well established the properties of larger structures, chain states or polymers can be predicted. We discuss here only the chain states of carbon (^{12}C-^{16}C). The available information and the predictions of chain states in the carbon isotopes with A = 12, 13, 14, 15, 16 are summarized in Fig. 7.

In the next sections the available information of each isotope is discussed following the presentations in Figs. 1, 6, 7; there excitation energies are shown on an energy scale, where the thresholds for example for the $^{12}C^{*}_{O_2^+}$ configuration (or for $\alpha+\alpha$), and a number (x) of nucleons, are aligned horizontally. The additional binding introduced by the valence neutrons as indicated by diagrams ("configurations") is than

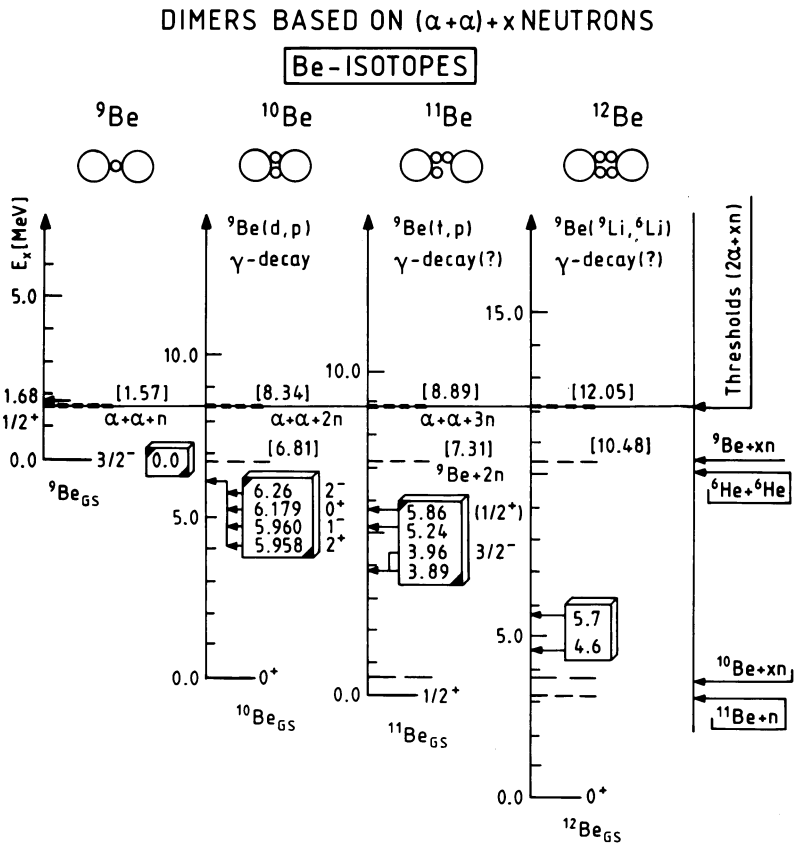


Fig. 1. Energy diagrams for dimers of the Beryllium isotopes. The excitation energies (and thresholds) are shown relative to the energy of the $(\alpha+\alpha+xn)$ threshold. Only the first possible isomeric two-center states are indicated (without rotational excitations) in boxes

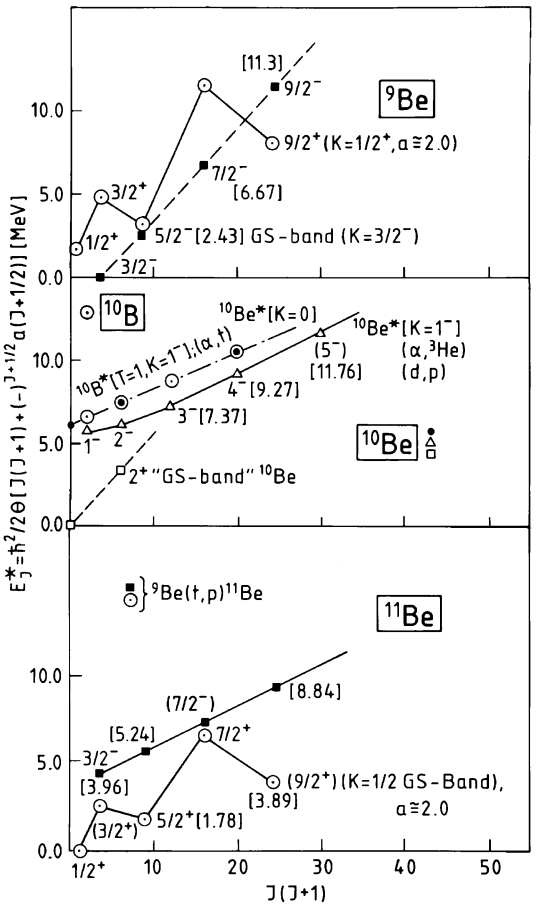
easily counted from this line. The expected excitation energy of the first chain state in each isotope is shown in a box with black corners. Other thresholds, which are important for the present discussion are shown as well, in particular those for (^{9}Be+$\alpha+xn$) and for (^{10}Be$^*+\alpha+xn$). All the other thresholds align in this representation and it is possible to discuss the decay channels of the chain states. Particularly strong clustering is in fact expected in the vicinity (i.e. also below) of the thresholds for cluster decay.

During the accomplishment of this work, two older references have been indicated to the author which contain similar ideas: i) Work by M. Seya, M. Kohno and S. Nagata [10] on the molecular orbital approach to the structure of ^{10}Be and ^{10}B; ii) The paper by D.H. Wilkinson on α-neutron rings and chains [11]. The first paper contains a similar but more quantitative discussion as [8]. The work of Seya et al. contains a detailed discussion of the molecular structure of the isotopes of Be and Boron. The assignment of configurations in the present paper is discussed in Chap. II.3, where the experimental information on the moments of inertia is summarised. In the paper by Wilkinson the groundstate of ^{10}Be, with a binding energy for two-neutrons of 8.5 MeV is used as a building block for more extended structures as opposed to the valence nucleon states at ca. 6 MeV in ^{10}Be* which are used in [8] and in the present work. A critique of both approaches will be given in the conclusions (Chap. IV).

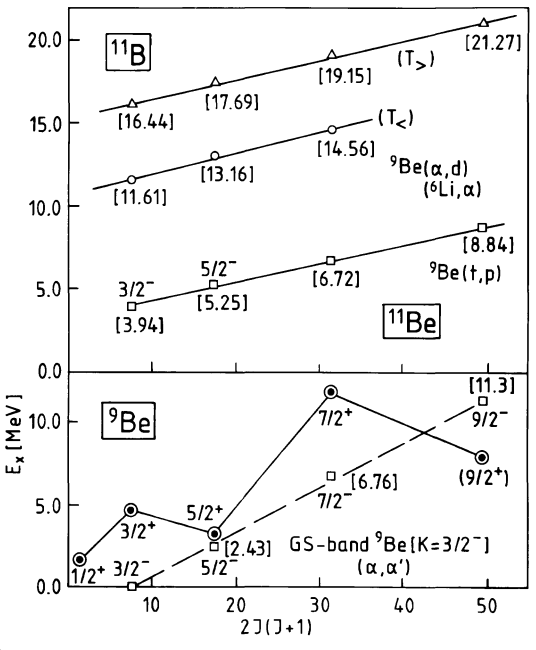
II Dimers

II.1 Beryllium isotopes

In the previous paper [8] we had shown that the cluster of states in ^{10}*Be* at ca 6.0 MeV excitation energy can be understood if the underlying structure is the two center $\alpha+\alpha$ potential with two valence particles in molecular orbits observed as basis states in ^{9}Be. The $3/2^-$ state of ^{9}Be corresponds here to the $K=3/2$, π^--binding orbital ($\pi_{3/2^-}$,g) (we use here the labeling of molecular orbital theory), the rotational band structure is illustrated in Fig. 2a; whereas the $1/2^+$ state corresponds to the $K=1/2$, σ-ungerade-binding orbital ($\sigma_{1/2^+}, u$), the $1/2^-$ state is described by the $K=1/2$, σ-gerade-anti-binding orbital ($\pi_{1/2^-}, g$). Thus *all* known states in ^{9}Be can be described by the valence nucleon basis states and the rotational bands built on these states (see Fig. 2). Using the Pauli-principle for two neutrons in these orbits (or a neutron and a proton in a $T=1$ state for ^{10}B) we obtain the well established four states in ^{10}Be with $J^\pi=0^+, 2^+; 1^-$ and 2^-, which will be the ground states of rotational bands also shown in Fig. 2a (the energies are in the box in Fig. 1). The ground state of ^{10}Be and its first excited state ($\Delta E(0^+-2^+)=3.36$ MeV) form an independent rotational band with a smaller deformation parameter (see Fig. 2 and the discussion below). The shape of ^{10}Be$_{GS}$ is actually much more compact compared to the negative parity states, as shown by Horiuchi [30]. The dimers have rotational excitations with a much larger moment of inertia ($\Delta E(0^+-2^+)=1.35$ MeV).



a



b

Fig. 2a, b. Excitation energies of states forming rotational bands of dimers based on $2\alpha + x$-nucleons. **a** for ^{9}Be, ^{10}Be, ^{10}B as function $J(J+1)$; **b** for ^{9}Be, ^{11}Be, ^{11}B as function of $2 \times J(J+1)$. Relevant reactions are shown in Figs. 3–6

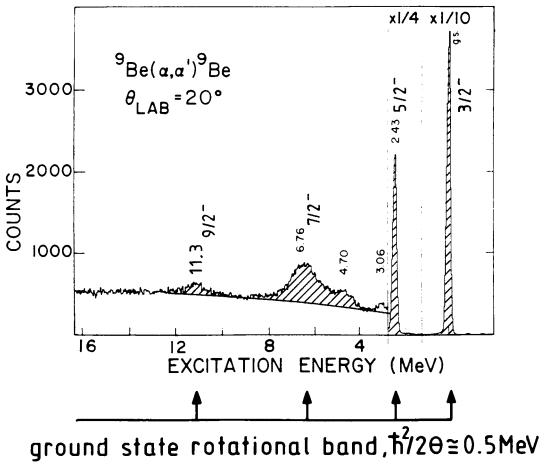


Fig. 3. Energy spectrum of the inelastic α-scattering on ^{9}Be, showing the population of the rotational band in ^{9}Be (adopted from [12])

Figure 1 shows the relevant information on binding energies including the heavier isotopes (^{11}Be and ^{12}Be). The excitation energies are shifted so as to have a common line for the relevant thresholds ($2\alpha + xn$) for all isotopes. The binding energy introduced by the addition of valence nucleons can thus be read from this line. Also the thresholds for the decay into other channels (with normal shapes) are indicated. We repeat here the evidence for rotational bands in ^{9}Be and ^{10}Be. This evidence is based mostly on inelastic α-scattering [12], (α, ^{3}He) reactions [12] as well as (d, p) reactions on ^{9}Be [14]. The strong population of states shows their direct relation to the ^{9}Be ground state, which has a large deformation. In Fig. 2 the different rotational bands which are obtained from the data are summarised and discussed at the end of Sect. II. In Fig. 3 we show the evidence for the rotational exitations in ^{9}Be from inelastic α-scattering [12]. For the rotational bands in ^{10}Be, Fig. 4 shows as an example the population of ^{10}Be states in ^{9}Be (d, p) and ^{9}Be (α, ^{3}He) reactions. The (α, ^{3}He) reaction is particularly selective at higher excitation energy; the very selective population of the $K = 1^-$ band is very conspicuous (the Q-values are very negative!). The same states are observed in the (d, p) reaction. The population of the excited 0^+ band is very weak, it may contribute to the observed peaks as well. In both cases the groundstate band 0^+, 2^+ (3.37 MeV), 4^+ (10.4 MeV ?, the position of this state is not clearly established) is populated with similar strength. A recent study of the ^{7}Li (α, p) ^{10}Be reaction by Hamada et al. [13] shows prominent populations of narrow states at higher excitations of 10–15 MeV, in this case the $K = 2^-$ states are all strongly excited.

The evidence for states in ^{11}Be corresponding to dimers can be obtained by looking into the population of states in two neutron transfer reactions like the ^{9}Be(t, p) reaction (Fig. 5a) or the (^{13}C, ^{11}C) reaction (Fig. 5b). Whereas these reactions should strongly populate such states, *they should not be observed in the one neutron stripping on ^{10}Be*, because the ground state does not have the appropriate structure. This is actually the case. If there is a rotational band structure based on the ^{11}Be, $J^\pi = 1/2^+$ ground state we must expect a similar drastic Coriolis coupling effect as for the $1/2^+$ band in ^{9}Be (see Fig. 2a). From the spin assignments given

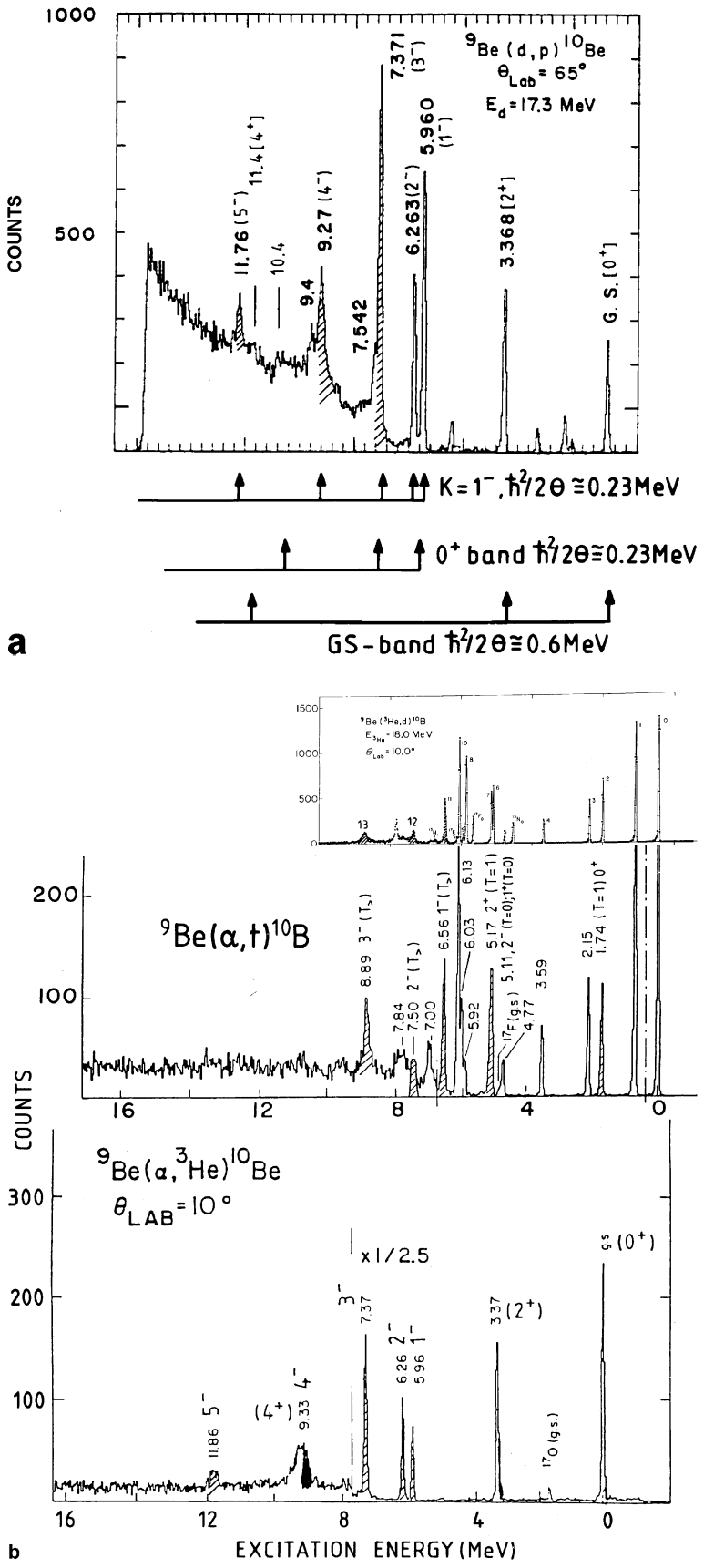


Fig. 4a,b. Single neutron stripping reactions on ^{9}Be populating strong single particle states in ^{10}Be, in particular the rotational bands of the dimers (adopted from [12, 14]). **a** the ^{9}Be(d, p)reaction; **b** the ^{9}Be(α, ^{3}He)reaction (in comparison with the reactions (α, t) and (^{3}He, d) leading to states in ^{10}B). The spectra are scaled and shifted so as to align the $T = 1$ states observed in ^{10}Be and ^{10}B

in the literature and from the population of the ^{11}Be states in the ^{9}Be(t, p) reaction (Fig. 5a) I can suggest the following sequence of states for the $K = 1/2^+$ band ($1/2^+$, 0.0 MeV; $5/2^+$, 1.78 MeV; $3/2^+$, 2.69 MeV; $9/2^+$, 3.88 MeV and $7/2^+$ at 6.510 MeV); they are shown in Fig. 2a together with the K = $3/2^-$ band starting at 3.96 MeV.

The (t, p) reactions shows very strong populations in the excitation region of 3.8–7.0 MeV [15, 16]. The angular distributions are characterised by $\ell = 1$ and 2 transfers. The most prominent narrow peaks seen in Fig. 5 are at 3.89 MeV, 3.96 MeV, 5.25 MeV, 6.72 MeV and 8.84 MeV. In the heavy ion transfer reaction (Fig. 5b) the same states are observed, with possible candidates for higher spin members.The population of these states has not been reported in (d, p) reactions. These states are narrow and will possibly show γ-transitions, which have not yet been studied. If the strongly excited state at 3.96 MeV is considered to be the lowest state of a dimer, the binding energy of three covalent valence neutrons amounts to approximately 5 MeV as expected from the molecular configuration; the other states align perfectly in a rotational band with $K = 3/2^-$ as shown in Fig. 2b (the ^{9}Be+2n threshold is 1.47 MeV above the next strongly excited state at 5.25 MeV). These states show a strong relation to the most strongly populated states in the ^{9}Be (d, p) ^{10}Be reaction [14], which are the negative parity states with $J^\pi = 2^-, 3^-, 4^-$ at 6.26, 7.37 and 9.27 MeV. Thus these states observed in ^{11}Be can be grouped into a rotational band with a moment of inertia similar to or slighty larger than that of the ^{10}Be-dimer moment of inertia. These suggestions for rotational bands in ^{10}Be and ^{11}Be are shown in Fig. 2, the dimers have larger moments of inertia than the ^{10}Be gs-band by almost a factor 2; their structure is discussed further below.

The state at 3.96 MeV has a total binding energy for three neutrons of 4.93 MeV in the covalent molecular bond, suggesting that the extra two neutrons contribute approximately 3.25 MeV of binding energy (relative to ^{9}Be) or 1.65 MeV per valence neutron. As discussed in Sect. II.3. the neutron-pair is coupled to spin 0^+ in the (t, p)reaction and populates states with the $(\sigma 1/2^+, u)^2$ configuration related to the ^{10}Be* ($K = 0$ band).

For *^{12}Be* there is little information available so far. If we fill 4 neutrons into the molecular orbitals ($\pi_{3/2^-}$) and ($\sigma_{1/2^+}$) we expect a rather compact shape for ^{12}Be similar to $^{10}\mathrm{Be}_{0^+,gs}$ due to the $(\sigma_{1/2^+})^2 \times (\pi_{3/2^-})^2$ configuration. In the two-center correlation diagram, see [8], the $\sigma_{1/2^+}$, configuration has a slope with an *increasing binding energy for larger deformation* (or two-center distance). Thus the fact that the moment of inertia is larger for the $^{12}\mathrm{Be}_{GS}$-band ($\Delta E(2^+ - 0^+) = 2.10$ MeV compared to the $^{10}\mathrm{Be}_{0^+}$-band can be understood. The ^{10}Be(t, p) two-neutron transfer reaction [9] will thus primarily populate this ground state band, and some states with an assignment 1^- for the $(\pi_{3/2^-})^2 \times (\sigma_{1/2^+}) \times (\sigma_{1/2^-})$ configuration, which could be for example the state at 2.70 MeV in ^{12}Be [9]. Starting with values of the n-binding energies obtained for ^{11}Be, and assuming that the fourth neutron can contribute approximately 1.5 MeV (like in ^{9}Be) the dimers in ^{12}Be could be observed from excitation energies of approximately 4.5 MeV (as indicated in Fig. 1). Thus rotational bands with moments of inertia similar to those observed in ^{10}Be* and ^{11}Be* (see Fig. 2) and larger are expected starting at an excitation energy of appr. 4.5 MeV. These could possibly be observed in inelastic scattering on ^{12}Be using a radioactive beam of ^{12}Be as done in [17], or in a ^{9}Be(^{9}Li, ^{6}Li)^{12}Be reaction and other three neutron stripping reactions on ^{9}Be. Similar to the case of ^{11}Be the rotational bands are expected to extend to high spin values because of the large covalent binding energy due to the presence of 4 valence nucleons.

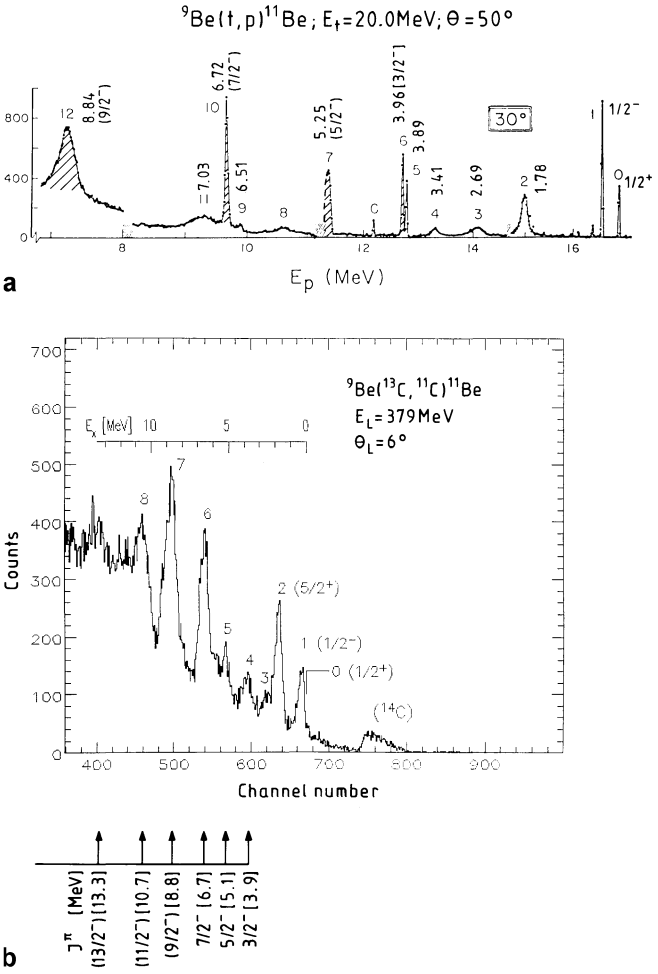


Fig. 5a, b. Spectra of two neutron stripping reactions on ^{9}Be populating strongly the rotational band of the dimer in ^{11}Be. **a** the (t, p)reaction (adopted from [14, 15]). The *dashed line* at 5.25 MeV indicates a not separated contaminant; **b** the (^{13}C, ^{11}C)reaction (Thesis M. Lucke-Petsch, FU-Berlin 1990). The positions of the possible members of a rotational band are indicated by *arrows*

Similar rotational band structure of excited states in the berylium isotopes is in fact also obtained by Kanada-Enyo and Horiuchi [30] in close agreement with the present result.

II.2 Boron isotopes

For *^{10}B* we start with the basic structure of ^{9}Be. By adding a proton we can form in ^{10}B the $T = 1$ analogue states of the four states in ^{10}Be*, the residual interaction between the two nucleons is weak in the $T = 1$ states. The analogues of these four states have been identified in ^{10}B in the excitation

energy region of 6.56–7.5 MeV. Their population has been observed as p+^{9}Be resonances [9]. Their population in (^{9}Be+ p) correlations used in high energy heavy ion collisions to determine temperatures of hot nuclear systems is discussed in [8].

Adding the proton to form the $T_<(T = 0)$ states (rotational band-heads) we will have again four states as in the ^{10}Be* case, and using the generalised Pauli principle the corresponding states which will form rotational bands are now with $J^\pi = 1^+, 3^+$ for $(\pi_{3/2^-})^2$ and 1^-, 2^- for the $(\pi_{3/2^-}) \times (\sigma_{1/2^+})$ configurations respectively. Now the nucleon-nucleon interaction is slightly more attractive and we expect that the corresponding states are lowered by ca. 2 MeV relative to the $T_>$ states.

The direct observation of states with dimer structure in ^{10}B is possible via one proton stripping reactions on ^{9}Be. The spectra of (^{3}He, d) and the (α, t) reactions (Refs. 18 and 12 respectively) are shown together with the one-neutron stripping reaction in Fig. 4b. The spectra are scaled and shifted in such a way, that the $T = 1$ states (^{10}Be and ^{10}B) are aligned. Particularly strong excitation of states is observed in the excitation energy region of 5–7 MeV. These states must be isospin analogues ($T_>$) and antianalogues ($T_<$) of the dimers in ^{10}Be. There are some larger shifts in energy for the analogues at the ^{9}Be + p threshold. We find by comparison of the (^{3}He, d) spectrum (numbered peaks) with the (α, t) and (α, ^{3}He) reactions for the $K = 1^-$ band states at 6.56 MeV (1^-) (state Nr. 11), at 7.50 MeV (2^-, Nr. 12) and at 8.89 MeV (3^-, Nr. 13). By comparing the ^{10}Be and ^{10}B spectra in Fig. 4b we further can select the strongly excited $T = 0$ states, which are partially known as resonances in α+^{6}Li: a resonance with $J^\pi = 3^+$ at 4.77 MeV, $J^\pi = 2^-$ at 5.11 MeV (state Nr. 6) and $J^\pi = 1^+$ at 5.10 MeV (Nr. 7), a (2^+)-state at 5.92 MeV (Nr. 8) and a state at 6.13 MeV (state Nr. 10, possibly a doublet). In [18] the authors report on the states Nr. 8, 9, 10, 11 "the extracted spectroscopic strength for $\ell = 1$ are much too large to be accommodated in any shell model description of ^{10}B". The same authors report that the inability of DWBA to fit the angular distributions remains a puzzle". I therefore suggest that the states Nr. 8, 10 and 11 are 1^- and 2^- and 3^+ states with dimer structure. The strongly excited states 8, 10 in the region of 6–6.5 MeV are the $T_<$ states with the spin 2^- and 1^- are populated with similar strength as the $T_>$ states (6.56 MeV, 1^-). In [18] the same angular distributions are observed for the states at 6.13 MeV and 6.56 MeV. Both are expected to be strongly influenced by coupled channel effects.

For the ^{11}B nucleus there are detailed studies of (^{3}He, p) and (α, d) reactions [19] on ^{9}Be, where the first can strongly populate states in ^{11}B with $T_> = 3/2$ and $T_< = 1/2$, and the second populates only $T = 1/2$ states. The $T_>$ states are analogues of the states in ^{11}Be already discussed. The state ($T = 3/2$) at 5.24 MeV of ^{11}Be will be located at approximately 17.50 MeV in ^{11}B. In fact strong transitions to states with very small width have been observed in the (^{3}He, p) reaction at high excitation energy. The strongest transitions in the work of [19] are in direct correspondence to the strong transitions in the ^{9}Be(t, p) reaction. I have made a classification into $T_<$ and $T_>$ states by comparing the (α, d) and (^{3}He, p) reactions. In Fig. 2b the corresponding energies are placed as rotational bands together with the result for ^{11}Be as a possible assignment to rotational band structure. The result is a surprisingly good agreement with the expected moments of inertia of all 3 isomeric bands (^{11}Be, ^{11}B ($T_<$ and $T_>$)).

For ^{12}B a study of the ^{9}Be(^{7}Li, α)^{12}B reaction at E_L = 20 MeV [20] has been reported. Very narrow states are observed at an excitation energy of 12.77 MeV and 13.33 MeV, which can have the expected structure of the dimers with one proton and three neutrons.

A survey of the possible structures in the boron isotopes in a similar scheme as in Fig. 1 is shown in Fig. 6. As a last isotope ^{13}B is shown, which can be constructed from the ^{12}Be*-dimer by adding a proton. In this nucleus again $T_>$ and $T_<$ states of the same structure can be formed. Their overlap with the normal (shell model) shapes should be very small. Thus these states in ^{12}B and ^{13}B are expected to be shape isomers with γ-transitions from high excitation energy; the observation of these γ-rays in these nuclei can thus be considered as a possible test for the proposed structures.

II.3 Moments of inertia and assignment of configurations to states in the isotopes ^{10}Be and ^{11}Be

The moments of inertia, θ, of the dimers are of course the most relevant points of information for the discussion of their shapes (in particular also the γ-ray transition probabilities between these states if they are observed). In Fig. 2 the excitation energies of states in the Beryllium and Boron isotopes are plotted in a way to place them in diagrams of rotational bands. I use here the simplest form for the rotational energy for a nucleus with moment of inertia, θ, as function of angular momention J : $E(J) = \hbar^2/2\theta[J(J+1) - K(K+1)]$ for different bands with $K > 1/2$. For $K = 1/2$, I use, incorporating the effect of Coriolis de-coupling [22]: $E(J) = \hbar^2/2\theta[J(J+1) + (-)^{J+1/2}\ a\ (J+1/2)]$; here, a, is the Coriolis decoupling parameter. The energy in this case can also be written as $E(J) = b[(J + (1 + \sigma a/2)]^2$, where σ is the signature, $(-)^{J+1/2}$, defining the reflection symmetry of the intrinsic state.

For the ^{9}Be* ($K = 1/2^+$) band I obtain the values $b(= \hbar^2/2\theta)$ = 0.35–0.46 [MeV] and $a \cong 2.0$; similar values are obtained for the $K = 1/2$ band of ^{11}Be shown in Fig. 2a (b is smaller $\sim$ 0.2–0.3 MeV), but a again is $\cong$ 2.0. This is a remarkable fact which points to strong structural similarity of the bands. Because a is given by $(-)^{j+1/2} \times (j + 1/2)$ (if only one orbital is considered) the fact must be interpreted that the original spin of the orbit, whose K-quantum number is considered, is $j = 3/2$. The Coriolis decoupling parameter in the two cases gives thus a strong support for the two-center correlation diagram interpretation of these states; the nucleons are in $K = 1/2$ orbits of the $P_{3/2}$ configurations of the separated nuclei, see also [8]).

Further we have the following results for the moments of inertia:

a) For the ^{9}Be, $K = 3/2^-$ and ^{10}Be$_{GS}$ bands (and ^{8}Be) we observe values for $\hbar^2/2\theta$ in the range of 0.5–0.6 MeV. If we use a simple formula for the moments of inertia, (μR^2), we have for $\mu = 2.0$ and $\mu = 2.22$ and $R = r_o(2 \cdot (4)^{1/3})$ a radial distance of the two masses

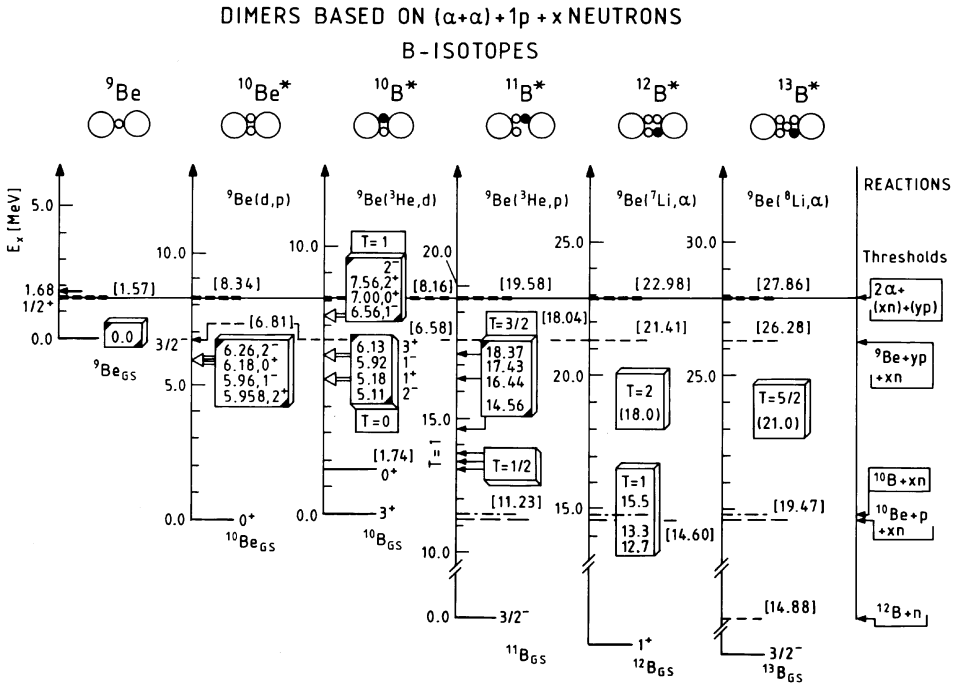


Fig. 6. Same as Fig. 1 for isotopes fo Boron

with $r_o \approx 1.3fm(2 \cdot A^{1/3})$. Thus these states still have a rather compact shape.

b) For the rotational bands of the excited dimer configurations in ^{10}Be*, ^{11}Be, ^{10}B and ^{11}B we observe values for $\hbar^2/2\theta$ of 0.23–0.25 MeV. This means that the distance between the two α-particles in the two-center diagram must be much larger and corresponds to $R \approx 4$ fm in the two center correlation diagram shown in [8].

Using this information from the rotational bands we resume the discussion of the structure of those states, which are supposed to have the structure of a dimer. For this purpose we use the two-center correlation diagram shown in [8]. The two lowest states at an internuclear distance of $R \approx 3.5$ fm for the two centers are the $(\pi_{3/2^-}, g)$ and $(\sigma_{1/2}, u^+)$ configurations. The $(\sigma_{1/2^+}, u)$ configuration crosses the $(\pi_{3/2^-}, g)$ at ca. $R \approx 4$ fm (see also [10]); it has a decreasing slope (increasing binding energy) for $R > 4$ fm, whereas the $(\pi_{3/2^-}, g)$ configuration shows an increase of the binding energy by approaching smaller distances ($R < 4$ fm) with a possible minimum at $R \approx 3$ fm.

As suggested by Seya et al. [10], the ground state of ^{10}Be (0^+) can be assumed to be the $(\pi 3/2^-, g)^2$, 0^+ configuration. The large (8.4 MeV) binding energy for 2 neutrons can be explained by the large overlap of the two paired orbits, which produces a *strong gain* in energy *by pairing and by a further decrease of the distance R* which is balanced by the repulsive α-α potential at small distances. The ^{10}Be ground state has a rather compact shape (see [30]).

The next excited state, the 2^+ state in ^{10}Be at 3.36 MeV, will be considered here as a purely radial (vibration) or rotational excitation (in contrast to [10]). The pick-up reaction [21] ^{10}Be (d, t) ^{9}Be shows in fact a dominant excitation of the $3/2^-$ ground state of ^{9}Be and about 1/20 of this cross section for the $1/2^+$ (1.68 MeV) state. We may thus give the following interpretation of the valence neutron structure of the two 0^+ states in ^{10}Be.

$$|^{10}\mathrm{Be}_{0^+}(\mathrm{GS}) >= (2.19)^{1/2}(\pi_{3/2^-} g)^2_{0^+} + (0.01)^{1/2}(\sigma_{1/2^+} u);$$

$$|^{10}\mathrm{Be}^*_{0^+}(6.26\ \mathrm{MeV}) >= a_0(\sigma_{1/2^+} u)^2_{0^+}$$

The other states at ca. 6 MeV energy in ^{10}Be than must have the following configurations:

$$|^{10}\mathrm{Be}_{2^+}(5.958) >= a_1(\pi_{3/2^-} g)^2_{2^+},$$

$$|^{10}\mathrm{Be}_{1^-}(5.960) >= a_2[(\pi_{3/2^-} g) \otimes (\sigma_{1/2^+} u)]_{1^-},$$

$$|^{10}\mathrm{Be}_{2^-}(6.263) >= a_3[(\pi_{3/2^-} g) \otimes (\sigma_{1/2^+} u)]_{2^-}.$$

These states are all strongly deformed with corresponding rotational bands. Seya et al. [10] indeed show how these states can be represented by Slater determinants of molecular orbitals and their energies can be calculated assuming these assignments. Their results are supported by the moments of inertia for the $K = 0^+$ and $K = 2^-$ bands presented here, which are *larger* than the ^{9}Be moments of inertia by almost a factor 2. The population of $(\sigma_{1/2^+}, u)$ molecular orbitals induces an increase of the two center distance because of the energy gain associated with an increasing R for these orbits. At smaller distances the "antibinding" orbital $(\sigma_{1/2^-}, g)$ would be the next orbit to be occupied, however, this is considered only for the heavier isotopes.

The second excited 2^+state appears much higher, because of the unfavourable overlap of the two $(\pi_{3/2^-} g)$-orbitals if they are coupled to spin 2^+. There should be in fact a further rotational band based on this state with states of 3^+ and 4^+ in the region of 7–9 MeV, which is possibly covered by the other states. This assignment is also supported by the observation of corresponding states in ^{10}B at the appropriate binding energies.

Concerning the configurations in ^{11}Be I can take the following assignments:

$$|^{11}\mathrm{Be}^*_{3/2^-}; (3.94) >= (\sigma_{1/2^+}, u)^2_{0^+} \otimes (\pi_{3/2^-} g)$$

$$\left|^{11}\mathrm{Be}_{1/2^+};\mathrm{GS}>=(\pi_{3/2^-}g)^2_{0^+}\otimes(\sigma_{1/2^+}u)$$

The latter configuration shows a less pronounced energy lowering of the $(\pi_{3/2^-}g)^2_{0^+}$ configuration than in the $^{10}\mathrm{Be}^+_0$ ground state; we attribute this to the influence of the $(\sigma_{1/2^+}u)$ configuration of the third neutron which attains its minimum energy at larger distances, off setting the effect of the $(\pi_{3/2^-}g)^2$ configurations.

The negative parity, $K = 1/2$, state is suggested to have the configuration

$$\left|^{11}\mathrm{Be}_{1/2^-};0.32\ \mathrm{MeV}>=(\sigma_{3/2}g)\otimes(\sigma_{1/2^-}g)^1.$$

This configuration, $(\sigma_{1/2^-}g)$, appears as the next at a smaller two-center distance of ca. 3 fm, where the minimum for the $^{10}\mathrm{Be}^+_0$ ground state is expected. This state has a spectroscopic amplitude for (in the deformed shell model) for the $p_{1/2}$ structure determined from the ^{10}Be (d, p)reaction of $(0.63)^{1/2}$ and is thus related to the more compact shape of the $^{10}\mathrm{Be}^+_0$ ground state.

For the *^{12}Be* nucleus, the information is rather scarce, however the most likely configuration for the ground state is (as already discussed in Sect. II.1.):

$$\left|^{12}\mathrm{Be}^+_0,\mathrm{GS}>=(\pi_{3/2^-}g)^2_{0^+}\otimes(\sigma_{1/2^+}u)^2_{0^+}\ .$$

The moment of inertia is expected to be larger than for $^{10}\mathrm{Be}^+_0$ because of the influence of the $(\sigma_{1/2^+}u)^2$ configurations which favours larger two-center distances. The first 2^+ state is observed at 2.1 MeV ($\hbar^2/2\theta \approx 0.35$ MeV), a candidate for the 4^+ state is at 5.7 MeV [23]. If we couple to the spin 2^+; $(\pi_{3/2^-}g)^2_{2^+}$, an excited band in ^{12}Be should be expected at an energy of approximately 5.0 MeV as suggested in Fig. 1. In the ^{12}C (^{14}C, ^{14}O) ^{12}Be reaction [23] the strongest state observed is at $E_x = 4.56$ MeV which we attribute however, in analogy to the population of states in the ^{14}C (with the same neutron number N=8, but with spherical shape) to a 3^- configuration $[(p_{3/2})^{-1}\otimes(d_{5/2})^{1}]_{3^-}$; at $E_x = 2.70$ MeV a weakly populated state appears which very likely has the configuration $[(\sigma_{1/2^+}u)^1\otimes(\sigma_{1/2^-}g)^1]_{1^-}$ (with $J^\pi = 1^-$), with the two lowest states of ^{11}Be as principal basis configuration.

The present systematics of the moments of inertia is strikingly confirmed by the calculations of Kanada-Enyo and Horiuchi [30] using the antisymmetrised molecular dynamics (AMD) approach. In this approach no model assumptions are made. Clustering is obtained in a natural way and rotational bands are obtained for the strongly deformed shapes. Similarly to the discussion in [8], the AMD calculations reflect the properties of the two-center correlation diagram in their densities which correspond to those of the $(\pi_{3/2})$ and $(\sigma_{1/2})$-orbitals.

III Isomeric chain states of the carbon isotopes

III.1 ^{12}C

The excited state of ^{12}C with spin parity 0^+ at 7.66 MeV is just 288 keV above the threshold for the decay into 3α-particles at 7.366 MeV. The threshold relative to the decay into ^{8}Be + α is 196 keV. Its width is determinated by the α-decay ($\Gamma\alpha = 8.3 \pm 1.0$ eV), the γ-decay branch has been measured and is $\Gamma\gamma/\Gamma\alpha = 4.13 \cdot 10^{-4}$ [9]. This state is also well known for its importance in the stellar burning process to bridge the gap from ^{4}He to ^{12}C; nucleosynthesis has to go through the 0^+_2 resonance in ^{12}C*. This state is used as a reference point in Fig. 7 for all carbon isotopes. Because of its peculiar properties it has been associated with α-particle cluster structure by Brink [3], and by many authors [4], in particular in the form of an α-particle chain [5]. This description is not completely uncontested, DeTakacsy and Das Gupta [24] for example argue that the bending mode for the three α-particles may be an important part of the wave function. Therefore it is often suggested that the 0^+_2 state at 7.66 MeV and the 0^+_3-state at 10.3 MeV are mixed to a certain degree. The bending modes of the pure α-particle chains are, however, certainly restricted once the valence-neutron bonds are added (if they correspond to a molecular covalent-binding configuration).

III.2 ^{13}C

The threshold in ^{13}C for the decay into ($^{12}\mathrm{C}^*_{0^+_2}+n$) is at 12.6 MeV. Using the binding energy of the neutron in the two-α-particle system (^{9}Be) we predict an isomeric state as a chain state with spin $3/2^-$ at ca 11.04 MeV. Another important threshold is for (^{9}Be + α), which is situated at 10.65 MeV. We can thus predict, that coupling of ^{9}Be with an α-particle should give a state at ca 11.0 MeV just above the ^{9}Be + α-threshold. Similar to the case of the 3α-state in ^{12}C, which is very close to the threshold, we expect a narrow state in ^{13}C. We find in fact in the compilations of Ajzenberg-Selove a state at 11.08 MeV which is very narrow and which decays by γ-emission (!) as well as n-and α-emission. This state in ^{13}C* at 11.08 MeV must be the corresponding state to the chain state in ^{12}C.

Population of this state and possible other states relevant for the chain configuration should be observed in various reactions starting with ^{9}Be as target. Davids [26] finds a narrow resonance ($\Gamma << 5$ keV) in the ^{9}Be(α, n_0) reaction at an energy $E_\alpha = 0.6$ MeV i.e. at ^{13}C* (11.08 MeV). The (^{6}Li, d) and (^{7}Li, t) reactions at $E_{\mathrm{Li}} = 23.8$ MeV on ^{9}Be studied by Ogloblin et al. [27] populate (among others) two states very strongly at 10.8 MeV and 12.0 MeV. Their angular distributions are typical for $\ell = 0$ behaviour with maxima at $\theta_{\mathrm{CM}} = 0°$. The absolute energy calibration of the solid state-counters was not better than 200–300 keV in these cases. Because there are no other states seen in this work with a large α-width in the vicinity of several MeV, we can assume that the same state is populated as in the α+^{9}Be resonance work, namely the 11.08 MeV state. The next state at ca 12 MeV is 1.2 MeV higher and much wider; it is very likely that this state corresponds to a rotational excitation with the same configuration (or to the $1/2^+$ state of the ^{9}Be substructure in the ^{13}C chain?).

The structure of the ^{13}C chain is not reflection symmetric; therefore a positive and negative parity band, which are slightly split as in the case of the excited 0^+ band in ^{20}Ne can be expected. They could be observed as resonances in α+^{9}Be scattering. States corresponding to the (^{9}Be*(1/2$^+$, 1.68 MeV) + α) configuration are expected to be observed as well.

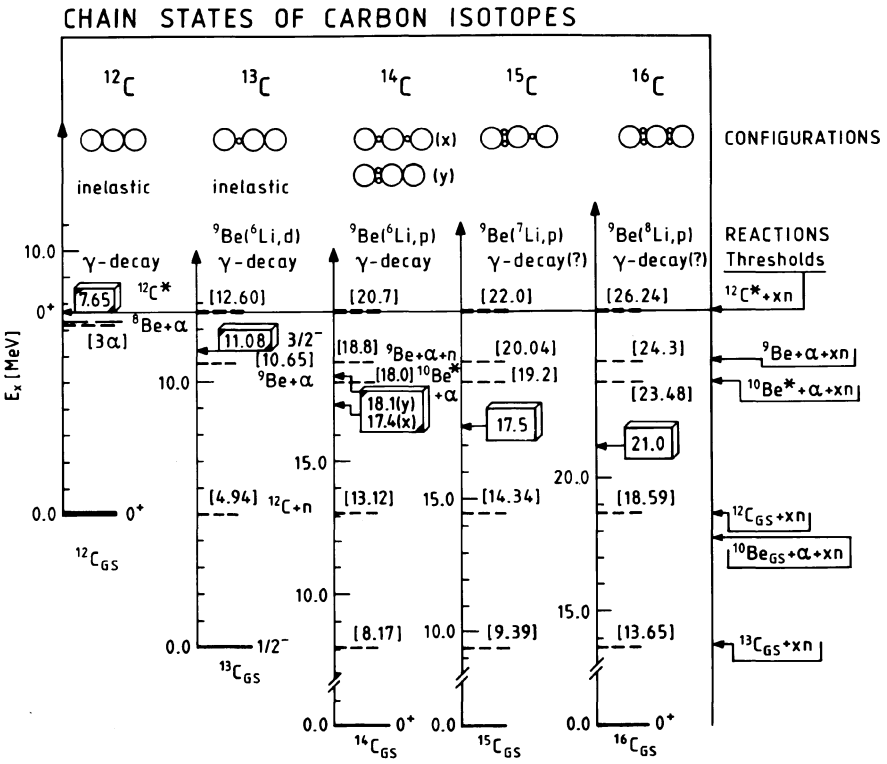


Fig. 7. Energy diagrams of molecular chain states in carbon isotopes. The energy scales are aligned to the same level for the decay threshold into 3α-particles and x-neutrons. The binding energy for the isomeric chain states can be read from this common reference line. Isomeric states are indicated in boxes with black corners, if properties (excitation energies, populating reactions) etc. are known. Predictions for rotational bandheads are given in boxes without black corners

III.3 ^{14}C

The threshold in ^{14}C for the ^{12}C*+$2n$ channel is quite high – at 20.7 MeV (see Fig. 7). Similarly the threshold for the configurations corresponding to the "unbound chain" (^{10}Be*+α) is high – at 18.0 MeV. We may thus expect resonances in the ^{10}Be* + α system just above 18.0 MeV; these would correspond to a chain state where the two valence neutrons are kept in one covalent bond (configuration "Y" in Fig. 7). We expect a series of 4 states connected to the group of states at 6.0 MeV in ^{10}Be* (0^+, 2^+, 1^-, 2^-) ranging up to 19.0 MeV. There is a clear possibility to observe γ-decay branches from and between those states just like for the 0_2^+ in ^{12}C, because the (^{9}Be+$\alpha+n$) channel is closed, other channels like (^{12}C+$2n$) or (^{13}C+$1n$) imply a very strong structural change; this observation would emphasize the property of a shape isomer. There are also interesting possibilities to study the relevant states of ^{14}C as $T = 1$ states of ^{14}N. Actually we find in the compilation of Ajzenberg-Selove [25] at an energy of 20.0–22.5 MeV, $T = 1$ states in ^{14}N with spins of 2^+ and 1^- which have γ-decay branches, and which could be analogues of chain states in ^{14}C. If we consider the more compact configuration of the $^{10}Be_0^+$ ground state, we will have to discuss states in the vicinity of the threshold of ^{10}Be+α at 12.012 MeV. These would also be observed in (^{7}Li, d) and (^{6}Li, p) reactions on ^{9}Be. These reactions have to be studied at higher energy to enhance the direct process and with good energy resolution.

The configuration where the valence bond is distributed equally (configuration X) is expected to come below the configurations (Y), because of the stronger binding effect for each valence neutron (2 × 1.66 MeV), we predict this state at ca 17.4 MeV. Reactions which could populate these states are the ^{9}Be(^{6}Li, p) reaction studied by F. Ajzenberg-Selove [28] and the ^{9}Be(^{7}Li, d) reaction [29]. The configuration (X) (it is reflection symmetric) is expected to have spin 0^+ and a rotational band sequence of 0^+, 2^+, 4^+. Again γ-decays from these states are a possible consequence of the isomeric structure, because channels with the same structural properties are closed. Figure 7 shows the relevant thresholds in ^{14}C.

III.4 ^{15}C

For ^{15}C with three valence neutrons, the isomeric structure can be based on the ^{10}Be* dimer and the ^{9}Be valence bond. Starting from the threshold for (^{12}C* + $3n$) at 22.0 MeV we can predict that states with this structure should be observed from approximately 17.5 MeV excitation energy. The group of 4 states of the ^{10}Be*-dimer with spins 0^+, 2^+, 1^-, 2^- will couple with the spin of $3/2^-$ of the valence neutron in the ^{9}Be bond and will form a dense cluster of states in this excitation energy region. Again states based on the more compact $^{10}Be_0^+$ groundstate could be formed, these would have to be expected at an energy of 11 MeV. Information on this nucleus is rather scarce in this energy regime.

Possible reactions, which can populate these states are the ^{9}Be(^{7}Li, p) reaction, inelastic scattering and final state interactions of neutron-rich fragments: e.g. ^{9}Be + 6He. More reactions can be conceived with radioactive (neutron-rich) beams.

III.5 ^{16}C

The ^{16}C chain isomer will consist of two bonds defined by the ^{10}Be* dimer.

The total binding energy of the two bonds is expected to be approximately 5 MeV ! The corresponding states are thus well below the thresholds for the decays into channels with the same structural properties (^{9}Be, ^{10}Be*). These states should have a small width for the decay into α's and neutrons and again the existence of γ-decays is a very interesting possibility. We actually expect a larger group of states due to the coupling of the two ^{10}Be* dimer-multiplets and the corresponding rotational excitation of the chain states. The more compact bond of the ^{10}Be$_{0}^{+}$ (GS) would give states in the region of ca. 12 MeV, which would bring the band head below the ^{13}C + 3-neutron threshold. These chain states can possibly be populated directly using radioactive beams, for example in a ^{9}Be(^{8}Li, p) reaction. Higher excitations of the chain will eventually decay into ^{10}Be* + ^{6}He.

III.6 Heavier carbon isotopes

It is interesting to speculate on the properties of the shape isomers (chain states, or polymers) of heavier isotopes in particular of ^{18}C and ^{20}C. The former will be based on the structure of ^{11}Be and will have two times 3 valence neutrons at the covalent bond positions; the total binding will be 10 MeV! A very stable chain configuration is expected at an excitation energy of a ca. 20 MeV. Similarly in ^{20}C such isomeric chain states must be expected, which are related to the excited states of ^{12}Be.

The extrapolation of these structures to chain states (polymers) in oxygen isotopes like ^{22}O up to ^{28}O is quite obvious. Rather exotic neutron-rich beams will be needed to populate such chain states. Recent calculations of Horiuchi et al. [30] indicate that for very neutron rich isotopes already the ground states show a strong clusterisation into α-particles and valence neutrons.

III.7 Conclusions

The present search on information on nuclear dimers and isomeric chain states (polymers) in neutron rich isotopes of light nuclei is based on the covalent molecular orbitals of neutrons in ^{9}Be and ^{10}Be. Some preliminary evidence for the existence of such states exists. In particular the level scheme of the Be-isotopes can almost exclusively be explained in this approach. This conclusion is also the main result of the work of Seya et al., however, the detailed information on moments of inertia was not used. It is conceivable that in the spirit of the heavier dimers like $^{10-12}$Be discussed here, isomeric chain states can be expected in ^{14}C, ^{15}C, ^{16}C as well as in ^{17}C, ^{18}C, ^{19}C and ^{20}C, because up to four neutrons can be placed into the covalent bonds with a binding character (using the information on the neutron-bonds of the relevant states of ^{10}Be*, ^{11}Be* and ^{12}Be* we find for example a binding energy relative to the $3\alpha+6n$ threshold of two times 4.9 MeV for two $3n$-bonds in ^{18}C). We may expect as a general rule that clusterisation and formation of chains will occur for very neutron rich nuclei close to the threshold to α-particle substructures (see [8]). The formation of such structures via condensation from a nuclear medium consisting of α-particles and neutrons may be expected to occur; the formation of chain states of neutron rich nuclei can possibly be observed in central nuclear collisions using very neutron rich beams (and targets), following the speculative suggestion of Wilkinson [11] on the formation of nuclear rings. Wilkinson started with the idea to form rings of α-particles bound by the ($\pi_{3/2^-}g$) bonds in order to have a possible explanation of the "Anomalon" phenomenon. However, as shown in the present work, the groundstate of ^{10}Be does not show the necessary properties because it is to compact.

The neutron rich chain states have corresponding isospin analogue states ($T_>$ and $T_<$ states) in the neighbouring isotopes, where one neutron is replaced by a proton. These states again may have detectable γ-ray transition probabilities, in particular to their isospin partners ($T_<$ states) with the same structure (see the case of the boron isotopes), but placed at lower excitation energy.

Also the question must be raised how accurately energies of these states can be calculated. There appear mainly two approaches, the molecular dynamics approach of Kanada-Enyo and Horiuchi et al. [30] or an explicit ("geometric") multi-center calculation with a LCNO basis in analogy to molecular structure calculations of atomic polymers.

These states offer fascinating possibilities for studies using radioactive beam facilities in particular for γ-spectroscopy if the γ-branches can be measured. For this purpose coincidences of γ-rays with ejectiles from binary reactions with the detection of both fragments must be studied [31]. Of particular interest are beams of neutron rich Lithium, Beryllium and Boron isotopes as well as neutron rich oxygen and neon isotopes. For reactions like (^{11}Be, α), (^{20}O, ^{12}C) or (^{9}Li, α) and (^{9}Li, p) the γ-ray transitions must be studied in coincidence with the lighter emitted particles using a silicon ball inside a γ-detector array [32]. Also various compound reactions using heavier projectiles will be very usefull for the study of γ-transitions by using charged particle-γ-coincidence techniques.

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