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CORRECTION

Correction to: Stable and convergent fully discrete interior–exterior coupling of Maxwell's equations

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Abstract
We correct a sign error in the paper [3] by the second and third authors, noted by the first author. This sign error in the definition of the Calderón operator has no effect on the theory presented in [3], but it does affect the implementation of the proposed numerical method.

Keywords Transparent boundary conditions · Boundary integral equations · Calderón operator

Mathematics Subject Classification 35Q61 · 65M60 · 65M38 · 65M12 · 65R20

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Introduction

In [3] we present a time-domain boundary integral formulation of an interior–exterior coupling of Maxwell's equations, with the help of a Calderón operator whose coercivity plays a fundamental role in proving the well-posedness of the proposed time-domain boundary integral equations and the stability of the numerical discretization.

The functional analytic setting of [3, Section 2.3] follows Buffa and Hiptmair [2]. The latter paper defines boundary integral operators in the Fourier domain, whereas

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[3] uses the Laplace domain (which fits better with convolution quadratures, cf. [3, Section 4]).

A sign error occurred while translating the definition of the potential operators from the Fourier to the Laplace domain. The effects of this sign error are restricted to Section 2.3 and the definition of the Calderón operator in [3], but otherwise all the results of [3] hold unchanged. On the other hand, for the implementation of the method the correct sign is crucial.

In the following we use the notation of [3].

Corrections

Let us recall the *time-harmonic Maxwell's equation*, obtained as the Laplace transform of the second order Maxwell's equation (with constant permeability μ and permittivity ε):

$$\varepsilon\mu s^2 u + \operatorname{curl}\operatorname{curl} u = 0 \qquad \text{in } \mathbb{R}^3 \setminus \Gamma, \tag{1}$$

with exterior normal ν. The complex parameter s of positive real part is the Laplace transform variable.

- Throughout [3] appropriate physical units should be assumed such that

$$\varepsilon\mu = 1, \tag{2}$$

that is, the wave speed is set to 1. In the original work [3], the dependence on $\varepsilon\mu$ was erroneous. The normalization (2) corresponds to a rescaling of time $t \to t/\sqrt{\varepsilon\mu}$ or of frequency $s \to s\sqrt{\varepsilon\mu}$. With the scaling (2), equation (1) becomes the time-harmonic Maxwell's equation $-\kappa^2 u + \operatorname{curl}\operatorname{curl} u = 0$ as in [2] on setting $s = -i\kappa$.

- The setting uses the following skew-hermitian pairing on $L^2(\Gamma)$:

$$[\,w,\ v]_\Gamma = \int_\Gamma (\,w \times \nu) \cdot \,v \,\mathrm{d}\sigma.$$

where the dot $\cdot$ stands for the Euclidean inner product on $\mathbb{C}^3$, i.e., $a \cdot b = \overline{a}^T b$ for $a, b \in \mathbb{C}^3$. The complex conjugation was not stated explicitly in [3] although it was actually used, e.g. in formula (2.3) and Lemma 3.1 of [3].

- The solution of (1) is then given by the correct representation formula:

$$u = -\mathcal{S}(s)\varphi + \mathcal{D}(s)\psi, \qquad x \in \mathbb{R}^3 \setminus \Gamma. \tag{3}$$

In [3, equation (2.5)] the first negative sign was erroneously missing.

- The correct jump relations are, with a different sign in the first identity:

$$[\![\gamma_N \circ \mathcal{S}(s)]\!] = -\operatorname{Id}, \qquad [\![\gamma_N \circ \mathcal{D}(s)]\!] = 0,$$
$$[\![\gamma_T \circ \mathcal{S}(s)]\!] = 0, \qquad [\![\gamma_T \circ \mathcal{D}(s)]\!] = \operatorname{Id}.$$

• The boundary integral operators V and K satisfy the relations

$$\begin{aligned} V(s) &= \{\gamma_T \circ \mathcal{S}(s)\} = \{\gamma_N \circ \mathcal{D}(s)\}, \\ K(s) &= \{\gamma_T \circ \mathcal{D}(s)\} = -\{\gamma_N \circ \mathcal{S}(s)\}. \end{aligned} \tag{4}$$

In [3] the negative sign in the last term of the second line was missing.

• The negative sign in (4) changes the signs in the expression for the averages of the traces using the operators V and K, see [3, equation (2.6)]. The correct relations are:

$$\begin{aligned} \{\gamma_T u\} &= -\{\gamma_T \mathcal{S}(s)\varphi\} + \{\gamma_T \mathcal{D}(s)\psi\} \\ &= -V(s)\varphi + K(s)\psi, \qquad \text{and} \\ \{\gamma_N u\} &= -\{\gamma_N \mathcal{S}(s)\varphi\} + \{\gamma_N \mathcal{D}(s)\psi\} \\ &= K(s)\varphi + V(s)\psi. \end{aligned} \tag{5}$$

The negative sign in the first equation was missing in [3, equation (2.7)].

• Due to the above formulas, the correct *Calderón operator* is given by

$$B(s) = \mu^{-1} \begin{pmatrix} -V(s) & K(s) \\ -K(s) & -V(s) \end{pmatrix}, \tag{6}$$

with a correct negative sign in the left upper block of $B(s)$ as opposed to [3, equation (3.1)]. The crucial first equality in the proof of Lemma 3.1 in [3] remains valid.

Validity of the results of [3]

The sign difference clearly does not influence the boundedness of the above operators as stated in [3], based on [1].

Most importantly, in the corrected setting the Calderón operator (6) still satisfies the crucial coercivity result [3, Lemma 3.1], with the proof given as in [3].

Thanks to this coercivity estimate for the Calderón operator B defined in (6) above, *all* the stability and convergence results of [3] remain valid, since the proofs depend on this coercivity result and not on the particular form of the Calderón operator.

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