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A Sobolev mapping property of the Bergman kernel

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Abstract. We prove that if D is a pseudoconvex domain with Lipschitz boundary having an exhaustion function ρ such that $-(-\rho)^{\eta}$ is plurisubharmonic, then the Bergman projection maps the Sobolev space W_s boundedly to itself for any $s < \eta/2$.

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1 Introduction

It was long an open problem whether in any smoothly bounded pseudoconvex domain the Bergman projection preserves smoothness up to the boundary. This question was finally resolved in the negative by M Christ in [Ch]. Christ's proof is based on a previous result by D Barrett [Ba], saying that for any $s > 0$, there is a smoothly bounded pseudoconvex domain such that the Bergman projection does not map the Sobolev space W_s to itself. (Barretts theorem is in turn inspired by an earlier result of C O Kiselman, [Ki], who constructed non-smooth pseudoconvex domains with this property.)

Among the abundance of results in the positive direction, the one which is most relevant to us here is the theorem of Boas and Straube, [Bo-St1], saying that if a smoothly bounded pseudoconvex domain has a plurisubharmonic defining function that is smooth up to the boundary, then the Bergman projection maps W_s to itself for any $s > 0$. (Their theorem is even a bit more general, requiering the defining function to be plurisubharmonic only at the boundary.)

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It was proved already by Diederich and Fornaess, [Di-Fo1], that in general such a plurisubharmonic defining function does not exist. However, Diederich and Fornaess also proved that for any smoothly bounded pseudoconvex domain there is a number $\eta > 0$ such that D has a defining function such that $-(-\rho)^\eta$ is plurisubharmonic. The main result of this note (Theorem 2.4) says that the bigger one can take the number η in the theorem of Diederich and Fornaess, the better regularity properties one has for the Bergman projection. More precisely, we show that the Bergman projection is bounded on W_s for any $s < \eta/2$. We also show that a similar result holds for the operator, K giving the L^2-minimal solution to the $\bar\partial$ problem.

The proof consist in showing that the Bergman projection and K satisfy a weighted L^2-estimate of a type first considered by Donnelly and Fefferman, [D-F], and after that generalized by many authors (see e g [Di-O], [McN 1], [B], [Del]). We will however not need to use the methods developed in these papers, but will instead give a selfcontained and simple proof of a generalized Donnelly-Fefferman estimate, using only Hörmanders theorem [H]. From this weighted estimate the Sobolev estimate follows from well known results in elliptic PDE:s. Actually, by more recent work in this area, these results hold also in domains of low regularity, so our main result holds also for domains with Lipschitz boundary.

The case $\eta = 1$ was previously considered in [Bon-Ch 1] and [Bon-Ch 2]. For $\eta = 1$ our main result follows from the theorem in [Bon-Ch 1], at least in the case of a boundary of class C^2.

This note is also related to a recent paper by Kohn, [Ko], who shows that the Bergman projection is bounded on the Sobolev spaces W_s for a range of s depending on the constant η in the Diederich-Fornaess theorem . The dependence $s = s(\eta)$ in Kohn's theorem is however not easy to compute and we have not been able to prove Theorem 2.4 from Kohn's result. On the other hand, Kohn's theorem has the advantage that $s(\eta)$ tends to ∞ as η tends to 1, thus giving back the Boas-Straube theorem in the limit. Finally, J McNeal, [McN 2], has kindly informed us that he also has obtained a proof of Theorem 2.4 (unpublished).

2 Weighted estimates and Sobolev estimates

First we need to introduce the set up and some terminology. Let D be a bounded pseudoconvex domain and let ϕ be a locally bounded realvalued function. We shall consider the weighted L^2-spaces

$$L^2(e^{-\phi}) = \{f; \int |f|^2 e^{-\phi} < \infty\}$$

of differential forms of various degrees. We let $\bar\partial_\phi^*$ be the adjoint of the $\bar\partial$-operator in $L^2(e^{-\phi})$, and let

$$\Box_\phi = \bar\partial\bar\partial_\phi^* + \bar\partial_\phi^*\bar\partial$$

be the associated complex Laplace operator. If $\phi = 0$ we shall omit subscripts and write simply $\bar\partial_\phi^* = \bar\partial^*$, $\Box_\phi = \Box$ etc. Let $N_{q,\phi}$ be the $\bar\partial$-Neumann operator on $(0,q)$-forms (cf [F-Ko]), solving

$$\Box_\phi N_{q,\phi}(f) = f$$

for any $(0,q)$-form f in $L^2(e^{-\phi})$ (in our later specific choices of weight function ϕ the $\bar\partial$-Neumann operator will always exist).

We denote by $B_{q,\phi}$ the Bergman operator, mapping a $(0,q)$-form in $L^2(e^{-\phi})$ to its orthogonal projection in the closed subspace of $\bar\partial$-closed forms. In particular, for $q = 0$, $B_{0,\phi}$ maps a function to a holomorphic function. Finally we put

$$K_{q,\phi}(f) = \bar\partial_\phi^* N_{q,\phi}(f).$$

By a classical result, if f is $\bar\partial$-closed, then

$$u = K_{q,\phi}(f)$$

is the solution to $\bar\partial u = f$ of minimal norm in $L^2(e^{-\phi})$.

We are now ready to state our first result.

Theorem 2.1. *Let $D \subset \mathbb{C}^n$ be a bounded pseudocomvex domain. Suppose $\psi \in PSH(D)$ satisfies*

$$i\partial\psi \wedge \bar\partial\psi \leq ri\partial\bar\partial\psi \tag{1}$$

where $r < 1$. Let ϕ be a plurisubharmonic function in D. Then $\bar\partial_\phi^* N_{q,\phi}$ maps $L^2_{0,q}(e^{\psi-\phi})$ boundedly to $L^2_{0,q-1}(e^{\psi-\phi})$, and $B_{q,\phi}$ maps $L^2_{0,q}(e^{\psi-\phi})$ boundedly to itself.

The condition (1) (introduced in [D-F]; see also [Gr]) is of crucial importance in this paper. An alternative formulation of (1) is that the norm of the form $\partial\psi$, measured in the metric with Kähler form $\partial\bar\partial\psi$ is smaller than r at any point. Yet another equivalent formulation is that the function

$$-e^{-\psi/r}$$

is plurisubharmonic. The last formulation also makes it clear how to interpret (1) in case ψ is not of class C^2, and shows that any ψ that satisfies (1) in this weak sense can, on any relatively compact subdomain, be approximated by a decreasing sequence of smooth functions satisfying the same thing.

Therefore it will be enough to prove Theorem 2.1 for smooth functions ψ although in the end we will apply it to functions that are not necessarily smooth. Similarily, in the proof of Theorem 2.1, we will also assume that ϕ is smooth; the general case then follows from a standard limiting procedure.

By definition the Bergman operator is bounded from $L^2(e^{-\phi})$ to itself. What Theorem 2.1 says is that B_q is also bounded for the (in general stronger) norms in $L^2_{0,q}(e^{\psi-\phi})$, as long as ψ satisfies (1). For a particular choice of ψ and for forms that satisfy certain elliptic equations, these norms will be equivalent to Sobolev norms (see Theorem 2.4).

To prove Theorem 2.1 we shall first establish a more precise result for the operator $\bar\partial^*_\phi N_{q,\phi}$, related to the Donnelly-Fefferman estimate. For this we have to recall an appropriate version of Hörmander's fundamental L^2-estimate for the $\bar\partial$-equation (cf [H]), and to formulate that theorem we first need some preparations from linear algebra.

Let

$$\Omega = \Sigma \Omega_{j\bar k} dz_j \wedge d\bar z_k$$

be a positive $(0,1)$-form. We shall use Ω to define a norm, $||f||_\Omega$, on $(0,q)$-forms in $\mathbb{C}^n$. This norm has three crucial properties. The first one is that if v is a $(0,1)$-form then

$$||f \wedge v||_\Omega \le ||v||_\Omega |f|,$$

where $|f|$ is the Euclidean norm of f. Next, $||\cdot||_\Omega$ is a decreasing function of Ω, and finally $||f||^2_\beta = (1/q)|f|^2$ if

$$\beta = \Sigma dz_j \wedge d\bar z_k$$

is the Kähler form of the Euclidean metric, and f is of bidegree $(0,q)$.

For $q = 1$ our norm is simply defined by

$$||f||^2_\Omega = \Sigma \Omega^{j\bar k} f_j \bar f_k$$

where $(\Omega^{j\bar k})$ is the inverse matrix of $(\Omega_{j\bar k})$. In other words, $||f||_\Omega$ is the norm of f measured in the Kähler metric defined by Ω. This is however not true for forms of higher degree where the definition is a bit more cumbersome. The reader who is primarily interested in the case of $(0,1)$-forms can skip the next paragraph.

We first define the norm $||f||^2_\Omega$ for forms of bidegree (n,q) and then define it on $(0,q)$-forms using the trivial identification of (n,q)- and $(0,q)$-forms in $\mathbb{C}^n$. The customary definition uses the formalism of Kähler Geometry. Let Λ denote the operator of interior multiplication with the Kähler form β, so that for any forms u and v

$$\langle \Lambda u, v\rangle = \langle u, \beta \wedge u\rangle,$$

where $\langle , \rangle$ denotes the Euclidean scalar product on (n,q)-forms. Then consider the quadratic form

$$\langle \Omega \wedge \Lambda h, h \rangle.$$

We put

$$||f||_\Omega = \sup\langle f, h\rangle$$

where the supremum is taken over all forms h with $\langle \Omega \wedge \Lambda h, h \rangle \leq 1$.

One can then verify that the norm $||f||_\Omega$ has the three properties stated just before. We refer the reader to [D] where this is done in detail, even in the context of forms with values in a vector bundle.

The version of Hörmander's theorem refered to above is the following.

Theorem 2.2. *Let D be a pseudoconvex domain in $\mathbb{C}^n$ and let ϕ be plurisubharmonic and of class C^2 in D. Let f be a $\bar\partial$-closed $(0,q)$-form in D. Then there is a solution to the equation*

$$\bar\partial u = f \tag{2}$$

satisfying the estimate

$$\int |u|^2 e^{-\phi} \leq \int ||f||^2_{i\partial\bar\partial\phi} e^{-\phi}. \tag{3}$$

In [H] this theorem is formulated with $2|f|^2/c$ instead of $||f||^2_{i\partial\bar\partial\phi}$, but the statement of Theorem 2 follows from basically the same proof. The precise estimate (3) for the solution is a special case of Theorem 4.1 in [D] and we refer the reader to that article for the proof.

Before continuing let us remark that one can view (3) as a complex Poincaré inequality. It is clear that (3) must be satified by the $L^2(e^{-\phi})$-minimal solution to (2), that is by the solution satisfying

$$\int u \cdot \bar h e^{-\phi} = 0 \tag{4}$$

for any $\bar\partial$-closed form h. Hence the theorem implies that if u is any form which is orthogonal in $L^2(e^{-\phi})$ to the space of $\bar\partial$-closed forms then u satisfies

$$\int |u|^2 e^{-\phi} \leq \int ||\bar\partial u||^2_{i\partial\bar\partial\phi} e^{-\phi}.$$

We are now ready to formulate our main weighted $\bar\partial$-estimate. It's content is that if ψ satisfies the crucial assumption (1), then we can improve the estimate in Hörmander's theorem by replacing $||f||_{i\partial\bar\partial(\phi)}$ by $||f||_{i\partial\bar\partial(\psi+\phi)}$ without having to change the weight function from ϕ to $\phi + \psi$. We may even make the weight function ϕ *less* plurisubharmonic by subtracting ψ.

Theorem 2.3. *Let D be a bounded pseudoconvex domain in $\mathbb{C}^n$. Let ϕ and ψ be plurisubharmonic and of class C^2 in D, and assume $\psi \geq 0$ satisfies (1) with $r < 1$. Let f be a $\bar\partial$-closed $(0, q)$-form in D belonging to $L^2(e^{\psi-\phi}) \subset L^2(e^{-\phi})$ and let u be the solution to the equation*

$$\bar\partial u = f \tag{5}$$

of minimal norm in $L^2(e^{-\phi})$ Then

$$\int |u|^2 e^{\psi-\phi} \leq C_r \int ||f||^2_{i\partial\bar\partial(\psi+\phi)} e^{\psi-\phi}. \tag{6}$$

Proof. Since $\psi \geq 0$ f lies in $L^2(e^{-\phi})$, so by Hörmander's theorem there is a solution to (1) in $L^2(e^{-\phi})$. Let u be the $L^2(e^{-\phi})$-minimal solution, so that (4) is satisfied. For a moment we assume that ψ is bounded. Put $v = ue^{\psi}$. Then clearly v is orthogonal to all closed forms in $L^2(e^{-\psi-\phi})$, so by the remark immediately before the theorem we have

$$\int |v|^2 e^{-\psi-\phi} \leq \int ||\bar\partial v||^2_{i\partial\bar\partial(\psi+\phi)} e^{-\psi-\phi}.$$

Recalling the definition of v

$$\int |u|^2 e^{\psi-\phi} \leq \int ||f + \bar\partial\psi \wedge u||^2_{i\partial\bar\partial(\psi+\phi)} e^{\psi-\phi} \leq$$

$$\leq (1 + 1/\epsilon) \int ||f||^2_{i\partial\bar\partial(\psi+\phi)} e^{\psi-\phi} + (1 + \epsilon) \int ||u \wedge \bar\partial\psi||^2_{i\partial\bar\partial(\psi+\phi)} e^{\psi-\phi}. \tag{7}$$

By the discussion preceeding Theorem 2.2

$$||u \wedge \bar\partial\psi||^2_{i\partial\bar\partial(\psi+\phi)} \leq ||u \wedge \bar\partial\psi||^2_{i\partial\bar\partial\psi} \leq r|u|^2.$$

Choosing ϵ so small that $(1 + \epsilon)r < 1$ we can thus absorb the last term in (7) in the left hand side, which immediately gives the theorem. This is proved under the assumption that ψ is bounded but the general case follows by exhausting with relatively compact subdomains.

The case $\phi = \psi$ in Theorem 2.3 is essentially the Donnelly-Fefferman estimate, see [D-F]. Results related to Theorem 2.3 can be found in [O], [Di-O], [Gr],[Del] [McN] and [B], to mention only a few examples. We have included the proof of Theorem 2.2 here first since we have not found exactly the statement we need in the literature, and secondly because it is the simplest proof we know of the Donnelly-Fefferman estimate.

Proof of Theorem 2.1. We shall carry out the proof in the case when ϕ and ψ are of class C^2 up to the boundary; the general case follows from a standard limiting procedure. Notice that since D is bounded we can replace ψ by $\tilde{\psi} = \psi + \delta|z|^2$ if δ is small enough and $\tilde{\psi}$ will still satisfy (1). Since $i\partial\bar{\partial}\tilde{\psi} \geq \delta\beta$, this implies the first part of Theorem 2.1 for $\bar{\partial}$-closed forms. For a non-closed form f we decompose

$$f = f_1 + f_2,$$

where f_1 is $\bar{\partial}$-closed and f_2 is orthogonal to the space of $\bar{\partial}$-closed forms in $L^2(e^{-\phi})$. Then $\bar{\partial}^*_\phi N_\phi(f) = \bar{\partial}^*_\phi N_\phi(f_1)$, so to prove boundedness of the operator $\bar{\partial}^*_\phi N_\phi$ on all forms it suffices to prove that the map $f \to f_1$ is bounded. But this map is precisely the Bergman operator, so to prove Theorem 2.1 completely it suffices to prove that the Bergman operator B_ϕ is bounded.

We then claim that

$$B_\phi(f) = e^{-\psi} B_{\phi+\psi}(e^\psi f) - \bar{\partial}^*_\phi N_\phi(\bar{\partial}(e^{-\psi} B_{\phi+\psi}(e^\psi f)).$$

Indeed, the right hand side is $\bar{\partial}$-closed and has the same scalar product as f against any $\bar{\partial}$-closed form. The first term, v, defines a bounded operator on $L^2(e^{\psi-\phi})$ since

$$\int |v|^2 e^{\psi-\phi} = \int |B_{\psi+\phi}(e^\psi f)|^2 e^{-\psi-\phi} \leq \int |e^\psi f|^2 e^{-\psi-\phi}$$
$$= \int |f|^2 e^{\psi-\phi}.$$

Next we note that the second term equals

$$\bar{\partial}^*_\phi N_\phi(-\bar{\partial}\psi \wedge v)$$

since $B_{\psi+\phi}(e^\psi f)$ is $\bar{\partial}$-closed. By (6) we have

$$\int |\bar{\partial}^*_\phi N_\phi(-\bar{\partial}\psi \wedge v)|^2 e^{\psi-\phi} \leq C_r \int ||\bar{\partial}\psi \wedge v||^2_{i\partial\bar{\partial}(\psi+\phi)} e^{\psi-\phi}.$$

But by our assumption on ψ

$$||\bar{\partial}\psi \wedge v||_{i\partial\bar{\partial}(\psi+\phi)} \leq ||\bar{\partial}\psi \wedge v||_{i\partial\bar{\partial}\psi} \leq r|v|,$$

and this completes the proof since we have already controlled v by f.

We shall now apply Theorem 2.1 to prove our Sobolev estimate, and from now on we shall take $\phi = 0$.

The crucial assumption in Theorem 2.1 is that ψ satisfy (1), which means precisely that the function

$$-e^{-\psi/r}$$

is plurisubharmonic. Now suppose that our domain D has an exhaustion function ρ, such that $-(-\rho)^\eta$ is plurisubharmonic. Let $\delta > 0$ satisfy $\delta < \eta$, and put $\psi = \delta \log \frac{-1}{\rho}$, so that ψ satisfies (1) with $r = \delta/\eta < 1$. Then Theorem 2.1 implies that the Bergman operator satisfies

$$\int |B(u)|^2(-\rho)^{-\delta} \leq C \int |u|^2(-\rho)^{-\delta},$$

and that $\bar\partial^* N$ satisfies a similar estimate. This implies our main result:

Theorem 2.4. *Let D be a bounded pseudoconvex domain with Lipschitz boundary. Assume there exists a function $\rho < 0$ in D such that*

$$cd(\cdot, \partial D) \leq -\rho \leq Cd(\cdot, \partial D)$$

and such that $-(-\rho)^\eta$ is plurisubharmonic. Then the operators $\bar\partial^* N_q$ and B_q map the Sobolev space W_s to itself for any $s < \eta/2$.

We will use repeatedly two facts from real analysis. The first is an embedding result, see [Gri] Thm 1.4.4.3, which says that the space W_s is continuously embedded in $L^2((-\rho))^{-2s}$ if $0 < s < 1/2$, and ρ is comparable to the distance to the boundary in a domain with Lipschitz boundary. The second result says that any *harmonic* function in $L^2((-\rho)^{-2s}))$ also lies in W_s , under the same assumptions. This fact is a consequence of Thm 4.2 in [J-K] together with the Lemma 1 in [Det].

Consider first the case of the Bergman operator $B = B_0$ on functions, and assume u is a function in W_s. Then by the embedding result, u lies in $L^2((-\rho))^{-2s}$, so by Theorem 2.1 $B(u)$ also lies in the latter space. Since $B(u)$ is holomorphic, hence harmonic, it follows that $B(u)$ belongs to W_s.

Next, let f be a $(0, q)$-form in W_s, with $q \geq 1$. Then by the same embedding result $f \in L^2((-\rho)^{-2s})$, so by Theorem 2.1 $B_q(f) \in L^2((-\rho))^{-2s}$. Note that

$$\bar\partial B_q(f) = 0 \text{ and } \bar\partial^* B_q(f) = \bar\partial^* f.$$

Hence $\Box B_q(f)$, which as a differential operator is the Laplacian on each component of f satisfies

$$\Box B_q(f) = \bar\partial\bar\partial^* f.$$

Since $f \in W_s$, $f = \Delta g$ with $g \in W_{s+2}$. (This follows since by Thm 1.4.3.1 in [Gri] f can be extended to a form with compact support in W_s on all of $\mathbb{C}^n$, so we may take g to be the Newtonian potential of this extension.) Hence

$$\Delta B_q(f) = \bar\partial\bar\partial^* f = \Delta v$$

where $v \in W_s$. Let $w = B_q(f) - v$ so that w is a form with harmonic coefficients. Since both $B_q(f)$ and v lie in $L^2((-\rho))^{-2s}$ by the embedding theorem, so does w. Since w has harmonic coefficients w lies in W_s, so $B_q(f)$ also belongs to W_s. We have thus proved that the Bergman operator is bounded on W_s in any degree.

It only remains to prove that if f is a $(0,q)$-form in W_s then $u = \bar\partial^* N_q(f)$ is also in W_s. Since $\bar\partial^* N_q(f) = \bar\partial^* N_q(B_q(f))$, and we already know that B_q is bounded on W_s we may as well assume from the start that $\bar\partial f = 0$. Then

$$\bar\partial u = f \text{ and } \bar\partial^* u = 0.$$

Thus

$$\Delta u = (\bar\partial\bar\partial^* + \bar\partial^*\bar\partial)u = \bar\partial^* f \in W_{s-1}.$$

By Thm 0.5 in [J-K] this implies that we can solve $\Delta g = \Delta u$ with $g \in W_{s+1} \subset W_s$, indeed the Green potential of Δu will do. By the embedding theorem both g and f belong to $L^2((-\rho))^{-2s}$, so by Theorem 2.1 u and $u - g$ also belong to this space. Since $u - g$ has harmonic coefficients it follows that $u - g$ lies in W_s. Hence this also holds for u and Theorem 2.4 is completely proved.

References

[B] Berndtsson, B. The extension theorem of Ohsawa-Takegoshi and the theorem of Donnelly-Fefferman. Ann. Inst. Fourier (Grenoble) **46** (1996), 1083–1094

[Bon-Ch 1] Bonami, A. and Charpentier, Ph. Une estimation Sobolev $\frac{1}{2}$ pour le projecteur de Bergman. C. R. Acad. Sci. Paris Sér. I Math. **307** (1988), 173–176

[Bon-Ch 2] Bonami, A. and Charpentier, Ph. Boundary values for the canonical solution to $\bar\partial$-equation and $W^{\frac{1}{2}}$ estimates. Preprint Bordeaux 1990

[Ba] Barrett, D. Behavior of the Bergman projection on the Diederich-Fornæss worm. Acta Math. **168** (1992), 1–10

[Bo-St 1] Boas, H., Straube and E. Sobolev estimates for the $\bar\partial$-Neumann operator on domains in C^n admitting a defining function that is plurisubharmonic on the boundary. Math. Z. **206** (1991), 81–88

[Bo-St 2] Boas, H. Straube and E. Equivalence of regularity for the Bergman projection and the $\bar\partial$-Neumann operator. Manuscripta Math. **67** (1990), 25–33

[Ch] Christ, M. Global C^∞ irregularity of the $\bar\partial$-Neumann problem for worm domains. J. Amer. Math. Soc. **9** (1996), 1171–1185

[Del] Delin, H. Pointwise estimates for the Bergman kernel using a weighted L^2-estimate for the $\bar\partial$-equation. To appear in Ann Inst Fourier

[Det] Detraz, J. Classes de Bergman de fonctions harmoniques, Bull. Soc. Math. France **109** (1981), 259–268

[Di-Fo 1] Diederich, K and Fornaess, J-E. Pseudoconvex domains: an example with nontrivial Nebenhülle. Math. Ann. **225** (1977), 275–292

[Di-Fo 2] Diederich, K and Fornaess, J-E. Pseudoconvex domains: bounded strictly plurisubharmonic exhaustion functions. Invent. Math. **39** (1977), 129–141

[Di-O] Diederich, K. and Ohsawa, T. An estimate for the Bergman distance on pseudoconvex domains. Ann. of Math. (2) **141** (1995), 181–190

[D] Demailly, J-P. Estimations L^2 pour l'opérateur $\bar{\partial}$ d'un fibré vectoriel holomorphe semi-positif au-dessus d'une variété kählérienne complète. Ann. Sci. École Norm. Sup. (4) **15** (1982), 457–511

[D-F] Donnelly, H. and Fefferman, C. Ann. of Math. (2) **118** (1983), 593–618

[F-Ko] Folland, G. and Kohn, J.J. The Neumann problem for the Cauchy-Riemann complex. Annals of mathematics Studies, Princeton University Press 1972

[Gri] Grisvard, P. Elliptic problems in nonsmooth domains. Monographs and Studies in Mathematics, 24. Pitman (Advanced Publishing Program), Boston, Mass.-London, 1985. xiv+410 pp

[Gr] Gromov, M. Kähler hyperbolicity and L_2-Hodge theory. J. Differential Geom. **33** (1991), 263–292

[H] Hörmander, L. An introduction to complex analysis in several variables. Third edition. North-Holland Mathematical Library, 7. North-Holland Publishing Co., Amsterdam-New York, 1990

[J-K] Jerison, D. and Kenig, K. The inhomogeneous Dirichlet problem in Lipschitz domains. J. Funct. Anal. **130** (1995), 161–219

[Ki] Kiselman, C.O. Study of the Bergman projection in certain Hartogs domains. Several complex variables and complex geometry, Part 3 (Santa Cruz, CA, 1989), 219–231, Proc. Sympos. Pure Math., 52, Part 3, Amer. Math. Soc., Providence, RI, 1991

[Ko] Kohn, J.J. Quantitative estimates for global regularity. Preprint 1998

[McN 1] McNeal, J. On large values of L^2 holomorphic functions. Math. Res. Lett. **3** (1996), 247–259

[McN 2] McNeal, J. Private communication

[O] Ohsawa, T. Recent applications of L^2 estimates for the operator $\overline{\partial}$. Proceedings of the International Congress of Mathematicians, Vol. I, II (Kyoto, 1990), 913–921, Math. Soc. Japan, Tokyo, 1991