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Global solutions for the Navier-Stokes equations in the rotational framework

Tsukasa Iwabuchi · Ryo Takada

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Abstract The existence of global unique solutions to the Navier-Stokes equations with the Coriolis force is established in the homogeneous Sobolev spaces $\dot{H}^s(\mathbb{R}^3)^3$ for $1/2 < s < 3/4$ if the speed of rotation is sufficiently large. This phenomenon is so-called the global regularity. The relationship between the size of initial datum and the speed of rotation is also derived. The proof is based on the space time estimates of the Strichartz type for the semigroup associated with the linearized equations. In the scaling critical space $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$, the global regularity is also shown.

1 Introduction

We consider the initial value problem for the Navier-Stokes equations with the Coriolis force

$$\begin{cases} \dfrac{\partial u}{\partial t} - \Delta u + \Omega e_3 \times u + (u \cdot \nabla)u + \nabla p = 0 & \text{in } \mathbb{R}^3 \times (0, \infty), \\ \operatorname{div} u = 0 & \text{in } \mathbb{R}^3 \times (0, \infty), \\ u(x, 0) = u_0(x) & \text{in } \mathbb{R}^3, \end{cases} \tag{NSC}$$

where $u = u(x, t) = (u_1(x, t), u_2(x, t), u_3(x, t))$ and $p = p(x, t)$ denote the unknown disturbance of velocity field and the unknown disturbance of pressure of

T. Iwabuchi (✉)
Department of Mathematics, Faculty of Science and Engineering, Chuo University,
Tokyo 112-8551, Japan
e-mail: iwabuchi@math.chuo-u.ac.jp

R. Takada
Department of Mathematics, Kyoto University, Kyoto 606-8502, Japan
e-mail: takada@math.kyoto-u.ac.jp

the fluid at the point $(x, t) \in \mathbb{R}^3 \times (0, \infty)$, respectively, while $u_0 = u_0(x) = (u_{0,1}(x), u_{0,2}(x), u_{0,3}(x))$ denotes the given initial velocity field satisfying the compatibility condition $\operatorname{div} u_0 = 0$. Here, $\Omega \in \mathbb{R}$ is the speed of rotation around the vertical unit vector $e_3 = (0, 0, 1)$. Note that (NSC) also demonstrates a rigid-body rotation in the geostatics.

The purpose of this paper is to show the existence and the uniqueness of the global solutions to (NSC) in the homogeneous Sobolev spaces $\dot{H}^s(\mathbb{R}^3)(s \geq 1/2)$. In particular, we obtain global solutions for large initial velocity u_0 if the speed of the rotation is sufficiently fast. For the existence of global solutions to (NSC), Chemin et al. [6,7] proved that for any initial data $u_0 \in L^2(\mathbb{R}^2)^2 + H^{\frac{1}{2}}(\mathbb{R}^3)^3$, there exists a positive parameter Ω_0 such that for every $\Omega \in \mathbb{R}$ with $|\Omega| \geq \Omega_0$ there exists a unique global solution. Babin et al. [2–4] showed the existence of global solutions and the regularity of the solutions to (NSC) for the periodic initial data with large $|\Omega|$. On the other hand, Giga et al. [12] showed the existence of global solutions for small initial data $u_0 \in FM_0^{-1}(\mathbb{R}^3)^3$, where the condition of smallness is independent of the speed of the rotation Ω, and $FM_0^{-1}(\mathbb{R}^3)$ is scaling critical to (NSC) with $\Omega = 0$. Indeed, for the solution u to (NSC) with $\Omega = 0$, let $u_\lambda(x, t) := \lambda u(\lambda x, \lambda^2 t)$ for $\lambda > 0$. Then, u_λ is also a solution to (NSC) with $\Omega = 0$ and we have $\|u_\lambda(\cdot, 0)\|_{FM_0^{-1}} = \|u(\cdot, 0)\|_{FM_0^{-1}}$ for all $\lambda > 0$. On such other results of global solutions for small initial data, Hieber and Shibata [13] studied in the Sobolev space $H^{\frac{1}{2}}(\mathbb{R}^3)$, Konieczny and Yoneda [19] studied in the Fourier-Besov space $F\dot{B}^{2-\frac{3}{p}}_{p,\infty}(\mathbb{R}^3)$ with $1 < p \leq \infty$. On the well-posedness for (NSC) with $\Omega = 0$ in the scaling critical spaces, we refer to Fujita and Kato [8], Kato [15], Kozono and Yamazaki [20], Koch and Tataru [18]. On the local existence of solutions to (NSC), we refer to the results by Giga et al. [10,11] and Sawada [21]. In our previous result [14], we see that the time-interval in which the local solution to integral equation exists can be taken arbitrary long for initial datum $u_0 \in \dot{H}^s(\mathbb{R}^3)^3$ with $1/2 < s < 5/4$, if the speed of rotation Ω is sufficiently large compared with the size of u_0.

In this paper, we establish the existence theorem on global solutions to (NSC) for the initial velocity u_0 in the homogeneous Sobolev spaces $\dot{H}^s(\mathbb{R}^3)$ $(1/2 \leq s < 3/4)$. In the case $s > 1/2$, the existence of global solutions is obtained if the speed of rotation Ω is large compared with the norm of initial data $\|u_0\|_{\dot{H}^s}$. On the other hand, in the critical case $s = 1/2$, the speed $|\Omega|$ to obtain the existence of global solutions is determined by each precompact set $K \subset \dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$, which the initial data belong to.

We consider the following integral equation:

$$u(t) = T_\Omega(t)u_0 - \int_0^t T_\Omega(t - \tau)\mathbb{P}\nabla \cdot (u \otimes u)d\tau, \tag{IE}$$

where $\mathbb{P} = (\delta_{ij} + R_i R_j)_{1 \leq i,j \leq 3}$ denotes the Helmholtz projection onto the divergence-free vector fields and $T_\Omega(\cdot)$ denotes the semigroup corresponding to the linear problem of (NSC), which is given explicitly by

$$T_\Omega(t)f = \mathcal{F}^{-1}\Big[\cos\Big(\Omega\frac{\xi_3}{|\xi|}t\Big)e^{-t|\xi|^2}I\widehat{f}(\xi) + \sin\Big(\Omega\frac{\xi_3}{|\xi|}t\Big)e^{-t|\xi|^2}R(\xi)\widehat{f}(\xi)\Big]$$

for $t \geq 0$ and divergence-free vector fields f. Here, I is the identity matrix in $\mathbb{R}^3$, R_j $(j = 1, 2, 3)$ is the Riesz transform and $R(\xi)$ is the skew-symmetric matrix symbol related to the Riesz transform, which is defined by

$$R(\xi) := \frac{1}{|\xi|}\begin{pmatrix} 0 & \xi_3 & -\xi_2 \\ -\xi_3 & 0 & \xi_1 \\ \xi_2 & -\xi_1 & 0 \end{pmatrix} \quad \text{for } \xi \in \mathbb{R}^3 \setminus \{0\}.$$

We refer to Babin et al. [1–3], Giga et al. [11] and Hieber and Shibata [13] for the derivation of the explicit form of $T_\Omega(\cdot)$.

Theorem 1.1 *Let $\Omega \in \mathbb{R}\setminus\{0\}$, and let s, p and θ satisfy*

$$\frac{1}{2} < s < \frac{3}{4}, \quad \frac{1}{3} + \frac{s}{9} < \frac{1}{p} < \frac{2}{3} - \frac{s}{3}, \tag{1.1}$$

$$\frac{s}{2} - \frac{1}{2p} < \frac{1}{\theta} < \frac{5}{8} - \frac{3}{2p} + \frac{s}{4}, \quad \frac{3}{4} - \frac{3}{2p} \leq \frac{1}{\theta} < 1 - \frac{2}{p}. \tag{1.2}$$

Then, there exists a positive constant $C = C(s, p, \theta) > 0$ such that for any initial velocity field $u_0 \in \dot{H}^s(\mathbb{R}^3)^3$ with

$$\|u_0\|_{\dot{H}^s} \leq C|\Omega|^{\frac{s}{2}-\frac{1}{4}} \quad \textit{and} \quad \operatorname{div} u_0 = 0, \tag{1.3}$$

there exists a unique global solution $u \in C([0,\infty); \dot{H}^s(\mathbb{R}^3))^3 \cap L^\theta(0,\infty; \dot{H}^s_p(\mathbb{R}^3))^3$ to (NSC).

Remark 1.2 (i) The existence of global solutions for small initial data $u_0 \in \dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$ was shown by Hieber and Shibata [13]. The size condition (1.3) on initial data can be regarded as a continuous extension of that in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$. Indeed, Hieber and Shibata [13] assumed the smallness condition $\|u_0\|_{H^{\frac{1}{2}}} \leq \delta$ for some $\delta > 0$, which corresponds to our condition (1.3) with $s = 1/2$.

(ii) The space $L^{\theta_0}(0,\infty; \dot{H}^{s_0}_{p_0}(\mathbb{R}^3))$ is scaling invariant to (NSC) in the case $\Omega = 0$ if θ_0, s_0 and p_0 satisfy

$$\frac{2}{\theta_0} + \frac{3}{p_0} = 1 + s_0. \tag{1.4}$$

On the first condition of (1.2), we see that

$$\frac{2}{\theta} + \frac{3}{p} < \frac{5}{4} + \frac{s}{2} < 1 + s \qquad \text{if } s > \frac{1}{2}.$$

Therefore, the space $L^\theta(0,\infty; \dot{H}^s_p(\mathbb{R}^3))$ in Theorem 1.1 includes more regular functions than those in the scaling invariant spaces. Besides, it is possible to show that the solutions in Theorem 1.1 are smooth. Indeed, by the smoothing effect of the semigroup $T_\Omega(t)$:

$$\|\nabla^\alpha T_\Omega(t) f\|_{L^2} \le C t^{-\frac{|\alpha|}{2}} \|f\|_{L^2} \quad \text{for any } \alpha \in (\mathbb{N} \cup \{0\})^3,$$

we can show that the solution u in Theorem 1.1 is in $C((0,\infty), H^k(\mathbb{R}^3))^3$ for any $k \in \mathbb{N}$.

(iii) In Theorem 1.1, it is possible to show that the gradient of pressure p is smooth. Indeed, ∇p is in $C((0,\infty); H^s(\mathbb{R}^3))^3$ for any $s \ge 0$ by the following fomula:

$$\nabla p = (-\Delta)^{-1} \nabla \Omega\Big(- \partial_{x_1} u_2 + \partial_{x_2} u_1 \Big) + \sum_{j,k=1}^{3} (-\Delta)^{-1} \nabla \Big(\partial_{x_j} u_k \partial_{x_k} u_j \Big),$$

the boundedness of the Riesz transform in the Sobolev space $H^s(\mathbb{R}^3)$ and the smoothness of the solution.

(iv) In the case of periodic boundary condition $\mathbb{T}^3$, it seems difficult to obtain the characterization (1.3) for the size condition on initial data due to the resonances in the nonlinear term and the lack of the dispersive effect. For the existence theorem of solutions to (NSC) in $\mathbb{T}^3$, we refer to Babin et al. [1–4], and Chemin et al. [7].

By Theorem 1.1 for the case $s > 1/2$, it is possible to obtain global solutions for initial data $u_0 \in \dot{H}^s(\mathbb{R}^3)^3$ if Ω satisfies

$$|\Omega| > C \|u_0\|_{\dot{H}^{\frac{1}{2}}}^{\frac{2}{s-\frac{1}{2}}}. \tag{1.5}$$

Therefore, the speed $|\Omega|$ of rotation to obtain global solutions is determined by the each bounded set in $\dot{H}^s(\mathbb{R}^3)$ if $s > 1/2$. We next consider the critical case $s = 1/2$.

Theorem 1.3 *For any* $u_0 \in \dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$ *with* $\operatorname{div} u_0 = 0$*, there exists* $\omega = \omega(u_0) > 0$ *such that for any* $\Omega \in \mathbb{R}$ *with* $|\Omega| > \omega$*, there exists a unique global solution* u *to (NSC) in* $C([0,\infty); \dot{H}^{\frac{1}{2}}(\mathbb{R}^3))^3 \cap L^4(0,\infty; \dot{H}_3^{\frac{1}{2}}(\mathbb{R}^3))^3$.

Remark 1.4 The space $L^4(0,\infty; \dot{H}_3^{\frac{1}{2}}(\mathbb{R}^3))$ in Theorem 1.3 is scaling invariant space in the case $\Omega = 0$ since $\theta_0 = 4$, $s_0 = 1/2$ and $p_0 = 3$ satisfy (1.4).

Since the condition (1.5) breaks down in the case $s = 1/2$, it is not clear whether the Coriolis parameter Ω to obtain global solutions for initial data $u_0 \in \dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$ can be characterized by the norm of initial data $\|u_0\|_{\dot{H}^{\frac{1}{2}}}$ such as (1.5). To overcome this difficulty, we consider a class of precompact subsets in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$. We also show the similar result on the existence of local solutions. In our previous work [14], we considered the case $s > 1/2$ and showed that the existence time $T > 0$ satisfies $T \ge c|\Omega|^\alpha \|u_0\|_{\dot{H}^s}^{-\beta}$ with some constants $c, \alpha, \beta > 0$. By this result, we see that for the time $T > 0$ and the bounded set B in $\dot{H}^s(\mathbb{R}^3)$, the sufficient speed Ω to obtain local solutions is determined by T and B if $s > 1/2$. For the case $s = 1/2$, we obtain the following as a corollary of Theorem 1.3.

Corollary 1.5 *(i) Let K be an arbitrary precompact set in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$. Then, there exists $\omega(K) > 0$ such that for any $\Omega \in \mathbb{R}$ with $|\Omega| > \omega(K)$ and for any $u_0 \in K$ with* $\operatorname{div} u_0 = 0$, *there exists a unique global solution u to (NSC) in $C([0,\infty); \dot{H}^{\frac{1}{2}}(\mathbb{R}^3))^3 \cap L^4(0,\infty; \dot{H}_3^{\frac{1}{2}}(\mathbb{R}^3))^3$.*

(ii) For any $T > 0$ and precompact set K in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$, there exists $\omega = \omega(T, K) > 0$ such that for any $\Omega \in \mathbb{R}$ with $|\Omega| > \omega$ and for any $u_0 \in K$ with $\operatorname{div} u_0 = 0$, *there exists a unique local solution u in $C([0,T); \dot{H}^{\frac{1}{2}}(\mathbb{R}^3))^3 \cap L^4(0,T; \dot{H}_3^{\frac{1}{2}}(\mathbb{R}^3))^3$ to (NSC).*

Remark 1.6 (i) For the original Navier-Stokes equations

$$\begin{cases} \dfrac{\partial u}{\partial t} - \Delta u + (u \cdot \nabla) u + \nabla p = 0 & \text{in } \mathbb{R}^3 \times (0,\infty), \\ \operatorname{div} u = 0 & \text{in } \mathbb{R}^3 \times (0,\infty), \\ u(x,0) = u_0(x) & \text{in } \mathbb{R}^3, \end{cases} \tag{NS}$$

it is known by the results of Brezis [5], Giga [9] and Kozono [16] that the existence time T of local solutions for initial data in $L^r(\mathbb{R}^3)$ $(3 < r < \infty)$ and $L^3(\mathbb{R}^3)$ is determined by the each bounded set B in $L^r(\mathbb{R}^3)$ $(3 < r < \infty)$ and the each precompact set K in $L^3(\mathbb{R}^3)$, respectively. Note that the space $L^3(\mathbb{R}^3)$ is a scaling critical space to (NS). On the other hand, the sufficint speed Ω to obtain global solutions is determined by the bounded sets and precompact sets in Theorem 1.1 and (i) of Corollary 1.5, respectively. Therefore, our theorems can be regarded as a counterpart of such results from the viewpoint of the Coriolis parameter Ω for the existence of global solutions.

(ii) For any precompact set K in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$, the constant $\omega(T,K) > 0$ in (ii) of Corollary 1.5 is increasing and bounded with respect to $T > 0$. Indeed, $\omega(T,K) > \omega(\widetilde{T},K)$ if $T > \widetilde{T}$ since a local solution on the time interval $[0,T)$ is also a solution on $[0,\widetilde{T})$. By (i) of Corollary 1.5 for global solutions, it suffices to take $|\Omega|$ sufficiently large to obtain global solutions and the lower bound $\omega(T,K)$ for local solutions does not diverge to infinity as $T \to \infty$.

This paper is organized as follows. In Sect. 2, we introduce propositions to prove theorems which are on linear estimates for the semigroup $T_\Omega(\cdot)$ and the bilinear estimate. In Sect. 3, we prove Theorem 1.1, Theorem 1.3 and Corollary 1.5.

2 Preliminaries

In what follows, we denote by $C > 0$ various constants and by $0 < c < 1$ various small constants. In order to introduce propositions to provetheorems, let us recall the definition of the homogeneous Besov spaces in brief. Let ϕ be a radial smooth function satisfying

$$\operatorname{supp}\widehat{\phi} \subset \left\{ \xi \in \mathbb{R}^3 \,|\, 2^{-1} \le |\xi| \le 2 \right\}, \quad \sum_{j\in\mathbb{Z}} \widehat{\phi}(2^{-j}\xi) = 1 \qquad \text{for any } \xi \in \mathbb{R}^3 \setminus \{0\}\,.$$

Let $\{\phi_j\}_{j\in\mathbb{Z}}$ be defined by

$$\phi_j(x) := 2^{3j}\phi(2^j x) \quad \text{for } j \in \mathbb{Z}, x \in \mathbb{R}^3.$$

Then, for $s \in \mathbb{R}$, $1 \le p, q \le \infty$, the homogeneous Besov space $\dot{B}^s_{p,q}(\mathbb{R}^3)$ is defined by the set of all tempered distributions $f \in \mathcal{S}'(\mathbb{R}^3)$ with

$$\|f\|_{\dot{B}^s_{p,q}} := \left\| \left\{ 2^{sj} \|\phi_j * f\|_{L^p(\mathbb{R}^3)} \right\}_{j\in\mathbb{Z}} \right\|_{\ell^q(\mathbb{Z})} < \infty.$$

Lemma 2.1 *[14] Let $2 \le p \le \infty$. There exists $C > 0$ such that*

$$\|\mathcal{F}^{-1} e^{\pm i \frac{\xi_3}{|\xi|}\Omega t} \mathcal{F} f\|_{\dot{B}^0_{p,2}} \le C \left\{ \frac{\log(e + |\Omega| t)}{1 + |\Omega| t} \right\}^{\frac{1}{2}(1-\frac{2}{p})} \|f\|_{\dot{B}^{3(1-\frac{2}{p})}_{\frac{p}{p-1},2}} \tag{2.1}$$

for all $\Omega \in \mathbb{R}$, $t > 0$, $f \in \dot{B}^{3(1-\frac{2}{p})}_{\frac{p}{p-1},2}(\mathbb{R}^3)$.

Lemma 2.2 *Let $1 < q \le 2 \le p < \infty$ satisfy $1/q \ge 1 - 1/p$. Then, there exists $C > 0$ such that*

$$\|T_\Omega(t) f\|_{L^p} \le C t^{-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})} \left\{ \frac{\log(e + |\Omega| t)}{1 + |\Omega| t} \right\}^{\frac{1}{2}(1-\frac{2}{p})} \|f\|_{L^q} \tag{2.2}$$

for all $\Omega \in \mathbb{R}$, $t > 0$, $f \in L^q(\mathbb{R}^3)$.

Proof By the continuous embedding $\dot{B}^0_{p,2}(\mathbb{R}^3) \hookrightarrow L^p(\mathbb{R}^3)$ and (2.1), we have

$$\|T_\Omega(t) f\|_{L^p} \le C \|T_\Omega(t) f\|_{\dot{B}^0_{p,2}} \le C \left\{ \frac{\log(e + |\Omega| t)}{1 + |\Omega| t} \right\}^{\frac{1}{2}(1-\frac{2}{p})} \|e^{t\Delta} f\|_{\dot{B}^{3(1-\frac{2}{p})}_{\frac{p}{p-1},2}}.$$

We obtain from Lemma 2.2 in [17] and the continuous embedding $L^q(\mathbb{R}^3) \hookrightarrow \dot{B}^0_{q,2}(\mathbb{R}^3)$

$$\|e^{t\Delta} f\|_{\dot{B}^{3(1-\frac{2}{p})}_{\frac{p}{p-1},2}} \le C t^{-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})} \|f\|_{\dot{B}^0_{q,2}} \le C t^{-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})} \|f\|_{L^q(\mathbb{R}^3)}.$$

Therefore, (2.2) is obtained.

Proposition 2.3 *[14] Let* $2 < p < 6, 2 < \theta < \infty$ *satisfy*

$$\frac{3}{4} - \frac{3}{2p} \leq \frac{1}{\theta} < 1 - \frac{2}{p}.$$

Then, there exists $C > 0$ *such that*

$$\|T_\Omega(\cdot)f\|_{L^\theta(0,\infty;L^p)} \leq C|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|f\|_{L^2}$$

for all $\Omega \in \mathbb{R}\setminus\{0\}$, $f \in L^2(\mathbb{R}^3)$.

Proposition 2.4 *For every* $f \in \dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$*, it holds that*

$$\lim_{|\Omega|\to\infty} \|T_\Omega(\cdot)f\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} = 0. \tag{2.3}$$

Proof Let $\mathcal{Z}(\mathbb{R}^3)$ be defined by

$$\mathcal{Z}(\mathbb{R}^3) := \left\{ f \in \mathcal{S}(\mathbb{R}^3) \,\middle|\, \int_{\mathbb{R}^3} f(x)dx = 0 \right\}.$$

Since $\mathcal{Z}(\mathbb{R}^3)$ is dense in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$, there exists $\{f_N\}_{N=1}^\infty \subset \mathcal{Z}(\mathbb{R}^3)$ such that $f_N \to f$ in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$ as $N \to \infty$. Then, we have from Proposition 2.3

$$\begin{aligned} \|T_\Omega(\cdot)f\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} &\leq \|T_\Omega(\cdot)(f_N - f)\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} + \|T_\Omega(\cdot)f_N\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} \\ &\leq C\|f_N - f\|_{\dot{H}^{\frac{1}{2}}} + \|T_\Omega(\cdot)f_N\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}. \end{aligned} \tag{2.4}$$

On the second term of the last right hand side, we take p satisfying $8/3 < p < 3$ and have from the embedding $\dot{H}_p^{-\frac{1}{2}+\frac{3}{p}}(\mathbb{R}^3) \hookrightarrow \dot{H}_3^{\frac{1}{2}}(\mathbb{R}^3)$, Proposition 2.3 and $3/4 - 3/2p < 1/4$

$$\begin{aligned} \|T_\Omega(\cdot)f_N\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} &\leq C\|T_\Omega(\cdot)f_N\|_{L^4(0,\infty;\dot{H}_p^{-\frac{1}{2}+\frac{3}{p}})} \\ &\leq C|\Omega|^{-\frac{1}{4}+\frac{3}{4}(1-\frac{2}{p})}\|f\|_{\dot{H}_2^{-\frac{1}{2}+\frac{3}{p}}} \\ &\to 0 \quad \text{as } |\Omega| \to \infty. \end{aligned} \tag{2.5}$$

Therefore, (2.3) is obtained by (2.4), (2.5) and the convergence $f_N \to f$ in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$ as $N \to \infty$.

Proposition 2.5 *Let* $2 < p < 3$ *and* $6/5 < q < 2$ *satisfy*

$$1-\frac{1}{p}\le\frac{1}{q}<\frac{1}{3}+\frac{1}{p}, \tag{2.6}$$

$$\max\left\{0,\frac{1}{2}-\frac{3}{2}\Big(\frac{1}{q}-\frac{1}{p}\Big)-\frac{1}{2}\Big(1-\frac{2}{p}\Big)\right\}<\frac{1}{\theta}\le\frac{1}{2}-\frac{3}{2}\Big(\frac{1}{q}-\frac{1}{p}\Big). \tag{2.7}$$

Then, there exists $C > 0$ *such that*

$$\Big\|\int_0^t T_\Omega(t-\tau)\mathbb{P}\nabla f(\tau)d\tau\Big\|_{L^\theta(0,\infty;\dot H^s_p)}\le C|\Omega|^{-\left\{\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})-\frac{1}{\theta}\right\}}\|f\|_{L^{\frac{\theta}{2}}(0,\infty;\dot H^s_q)} \tag{2.8}$$

for all $s\in\mathbb{R}$, $\Omega\in\mathbb{R}\setminus\{0\}$, $f\in L^{\frac{\theta}{2}}(0,\infty;\dot H^s_q(\mathbb{R}^3))$.

Proof We only consider the case $s = 0$ for simplicity since it is possible to treat the case $s \neq 0$ similarly. By Lemma 2.2, we have

$$\begin{aligned}&\Big\|\int_0^t T_\Omega(t-\tau)\mathbb{P}\nabla f(\tau)d\tau\Big\|_{L^\theta(0,\infty;L^p)}\\&\le C\Big\|\int_0^t (t-\tau)^{-\frac{1}{2}-\frac{3}{2}\left(\frac{1}{q}-\frac{1}{p}\right)}\left\{\frac{\log(e+|\Omega||t-\tau|)}{1+|\Omega||t-\tau|}\right\}^{\frac{1}{2}\left(1-\frac{2}{p}\right)}\|f(\tau)\|_{L^q}d\tau\Big\|_{L^\theta(0,\infty)}.\end{aligned}$$

In the case $1/\theta = 1/2-3(1/q-1/p)/2$, we have from Hardy-Littlewood-Sobolev's inequality

$$\begin{aligned}&\Big\|\int_0^t (t-\tau)^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}\left\{\frac{\log(e+|\Omega||t-\tau|)}{1+|\Omega||t-\tau|}\right\}^{\frac{1}{2}(1-\frac{2}{p})}\|f(\tau)\|_{L^q}d\tau\Big\|_{L^\theta(0,\infty)}\\&\le\Big\|\int_0^t (t-\tau)^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}\|f(\tau)\|_{L^q}d\tau\Big\|_{L^\theta(0,\infty)}\\&\le C\|f\|_{L^{\frac{\theta}{2}}(0,\infty;L^q)}.\end{aligned}$$

In the case $1/\theta < 1/2-3(1/q-1/p)/2$, we have from Hausdorff-Young's inequality with $1/\theta = 1/r + 2/\theta - 1$

$$\begin{aligned}
&\Big\| \int_0^t (t-\tau)^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})} \left\{ \frac{\log(e+|\Omega||t-\tau|)}{1+|\Omega||t-\tau|} \right\}^{\frac{1}{2}(1-\frac{2}{p})} \|f(\tau)\|_{L^q} d\tau \Big\|_{L^\theta(0,\infty)} \\
&\le \Big\| t^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{q}-\frac{1}{p})} \left\{ \frac{\log(e+|\Omega|t)}{1+|\Omega|t} \right\}^{\frac{1}{2}(1-\frac{2}{p})} \Big\|_{L^r(0,\infty)} \|f\|_{L^{\frac{\theta}{2}}(0,\infty;L^q)} \\
&= C|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})} \|f\|_{L^{\frac{\theta}{2}}(0,\infty;L^q)}.
\end{aligned}$$

Therefore, (2.8) is obtained.

Proposition 2.6 *There exists a positive constant C such that*

$$\Big\| \int_0^t T_\Omega(t-\tau)\nabla f(\tau) d\tau \Big\|_{L^\infty(0,\infty;\dot{H}^s)\cap L^4(0,\infty;\dot{H}^s_3)} \le C\|f\|_{L^2(0,\infty;\dot{H}^s)} \tag{2.9}$$

for all $s \in \mathbb{R}, \Omega \in \mathbb{R}$, $f \in L^2(0,\infty;\dot{H}^s(\mathbb{R}^3))$.

Proof For simplicity, we show (2.9) in the case $s = 0$ since it is possible to treat the case $s \neq 0$ similarly. On the $L^\infty(0,\infty;L^2)$ norm, we have from Plancherel's theorem and Hölder's inequality

$$\begin{aligned}
\Big\| \int_0^t T_\Omega(t-\tau)\nabla f(\tau) d\tau \Big\|_{L^2} &\le C \Big\| \int_0^t e^{-(t-\tau)|\xi|^2} |\xi| |\widehat{f}(\tau)| d\tau \Big\|_{L^2} \\
&\le C \Big\| \|e^{-(t-\tau)|\xi|^2}\|_{L^2_\tau(0,t)} |\xi| \|\widehat{f}(\tau)\|_{L^2_\tau(0,t)} \Big\|_{L^2} \\
&\le C\|\widehat{f}\|_{L^2(0,\infty;L^2)} \\
&= C\|f\|_{L^2(0,\infty;L^2)}.
\end{aligned} \tag{2.10}$$

On the $L^4(0,\infty;L^3(\mathbb{R}^3))$ norm, we have from (2.2) and Hardy-Littlewood-Sobolev's inequality

$$\begin{aligned}
&\Big\| \int_0^t T_\Omega(t-\tau)\nabla f(\tau) d\tau \Big\|_{L^4(0,\infty;L^3(\mathbb{R}^3))} \\
&\le C \Big\| \int_0^t (t-\tau)^{-\frac{1}{2}-\frac{3}{2}(\frac{1}{2}-\frac{1}{3})} \|f(\tau)\|_{L^2} d\tau \Big\|_{L^4(0,\infty)} \\
&\le C\|f\|_{L^2(0,\infty;L^2)}.
\end{aligned} \tag{2.11}$$

Therefore, (2.9) is obtained by (2.10) and (2.11).

Lemma 2.7 *Let s, p satisfy*

$$0 \le s < 3, \quad \frac{s}{3} < \frac{1}{p} < \frac{1}{2} + \frac{s}{6},$$

and let q satisfy

$$\frac{1}{q} = \frac{2}{p} - \frac{s}{3}.$$

Then, there exists $C > 0$ such that

$$\|fg\|_{\dot{H}^s_q} \le C\|f\|_{\dot{H}^s_p}\|g\|_{\dot{H}^s_p}. \tag{2.12}$$

Proof Let r satisfy $1/q = 1/p + 1/r$. In the Sobolev spaces, it is known that

$$\|fg\|_{\dot{H}^s_q} \le C\|f\|_{\dot{H}^s_p}\|g\|_{L^r} + C\|f\|_{L^r}\|g\|_{\dot{H}^s_p}.$$

By the continuous embedding $\dot{H}^s_p(\mathbb{R}^3) \hookrightarrow L^r(\mathbb{R}^3)$, we obtain (2.12).

3 Proof of theorems

We prove Theorem 1.1 and Corollary 1.5 only. It is possible to show Theorem 1.3 in the analogous way to the proof of (i) of Corollary 1.5 since the set $\{u_0\} \subset \dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$ is compact for each $u_0 \in \dot{H}^{\frac{1}{2}}(\mathbb{R}^3)^3$.

Proof of Theorem 1.1 Since the assumption on θ and p in Proposition 2.3 is satisfied by (1.1) and (1.2), there exists $C_0 > 0$ such that

$$\|T_\Omega(\cdot)u_0\|_{L^\theta(0,\infty;\dot{H}^s_p)} \le |\Omega|^{-\frac{1}{\theta}+\frac{3}{4}\left(1-\frac{2}{p}\right)}C_0\|u_0\|_{\dot{H}^s}.$$

Let $\Psi(u)$ and Y be defined by

$$\Psi(u)(t) := T_\Omega(t)u_0 - \int_0^t T_\Omega(t-\tau)\mathbb{P}\nabla\cdot(u\otimes u)(\tau)d\tau, \tag{3.1}$$

$$Y := \left\{ u \in L^\theta(0,\infty;\dot{H}^s_p(\mathbb{R}^3))^3 \mid \|u\|_{L^\theta(0,\infty;\dot{H}^s_p)} \le 2C_0|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}\left(1-\frac{2}{p}\right)}\|u_0\|_{\dot{H}^s}, \right.$$
$$\left. \operatorname{div} u = 0 \right\}.$$

Let q satisfy $1/q = 2/p - s/3$. Since the assumptions on s, p, q and θ in Proposition 2.5 and Lemma 2.7 are satisfied by (1.1) and (1.2), for any $u, v \in Y$, we have from Proposition 2.3, Proposition 2.5 and Lemma 2.7

$$\begin{aligned}
&\|\Psi(u)\|_{L^\theta(0,\infty;\dot{H}^s_p)}\\
&\le C_0|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|u_0\|_{\dot{H}^s} + C|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}\|u\otimes u\|_{L^{\frac{\theta}{2}}(0,\infty;\dot{H}^s_q)}\\
&\le C_0|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|u_0\|_{\dot{H}^s} + C|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}\|u\|^2_{L^\theta(0,\infty;\dot{H}^s_p)}\\
&\le C_0|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|u_0\|_{\dot{H}^s} + C_1|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})+2\left\{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})\right\}}\|u_0\|^2_{\dot{H}^s}\\
&\le C_0|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|u_0\|_{\dot{H}^s} + C_1|\Omega|^{-\frac{s}{2}+\frac{1}{4}}|\Omega|^{-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|u_0\|^2_{\dot{H}^s}, \qquad (3.2)\\
&\|\Psi(u)-\Psi(v)\|_{L^\theta(0,\infty;\dot{H}^s_p)}\\
&= \left\| \int_0^t T_\Omega(t-\tau)\mathbb{P}\nabla\cdot\{u\otimes(u-v)(\tau)+(u-v)\otimes v(\tau)\}\,d\tau \right\|_{L^\theta(0,\infty;\dot{H}^s_p)}\\
&\le C|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}\|u\otimes(u-v)+(u-v)\otimes v\|_{L^{\frac{\theta}{2}}(0,\infty;\dot{H}^s_q)}\\
&\le C|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})}(\|u\|_{L^\theta(0,\infty;\dot{H}^s_p)}+\|v\|_{L^\theta(0,\infty;\dot{H}^s_p)})\|u-v\|_{L^\theta(0,\infty;\dot{H}^s_p)}\\
&\le C_2|\Omega|^{\frac{1}{\theta}-\frac{1}{2}+\frac{3}{2}(\frac{1}{q}-\frac{1}{p})-\frac{1}{\theta}+\frac{3}{4}(1-\frac{2}{p})}\|u_0\|_{\dot{H}^s}\|u-v\|_{L^\theta(0,\infty;\dot{H}^s_p)}\\
&= C_2|\Omega|^{\frac{1}{4}+\frac{3}{2q}-\frac{3}{p}}\|u_0\|_{\dot{H}^s}\|u-v\|_{L^\theta(0,\infty;\dot{H}^s_p)}\\
&= C_2|\Omega|^{-\frac{s}{2}+\frac{1}{4}}\|u_0\|_{\dot{H}^s}\|u-v\|_{L^\theta(0,\infty;\dot{H}^s_p)}.
\end{aligned}$$

If Ω, u_0 satisfy

$$C_1|\Omega|^{-\frac{s}{2}+\frac{1}{4}}\|u_0\|_{\dot{H}^s} \le C_0, \quad C_2|\Omega|^{-\frac{s}{2}+\frac{1}{4}}\|u_0\|_{\dot{H}^s} \le \frac{1}{2},$$

then, it is possible to apply Banach's fixed point theorem in Y and we obtain $u \in Y$ with

$$u(t) = T_\Omega(t)u_0 - \int_0^t T_\Omega(t-\tau)\mathbb{P}\nabla\cdot(u\otimes u)d\tau.$$

Here, we show that the solution $u \in Y$ satisfies $u(t) \in \dot{H}^s(\mathbb{R}^3)^3$ for all $t \ge 0$. On the linear part, it is easy to see that $T_\Omega(t)u_0 \in \dot{H}^s(\mathbb{R}^3)^3$ for any $t \ge 0$. On the nonlinear part, let $1/q = 2/p - s/3$ and we have from Lemma 2.2, Lemma 2.7 and Hölder's inequality

$$
\begin{aligned}
&\Big\| \int_0^t T_\Omega(t-\tau)\mathbb{P}\nabla\cdot(u\otimes u)(\tau)d\tau \Big\|_{\dot{H}^s}\\
&\le C\int_0^t (t-\tau)^{-\frac12-\frac32(\frac1q-\frac12)}\|(u\otimes u)(\tau)\|_{\dot{H}^s_q}d\tau\\
&\le C\int_0^t (t-\tau)^{-\frac12-\frac32(\frac1q-\frac12)}\|u(\tau)\|^2_{\dot{H}^s_p}d\tau \qquad (3.3)\\
&\le C\Big\|(t-\cdot)^{-\frac12-\frac32(\frac1q-\frac12)}\Big\|_{L^{\frac{\theta}{\theta-2}}(0<\tau<t)}\Big\|\|u(\tau)\|^2_{\dot{H}^s_p}\Big\|_{L^{\frac{\theta}{2}}(0,\infty)}\\
&\le Ct^{\frac{\theta-2}{\theta}\left[1-\frac{\theta}{\theta-2}\left\{-\frac12-\frac32(\frac1q-\frac12)\right\}\right]}\|u\|^2_{L^\theta(0,\infty;\dot{H}^s_p)}.
\end{aligned}
$$

Here, we note on the integrability at $\tau = t$ that

$$
\frac{\theta}{\theta-2}\left\{\frac12+\frac32\left(\frac1q-\frac12\right)\right\}<1 \quad \text{if and only if} \quad \frac1\theta<\frac58-\frac{3}{2p}+\frac{s}{4}.
$$

Therefore, we obtain $u(t)\in\dot{H}^s(\mathbb{R}^3)^3$ and we also see $u\in C([0,\infty),\dot{H}^s(\mathbb{R}^3))^3$. □

Proof of (i) of Corollary 1.5 Let $\delta>0$ be an arbitrary positive number to be determined later. Since K is precompact in $\dot{H}^{\frac12}(\mathbb{R}^3)^3$, the closure of K is compact. Hence there exist a natural number $N(\delta,K)$ and $\{f_j\}_{j=1}^{N(\delta,K)}\subset\dot{H}^{\frac12}(\mathbb{R}^3)^3$ such that

$$
K\subset\cup_{j=1}^{N(\delta,K)}B(f_j,\delta),
$$

where $B(f,\delta)$ denotes a ball in $\dot{H}^{\frac12}(\mathbb{R}^3)^3$ with center being f and radius δ. By Proposition 2.4, there exists $\omega_0(\delta,K)>0$ such that we have

$$
\sup_{j=1,2,\dots,N(\delta,K)}\|T_\Omega(\cdot)f_j\|_{L^4(0,\infty;\dot{H}^{\frac12}_3)}\le\delta
$$

for all $\Omega\in\mathbb{R}$ with $|\Omega|>\omega_0(\delta,K)$. Then, for any $f\in K$, there exists $j\in\{1,2,\dots,N(\delta,K)\}$ such that $f\in B(f_j,\delta)$ and we have from Proposition 2.3

$$
\begin{aligned}
\|T_\Omega(\cdot)f\|_{L^4(0,\infty;\dot{H}^{\frac12}_3)} &\le \|T_\Omega(\cdot)(f_j-f)\|_{L^4(0,\infty;\dot{H}^{\frac12}_3)}+\|T_\Omega(\cdot)f_j\|_{L^4(0,\infty;\dot{H}^{\frac12}_3)}\\
&\le C\|f_j-f\|_{\dot{H}^{\frac12}}+\delta\\
&\le C\delta.
\end{aligned}
$$

Therefore, there exists a positive constant $C_1>0$

$$
\sup_{f\in K}\|T_\Omega(\cdot)f\|_{L^4(0,\infty;\dot{H}^{\frac12}_3)}\le C_1\delta \qquad (3.4)
$$

for all $\Omega \in \mathbb{R}$ with $|\Omega| > \omega_0(\delta, K)$. Then, let the space X be defined by

$$X := \left\{ u \in C([0,\infty), \dot{H}^{\frac{1}{2}}(\mathbb{R}^3))^3 \mid \|u\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} \le 2C_1\delta,\ \operatorname{div} u = 0 \right\},$$
$$d(u,v) := \|u-v\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}.$$

Let Ψ be defined by (3.1). For any $u \in X$, we have from Proposition 2.6, Lemma 2.7 and Hölder's inequality

$$\begin{aligned}\|\Psi(u)\|_{L^\infty(0,\infty;\dot{H}^{\frac{1}{2}})} &\le C\|u_0\|_{\dot{H}^{\frac{1}{2}}} + C\|u\otimes u\|_{L^2(0,\infty;\dot{H}^{\frac{1}{2}})}\\ &\le C\|u_0\|_{\dot{H}^{\frac{1}{2}}} + C\left\| \|u\|^2_{\dot{H}_3^{\frac{1}{2}}} \right\|_{L^2(0,\infty)}\\ &\le C\|u_0\|_{\dot{H}^{\frac{1}{2}}} + C\|u\|^2_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}. \end{aligned} \tag{3.5}$$

We also have from Proposition 2.6, Lemma 2.7 and Hölder's inequality

$$\begin{aligned}\|\Psi(u)\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} &\le \|T_\Omega(\cdot)u_0\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} + C\|u\otimes u\|_{L^2(0,\infty;\dot{H}^{\frac{1}{2}})}\\ &\le \|T_\Omega(\cdot)u_0\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} + C\left\| \|u\|^2_{\dot{H}_3^{\frac{1}{2}}} \right\|_{L^2(0,\infty)}\\ &\le \|T_\Omega(\cdot)u_0\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} + C_2\|u\|^2_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}. \end{aligned} \tag{3.6}$$

Similarly, we also have for $u, v \in X$

$$\begin{aligned}&\|\Psi(u)-\Psi(v)\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}\\ &\le C_3\Big(\|u\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} + \|v\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}\Big)\|u-v\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}. \end{aligned} \tag{3.7}$$

Here, since δ is an arbitrary positive number, let $\delta > 0$ satisfy

$$\delta < \min\left\{\frac{1}{4C_1C_2}, \frac{1}{8C_1C_3}\right\},$$

where C_1, C_2 and C_3 is the constants in (3.4), (3.6) and (3.7), respectively. Then, we have from (3.4), (3.5), (3.6) and (3.7)

$$\begin{aligned}&\|\Psi(u)\|_{L^\infty(0,\infty;\dot{H}^{\frac{1}{2}})} < \infty\\ &\|\Psi(u)\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} \le 2C_1\delta,\\ &\|\Psi(u)-\Psi(v)\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})} \le \frac{1}{2}\|u-v\|_{L^4(0,\infty;\dot{H}_3^{\frac{1}{2}})}, \end{aligned}$$

for all $u, v \in X, \Omega \in \mathbb{R}$ with $|\Omega| > \omega_0(\delta, K)$. Therefore, it is possible to apply Banach's fixed point theorem to obtain the global solutions. □

Proof of (ii) of Corollary 1.5 By the same argument on the precompact set K in $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$ as that of proof of (i) of Corollary 1.5, we see that for any $T > 0$ and $\delta > 0$, there exist $\omega(T, K) > 0$ and $C_1 > 0$ such that

$$\sup_{f\in K} \|T_\Omega(\cdot)f\|_{L^4(0,T;\dot{H}_3^{\frac{1}{2}})} \le C_1\delta,$$

for all $\Omega \in \mathbb{R}$ with $|\Omega| > \omega(T, K)$. Then, we can obtain the similar estimate as (3.5), (3.6) and (3.7) in which time interval $(0, \infty)$ is replaced with $(0, T)$. It is possible to apply Banach's fixed point theorem in the space

$$X := \left\{ u \in C([0, T), \dot{H}^{\frac{1}{2}}(\mathbb{R}^3))^3 \mid \|u\|_{L^4(0,T;\dot{H}_3^{\frac{1}{2}})} \le 2C_1\delta,\ \operatorname{div} u = 0 \right\},$$
$$d(u, v) := \|u - v\|_{L^4(0,T;\dot{H}_3^{\frac{1}{2}})}$$

and obtain local solutions. □

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