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Residual $\bar\partial$-cohomology and the complex Radon transform on subvarieties of $\mathbb{C}P^n$

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Abstract We show that the complex Radon transform realizes an isomorphism between the quotient-space of residual $\bar\partial$-cohomologies of a locally complete intersection algebraic subvariety in a linearly concave domain of $\mathbb{C}P^n$ and the space of holomorphic solutions of the associated homogeneous system of differential equations with constant coefficients in the dual domain in $(\mathbb{C}P^n)^*$.

1 Introduction

In this article we consider two related problems: the first one is the description of infinite-dimensional spaces of $\bar\partial$-cohomologies of subvarieties in linearly concave domains of $\mathbb{C}P^n$ in terms of inverse Radon transform of the spaces of holomorphic solutions of associated systems of differential equations in dual domains, and the second one is the realization of the spaces of holomorphic solutions of systems of linear differential equations in convex domains by Radon transforms of $\bar\partial$-cohomologies of associated subvarieties in dual domains.

The study of these problems was started by Martineau in [24,25] and was continued in the papers [2,7,11,16–18]. The main result of Martineau in [24] was interpreted in [11] as the existence of an isomorphism defined by the complex Radon

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transform between the space of $(n, n-1)\bar{\partial}$-cohomologies of a linearly concave domain $D \subset \mathbb{C}P^n$ and the space of holomorphic functions on the dual linearly convex domain $D^* \subset (\mathbb{C}P^n)^*$.

We begin by describing the result that was produced by the study of the problems mentioned above in [16,18] for the case of complex submanifolds in linearly concave domains in $\mathbb{C}P^n$.

Let $(z_0, \ldots, z_n)$ and $(\xi_0, \ldots, \xi_n)$ be the homogeneous coordinates of points $z \in \mathbb{C}P^n$ and $\xi \in (\mathbb{C}P^n)^*$. Let $\langle \xi \cdot z \rangle \overset{\text{def}}{=} \sum_{k=0}^n \xi_k \cdot z_k$, and let $\mathbb{C}P_\xi^{n-1}$ denote the hyperplane

$$\mathbb{C}P_\xi^{n-1} = \{z \in \mathbb{C}P^n : \langle \xi \cdot z \rangle = 0\}.$$

Following [11,24] we call a domain $D \subset \mathbb{C}P^n$ a linearly concave domain, if there exists a continuous family of hyperplanes $\mathbb{C}P^{n-1}(z) \subset D$ defined for $z \in D$ and satisfying $z \in \mathbb{C}P^{n-1}(z)$. We notice that in the original definition of linearly concave domains in [24] the continuity of the family was not required, but the main results of [11,16,18,24,25] are valid only under the assumption of existence of such family.

The following theorem was obtained in [16].

Theorem 1 *Let D be a linearly concave domain in $\mathbb{C}P^n$, $n \geq 2$, and let $D^* \subset (\mathbb{C}P^n)^*$ be the dual domain*

$$D^* = \{\xi \in (\mathbb{C}P^n)^* : \mathbb{C}P_\xi^{n-1} \subset D\}.$$

Let V be a $(n-m)$-dimensional connected algebraic manifold of the form

$$V = \{z \in \mathbb{C}P^n : P_1(z) = \ldots = P_r(z) = 0\},$$

where homogeneous polynomials $P_1, \ldots, P_r$ are such that everywhere on V

$$rank\,[grad\, P_1, \ldots, grad\, P_r] = m.$$

Let $V_D = V \cap D$, let $Z^{(n-m,n-m-1)}(V_D)$ denote the space of $\bar{\partial}$-closed smooth forms on V_D of bidegree $(n-m, n-m-1)$, and let $H^0(D^*)$ and $H^{(1,0)}(D^*)$ denote the spaces of holomorphic functions and respectively holomorphic 1-forms on D^*.
Then the Radon transform

$$\mathcal{R}_V : Z^{(n-m,n-m-1)}(V_D) \to H^{(1,0)}(D^*)$$

defined by the formula

$$\mathcal{R}_V[\phi](\xi) = \sum_{j=0}^n \left(\int\limits_{z \in \mathbb{C}P_\xi^{n-1} \cap V} \langle \xi \cdot dz \rangle \rfloor\, z_j \phi \right) d\xi_j \tag{1}$$

induces a continuous linear operator on the space of cohomologies

$$\mathcal{R}_V : H^{(n-m,n-m-1)}(V_D) \to H^{(1,0)}(D^*).$$

Moreover, the following properties are satisfied:

(i) *the subspace $Ker\mathcal{R}_V \subset H^{(n-m,n-m-1)}(V_D)$ is finite-dimensional and consists of restrictions to V_D of $\bar{\partial}$-cohomologies from $H^{(n-m,n-m-1)}(V)$,*

(ii) *the image of $\mathcal{R}_V$ is the following subspace in $H^{(1,0)}(D^*)$*

$$\mathcal{R}_V(H^{(n-m,n-m-1)}(V_D)) = \Big\{ f \in H^{(1,0)}\left(D^*\right) : f = dg \text{ with } g \in H^0\left(D^*\right) \text{ such that } \left\{ P_k\left(\frac{\partial}{\partial \xi}\right) g = 0 \right\}_1^r \Big\}. \tag{2}$$

Remarks

- If $V \subset \mathbb{C}P^n$ is a smooth complete intersection, and $D \subset \mathbb{C}P^n$ is a linearly concave domain, then in ([16] Theorem 5.1) an explicit inversion formula for R_V is obtained in the spirit of explicit fundamental principle of [3].
- For $m = n - 1$ the statement (i) of Theorem 1 is a corollary of the inverse Abel theorem (see Saint-Donat [33], Griffiths [12]). For $m < n - 1$ and V—complete intersection, the statement (i) of Theorem 1 is a consequence of Theorem 3.3 from [18].
- In the statement (ii) of Theorem 1 if $\phi \in Z^{(n-m,n-m-1)}(V_D)$ is such that $\phi = \partial\psi$ for $\psi \in Z^{(n-m-1,n-m-1)}(V_D)$, then g is the image of ψ under the map introduced by Andreotti and Norguet (see [1,26]).

The main result of this article is a natural generalization of Theorem 1 to the case of an arbitrary locally complete intersection in a linearly concave domain. In order to formulate this theorem we need to introduce some additional definitions and notations.

Throughout the whole article we will denote by $D \subset \mathbb{C}P^n$ a linearly concave domain and by $G = \mathbb{C}P^n \backslash D$ its complement. We will also denote by D_δ linearly concave subdomains of D with smooth boundaries bD_δ such that

$$D_\delta \subset D_\nu \quad \text{for } \nu < \delta, \quad \text{and} \quad \bigcup_\delta D_\delta = D.$$

The existence of a sequence of subdomains with the above properties is proved in Proposition 2.4. We will denote by $G_\delta = \mathbb{C}P^n \backslash D_\delta \supset G$ and by $\mathring{G} = G \backslash bG$.

Definition 1.1 (Locally Complete Intersections) An analytic subvariety $V \subset \mathbb{C}P^n$ is called a locally complete intersection subvariety in $\mathbb{C}P^n$ of pure dimension $n - m$ if there exist a finite open cover $\{U_\alpha\}_{\alpha=1}^N$ of $\mathbb{C}P^n$ and collections of holomorphic functions $\{F_k^{(\alpha)}\}$ in U_α, such that

$$V \cap U_\alpha = \{z \in U_\alpha : \ F_1^{(\alpha)}(z) = \cdots = F_m^{(\alpha)}(z) = 0\} \tag{3}$$

with the structure sheaf $\mathcal{O}/\mathcal{I}$, where $\mathcal{O}$ is the structure sheaf of $\mathbb{C}P^n$, and $\mathcal{I}$ is the sheaf of ideals defined by $\{F_k^\alpha\}_{k=1}^m$.

In our construction of $\bar\partial$-closed residual currents on a locally complete intersection variety V we will use a special vector bundle, the so-called *conormal vector bundle*. To describe this bundle we consider a domain $U \subset \mathbb{C}P^n$, a finite cover $\{U_\alpha\}_{\alpha=1}^N$ of U, and $V \subset U$—a locally complete intersection subvariety in U of pure dimension $n-m$, locally defined in U_α by the holomorphic vector function

$$\mathbf{F}^{(\alpha)}(z) = \begin{bmatrix} F_1^{(\alpha)}(z) \\ \vdots \\ F_m^{(\alpha)}(z) \end{bmatrix},$$

i.e.

$$V \cap U_\alpha = \{z \in U_\alpha : \ F_1^{(\alpha)}(z) = \cdots = F_m^{(\alpha)}(z) = 0\}.$$

Definition 1.2 (Conormal and Dualizing Bundles) The conormal vector bundle $N(V)$ on a locally complete intersection subvariety V is defined by the nondegenerate holomorphic transition matrices $A_{\alpha\beta}^{-1}(z) \in H\left(U_{\alpha\beta}\right)$ such that

$$\mathbf{F}^{(\alpha)}(z) = A_{\alpha\beta}(z) \cdot \mathbf{F}^{(\beta)}(z) \tag{4}$$

on $U_{\alpha\beta} = U_\alpha \cap U_\beta$.

Following [14,15] we define the dualizing bundle on a locally complete intersection subvariety V as

$$\omega_V^\circ = \omega_{\mathbb{C}P^n} \otimes \det N(V)^{-1} \tag{5}$$

where $\omega_{\mathbb{C}P^n}$ is the canonical bundle on $\mathbb{C}P^n$.

Remark Adjunction formula (see Proposition 8.20 in Ch. II of [15]) shows that for a nonsingular V the bundle defined in (5) coincides with the canonical bundle ω_V, implicitly used in Theorem 1, making ω_V° a natural generalization of the canonical bundle for locally complete intersection subvarieties of $\mathbb{C}P^n$.

We define further the spaces of residual currents and of residual $\bar\partial$-cohomologies on V_D, where $V \subset \mathbb{C}P^n$ is a locally complete intersection subvariety, and D a domain in $\mathbb{C}P^n$. In what follows we denote by $\mathcal{E}$ the space of infinitely differentiable functions.

Definition 1.3 (Residual currents) For a subvariety $V \subset \mathbb{C}P^n$ of the pure dimension $n-m$ locally satisfying (3) we say that an $(n, m+q)$ current ϕ with support in V is a residual current $\phi \in C^{(0,q)}(V_D, \omega_V^\circ)$ if there exists a finite collection of open neighborhoods $\{U_\alpha \subset \mathbb{C}P^n\}_{\alpha=1}^N$ and differential forms $\Phi_\alpha \in \mathcal{E}^{(n,q)}(U_\alpha \cap D)$, such that

$$\begin{cases} \bigcup_{\alpha=1}^N U_\alpha \supset V, \\ \langle \phi, \psi \rangle = \int_{U_\alpha} \psi \wedge \Phi_\alpha \wedge \bar{\partial}\left(\frac{1}{F_1^{(\alpha)}}\right) \wedge \cdots \wedge \bar{\partial}\left(\frac{1}{F_m^{(\alpha)}}\right) \stackrel{\text{def}}{=} \lim_{t\to 0} \int_{T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \frac{\psi \wedge \Phi_\alpha}{\prod_{k=1}^m F_k^{(\alpha)}}, \\ \Phi_\alpha = (\det A_{\alpha\beta}) \cdot \Phi_\beta + \sum_{k=1}^m F_k^{(\alpha)} \cdot \Omega_k^{(\alpha\beta)} \text{ on } U_\alpha \cap U_\beta \cap D, \end{cases} \tag{6}$$

where $\psi \in \mathcal{E}_c^{(0,n-m-q)}(U_\alpha \cap D)$ is a smooth form with compact support in $U_\alpha \cap D$,

$$T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t) = \{|F_1^{(\alpha)}(z)| = \epsilon_1(t), \ldots, |F_m^{(\alpha)}(z)| = \epsilon_m(t)\}$$

is a family of tubular varieties depending on the real parameter t, the limit in the right-hand side is taken along an admissible path $\{\epsilon_k(t)\}_1^m$ in the sense of Coleff–Herrera–Lieberman [4,20], i.e. an analytic map $\epsilon : [0,1] \to \mathbb{R}^m$ satisfying the conditions

$$\begin{cases} \lim_{t\to 0} \epsilon_m(t) = 0, \\ \lim_{t\to 0} \frac{\epsilon_j(t)}{\epsilon_{j+1}^l(t)} = 0, \quad \text{for any } l \in \mathbb{N}, \end{cases} \tag{7}$$

$A_{\alpha\beta}$ are holomorphic matrices from (4), and $\Omega_k^{(\alpha\beta)} \in \mathcal{E}(U_\alpha \cap U_\beta \cap D)$.
A residual current $\phi \in C^{(0,q)}(V_D, \omega_V^\circ)$ is called $\bar{\partial}$-closed—$\phi \in Z^{(0,q)}(V_D, \omega_V^\circ)$, if the following condition is satisfied

$$\bar{\partial}\Phi_\alpha = \sum_{k=1}^m F_k^{(\alpha)} \cdot \Omega_k^{(\alpha)} \text{ on } U_\alpha \cap D, \tag{8}$$

where $\Omega_k^{(\alpha)} \in \mathcal{E}(U_\alpha \cap D)$.

Remarks

- Condition (7), though looking technical, can not be replaced by a simpler condition $\epsilon_j(t) \to 0, t \to 0, j = 1, \ldots, m$, as was shown by Passare and Tsikh in [29].
- Notation in the definition above is substantiated by the fact that the collection

$$\left\{\Phi_\alpha \wedge \bar{\partial}\left(\frac{1}{F_1^{(\alpha)}}\right) \wedge \cdots \wedge \bar{\partial}\left(\frac{1}{F_m^{(\alpha)}}\right)\right\}_{\alpha=1}^N$$

  naturally defines a current of type $(0,q)$ on V_D with coefficients in holomorphic bundle ω_V° defined in (5).

Definition 1.4 (Residual $\bar{\partial}$-cohomologies) A $\bar{\partial}$-closed residual current $\phi \in Z^{(0,q)}(V_D, \omega_V^\circ)$ is called $\bar{\partial}$-exact ($\phi \in B^{(0,q)}(V_D, \omega_V^\circ)$) if there exists a residual current $\psi \in C^{(0,q-1)}(V_D, \omega_V^\circ)$ such that $\bar{\partial}\psi = \phi$.

Therefore

$$B^{(0,q)}(V_D, \omega_V^\circ) \subseteq Z^{(0,q)}(V_D, \omega_V^\circ),$$

and the spaces of residual $\bar{\partial}$-cohomologies of V_D of the type $(0, q)$:

$$H^{(0,q)}(V_D, \omega_V^\circ) = Z^{(0,q)}(V_D, \omega_V^\circ)/B^{(0,q)}(V_D, \omega_V^\circ)$$

are well defined.

Before defining the complex Radon transform we introduce an additional notation. We denote by S_V the following set of hyperplanes

$$S_V = \{\xi \in D^* : \dim_{\mathbb{C}}(V \cap \mathbb{C}P_\xi^{n-1}) \neq n - m - 1\}.$$

Using the arguments similar to those in the proof of Bertini's theorem (see [15]) we obtain that S_V is a subset of an analytic set in D^*.

In the definitions below we define the complex Radon transform of residual currents and the Fantappié transform of linear functionals in $H^0(V, \mathcal{O}/\mathcal{I})'$.

Definition 1.5 (Complex Radon Transform) Let $V \subset \mathbb{C}P^n$ be a locally complete intersection subvariety of pure dimension $n-m$. Then we define the Radon transform

$$\mathcal{R}_V : Z^{(0,n-m-1)}(V_D, \omega_V^\circ) \to H^{(1,0)}(D^* \backslash S_V)$$

on the space of $\bar{\partial}$-closed residual currents by the formula (see Proposition 2.5)

$$\mathcal{R}_V[\phi](\xi) = \frac{1}{(2\pi i)^{m+1}} \sum_{j=0}^{n} \left(\sum_{\alpha=1}^{N} \int_D \vartheta_\alpha(z) \cdot z_j \cdot \Phi_\alpha^{(n,n-m-1)}(z) \right.$$
$$\left. \wedge \bar{\partial}\left(\frac{1}{\langle \xi \cdot z \rangle}\right) \bigwedge_{k=1}^{m} \bar{\partial}\left(\frac{1}{F_k^{(\alpha)}(z)}\right)\right) d\xi_j, \tag{9}$$

where $\{\vartheta_\alpha\}_1^N$ is a partition of unity subordinate to a finite cover $\{U_\alpha\}_1^N$ of D by open subdomains in $\mathbb{C}P^n$, and the forms

$$\left\{ \Phi_\alpha^{(n,n-m-1)} \bigwedge_{k=1}^{m} \bar{\partial}\left(\frac{1}{F_k^{(\alpha)}}\right) \right\}_{\alpha=1}^{N}$$

are the local representatives of the current ϕ.

Definition 1.6 (Fantappié Transform) Let $V \subset \mathbb{C}P^n$ be a locally complete intersection subvariety, let G be a linearly convex compact in $\mathbb{C}P^n$, and let $\mathcal{I}$ be the sheaf of ideals, associated with V. We define the Fantappié transform of a linear functional $\mu \in H^0(G, \mathcal{O}/\mathcal{I})'$ by the formula

$$\mathcal{F}_V[\mu](\xi) = \sum_{j=0}^{n} \mu\left(\frac{z_j}{\langle \xi \cdot z \rangle}\right) d\xi_j, \tag{10}$$

where $\xi \in D^* = (\mathbb{C}P^n \backslash G)^*$.

The theorem below is the main result of the present article. In this theorem we describe the action of the Fantappié and complex Radon transforms on the spaces of residual cohomologies of linearly concave locally complete intersection subvarieties of $\mathbb{C}P^n$.

Theorem 2 *Let*

$$V = \{z \in \mathbb{C}P^n : \ P_1(z) = \cdots = P_r(z) = 0\} \tag{11}$$

be a locally complete intersection subvariety of pure dimension $(n-m)$ with the structure sheaf $\mathcal{O}/\mathcal{I}$, where $\mathcal{I}$ is the sheaf of ideals defined by homogeneous polynomials $\{P_k\}_1^r$, $r \geq m$. Let $D \subset \mathbb{C}P^n$ be a linearly concave domain, and let D^* be its dual domain.

Then transform $\mathcal{R}_V$ defined in (9) induces a continuous linear operator on the space of residual $\bar{\partial}$-cohomologies

$$\mathcal{R}_V : H^{(0,n-m-1)}(V_D, \omega_V^{\circ}) \to H^{(1,0)}(D^*),$$

and transform $\mathcal{F}_V$ defined in (10) induces a continuous linear operator

$$\mathcal{F}_V : \ H^0(G, \mathcal{O}/\mathcal{I})' \to H^{(1,0)}(D^*).$$

Moreover, transforms $\mathcal{R}_V$ and $\mathcal{F}_V$ satisfy the following properties:

(i) *$Ker\,\mathcal{F}_V = \{0\}$, $Ker\,\mathcal{R}_V \subset H^{(0,n-m-1)}(V_D, \omega_V^{\circ})$ is finite-dimensional and consists of restrictions to V_D of classes of residual $\bar{\partial}$-cohomologies from $H^{(0,n-m-1)}(V, \omega_V^{\circ})$,*

(ii) *the images of $\mathcal{F}_V$ and $\mathcal{R}_V$ are the following subspaces in $H^{(1,0)}(D^*)$:*

$$\begin{aligned} \textit{Image}\ \mathcal{F}_V = \Bigg\{ & f \in H^{(1,0)}(D^*) : f = dg \ \textit{with}\ g \in H^0(D^*) \ \textit{such that} \\ & \left\{ P_k\left(\frac{\partial}{\partial \xi}\right) g = 0 \right\}_1^r \Bigg\}, \end{aligned} \tag{12}$$

$$\begin{aligned} \textit{Image}\ \mathcal{R}_V = \{ & f \in \textit{Image}\ \mathcal{F}_V : f = \mathcal{F}_V[\mu], \ \textit{where}\ \mu(h) = 0 \ \textit{for} \\ & \forall h \in H^0(\mathbb{C}P^n, \mathcal{O}/\mathcal{I})\}, \end{aligned}$$

(iii) *if V is connected in the sense that* $\dim H^0(V, \mathcal{O}/\mathcal{I}) = 1$, *then*

$$Image\ \mathcal{R}_V = Image\ \mathcal{F}_V,$$

(iv) *for a functional $\mu \in H^0(G, \mathcal{O}/\mathcal{I})'$ defined for $h \in H^0(G, \mathcal{O}/\mathcal{I})$ through the residual current $\phi = \{\Phi_\alpha\}$ by the formula*

$$\mu(h) = \sum_{\alpha=1}^{N} \int_{bD_\delta} \vartheta_\alpha(z) h(z) \Phi_\alpha(z) \bigwedge_{k=1}^{m} \bar\partial\left(\frac{1}{F_k^{(\alpha)}(z)}\right),$$

the following equality holds

$$\mathcal{R}_V[\varphi](\xi) = \left(\frac{1}{2\pi i}\right)^{m+1} \mathcal{F}_V[\mu](\xi). \tag{13}$$

Remark
- The statements in (ii) of Theorem 2 can be interpreted as versions of the Ehrenpreis "fundamental principle" for systems of partial differential equations (see [8,10,27]) in terms of Fantappié and complex Radon transforms instead of Fourrier-Laplace transform.
- If V is an arbitrary, not necessarily reduced, complete intersection in $\mathbb{C}P^n$ and D is a linearly concave domain in $\mathbb{C}P^n$, then in [19] an explicit inversion formula for Radon transform $\mathcal{R}_V$ is obtained together with a formula for solutions of appropriate boundary value problem for the corresponding system of homogeneous differential equations with constant coefficients in D^*.
- For the case $m = n-1$ the statement (i) of Theorem 2 for Radon transform follows from the result of Fabre [9].
- If V is a complete intersection in $\mathbb{C}P^n$, then the property of V to be connected in the sense of (iii) is always satisfied (see Ex. 5.5 §III.5 in [15]).
- Theorem 2 admits a generalization for analytic subvarieties of a linearly concave domain D. If $m < n-1$, then an analytic subvariety $V' \subset D$ of D is a trace of an algebraic subvariety $V \subset \mathbb{C}P^n$ (see [32,35]), and an appropriate version of Theorem 2 applies. If $m = n-1$, then $V' \subset D$ is a trace of an algebraic curve $V \subset \mathbb{C}P^n$ if there exists a form $\phi \in Z^{(0,1)}(V', \omega_V^\circ)$, such that $\phi \neq 0$ almost everywhere on V and $R_{V'}[\phi] \equiv 0$ (see [9,12]).

In Sect. 2 we prove the correctness of Definition 1.5 and some properties of $\mathcal{R}_V$ and $\mathcal{F}_V$, and in Sects. 3 and 4 we prove propositions representing different parts of Theorem 2.

2 Properties of residual currents

In this section we describe some properties of residual currents used in the proof of Theorem 2 and prove some properties of the Radon transform defined by formula (9). In the proposition below we describe the dependence of a local formula for a residual current on the choice of a basis of the ideal for the case of a complete intersection.

Proposition 2.1 *Let $U \subset \mathbb{C}P^n$ be a domain in $\mathbb{C}P^n$ and let $V \subset U$ be a complete intersection subvariety of pure dimension $n-m$ in U, defined by two different collections of holomorphic functions $\mathbf{F} = \{F_k\}_1^m$ and $\mathbf{P} = \{P_k\}_1^m$ such that*

$$\mathbf{F} = A \cdot \mathbf{P} \tag{14}$$

where $A(z)$ is a nondegenerate holomorphic matrix-function.

Let $\{\epsilon(t)\}$ be an admissible path, and let

$$T^{\epsilon}_{\{\mathbf{F}\}}(t) = \{z \in U : |F_1(z)| = \epsilon_1(t), \dots, |F_m(z)| = \epsilon_m(t)\},$$
$$T^{\epsilon}_{\{\mathbf{P}\}}(t) = \{z \in U : |P_1(z)| = \epsilon_1(t), \dots, |P_m(z)| = \epsilon_m(t)\},$$

be the corresponding tubular varieties.

Then for an arbitrary $\gamma \in \mathcal{E}_c^{(n,n-m)}(U)$ we have the following equality

$$\lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{P}\}}(t)} \frac{\gamma(z)}{\prod_{k=1}^m P_k(z)} = \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{F}\}}(t)} \frac{\det A(z) \cdot \gamma(z)}{\prod_{k=1}^m F_k(z)}. \tag{15}$$

Proof In the proof of Proposition 2.1 we will use the following proposition describing the transformation of the Grothendieck's residue under the change of basis in the ideal for the case of isolated point in $\mathbb{C}^n$.

Proposition 2.2 [13,36] *Let $U \in \mathbb{C}^n$ be a neighborhood of the origin $\{0\} \in \mathbb{C}^n$ and let $\mathbf{P} = \{P_1, \dots, P_n\}$ and $\mathbf{F} = \{F_1, \dots, F_n\}$ be two different collections of holomorphic functions on U having $\{0\}$ as an isolated zero, and satisfying (14) with a nondegenerate holomorphic matrix-function $A(z)$ on U.*

Then for an arbitrary function $h \in \mathcal{E}_c(U)$ we have the following equality

$$\lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{P}\}}(t)} \frac{h(z)}{\prod_{k=1}^n P_k(z)} = \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{F}\}}(t)} \frac{\det A(z) \cdot h(z)}{\prod_{k=1}^n F_k(z)}. \tag{16}$$

□

To prove equality (15) we use the fibered residual currents from [4]. Namely, we consider a polydisk $\mathcal{P}^n = \{|z_i| < 1,\ i = 1, \dots, n\} \subset U$ such that the restriction of the projection

$$\pi : \mathcal{P}^n \to \mathcal{P}^{n-m},$$

defined by the formula $\pi(z_1, \dots, z_n) = (z_{m+1}, \dots, z_n)$, to $V \cap \mathcal{P}$ is a finite proper covering. Then we use Theorem 1.8.3 from [4] and obtain the existence of a holomorphic function g on $\mathcal{P}^n$ such that $\dim\{V \cap \{|g(z)| = 0\}\} \le n-m-1$, and

$$\lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{P}\}}(t)} \frac{\gamma(z)}{\prod_{k=1}^m P_k(z)} = \lim_{\delta\to 0} \int\limits_{V\cap\{|g(z)|>\delta\}} \operatorname{res}_{\{\mathbf{P},\pi\}}(\gamma, z), \tag{17}$$

where

$$\operatorname{res}_{\{\mathbf{P},\pi\}}(\gamma, z) = \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\widetilde{\mathbf{P}}\}}(t)} \frac{\widetilde{\gamma}(z_{m+1}, \ldots, z_n)}{\prod_{k=1}^{n} \widetilde{P}_k(z)},$$

$$\widetilde{\gamma}(z_{m+1}, \ldots, z_n) = \gamma\Big|_{\pi^{-1}(z_{m+1},\ldots,z_n)}, \ \widetilde{P}_k = P_k\Big|_{\pi^{-1}(z_{m+1},\ldots,z_n)},$$

and $z \in V \cap \pi^{-1}(z_{m+1}, \ldots, z_n)$.

Applying Proposition 2.2 to the right-hand side of equality (17) we obtain equality (15). □

The next proposition is a reformulation of Theorem 1.7.6(2) from [4], which will be used in the article.

Proposition 2.3 *Let U be a domain in $\mathbb{C}^n$, and let*

$$V = \{z \in U : \ F_1(z) = \cdots = F_m(z) = 0\}$$

be a complete intersection in U. If a differential form $\Phi \in \mathcal{E}_c^{(n,n-m)}(U)$ with compact support in U admits a representation

$$\Phi = \sum_{k=1}^{m} F_k \cdot \Phi_k,$$

where forms $\Phi_k \in \mathcal{E}^{(n,n-m)}(U)$ have compact support in U, then

$$\int\limits_U \Phi \bigwedge_{k=1}^{m} \bar{\partial}\left(\frac{1}{F_k}\right) = 0.$$

□

In the proposition below we prove the existence of a family of smoothly bounded linearly concave domains approximating D. Existence of such family provides a convenient tool in many constructions of the present article.

Proposition 2.4 *Let a linearly concave domain $D \subset \mathbb{C}P^n$ admit continuos map $\eta : D \to D^*$ satisfying condition $\langle \eta(z) \cdot z \rangle = 0$. Then there exist a sequence of real numbers $\{\delta_n\}_1^{\infty}$ such that $\delta_n > \delta_m$ for $n < m$ and* $\lim_{n\to\infty} \delta_n = 0$, *and of smoothly bounded linearly concave domains*

$$D_{\delta_n} \subset D = \{z \in D : \rho_{\delta_n}(z) < 0\} \tag{18}$$

satisfying

$$D_{\delta_n} \subset D_{\delta_m} \ \ \textit{for } m > n, \quad \textit{and} \ \bigcup_{n=1}^{\infty} D_{\delta_n} = D. \tag{19}$$

Proof We construct a sequence of smoothly bounded linearly concave domains satisfying (19) in two steps. On the first step we construct a family of domains exhausting D^*. We consider the function $\rho^*(\xi) = \mathrm{dist}(\xi, bD^*)$ on D^*, and averaging this function with the kernel $K_\delta(\zeta) = \delta^{-2n} \cdot K(\zeta/\delta)$, where

$$K(\zeta) = \begin{cases} Ce^{1/(|\zeta|^2-1)} & \text{if } |\zeta| < 1, \\ 0 & \text{if } |\zeta| \geq 1, \end{cases}$$

and $C = (\int_{|\zeta|\leq 1} e^{1/(|\zeta|^2-1)} d\zeta)^{-1}$, obtain a smooth function

$$\rho^*_\delta(\xi) = \int \rho^*(\zeta) K_\delta(\xi - \zeta) d\zeta$$

on the set $\{\xi \in D^* : \rho^*(\xi) > \delta\}$. We define then for $\nu < \delta/2$

$$D^*_{\delta,\nu} = \{\xi \in D^* : \rho^*_\delta(\xi) > 3\delta - \nu\}.$$

To see that

$$\{\xi \in D^* : \rho^*(\xi) > 4\delta\} \subset D^*_{\delta,\nu} \subset \{\xi \in D^* : \rho^*(\xi) > \delta\} \tag{20}$$

for $\nu < \delta/2$ we use the inequality

$$\begin{aligned} |\rho^*_\delta(\xi) - \rho^*(\xi)| &= \left| \int (\rho^*(\zeta) - \rho^*(\xi)) K_\delta(\xi - \zeta) d\zeta \right| \\ &\leq \delta^{-2n} \int |\rho^*(\zeta) - \rho^*(\xi)| K\left(\frac{\xi - \zeta}{\delta}\right) d\zeta \\ &= \int\limits_{|u|\leq 1} |\rho^*(\xi + \delta \cdot u) - \rho^*(\xi)| K(u) du \leq \delta. \end{aligned}$$

Relation (20) shows that the family of domains $D^*_{\delta,\nu}$ exhausts the domain D^*. On the second step we consider the domain

$$W^*_\delta = \{\xi \in D^* : \rho^*(\xi) > \delta\},$$

and apply a smoothing procedure, similar to the described above, to the continuous family of hyperplanes $\eta : D \to D^*$ restricted to the domain $\eta^{-1}(W^*_\delta)$. For z in the domain

$$\eta^{-1}(W^*_\delta) \cap U_j = \{z \in \eta^{-1}(W^*_\delta) : z_j \neq 0\}$$

we define

$$\eta^i_{j,\delta'}(z) = \begin{cases} \int \eta_i(\zeta) K_{\delta'}(z-\zeta) d\zeta & \text{if } i \neq j, \\ \eta^j_{j,\delta'}(z) = -\sum_{i\neq j} \eta^i_{j,\delta'}(z) \dfrac{z_i}{z_j}, \end{cases}$$

for $\delta' > 0$ small enough, and set

$$\eta_{\delta'}(z) = (\eta_{0,\delta'}(z), \ldots, \eta_{n,\delta'}(z)),$$

where

$$\eta_{k,\delta'}(z) = \sum_{j=0}^{n} \vartheta_j(z) \cdot \eta^k_{j,\delta'}(z),$$

and $\{\vartheta_j\}_{j=0}^n$ is a partition of unity subordinate to the cover $\{U_j\}$ of $\mathbb{C}P^n$.
We notice that for every $j \in (0, \ldots, n)$ we have

$$\sum_{k=0}^{n} z_k \cdot \eta^k_{j,\delta'}(z) = 0,$$

and therefore

$$\begin{aligned} \sum_{k=0}^{n} z_k \cdot \eta_{k,\delta'}(z) &= \sum_{k=0}^{n} z_k \cdot \left(\sum_{j=0}^{n} \vartheta_j(z) \cdot \eta^k_{j,\delta'}(z) \right) \\ &= \sum_{j=0}^{n} \vartheta_j(z) \cdot \left(\sum_{k=0}^{n} z_k \cdot \eta^k_{j,\delta'}(z) \right) = 0. \end{aligned}$$

Then we obtain a continuous and smooth in a neighborhood of $\eta^{-1}(D^*_\delta)$ family of hyperplanes $\eta_{\delta'}(z) \in D^*$ such that $z \in \eta_{\delta'}(z)$ for

$$z \in \eta^{-1}\{\xi \in D^* : \rho^*(\xi) > \delta\}.$$

We define

$$D'_{\delta,\nu} = \{z \in D : \mathbb{C}P^{n-1}(z) \subset D^*_{\delta,\nu}\} = \{z \in D : \rho_\delta(z) \overset{\text{def}}{=} 3\delta - \nu - \rho^*_\delta(\eta_{\delta'}(z)) < 0\},$$

and applying the Sard's theorem find $\nu' < \delta/2$ such that $D_\delta = D'_{\delta,\nu'}$ has smooth boundary.

Sequences $\{\delta_n\}_1^\infty$ and $\{D_{\delta_n}\}_1^\infty$ satisfying (19) can be chosen as subsequences corresponding to an arbitrary sequence of decreasing δ_n tending to zero as $n \to \infty$ based on the exhaustion property. To construct an "explicit" sequence $\{D_{\delta_n}\}_1^\infty$ satisfying

(19) we can choose for example the sequence $\{\delta_n = \delta_1/8^{n-1}\}_1^{\infty}$. The numbers δ'_n can be chosen so that

$$|\rho^*_{\delta_n}(\eta_{\delta'_n}(z)) - \rho^*_{\delta_n}(\eta(z))| < \frac{\delta_n}{16} \tag{21}$$

and therefore $|\rho^*_{\delta_n}(\eta_{\delta'}(z)) - \rho^*(\eta(z))| \le \frac{17}{16}\delta_n$, for $z \in \eta^{-1}\{\xi \in D^* : 4\delta_n > \rho^*(\xi) > \delta_n\}$.

The boundary of the domain

$$D'_{\delta_n,\nu_n} = \{z \in D : 3\delta_n - \nu_n - \rho^*_{\delta_n}(\eta_{\delta'_n}(z)) < 0\}$$

will satisfy the condition $\rho^*_{\delta_n}(\eta_{\delta'_n}(z)) = 3\delta_n - \nu_n$, and for $z \in bD'_{\delta_n,\nu_n}$ we will have using (21)

$$\frac{57}{16}\delta_n \ge \rho^*(\eta(z)) \ge \frac{21}{16}\delta_n.$$

Since $\frac{57}{16\cdot 8} < \frac{21}{16}$ we obtain that $bD'_{\delta_n,\nu_n} \cap bD'_{\delta_{n+1},\nu_{n+1}} = \emptyset$, and therefore the sequence D_{δ_n} is strictly monotonous. □

In the next proposition we prove a useful boundary formula for the Radon transform. As a corollary of this formula we obtain that Definition (9) of the Radon transform $\mathcal{R}_V$ coincides with the standard definition of Radon transform for the case of a differential form on a nonsingular variety V.

Proposition 2.5 *Let $D \subset \mathbb{C}P^n$ be a linearly concave domain, let $\{U_\alpha\}_{\alpha=1}^N$ be a finite cover of D, let $\{\vartheta_\alpha\}_{\alpha=1}^N$ be a partition of unity subordinate to the cover $\{U_\alpha\}_{\alpha=1}^N$, and let $V \subset D$ be a locally complete intersection subvariety of pure dimension $n-m$, locally defined in U_α by the holomorphic functions $\{F_k^{(\alpha)}\}_{k=1}^m$.*

Then for a $\bar{\partial}$-closed residual current ϕ defined locally by the differential forms

$$\Phi_\alpha \in \mathcal{E}^{(n,n-m-1)}(U_\alpha)$$

and a subdomain $D_\delta \subset D$ with smooth boundary bD_δ the following equality holds

$$\begin{aligned}
&\sum_{\alpha=1}^N \int_D \vartheta_\alpha(z)\cdot z_j \cdot \Phi_\alpha(z) \wedge \bar{\partial}\left(\frac{1}{\langle \xi \cdot z\rangle}\right) \bigwedge_{k=1}^m \bar{\partial}\left(\frac{1}{F_k^{(\alpha)}(z)}\right) \\
&= \sum_{\alpha=1}^N \lim_{\tau\to 0}\lim_{t\to 0} \int_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)},\tau\}}(t)} \vartheta_\alpha(z) \frac{z_j\cdot\Phi_\alpha(z)}{\langle \xi\cdot z\rangle \cdot \prod_{k=1}^m F_k^{(\alpha)}(z)} \\
&= \sum_{\alpha=1}^N \lim_{t\to 0} \int_{bD_\delta \cap T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z) \frac{z_j\cdot\Phi_\alpha(z)}{\langle \xi\cdot z\rangle \cdot \prod_{k=1}^m F_k^{(\alpha)}(z)},
\end{aligned} \tag{22}$$

where

$$T^{\epsilon}_{\{\mathbf{F}^{(\alpha)},\tau\}}(t)=\Big\{z\in U_\alpha:\ \{|F_k^{(\alpha)}(z)|=\epsilon_k(t)\}_{k=1}^m,\ \chi(\xi,z)\overset{\text{def}}{=}\sum_{\alpha=1}^N\vartheta_\alpha(z)\cdot|\langle\xi\cdot z^{(\alpha)}\rangle|=\tau\Big\}$$

with admissible path $\{\epsilon_k(t)\}_{k=1}^m$.

Under the hypotheses of Theorem 2 the transform $\mathcal{R}_V$ *from (9) maps the* $\bar\partial$*-closed residual currents on* D *with support on* V_D *into holomorphic forms on* D^**, and induces a linear map on the spaces of cohomologies.*

In the proof of Proposition 2.5 we will use the following two lemmas.

Lemma 2.6 *Let* $U\subset\mathbb{C}P^n$ *be a domain in* $\mathbb{C}P^n$*, let* $\{U_\alpha\}_{\alpha=1}^N$ *be a finite cover of* U*, and let* $V\subset U$ *be a locally complete intersection subvariety in* U *of pure dimension* $n-m$*, locally defined in* U_α *by holomorphic functions* $\{F_k^{(\alpha)}\}_1^m$*. Let* ω *be a* $\bar\partial$*-closed residual current with support on* V *locally defined by the differential forms* $\Omega_\alpha\in\mathcal{E}^{(n,n-m-1)}(U_\alpha)$*.*

Then for an arbitrary function $\eta\in\mathcal{E}_c(U)$ *we have*

$$\sum_{\alpha=1}^N\lim_{t\to0}\int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\eta(z)\bar\partial\vartheta_\alpha(z)\wedge\frac{\Omega_\alpha(z)}{\prod_{k=1}^m F_k^{(\alpha)}(z)}=0. \tag{23}$$

Proof To prove equality (23) we apply the Stokes' formula, and using equality

$$\bar\partial\Omega_\alpha=\sum_{k=1}^m F_k^{(\alpha)}\cdot\Omega_k^{(\alpha)}$$

for $i=1\dots N$ and Proposition 2.3 obtain

$$\begin{aligned}
&\sum_{\alpha=1}^N\lim_{t\to0}\int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\eta(z)\bar\partial\vartheta_\alpha(z)\wedge\frac{\Omega_\alpha(z)}{\prod_{k=1}^m F_k^{(\alpha)}(z)}\\
&=-\sum_{\alpha=1}^N\lim_{t\to0}\int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\vartheta_\alpha(z)\bar\partial\eta(z)\wedge\frac{\Omega_\alpha(z)}{\prod_{k=1}^m F_k^{(\alpha)}(z)}\\
&\quad-\sum_{\alpha=1}^N\lim_{t\to0}\int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\vartheta_\alpha(z)\eta(z)\wedge\frac{\bar\partial\Omega_\alpha(z)}{\prod_{k=1}^m F_k^{(\alpha)}(z)}\\
&=-\omega\left(\sum_{\alpha=1}^N\vartheta_\alpha\cdot\bar\partial\eta\right)=-\omega(\bar\partial\eta)=\pm\bar\partial\omega(\eta)=0.
\end{aligned}$$

□

Lemma 2.7 *Let $D \subset \mathbb{C}P^n$ be a linearly concave domain, let $V \subset D$ be a locally complete intersection subvariety of pure dimension $n-m$, locally defined in U_α by holomorphic functions $\{F_k^{(\alpha)}\}_1^m$.*

Then for a fixed $\xi \in D^*\backslash S_V$ and a $\bar\partial$-closed residual current ω defined locally by the differential forms

$$\Omega_\alpha \in \mathcal{E}^{(n,n-m-1)}(U_\alpha\backslash\{\langle\xi\cdot z\rangle = 0\})$$

the expression

$$\sum_{\alpha=1}^N \lim_{t\to 0} \int\limits_{T^\epsilon_{\{\mathbf{F}^{(\alpha)},\tau\}}(t)} \vartheta_\alpha(z)\frac{\Omega_\alpha(z)}{\prod_{k=1}^m F_k^{(\alpha)}(z)} \tag{24}$$

is well defined, does not depend on τ, and the following equality holds

$$\sum_{\alpha=1}^N \lim_{t\to 0} \int\limits_{T^\epsilon_{\{\mathbf{F}^{(\alpha)},\tau\}}(t)} \vartheta_\alpha(z)\frac{\Omega_\alpha(z)}{\prod_{k=1}^m F_k^{(\alpha)}(z)} = \sum_{\alpha=1}^N \lim_{t\to 0} \int\limits_{bD_\delta\cap T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z)\frac{\Omega_\alpha(z)}{\prod_{k=1}^m F_k^{(\alpha)}(z)}. \tag{25}$$

Proof We fix a sufficiently small $\mu > 0$ and consider for an arbitrary $\tau > 0$ such that $\tau < \mu$ a family of nonnegative functions $\eta_\nu \in \mathcal{E}(D)$ such that

$$\eta_\nu(z) = \begin{cases} 0 & \text{if } \chi(\xi,z) < \tau-\nu, \\ 1 & \text{if } \tau+\nu < \chi(\xi,z) < \mu, \\ 0 & \text{if } \chi(\xi,z) > 2\mu, \end{cases}$$

and such that $\eta_\nu(z) = \eta(z)$ for z with $\chi(\xi,z) > \mu$, where $1 \geq \eta(z) \geq 0$ is a fixed smooth function.

Applying then the Stokes' formula we obtain

$$\begin{aligned}
&\sum_{\alpha=1}^N \lim_{t\to 0} \int\limits_{T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \eta_\nu(z)\bar\partial\vartheta_\alpha(z)\wedge\frac{\Omega_\alpha(z)}{\prod_{k=1}^m F_k^{(\alpha)}(z)} \\
&\quad+\sum_{\alpha=1}^N \lim_{t\to 0} \int\limits_{T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z)\bar\partial\eta_\nu(z)\wedge\frac{\Omega_\alpha(z)}{\prod_{k=1}^m F_k^{(\alpha)}(z)} \\
&\quad+\sum_{\alpha=1}^N \lim_{t\to 0} \int\limits_{T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z)\eta_\nu(z)\wedge\frac{\bar\partial\Omega_\alpha(z)}{\prod_{k=1}^m F_k^{(\alpha)}(z)} = 0,
\end{aligned}$$

which we transform using Proposition 2.3 and Lemma 2.6 into

$$\sum_{\alpha=1}^{N}\lim_{t\to 0}\int_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\vartheta_{\alpha}(z)\bar{\partial}\eta_{\nu}(z)\wedge\frac{\Omega_{\alpha}(z)}{\prod_{k=1}^{m}F_k^{(\alpha)}(z)}=0,$$

and then further into

$$\begin{aligned}&-\sum_{\alpha=1}^{N}\lim_{t\to 0}\int_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)\cap\{\tau-\epsilon<\chi(\xi,z)<\tau+\epsilon\}}\vartheta_{\alpha}(z)\bar{\partial}\eta_{\nu}(z)\wedge\frac{\Omega_{\alpha}(z)}{\prod_{k=1}^{m}F_k^{(\alpha)}(z)}\\&=\sum_{\alpha=1}^{N}\lim_{t\to 0}\int_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)\cap\{\mu<\chi(\xi,z)<2\mu\}}\vartheta_{\alpha}(z)\bar{\partial}\eta(z)\wedge\frac{\Omega_{\alpha}(z)}{\prod_{k=1}^{m}F_k^{(\alpha)}(z)}\end{aligned}$$

for arbitrary small $\tau > 0$.
Considering then the limit of the equality above as $\nu \to 0$ we obtain the equality

$$\begin{aligned}&\sum_{\alpha=1}^{N}\lim_{t\to 0}\int_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)},\tau\}}(t)}\vartheta_{\alpha}(z)\frac{\Omega_{\alpha}(z)}{\prod_{k=1}^{m}F_k^{(\alpha)}(z)}\\&=\sum_{\alpha=1}^{N}\lim_{t\to 0}\int_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)\cap\{\mu<\chi(\xi,z)<2\mu\}}\vartheta_{\alpha}(z)\bar{\partial}\eta(z)\wedge\frac{\Omega_{\alpha}(z)}{\prod_{k=1}^{m}F_k^{(\alpha)}(z)}.\end{aligned}\tag{26}$$

The limit in the right-hand side of (26) exists according to the following proposition, which is a reformulation of item (2) of Theorem 1.7.2 from [4]. □

Proposition 2.8 *Let U be a relatively compact domain in $\mathbb{C}^n$, let*

$$V=\{z\in U: F_1(z)=\cdots=F_m(z)=0\}$$

be a complete intersection subvariety in U, let $\beta\in\mathcal{E}_c^{(n,n-m)}(U)$ be a differential form with compact support in U, and let $T^{\epsilon}_{\{\mathbf{F}\}}(t)$ be an admissible path. Then the following limit

$$\lim_{t\to 0}\int_{T^{\epsilon}_{\{\mathbf{F}\}}(t)}\frac{\beta(z)}{\prod_{k=1}^{m}F_k(z)}$$

exists.

From the form of the integral in the right-hand side of (26) we conclude that it doesn't depend on the choice of τ, and therefore the same is true for the left-hand side of this equality.

To prove equality (25) we change the choice of the family of functions η_ν to the following:

$$\eta_\nu(z) = \begin{cases} 0 & \text{if } \chi(\xi, z) < \tau - \nu \text{ or } \rho_\delta(z) > \nu, \\ 1 & \text{if } \chi(\xi, z) > \tau + \nu \text{ and } \rho_\delta(z) < -\nu, \end{cases}$$

where ρ_δ is the function from Proposition 2.4.
Applying then the Stokes' formula we obtain the equality

$$\sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} \eta_\nu(z)\bar\partial\vartheta_\alpha(z) \wedge \frac{\Omega_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)}$$
$$+ \sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z)\bar\partial\eta_\nu(z) \wedge \frac{\Omega_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)}$$
$$+ \sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z)\eta_\nu(z) \wedge \frac{\bar\partial\Omega_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)} = 0,$$

and then using Lemma 2.6 and Proposition 2.3

$$-\sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)\cap\{\tau-\nu<\chi(\xi,z)<\tau+\nu\}} \vartheta_\alpha(z)\bar\partial\eta_\nu(z) \wedge \frac{\Omega_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)}$$
$$= \sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)\cap\{-\nu<\rho_\delta(z)<\nu\}} \vartheta_\alpha(z)\bar\partial\eta_\nu(z) \wedge \frac{\Omega_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)}.$$

Passing to the limit as $\nu \to 0$ we obtain equality (25).

Proof of Proposition 2.5 Equality (22) is an immediate corollary of equality (25). In view of (22) formula (9) defines a bounded holomorphic function on the intersection of an arbitrary compact set in D^* with $D^*\backslash S_V$. Therefore, since S_V is a subset of an analytic set in D^*, there exists a unique extension of this function to D^*.

To prove that $\mathcal{R}_V[\phi] = 0$ for a $\bar\partial$-exact residual current $\phi^{n,n-1}$ we assume the existence of a current $\psi \in \mathcal{K}^{(n,n-2)}(D)$ such that equality

$$\langle \gamma^{(0,1)},\ \phi^{(n,n-1)} \rangle = \langle \bar\partial\gamma^{(0,1)},\ \psi^{(n,n-2)} \rangle$$

is satisfied for an arbitrary $\gamma^{(0,1)} \in \mathcal{E}_c^{(0,1)}(D)$.

Then, using formula (22) and Proposition 2.3 we obtain

$$\begin{aligned}
\mathcal{R}_V[\phi](\xi) &= \frac{1}{(2\pi i)^{m+1}}\sum_{j=0}^{n}\left(\sum_{\alpha=1}^{N}\lim_{t\to 0}\int\limits_{bD_\delta\cap T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\vartheta_\alpha(z)\frac{z_j\cdot\Phi_\alpha(z)}{\langle\xi\cdot z\rangle\cdot\prod_{k=1}^{m}F_k^{(\alpha)}(z)}\right)d\xi_j\\
&= \frac{1}{(2\pi i)^{m+1}}\sum_{j=0}^{n}\left(\sum_{\alpha=1}^{N}\lim_{\nu\to 0}\lim_{t\to 0}\int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\vartheta_\alpha(z)\bar\partial\eta_\nu(z)\wedge\frac{z_j\cdot\Phi_\alpha(z)}{\langle\xi\cdot z\rangle\cdot\prod_{k=1}^{m}F_k^{(\alpha)}(z)}\right)d\xi_j\\
&\quad+\frac{1}{(2\pi i)^{m+1}}\sum_{j=0}^{n}\left(\sum_{\alpha=1}^{N}\lim_{\nu\to 0}\lim_{t\to 0}\int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\vartheta_\alpha(z)\eta_\nu(z)\frac{z_j\cdot\bar\partial\Phi_\alpha(z)}{\langle\xi\cdot z\rangle\cdot\prod_{k=1}^{m}F_k^{(\alpha)}(z)}\right)d\xi_j\\
&= \frac{1}{(2\pi i)^{m+1}}\sum_{j=0}^{n}\left(\sum_{\alpha=1}^{N}\lim_{\nu\to 0}\left\langle\frac{z_j}{\langle\xi\cdot z\rangle}\cdot\bar\partial\eta_\nu,\ \phi\right\rangle\right)d\xi_j\\
&= \frac{1}{(2\pi i)^{m+1}}\sum_{j=0}^{n}\left(\sum_{\alpha=1}^{N}\lim_{\nu\to 0}\left\langle\frac{z_j}{\langle\xi\cdot z\rangle}\cdot\bar\partial^2\eta_\nu,\ \psi\right\rangle\right)d\xi_j=0,
\end{aligned}$$

where

$$\eta_\nu(z)=\begin{cases}1 & \text{if } \rho_\delta(z)>\nu,\\ 0 & \text{if } \rho_\delta(z)<-\nu.\end{cases}$$

The equality above allows to define the Radon transform $\mathcal{R}_V[\phi]=0$ for an arbitrary $\bar\partial$-exact current ϕ with support on $V\cap D$. □

In the proposition below we prove the inclusion of the images of $\mathcal{F}_V$ and $\mathcal{R}_V$ in the space of solutions in the right-hand side of (12).

Proposition 2.9 *Radon and Fantappié transforms defined in* (9) *and* (10) *satisfy the following properties:*

$$\begin{aligned}
&\mathit{Image}\ \mathcal{F}_V\\
&\subseteq\left\{f\in H^{(1,0)}(D^*) : f=dg \text{ with } g\in H^0(D^*) \text{ such that } \left\{P_k\left(\frac{\partial}{\partial\xi}\right)g=0\right\}_1^r\right\},\\
&\mathit{Image}\ \mathcal{R}_V\\
&\subseteq\{f\in \mathit{Image}\ \mathcal{F}_V :\ f=\mathcal{F}_V[\mu],\ \text{where } \mu(h)=0 \text{ for } \forall h\in H^0(\mathbb{C}P^n,\mathcal{O}/\mathcal{I})\}.
\end{aligned}\tag{27}$$

Proof For the Fantappié transform of a linear functional $\mu\in H^0(G,\mathcal{O}/\mathcal{I})'$ we have

$$\mathcal{F}[\mu]=\sum_{j=0}^{n}\mu\left(\frac{z_j}{\langle\xi\cdot z\rangle}\right)d\xi_j=d_\xi g(\xi),$$

where

$$g(\xi) = \mu\left(\log\langle\xi \cdot z\rangle\right). \tag{28}$$

We notice that analytic function $\log\langle\xi \cdot z\rangle$, and therefore $g(\xi)$, is well defined on $D^*(z)$. It is a corollary of the contractibility of

$$D^*(z) = \{\xi \in D^* : \langle\xi \cdot z\rangle = 0\}$$

for any $z \in D$ under the condition of existence of a continuous family of hyperplanes covering the whole D. Namely, as it was proved in [11], the existence of such family implies the isomorphism

$$H(\mathbb{C}P^n \backslash D)' \cong H(D^*),$$

and then the result in [37,38] and the isomorphism above imply the contractibility of $D^*(z)$.
For g defined in (28) we have

$$P_j\left(\frac{\partial}{\partial\xi}\right)(g) = (-1)^{\deg P_j - 1}(\deg P_j - 1)!\mu\left(\frac{P_j(z)}{\langle\xi \cdot z\rangle^{\deg P_j}}\right) = 0,$$

which concludes the proof of inclusion for $\mathcal{F}$.
To prove the inclusion for the image of $\mathcal{R}_V$ we consider for an arbitrary residual current $\phi \in Z^{(0,n-m-1)}\left(V_D, \omega_V^\circ\right)$ the analytic functional on $H(G)$

$$\mu^\phi(h) = \sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{bD_\delta \cap T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z)\frac{h(z)\cdot\Phi_\alpha(z)}{\prod_{k=1}^m F_k^{(\alpha)}(z)}.$$

From Proposition 2.3 we obtain that $\mu^\phi(h) = 0$ for any $h \in H^0(G, \mathcal{I})$, and therefore μ^ϕ defines a functional on $H^0(G, \mathcal{O}/\mathcal{I})$. From equality (22) we obtain equality

$$\mathcal{F}_V[\mu^\phi] = (2\pi i)^{m+1}\,\mathcal{R}_V[\phi],$$

which implies the inclusion

$$\text{Image } \mathcal{R}_V \subseteq \text{Image } \mathcal{F}_V$$

and equality (13).
To conclude the proof of inclusion for the image of $\mathcal{R}_V$ we have to prove equality

$$\mu^\phi(h) = 0 \tag{29}$$

for an arbitrary $h \in H^0\left(\mathbb{C}P^n, \mathcal{O}/\mathcal{I}\right)$. To prove this equality we assume that in every U_α function h is defined in some neighborhood of $V \cap U_\alpha$ and consider a sequence of nonnegative functions $\eta_\nu \in \mathcal{E}_c(D)$ approximating the characteristic function of D_δ as $\nu \to 0$. Then, applying the Stokes' formula in each U_α we obtain the equality

$$\sum_{\alpha=1}^{N} \lim_{t\to 0} \int_{D\cap T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z)\bar{\partial}\eta_\nu(z) \wedge \frac{h(z)\cdot\Phi_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)} + \sum_{\alpha=1}^{N} \lim_{t\to 0} \int_{D\cap T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} \eta_\nu(z)\bar{\partial}\vartheta_\alpha(z) \wedge \frac{h(z)\cdot\Phi_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)} = 0,$$

which is transformed into equality

$$\sum_{\alpha=1}^{N} \lim_{t\to 0} \int_{D\cap T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z)\bar{\partial}\eta_\nu(z) \wedge \frac{h(z)\cdot\Phi_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)} = 0$$

after application of Lemma 2.6.

Passing to the limit as $\nu \to 0$ in the equality above we obtain equality (29). □

3 Kernels of $\mathcal{R}_V$ and $\mathcal{F}_V$

In this section we describe the kernels of $\mathcal{F}_V$ and $\mathcal{R}_V$. In the next proposition we prove the triviality of the kernel of $\mathcal{F}_V$.

Proposition 3.1 *For the Fantappié transform defined in* (10) *we have*

$$Ker\ \mathcal{F}_V = \{0\}. \tag{30}$$

Proof To prove property (30) we use the linear concavity of D and contractibility of $D^*(z)$ for every $z \in D$, and obtain as in Proposition 2.9 the connectedness of D^*. Then using the connectedness of D^* and the Cauchy–Fantappié–Leray integral formula on G (see [21]) we obtain the density of the set of functions

$$\left\{ \frac{1}{\xi_0 + \sum_{j=1}^{n} \xi_j u_j} \right\}_{\xi\in D^*}$$

in $H(G)$, where we used the assumption $D \supset \{z_0 = 0\}$ and changed variables in G to $u_j = z_j/z_0$.

Then from equality $\mathcal{F}_V[\mu](z_0/\langle\xi\cdot z\rangle) = 0$ we obtain the equality $\mu = 0$. □

In the proposition below we prove the necessity of the condition on Ker $\mathcal{R}_V$ in the statement (i) of Theorem 2.

Proposition 3.2 *Let $V \subset \mathbb{C}P^n$ be a locally complete intersection subvariety, let $D \subset \mathbb{C}P^n$ be a linearly concave domain. If a residual current $\phi \in Z^{(0,n-m-1)}(V_D, \omega_V^\circ)$ admits an extension to $\mathbb{C}P^n$ as a $\bar{\partial}$-closed residual current supported on V, then $\mathcal{R}_V[\phi] = 0$.*

Proof Let $\phi \in Z^{(0,n-m-1)}(V_D, \omega_V^\circ)$ be the restriction of a $\bar{\partial}$-closed residual current on V. Then from equality (25) in Lemma 2.7 we obtain

$$\mathcal{R}_V[\phi](\xi) = \frac{1}{(2\pi i)^{m+1}} \sum_{j=0}^{n} \left(\sum_{\alpha=1}^{N} \lim_{t \to 0} \int_{bD_\delta \cap T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z) \frac{z_j \cdot \Phi_\alpha(z)}{\langle \xi \cdot z \rangle \cdot \prod_{k=1}^m F_k^{(\alpha)}(z)} \right) d\xi_j.$$

We choose an open domain $U_1 \subset G$ from the cover $\cup_{\alpha=1}^N U_\alpha$ of G such that

$$U_1 = \{z \in G : \tau(z) < 0\}$$

for a function $\tau \in \mathcal{E}(G)$. Then we consider for a fixed $\mu > 0$ a family of smooth nonnegative functions η_ν with compact support such that

$$\eta_\nu(z) = \begin{cases} 0 & \text{if } \tau(z) < -\mu - \nu, \text{ or } \rho_\delta(z) < -\nu \\ 1 & \text{if } \tau(z) > -\mu + \nu, \text{ and } \rho_\delta(z) > \nu. \end{cases}$$

As in Lemma 2.6 we have the equality

$$\sum_{\alpha=1}^{N} \lim_{t \to 0} \int_{T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z) \bar{\partial}\eta_\nu(z) \wedge \frac{z_j \cdot \Phi_\alpha(z)}{\langle \xi \cdot z \rangle \cdot \prod_{k=1}^m F_k^{(\alpha)}(z)} = 0,$$

which, after passing to the limit as $\nu \to 0$ implies the equality

$$\sum_{\alpha=1}^{N} \lim_{t \to 0} \int_{bD_\delta \cap T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z) \frac{z_j \cdot \Phi_\alpha(z)}{\langle \xi \cdot z \rangle \cdot \prod_{k=1}^m F_k^{(\alpha)}(z)}$$
$$= \sum_{\alpha=1}^{N} \lim_{t \to 0} \int_{\{z \in U_1 :\ \tau(z) = -\mu\} \cap T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z) \frac{z_j \cdot \Phi_\alpha(z)}{\langle \xi \cdot z \rangle \cdot \prod_{k=1}^m F_k^{(\alpha)}(z)}.$$

Choosing the partition of unity such that $\vartheta_1\Big|_{\{z\in U_1:\ \tau(z)\leq -\mu\}} \equiv 1$ we obtain

$$\mathcal{R}_V[\phi](\xi) = \frac{1}{(2\pi i)^{m+1}}\sum_{j=0}^{n}\left(\sum_{\alpha=1}^{N}\lim_{t\to 0}\int_{bD_\delta\cap T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\vartheta_\alpha(z)\frac{z_j\cdot\Phi_\alpha(z)}{\langle\xi\cdot z\rangle\cdot\prod_{k=1}^{m}F_k^{(\alpha)}(z)}\right)d\xi_j$$
$$= \frac{1}{(2\pi i)^{m+1}}\sum_{j=0}^{n}\left(\lim_{t\to 0}\int_{\{z\in U_1:\ \tau(z)=-\mu\}\cap T^{\epsilon}_{\{\mathbf{F}^{(1)}\}}(t)}\frac{z_j\cdot\Phi_1(z)}{\langle\xi\cdot z\rangle\cdot\prod_{k=1}^{m}F_k^{(1)}(z)}\right)d\xi_j. \tag{31}$$

Then applying the Stokes' formula to the form

$$\frac{z_j\cdot\Phi_1(z)}{\langle\xi\cdot z\rangle\cdot\prod_{k=1}^{m}F_k^{(1)}(z)}$$

on the manifold

$$\{z\in U_1:\ \tau(z)<-\mu\}\cap T^{\epsilon}_{\{\mathbf{F}^{(1)}\}}(t)$$

with the boundary

$$\{z\in U_1:\ \tau(z)=-\mu\}\cap T^{\epsilon}_{\{\mathbf{F}^{(1)}\}}(t),$$

and using Proposition 2.3 we obtain $\mathcal{R}_V[\phi]=0$. □

Remark The referee has drawn our attention to the fact that Proposition 3.2 must be valid for any current $\phi\in Z^{(0,n-m-1)}(V_D,\omega_V^{\circ})$ admitting an extension to $\mathbb{C}P^n$ as a $\bar\partial$-closed current. This is indeed true and can be reduced to the following statement: *If a current $\phi\in Z^{(0,n-m-1)}(V_D,\omega_V^{\circ})\subset\Gamma(D,\mathcal{K}^{(n,n-1)})$ is $\bar\partial$-cohomologically equivalent to a $\bar\partial$-closed form $\Phi\in C^{(n,n-1)}(D)$, then $\mathcal{R}_V[\phi]=\mathcal{R}[\Phi]$, where $\mathcal{R}[\Phi]$ is the standard Radon transform of Φ defined using the manifold of incidence*

$$\{(\xi,z)\in D^*\times D:\langle\xi\cdot z\rangle=0\}.$$

(*See similar statement for a reduced V on p. 242 in* [16].)

In the next Proposition we prove the sufficiency of the condition in the statement (i) of Theorem 2.

Proposition 3.3 *If a $\bar\partial$-closed residual current ϕ on V_D satisfies $\mathcal{R}_V[\phi]=0$, then ϕ is the restriction to V_D of a $\bar\partial$-closed residual current on V.*

Proof Without loss of generality we may assume that the $\bar{\partial}$-closed forms Φ_α associated with ϕ are defined in some linearly concave domain $D_{-\delta} \supset D$. We fix ν such that $\delta > \nu > 0$ and extend current ϕ into $\mathbb{C}P^n$ by extending the forms Φ_α by the formula $\vartheta\Phi_\alpha$, where

$$\vartheta(z) = \begin{cases} 1 & \text{if } z \in D_{-\nu}, \\ 0 & \text{if } z \notin D_{-\delta}, \end{cases}$$

is a smooth function. Then we consider current ψ defined on $\mathring{G}$ by the formula

$$\psi(f) = \sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z) f(z) \frac{\bar{\partial}\vartheta(z) \wedge \Phi_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)} \tag{32}$$

for $f \in \mathcal{E}_c(\mathring{G})$.

Using the Stokes' formula, Lemma 2.6, and Proposition 2.3 we obtain the following equality

$$\begin{aligned}
&\sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z) f(z) \frac{\bar{\partial}\vartheta(z) \wedge \Phi_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)} \\
&\quad = -\sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z) \bar{\partial} f(z) \wedge \frac{\vartheta(z)\Phi_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)} \\
&\qquad -\sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} f(z)\vartheta(z)\bar{\partial}\vartheta_\alpha(z) \wedge \frac{\Phi_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)} \\
&\qquad -\sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} f(z)\vartheta(z)\vartheta_\alpha(z) \wedge \frac{\bar{\partial}\Phi_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)} \\
&\quad = -\sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z) \bar{\partial} f(z) \wedge \frac{\vartheta(z)\Phi_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)},
\end{aligned}$$

i.e. ψ is a current with compact support in $\mathring{G}$ satisfying the condition

$$\psi = \bar{\partial}\left(\vartheta\phi\right).$$

Considering the extension of ψ to the space of holomorphic functions on $\mathring{G}$ and using the Stokes' formula we obtain

$$\sum_{\alpha=1}^{N}\lim_{t\to 0}\int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\vartheta_{\alpha}(z)h(z)\frac{\bar{\partial}\vartheta(z)\wedge\Phi_{\alpha}(z)}{\prod_{k=1}^{m}F_k^{(\alpha)}(z)}$$
$$=\sum_{\alpha=1}^{N}\lim_{t\to 0}\int\limits_{bD_{-\nu}\cap T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\vartheta_{\alpha}(z)h(z)\frac{\Phi_{\alpha}(z)}{\prod_{k=1}^{m}F_k^{(\alpha)}(z)} \tag{33}$$

for a holomorphic $h\in H(\mathring{G})$.

Using condition $\mathcal{R}_V[\phi]=0$ and introducing variables

$$u_j=\frac{z_j}{z_0}\ \text{for}\ \ j=1,\ldots,n,$$

in the neighborhood $\{z_0\neq 0\}$ we obtain the equality

$$\mathcal{R}_V[\phi]_0(\xi)=\frac{1}{(2\pi i)^{m+1}}\sum_{\alpha=1}^{N}\lim_{t\to 0}\int\limits_{bD_{-\nu}\cap T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\vartheta_{\alpha}(u)\frac{\Phi_{\alpha}(u)}{(\xi_0+\sum_{l=1}^{n}\xi_l\cdot u_l)\cdot\prod_{k=1}^{m}F_k^{(\alpha)}(u)}=0$$

for arbitrary $\xi\in D^*$.

From the linear concavity of D and contractibility of $D^*(z)$ for every $z\in D$, which we pointed out above in Proposition 3.1, we obtain the connectedness of D^*. Then again using the connectedness of D^* and the Cauchy–Fantappié–Leray integral formula on G (see [21]) we obtain the density of the set of functions

$$\left\{\frac{1}{\xi_0+\sum_{j=1}^{n}\xi_j u_j}\right\}_{\xi\in D^*_{\delta}}$$

in $H(\mathring{G})$. Then the equality

$$\sum_{\alpha=1}^{N}\lim_{t\to 0}\int\limits_{bD_{-\nu}\cap T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\vartheta_{\alpha}(u)\frac{h(u)\Phi_{\alpha}(u)}{\prod_{k=1}^{m}F_k^{(\alpha)}(u)}=0$$

holds for an arbitrary $h\in H(\mathring{G})$, which implies, according to (33), the equality

$$\psi(h)=\sum_{\alpha=1}^{N}\lim_{t\to 0}\int\limits_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)}\vartheta_{\alpha}(z)h(z)\frac{\bar{\partial}\vartheta(z)\wedge\Phi_{\alpha}(z)}{\prod_{k=1}^{m}F_k^{(\alpha)}(z)}=0. \tag{34}$$

From the Serre–Malgrange duality (see [23,34]) one can obtain (see [5], §2, Lemma 2.2) that

$$H^0(\mathring{G}, \mathcal{O}/\mathcal{I})' = \Gamma_c(\mathring{G}, \mathcal{K}_{\mathcal{I}}^{(n,n)})/\bar{\partial}\{\Gamma_c(\mathring{G}, \mathcal{K}_{\mathcal{I}}^{(n,n-1)})\}, \tag{35}$$

where $\mathcal{I}$ is the sheaf of ideals defined by the polynomials $\{P_1, \ldots, P_r\}$ and $\mathcal{K}_{\mathcal{I}}^{(p,q)}$ is the sheaf of germs of currents $\gamma^{(p,q)}$ on $\mathring{G}$ with compact support in V such that for any open subset $U \subset \mathring{G}$ the current γ satisfies

$$\gamma(g \cdot f) = 0$$

for any $g \in H^0(U, \mathcal{I})$ and $f \in \mathcal{E}_c^{(n-p,n-q)}(U)$.

From equality (35) applied to the current ψ defined in (32) using (34), we obtain the existence of $\beta \in \Gamma_c(\mathring{G}, \mathcal{K}_{\mathcal{I}}^{(n,n-1)})$ satisfying

$$\bar{\partial}\beta = \psi,$$

and therefore, the current $\beta - \vartheta\phi$ is an extension of the current ϕ into G as a $\bar{\partial}$-closed current. The existence of such current is precisely the appropriate modification of the statement of Theorem 2 mentioned in the remark to this theorem. Namely, if $m < n - 1$, $V \subset D$ is a locally complete intersection in D, and a $\bar{\partial}$-closed residual current ϕ on V_D satisfies $\mathcal{R}_V[\phi] = 0$, then ϕ admits a $\bar{\partial}$-closed extension to a current γ on $\mathbb{C}P^n$ satisfying

$$\gamma(g \cdot f) = 0$$

for any $g \in H^0(U, \mathcal{I})$ and $f \in \mathcal{E}_c^{(n-p,n-q)}(U)$.

If V is a locally complete intersection in $\mathbb{C}P^n$, then a residual current extension can be found. In this case using the partition of unity $\{\vartheta_\alpha\}_{\alpha=1}^N$ we rewrite the last equality as

$$\sum_{\alpha=1}^{N} \bar{\partial}(\vartheta_\alpha \beta) = \psi$$

with currents $\vartheta_\alpha \beta$ having compact supports in U_α and satisfying

$$\vartheta_\alpha \beta(g \cdot f^{(0,1)}) = 0$$

for any $g \in H^0(U, \mathcal{I})$ and $f^{(0,1)} \in \mathcal{E}_c^{(0,1)}(U)$.

Using then the result of Dickenstein–Sessa [6] motivated by Palamodov [28] (see also Theorem 3.4 from [5]) we obtain the existence in $\{U_\alpha\}_{\alpha=1}^N$ of a collection of residual currents θ_α with compact support in U_α of the form

$$\theta_\alpha(f) = \lim_{t \to 0} \int_{T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \frac{f(u) \wedge \Theta_\alpha(u)}{\prod_{k=1}^m F_k^{(\alpha)}(u)},$$

where Θ_α are $\bar\partial$-closed forms of type $(n, n-m-1)$ in some neighborhood of $U_\alpha \cap V$ with compact support in U_α, such that

$$\bar\partial(\vartheta_\alpha\beta - \theta_\alpha) = 0.$$

Therefore, the current $\vartheta\phi - \theta$ is an extension of current ϕ into G as a $\bar\partial$-closed residual current. □

4 Images of $\mathcal{F}_V$ and $\mathcal{R}_V$

In this section we complete the proof of Theorem 2 by proving the second part of statement (ii), namely the inclusions

$$\begin{aligned}&\text{Image } \mathcal{F}_V \supseteq \left\{ f \in H^{(1,0)}(D^*) : f = dg \text{ with } g \in H^0(D^*) \text{ such that } \left\{ P_k\left(\frac{\partial}{\partial\xi}\right) g = 0 \right\}_1^r \right\},\\ &\text{Image } \mathcal{R}_V \supseteq \{ f \in \text{Image } \mathcal{F}_V : \ f = \mathcal{F}_V[\mu], \text{ where } \mu(h) = 0 \text{ for } \forall h \in H^0(\mathbb{C}P^n, \mathcal{O}/\mathcal{I})\},\end{aligned} \tag{36}$$

and statement (iii) of this theorem.

In the proposition below we prove the inclusion above for the image of the Fantappié transform.

Proposition 4.1 *Under the hypotheses of Theorem 2 for any $f = dg \in H^{(1,0)}(D^*)$ with g satisfying equations*

$$P_1\left(\frac{\partial}{\partial\xi}\right) g = \cdots = P_r\left(\frac{\partial}{\partial\xi}\right) g = 0, \tag{37}$$

there exists a linear functional $\mu \in H^0(G, \mathcal{O}/\mathcal{I})'$, such that $\mathcal{F}_V[\mu] = f$.

Proof To prove the proposition we use the following version of the Martineau's (see [25]) inversion formula from [11].

Proposition 4.2 (Generalized Martineau inversion formula [11,25].) *Let $D \subset \mathbb{C}P^n$ be a linearly concave domain such that $D^* \subset \{\xi_0 \neq 0\}$, and let $g \in H^0(D^*)$ be a holomorphic function of homogeneity* 0 *on D^*.*

Let μ^g be the linear functional on $H(G)$ defined by the formula (see [11,25]*)*

$$\mu^g(h) = \int_{bG_\nu} h \cdot \Omega_g, \tag{38}$$

where

$$\Omega_g(z) = \frac{(-1)}{(2\pi i)^n} \frac{\partial^n g}{\partial \xi_0^n}(\eta(z))\omega'(\eta(z)) \bigwedge_{j=1}^{n} d\left(\frac{z_j}{z_0}\right),$$

$$\omega'(\eta) = \sum_{j=1}^{n} (-1)^j \eta_j d\eta_1 \wedge \overset{j}{\overset{\vee}{\cdots}} \wedge d\eta_n,$$

and a map $\eta : bG_\nu \to D^*$ *satisfies* $\langle \eta(z) \cdot z \rangle = 0$ *for* $z \in bG_\nu$.

Then the following equality holds:

$$\mathcal{F}[\mu^g](\xi) = dg(\xi), \tag{39}$$

or

$$\frac{(-1)}{(2\pi i)^n} \int_{bG_\nu} \frac{z_k}{\langle \xi \cdot z \rangle} \frac{\partial^n g}{\partial \xi_0^n}(\eta(z))\omega'(\eta(z)) \bigwedge_{j=1}^{n} d\left(\frac{z_j}{z_0}\right) = \frac{\partial g}{\partial \xi_k}(\xi) \ \textit{for}\ k = 0, \ldots, n$$

for $\xi \in D^*$. □

Using Proposition 4.2 we construct for an arbitrary $g \in H^0(D^*)$ the current μ^g satisfying equality (39). To prove that $\mu^g(h) = 0$ for any $h \in H^0(G, \mathcal{I})$, and that therefore μ^g defines a functional on $H^0(G, \mathcal{O}/\mathcal{I})$ we use the assumption on g, to obtain the equality

$$(-1)^{1+\deg P_k} (\deg P_k)! \cdot \int_{bG_\nu} \frac{z_0}{\langle \xi \cdot z \rangle^{1+\deg P_k}} P_k(z)\Omega_g(z) = P_k\left(\frac{\partial}{\partial \xi}\right)\left[\frac{\partial g}{\partial \xi_0}\right] = 0.$$

Then from the connectedness of D^* (see discussion in Proposition 3.1), and therefore the density of the set of functions

$$\left\{ \frac{z_0}{\langle \xi \cdot z \rangle^{1+\deg P_k}} \right\}_{\xi \in D^*}$$

in the space $H^0(G)$, we obtain the equality

$$\mu^g(h \cdot P_k) = \frac{(-1)}{(2\pi i)^n} \int_{bG_\nu} h(z) \cdot P_k(z) \cdot \frac{\partial^n g}{\partial \xi_0^n}(\eta(z))\omega'(\eta(z)) \bigwedge d\left(\frac{z_j}{z_0}\right) = 0 \tag{40}$$

for an arbitrary $h \in H^0(G)$. □

We prove the second inclusion from (36) and statement (iii) of Theorem 2 using the following proposition.

Proposition 4.3 *Under the hypotheses of Theorem 2 for any $f = dg \in H^{(1,0)}(D^*)$ with g satisfying equations (37) and μ^g constructed in Proposition 4.1 satisfying*

$$\mathcal{F}[\mu^g] = dg, \text{ and } \mu^g(h) = 0 \text{ for } \forall h \in H^0(\mathbb{C}P^n, \mathcal{O}/\mathcal{I}), \tag{41}$$

there exists a residual current $\phi \in Z^{(0,n-m-1)}(V_D, \omega_V^\circ)$, such that $\mathcal{R}_V[\phi] = f$. Such current in particular exists if V is connected in the sense that $\dim H^0(V, \mathcal{O}/\mathcal{I}) = 1$.

Proof To construct a $\bar\partial$-closed residual current with support on V_D, such that its Radon transform coincides with dg, we need an identification described below.

First we consider the following equality of Hartshorne (see [15], Ch.III, Corollary 7.7, Theorem 7.11), specifying the results of Serre [34], Grothendieck [14], Ramis, Ruget, Verdier [30,31] for locally complete intersections

$$H^0(V, \mathcal{O}/\mathcal{I})' \cong H^{n-m}(V, \omega_V^\circ), \tag{42}$$

where $\mathcal{I}$ is the sheaf of germs of ideals corresponding to V, and $\omega_V^\circ = \omega_{\mathbb{C}P^n} \otimes \det N(V)^{-1}$ is the *dualizing sheaf* of V defined earlier in (5).

Using the exactness of the $\bar\partial$-complex of sheaves

$$0 \to \mathcal{O}/\mathcal{I} \otimes \omega_{\mathbb{C}P^n} \to \mathcal{O}/\mathcal{I} \otimes \mathcal{E}^{(n,0)} \xrightarrow{\bar\partial} \cdots \xrightarrow{\bar\partial} \mathcal{O}/\mathcal{I} \otimes \mathcal{E}^{(n,n)} \to 0,$$

which follows from the Malgrange's theorem on $\mathcal{O}$-flatness of $\mathcal{E}$ (see [22], $n°25$, Th. 2), we obtain the equality

$$\begin{aligned} H^{n-m}(V, \omega_V^\circ) &\cong H_{\bar\partial}^{n-m}(V, \omega_V^\circ) \\ &\cong \frac{\{\phi \in \mathcal{E}^{(n,n-m)}(U, \det N(V)^{-1}) : \ \bar\partial\phi \in \mathcal{I} \otimes \mathcal{E}^{(n,n-m+1)}(U, \det N(V)^{-1})\}}{\{\phi \in \bar\partial\mathcal{E}^{(n,n-m-1)}(U, \det N(V)^{-1}) + \mathcal{I} \otimes \mathcal{E}^{(n,n-m)}(U, \det N(V)^{-1})\}} \end{aligned} \tag{43}$$

for a small enough neighborhood $U \supset V$.

On the other hand, for any representative $\Phi \in H_{\bar\partial}^{n-m}\left(V, \omega_V^\circ\right)$ using the Coleff-Herrera theory we can construct a linear functional on $H^0\left(V, \mathcal{O}/\mathcal{I}\right)$ by the formula

$$\langle \phi, h \rangle = \sum_{\alpha=1}^{N} \lim_{t \to 0} \int_{T^{\epsilon}_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z) \frac{h(z)\Phi_\alpha(z)}{\prod_{k=1}^{m} F_k^{(\alpha)}(z)}, \tag{44}$$

explicitly defining the isomorphism in (42).

Continuing then with the construction of the sought current we observe that for an arbitrary fixed $\delta > 0$ and the analytic functional μ^g on $H(\mathring{G}_\delta)$ defined in (38) we can use equality (35) and obtain the existence of a current $\psi^{(\delta)} \in \Gamma_c(\mathring{G}_\delta, \mathcal{K}_{\mathcal{I}}^{(n,n)})$ with support in $V \cap \mathring{G}_\delta$, coinciding with the analytic functional μ^g on $H(\mathring{G}_\delta)$ defined in (38). Considering current $\psi^{(\delta)}$ as a current on V and using equality (40) we obtain the existence of a $\bar{\partial}$-closed differential form

$$\Psi^{(\delta)} \in \mathcal{E}^{(0,n-m)}\left(V, \omega_V^\circ\right)$$

corresponding to μ^g by equality (42) and such that

$$\psi^{(\delta)}(h) = \sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z) \frac{h(z)\Psi^{(\delta)}_\alpha(z)}{\prod_{k=1}^m F_k^{(\alpha)}(z)} \tag{45}$$

by equality (44).

Using condition (41) for functional μ^g and equality (43) we obtain that the functional in (45) is equal to zero, i.e. $\psi^{(\delta)} = 0$ in $H^{n-m}_{\bar{\partial}}(V, \omega_V^\circ)$. Therefore, there exists an element

$$\Theta^{(\delta)} \in \mathcal{E}^{(0,n-m-1)}(U, \omega_V^\circ)$$

in some neighborhood U of V such that

$$\bar{\partial}\Theta^{(\delta)}|_V = \Psi^{(\delta)}|_V.$$

Since $\Psi^{(\delta)}$ has a support in G_δ, it follows that the restriction of the form $\Theta^{(\delta)}$ to D_δ is a $\bar{\partial}$-closed form on $V \cap D_\delta$, and the current

$$\theta^{(\delta)}(\gamma^{(0,1)}) = \sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(\zeta) \frac{\gamma \wedge \Theta^{(\delta)}_\alpha(\zeta)}{\prod_{k=1}^m F_k^{(\alpha)}(\zeta)}$$

is a $\bar{\partial}$-closed closed residual current in D_δ with support in $V \cap D_\delta$.

Applying the Radon transform to the current $\theta^{(\delta)}$ and using equality (25) we obtain the equality

$$\begin{aligned}
\mathcal{R}_V[(2\pi i)^{m+1} \cdot \theta^{(\delta)}] &= \sum_{j=0}^{n} \left(\sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{bD_\delta \cap T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \vartheta_\alpha(z) \frac{z_j \cdot \Theta^{(\delta)}_\alpha(z)}{\langle \xi \cdot z\rangle \cdot \prod_{k=1}^m F_k^{(\alpha)}(z)} \right) d\xi_j \\
&= \sum_{j=0}^{n} \left(\sum_{\alpha=1}^{N} \lim_{t\to 0} \int\limits_{G \cap T^\epsilon_{\{\mathbf{F}^{(\alpha)}\}}(t)} \frac{z_j \cdot \Psi^{(\delta)}_\alpha(z)}{\langle \xi \cdot z\rangle \cdot \prod_{k=1}^m F_k^{(\alpha)}(z)} \right) d\xi_j \\
&= \sum_{j=0}^{n} \psi^{(\delta)} \left(\frac{z_j}{\langle \xi \cdot z\rangle} \right) d\xi_j = \sum_{j=0}^{n} \mu^g \left(\frac{z_j}{\langle \xi \cdot z\rangle} \right) d\xi_j \\
&= \mathcal{F}[\mu^g](\xi) = dg(\xi).
\end{aligned} \tag{46}$$

Using the same arguments as above we construct currents $\psi^{(\delta')}$ and $\theta^{(\delta')}$ for an arbitrary $\delta' < \delta$. Then, from (46) we obtain the equality

$$\mathcal{R}_V[\theta^{(\delta)} - \theta^{(\delta')}](\xi) = 0$$

for $\xi \in D^*_\delta$, and therefore, applying Proposition 3.3 to the current $\theta^{(\delta)} - \theta^{(\delta')}$ on D_δ we obtain the existence of a $\bar\partial$-closed current $\omega^{(\delta)}$ on V, such that

$$\theta^{(\delta)} + \omega^{(\delta)}|_{V\cap D_\delta} = \theta^{(\delta')},$$

and therefore

$$\bar\partial\theta^{(\delta)} = \bar\partial\theta^{(\delta')} = \psi^{(\delta')}. \tag{47}$$

The equality above shows that the support of $\bar\partial\theta^{(\delta)}$ belongs to G_ν with arbitrary $\nu > 0$, i.e. the restriction of the constructed residual current $\theta^{(\delta)}$ to D is a $\bar\partial$-closed current satisfying (46).

This completes the proof of the second inclusion in (36).

To prove statement (iii) of Theorem 2 we notice that if $\dim H^0(\mathbb{C}P^n, \mathcal{O}/\mathcal{I}) = 1$, then using equality

$$\begin{aligned}\mu^g(1) &= \mu^g\left(\frac{z_0}{1\cdot z_0 + 0\cdot z_2 + \cdots + 0\cdot z_n}\right) = [\mathcal{F}\mu^g]_0(1, 0, \ldots, 0)\\ &= \frac{\partial g}{\partial\xi_0}(1, 0, \ldots, 0) = 0,\end{aligned}$$

we obtain that functional μ^g is equal to zero on $H^0(\mathbb{C}P^n, \mathcal{O}/\mathcal{I})$, and therefore using equality (43) we obtain that $\psi^{(\delta)} = 0$ in $H^{n-m}_{\bar\partial}(V, \omega^\circ_V)$. The rest of the proof in this case goes exactly as in the proof of (36). □

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