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ORIGINAL PAPER

Alternative feeder concepts in urban distribution networks

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Abstract

The paper considers various basic feeder schemes of urban distribution networks from the standpoint of economy, reliability, and technical limits. Presented are the mathematical models for the analysis of feeders' annual costs, reliability performances, voltage drops and cable loading in normal and critical emergency situations. The suggested approach takes into account the uncertainty of various basic data by modeling them as grey inputs, which can have any value within the intervals of values experienced in practice. It was implied that adequate statistical data for the construction of probability distribution functions for uncertain parameters are missing, which is often the case. The proposed mathematical model could serve as a useful tool in planning new or expanding existing distribution networks. As a numerical illustration, the presented calculation methods have been applied for the analysis of the performances of the considered feeder schemes in the Belgrade area.

Keywords Urban distribution networks · Feeder concepts · Total costs · Technical limits · Reliability

1 Introduction

The majority of distribution systems operate with a radial configuration in order to simplify the exploitation and the protection systems for eliminating the consequences of system failures. The urban distribution systems are often constructed based upon the *n*-1 security concept that allows the back feed of network portions in case of outages of some network branches. The practical task is then to find the radial configuration that satisfies best an imposed objective.

The problem of feeder reconfiguration for loss reduction has been early recognized, and various approaches to simplify the search pattern and associated load flow calculations have been elaborated [1–4]. They differ in the kind of approximations made in calculating the load flows after stepwise reformations of network configurations, and in the criteria applied in searching for better solutions. In [5], the minimum loss configuration has been searched for using the simulated annealing approach. The Benders decomposition method has been applied by introducing a regression equation approximating the interrelationship of losses and node loads [6]. In [7–9] the minimum loss trees have been searched for by applying different enhanced versions of genetic algorithms reducing the computation time. An approach based on sequential opening of minimum loaded branches has been also applied in the search of minimum loss trees [10].

Several recent papers have considered the optimization of distribution network configuration as a multi-objective problem by including some other aspects of network operation besides power loss. Optimum trees with regard to loss and reliability performances, separately or combined, have been searched for by applying the binary particle swarm method and Pareto approach [11]. A similar approach, applying genetic algorithm and the max min criterion for selection of the best power loss and reliability solution, has been presented in [12].

The effects of uncertainties in loads, failure, and repair rates in the planning and the reconfiguration of a distribution network have been accounted for by applying the point estimate method [13, 14]. The application of point estimate method implies that the probability distribution functions of uncertain data can be predicted from experience, which is not always the case. Paper [15] has analyzed the power flows in distribution networks by treating the loads as grey variables. For the analysis of power flows in radial distribution networks under uncertainties, the interval constraint

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propagation method has been proposed [16]. An optimization method for power flow analysis under interval uncertainty was also presented in [17]. In [18] a method for optimal reconfiguration of distribution networks is developed, based on the representation of uncertainties using also the interval analysis. The same approach in modeling the uncertain inputs was applied for distribution system planning [19].

The reliability criteria to be applied in the operation and planning of power systems have been discussed in [20] and [21]. Methods used for the evaluation of the loading capacity of single cables and pairs of parallel cables, buried in trenches with mixture beddings, have been presented in [22] and [23].

As we can conclude from the previous overview, various aspects concerning the configuration and operation of distribution systems have been addressed in the available papers. They considered specific examples from the published papers or engineering practice. However, the costs, reliability and technical limits of typical, widely used distribution feeders' solutions were not the object of a detailed analysis so far. This paper tries to cover this gap by developing mathematical models for such an analysis. Also, new feeder solutions with reserve cables are suggested and investigated. The uncertainties of some input data were accounted for by modeling them as grey variables, which may have any value within an interval of values, assessed from practical experience. The application of such an approach gives the intervals of possible values of analyzed variables. It is fully justified in cases when sufficient experienced or measured data for uncertain inputs are missing for construction of adequate probability distribution functions. The proposed mathematical models may help to the distribution network planners in the choice of the adequate feeder solutions in various stages of the distribution network development. For numerical illustration, the presented mathematical models were applied for the analysis of feeder solutions in the circumstances characteristic for the 10 kV Belgrade distribution network. The results of this analysis have clearly shown the technical characteristics, limits and costs of all considered solutions in normal and fault conditions.

Section 2 of the paper presents the analyzed feeder schemes, and derives the expressions for the calculation of their annual capital, losses, and undelivered energy costs. The constraints, concerning the loading of cables, voltage drops and reliability indices, are also defined. The proposed mathematical models are in Sect. 3 used for the comparative analysis of the considered feeder schemes, when applied in the 10 kV distribution cable network in Belgrade city area. The Conclusions section discusses the calculation results obtained in Sect. 3, regarding the behavior of the analyzed schemes in normal and fault conditions, and their application limits. The "Appendix" provides the basic relationships for grey variables, used in the paper.

2 Distribution network feeders

2.1 Feeder schemes

The analyzed feeder schemes, that could be applied as basic elements of medium voltage urban distribution systems, are presented in Figs. 1 and 2 The most commonly applied are schemes *B* in Fig. 1 and, not so often, scheme *D* in Fig. 2. Scheme *A* is usually treated as a temporary solution, used only in early phases of network development. New schemes

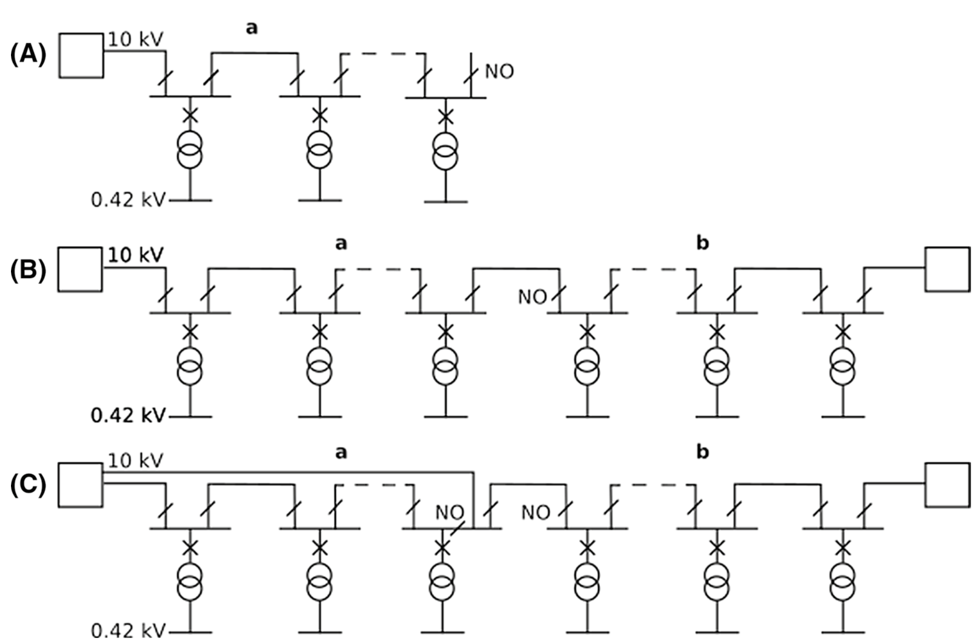

Fig. 1 Solutions for one and two feeders, NO – normally open

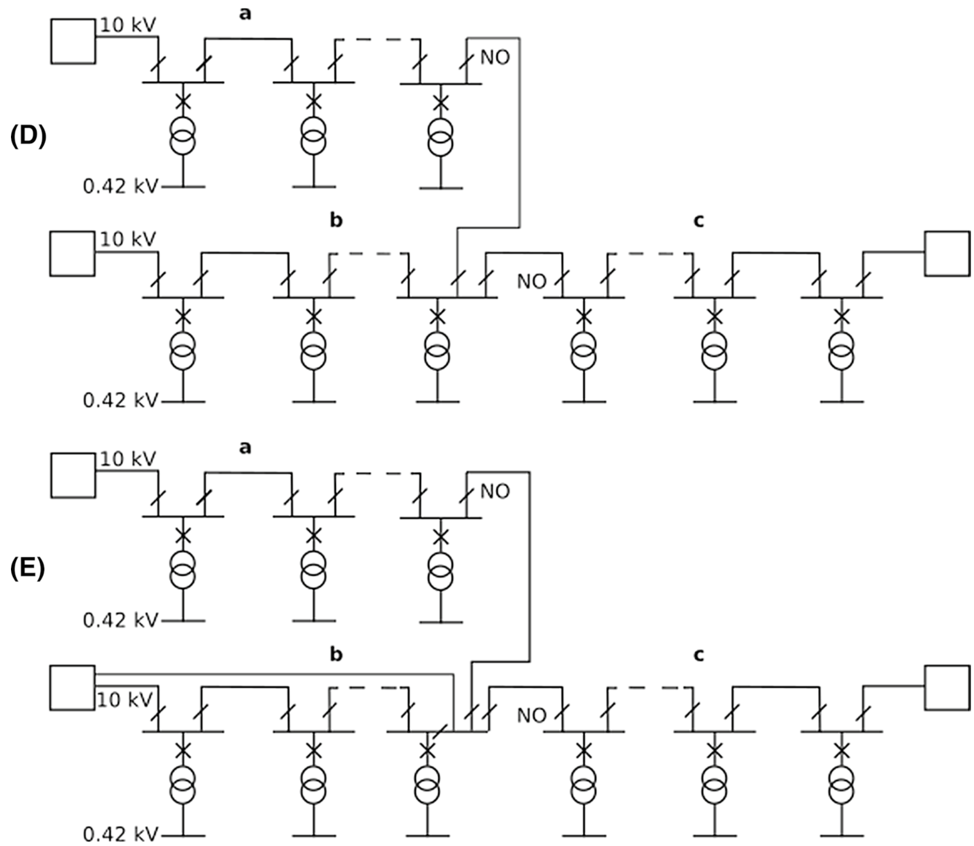

Fig. 2 Solutions for three feeders, NO – normally open

C and *E*, having a reserve cable, are here considered as potentially reliability improved versions of schemes *B* and *D*. The additional cable is laid in the same trench with the feeder. It is normally unloaded and ready to supply the curtailed customer transformer stations (TSs), in case of the failure of an operating feeder. The advantages and drawbacks of these schemes, when compared to their corresponding simpler versions, will be analyzed further on, by taking into account all relevant aspects.

2.2 Annual costs

The capital costs of transformers and switchgear per customer transformer station (TS) for considered schemes can be determined by applying the following expressions:

$$\begin{aligned}
&\mathit{Scheme\ A}: && C_{ts} = \frac{a_g C_g}{n} + 2\,a_s C_s + a_B C_B + a_T C_T \\
&\mathit{Scheme\ B}: && C_{ts} = \frac{a_g C_g}{n} + 2 a_s C_s + a_B C_B + a_T C_T \\
&\mathit{Scheme\ C}: && C_{ts} = \frac{3}{2n} a_g C_g + \frac{4n+1}{2n} a_s C_s + a_B C_B + a_T C_T \\
&\mathit{Scheme\ D}: && C_{ts} = \frac{a_g C_g}{n} + \frac{6n+1}{3n} a_s C_s + a_B C_B + a_T C_T \\
&\mathit{Scheme\ E}: && C_{ts} = \frac{4}{3n} a_g C_g + \frac{6n+2}{3n} a_s C_s + a_B C_B + a_T C_T
\end{aligned} \tag{1}$$

In (1), *C*, *a,* and *n* are annual cost, annual capital recovery cost factor and number of the customer TSs, respectively. Indices *ts, g, s, B,* and *T* are related to total considered cost per customer TS, source substation switchgear, switch, customer TS circuit breaker, and customer TS transformer. Costs include the cost of occupied space. Cost of scheme *A* will be the same as this for scheme *B*, if two switches are installed in the end customer TS. It is usually the most reasonable solution, keeping in mind possible future network development.

Cable and trench annual costs per consumer TS are

$$\begin{aligned}
&\mathit{Scheme\,A}: && C_{ct} = a_c\, l\, c_c \\
&\mathit{Scheme\,B}: && C_{ct} = a_c \left(l + \frac{L_c}{2n} \right) c_c \\
&\mathit{Scheme\,C}: && C_{ct} = \frac{a_c}{2} \left(l + \frac{L_c}{n} \right) c_c + a_c\, l\, c_{cc} \\
&\mathit{Scheme\,D}: && C_{ct} = a_c \left(l + \frac{2L_c}{3n} \right) c_c \\
&\mathit{Scheme\,E}: && C_{ct} = \frac{2a_c}{3} \left[\left(l + \frac{L_c}{n} \right) c_c + l\, c_{cc} \right]
\end{aligned} \tag{2}$$

In (2), *C, c, l,* L_c and *a* are cost per customer TS, cost per unit of cable length, cable section length between two neighboring customer TSs, length of the cable connecting feeders, and annual capital recovery cost. Indices *ct, c, cc* indicate cable total, single cable, and two cables in the same trench.

The energy losses per customer TS per hour are equal

$$G = \frac{3R}{n}\sum_{j=1}^{n}(\beta Ij)^2 = \frac{3R}{n}(\beta I)^2\sum_{j=1}^{n}j^2 = R(\beta I)^2(n+1)(n+0.5) \tag{3}$$

as [24]

$$\sum_{j=1}^{n}j^2 = \frac{n}{6}(n+1)(2n+1) \tag{4}$$

In (3) I and β designate the maximum customer TS load current and the corresponding loss factor. R is the resistance of the cable section length between neighboring customers.

The cost of losses is

$$C_L = G\,c_L \tag{5}$$

where c_L denotes the cost per unit of losses.

The reliability cost depends on the frequency and duration of the failures, that can occur in the considered schemes.

Failure rate and unavailability of all considered schemes, due to customer TS failures, are

$$\begin{aligned}\lambda_{cs} &= 2\lambda_s + \lambda_{cb}\\ U_{cs} &= 2\lambda_s r_s + \lambda_{cb} r_{cb}\end{aligned} \tag{6}$$

In (6) λ, U and r are general symbols for failure rates, unavailability, and repair duration, respectively. Indices *cs, s* and *cb* are related to customer TS, switch, and circuit breaker.

Failure rate and unavailability of scheme *A* per customer TS can be determined using the following relationships

$$\lambda_R = \lambda_{ss} + \lambda_T + n\,(\lambda_c l + \lambda_{cs}) \tag{7}$$

$$U_R = \lambda_{ss} r_s + \lambda_T r_T + n\,(\lambda_c\,l + \lambda_{cs})r_m + (\lambda_c\,l\,r_c + U_{cs})(n+1)/2$$

by bearing in mind that

$$\frac{1}{n}\sum_{j=1}^{n} j = (n+1)/2$$

Symbols λ_{ss}, λ_T, λ_c designate failure rates of the switchgear in the source substation, customer transformer, and cable per 1 m length. Symbols r_{ss}, r_T, r_c denote repair durations of the above mentioned equipment, whereas r_m is the manipulation time needed for the separation of the damaged component from the feeder.

Failure rate and unavailability per consumer TS for scheme *B* are

$$\begin{aligned}\lambda_R &= \lambda_{ss} + \lambda_T + n\,(\lambda_c l + \lambda_{cs}) + \lambda_c L_c/2\\ U_R &= \lambda_{ss} r_m + \lambda_T r_T + n\,(\lambda_c l + \lambda_{cs})r_m + \lambda_c L_c r_m/2 + U_{cs}\end{aligned} \tag{8}$$

with the same meaning of symbols as in relationship (7). As mentioned before, L_c is the length of the cable connecting feeders.

The failure rate and unavailability per customer station for feeder scheme *C* are higher from these for scheme *B* for $\lambda_s/2$ and $\lambda_s r_m/2$, respectively.

The considered reliability indices for scheme *D* are:

$$\begin{aligned}\lambda_R &= \lambda_{ss} + \lambda_T + n\,(\lambda_c l + \lambda_{cs}) + (2\lambda_c L_c + \lambda_s)/3\\ U_R &= \lambda_{ss} r_m + \lambda_T r_T + n\,(\lambda_c l + \lambda_{cs})r_m + (2\lambda_c L_c + \lambda_s)r_m/3 + U_{cs}\end{aligned} \tag{9}$$

The failure rate and unavailability per customer TS for scheme *E* are higher than these indices for scheme *D* for $\lambda_s/3$ and $\lambda_s r_m/3$.

As we can see, the reliability indices of schemes *B, C, D,* and *E* differ very slightly from one other.

It should be noted that the *System Average Interruption Frequency Index* (*SAIFI)* per customer TS equals λ_R. Also, the *System Average Interruption Duration Index* (*SAIDI)* per customer TS equals U_R, if this is determined in hours per year.

The reliability cost was taken to be the lost income of energy providers because of undelivered energy, caused by supply interruptions. This cost per customer TS is

$$C_{M1} = \sqrt{3}\;U_n\,\alpha\,I\,c_e\,U_n, \tag{10}$$

with U_n, α, I and c_e being the rated voltage, load factor, maximum customer TS current, and cost of unit of energy supplied, respectively.

To minimize the supply curtailment effects upon the customers, the adopted reliability cost approach is complemented by the restrictions concerning maximum tolerable values of the *SAIDI* and *SAIFI* indices.

2.3 Constraints

All considered schemes should in normal operation satisfy the condition

$$n\,I \le J \tag{11}$$

with J designating the maximum allowable loading of cable. The most critical situation in cable loading for schemes *B, C, D* and *E* occurs in the case when the first section of the associated cable, that should be reserved, is damaged and under repair. In this case, the loading constraint for schemes *B* and *D* should be

$$2\,n\,I \le J \tag{12}$$

For schemes C and E the expression (11) is valid also in the worst failure location case, because the reserve cable supplies only the customers of the damaged feeder.

The maximum voltage drops at all considered schemes should not, in normal operating conditions, exceed the allowed value $\Delta U_{\max}$

$$\Delta U = \sqrt{3}\sqrt{R^2 + X^2}\, I\, l\, n\,(n+1)/2 \le \Delta U_{\max}, \tag{13}$$

with R and X designating the cable resistance and reactance per unit length.

The highest voltage drop for schemes B, C, D and E occurs in case of the worst failure location on the reserved cable. In this case, as simple analysis shows, the maximum voltage drop will for schemes B and D be

$$\Delta U = \sqrt{3}\sqrt{R^2 + X^2}\, I\, n\,(2\, l\, n + L_c) \tag{14}$$

This voltage drop for schemes C and E is

$$\Delta U = \sqrt{3}\sqrt{R^2 + X^2}\, I\, n\,\left[(3n-1)\, l/2 + L_c\right] \tag{15}$$

The reliability constraints for all schemes are

$$\lambda_R \le SAIFI_{\max} \tag{16}$$

$$U_R \le SAIDI_{\max} \tag{17}$$

with $SAIFI_{max}$ and $SAIDI_{max}$ being the maximum allowed values of the considered indices.

3 Numerical analysis

3.1 Feeders data

Some parameters of the presented mathematical model for the analysis of various feeder concepts should be considered as uncertain, with values being within certain intervals, predicted based upon exploitation experience. To take into account these circumstances, in following analyzes each uncertain parameter G is treated as grey input [17–19], that can have all values within a defined interval. This is formally written as $G = [\underline{G}, \ \overline{G}]$, with $\underline{G}$ and $\overline{G}$ designating the lower and upper bounds of G. In the numerical analyzes that follow, we have introduced p possibility lower $\underline{G}_p$ and upper $\overline{G}_p$ bounds of G, for a reasonable comparison of considered feeder schemes.

The data for variables and parameters figuring in the expressions presented before, characteristic for the Belgrade 10 kV distribution network, are listed in Table 1.

The maximum allowable numbers of customer TSs, regarding various technical constraints discussed before, depend considerably upon l. Therefore, we have further on determined these maximum values for three characteristic section lengths. The maximum allowable voltage drop is 1.1 kV. According to European experience [21], the preferable values for reliability indices are $SAIDI < 400$ min./cust. yr. and SAIFI < 3 inter./cust.yr, which were achieved in 2016 by many European Union countries. The maximum allowable $SADI$ and $SAIFI$ values are taken to be 3 h. / cust. yr. and 1 inter./cust. yr, respectively.

The XLPE insulated 10 kV XHE 49-A cables are considered, widely used in urban distribution networks in the authors' country. Cables are with stranded phase aluminum conductors, insulated by cross-linked polyethylene, with copper screen, and outer sheet made of polyethylene, The cables are laid in the ground in trefoil formation. The conductors' cross Sects. 120, 150, 185 and 240 mm^2 have been considered for comparison from various aspects, as these cross sections are used usually. Cables in schemes C and E, laid in the same trench, are spaced 7 cm. Figure 3 displays the cross section of the trenches for two cables.

Table 1 Input data

U_n, kV	S_{TS}, kVA	$\cos\varphi$	I_{TS}, A	α
10	630	0.95 ind	[20, 32]	[0.5, 0.8]
R, Ω/m	X, Ω/m	L_c / l	l, m	λ_c, fl/yr m
$1.62 \cdot 10^{-4}$	$1.02 \cdot 10^{-4}$	[1, 2]	[200, 400]	$[5, 7.5] \cdot 10^{-5}$
λ_s, fl/yr	λ_{cb}, fl/yr	λ_T, fl/yr	λ_{ss}, fl/yr	r_c, h
$[7,\ 10.5] \cdot 10^{-5}$	$[1.5,\ 2.25] \cdot 10^{-3}$	[0.02, 0.03]	[0.1, 0.15]	[9, 13]
r_{cb}, h	r_s, h	r_T, h	r_m, h	r_{ss}, h
[2, 2.5]	[3.5, 5]	[7,10]	[0.6, 0.9]	0.1
C_{ss}, EUR	C_T, EUR	C_B, EUR	C_s, EUR	c_E, EUR/kWh
24,000	9000	7000	4000	0.1
c_c, EUR/m	c_{cc}, EUR/m	a*, per year	c_L, EUR/kWh	β
39	27	0.1	0.1	[0.32, 0.69]

*for customer transformers, cables and switching equipment

For boundary values of β from Table 1, that are typical for Belgrade area due to various methods of heating, we have determined the intervals of maximum allowable load currents for all considered cross sections. This analysis has been performed using the finite elements method and trench material and environment data, as presented in [22] and [23].

3.2 Annual costs

The considered feeder schemes have been compared regarding annual costs per customer TS. Table 2 presents the mean ($\overline{C}_{0.5}$) and 0.9 possibility upper bound ($\overline{C}_{0.9}$) of annual costs of the considered schemes per customer transformer station. These costs are taken to be representative for the practical assessment of the considered feeder schemes. The calculations were performed for feeders' sections length ranging from 200 to 400 m, treated as a grey input, and for various numbers of customers TSs per feeder, most often encountered in urban distribution networks in the authors' country.

From the presented results we can see that the annual costs per customer TS for all schemes are lower the higher is the cable phase cross section. This can be explained by the fact that the decrement of losses cost with higher cable cross sections over compensates the difference in cable prices.

The performed calculations show that scheme *A* is the cheapest one in all considered cases, closely followed by schemes *B* and *D*. The most expensive are schemes *C* and *E*. The costs of schemes *B* and *D* differ from one other negligibly in all cases. The same holds for schemes *C* and *E*. It can be seen that the differences between the costs of schemes *A* and *B* decrease with higher n. This is because of the increments of the reliability cost of scheme *A* and decrements of the cost per customer TS of the cable section connecting feeders in scheme *B* when n increases.

As can be noticed, the mean costs of all schemes are approximately 20% to 30% lower than their 0.9 possibility upper bounds. That means that the uncertainties can considerably affect the costs and should be accounted for. However, the average costs can serve for a rough comparison of schemes.

3.3 Constraints for considered schemes

To have a reserve on the safe side in cable loading, the maximum allowable number of customer TSs per feeder was determined to satisfy the condition

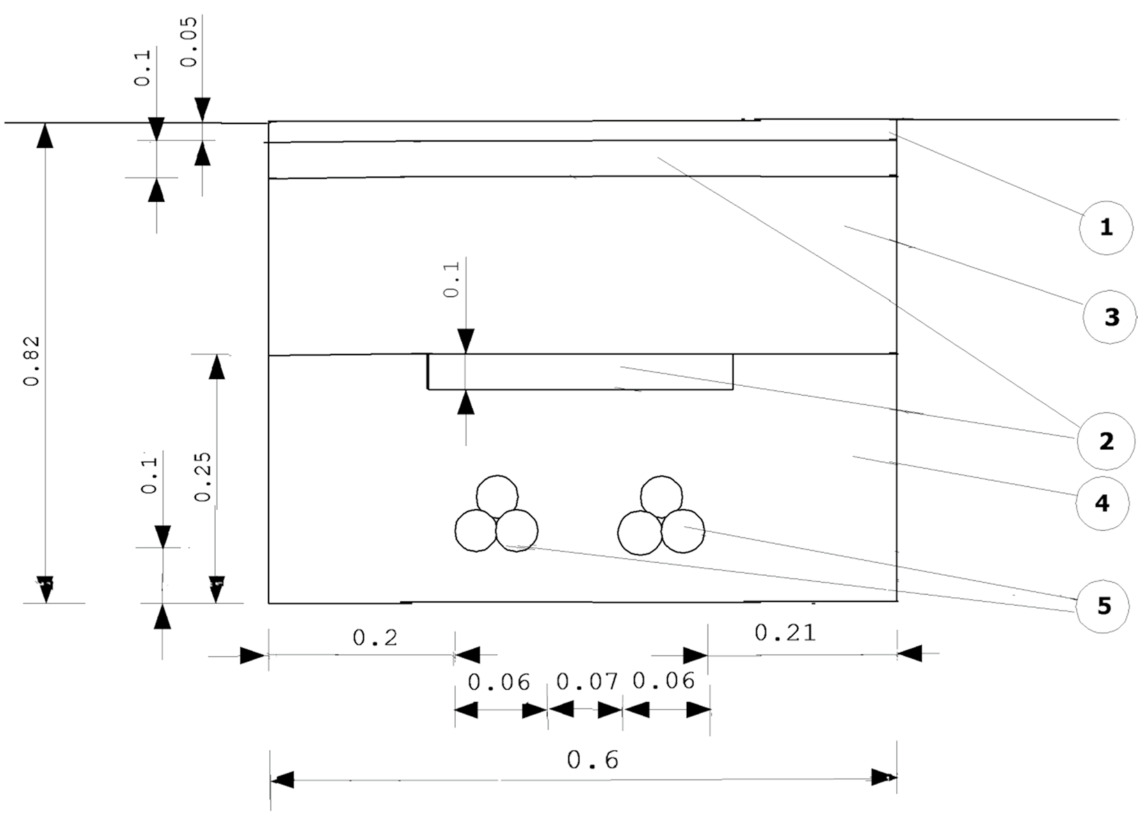

Fig. 3 Cross section of the trench for two cables 1 – asphalt cover, 2 – concrete covers, 3 – backfilling material, 4 – special mixture bedding, 5 – cables (all measures are in m)

Table 2 Annual costs of schemes per customer TS

n	C_A, k\$/n		C_B, k\$/n		C_C, k\$/n		C_D, k\$/n		C_E, k\$/n	
	$\overline{C}_{0.5}$	$\overline{C}_{0.9}$	$\overline{C}_{0.5}$	$\overline{C}_{0.9}$	$\overline{C}_{0.5}$	$\overline{C}_{0.9}$	$\overline{C}_{0.5}$	$\overline{C}_{0.9}$	$\overline{C}_{0.5}$	$\overline{C}_{0.9}$
	$q=120$ mm^2									
5	5004	6034	5160	6260	5679	6854	5278	6426	5604	6790
6	5291	6588	5411	6760	5888	7315	5519	6914	5814	7247
7	5658	7265	5751	7395	6200	7926	5853	7544	6126	7855
8	6097	8055	6169	8154	6597	8667	6268	8300	6524	8594
	$q=150$ mm^2									
5	4859	5768	5019	6001	5509	6557	5139	6169	5445	6508
6	5078	6206	5203	6384	5650	6902	5312	6541	5587	6849
7	5368	6749	5465	6885	5884	7378	5568	7036	5821	7321
8	5719	7388	5794	7491	6193	7967	5894	7639	6131	7907
	$q=185$ mm^2									
5	4700	5494	4861	5727	5365	6302	4980	5896	5297	6247
6	4860	5829	4984	6006	5447	6543	5093	6163	5379	6484
7	5080	6251	5177	6387	5611	6899	5280	6538	5544	6836
8	5352	6753	5427	6856	5841	7351	5527	7004	5774	7285
	$q=240$ mm^2									
5	4587	5283	4753	5524	5257	6099	4873	5695	5190	6046
6	4692	5524	4821	5708	5283	6245	4931	5867	5217	6187
7	4849	5837	4950	5979	5384	6490	5054	6131	5317	6429
8	5049	6215	5127	6322	5541	6817	5229	6472	5475	6753

$$n_{\max}\overline{I}_{0.9} \le \underline{J}_{0.9} \tag{18}$$

The obtained calculation results are presented in Table 3.

Schemes *B* and *D* can provide the supply of all customers at the worst failure location if the number of customers per feeder is not greater than a half of n_{max} values, given in Table 3 for normal operation.

The maximum allowable numbers of customers' TSs, concerning the voltage drop criterion, have been determined by comparing the 0.9 possibility upper bounds of voltage drops for various n to the maximum allowed voltage drop upper limit being 1.1 kV. This analysis was performed both for normal operation and for the most critical fault location on the reserved feeder. The results obtained for normal operation are presented in Table 4. These results are the same for all schemes. Table 5 shows the maximum allowed number of customers' TSs per feeder in the case of the worst failure location on the reserved feeder.

We can see that the allowed numbers of customers' TSs per feeders for schemes *B* and *D* are comparatively low, particularly for smaller conductor cross sections and high distances between neighboring customers' TSs. However, this is not the case for schemes *C* and *E*. They can provide the emergency supply to all customers for the most cases that could be encountered in practice.

As discussed before, all considered schemes should in normal operation satisfy the reliability constraints. We have determined the maximum n values for which the 0.9 possibility upper bounds of grey *SAIFI* and *SAIDI* indices, determined by (16) and (17), do not exceed the maximum allowable *SAIFImax* and *SAIDImax* values. It was established that the critical reliability index for scheme *A* is *SAIDI* index. For all remaining feeder schemes as critical appears *SAIFI* index. The results of the mentioned calculations are presented in Table 6.

Table 3 Maximum allowable number of customer TSs regarding loading of cables

q mm^2	All schemes, normal operation				Schemes *C* and *E*, worst failure location			
	J A	$\underline{J}_{0.9}$ A	$\overline{I}_{0.9}$ A	n_{max}	J A	$\underline{J}_{0.9}$ A	$\overline{I}_{0.9}$ A	n_{max}
120	[356, 377]	358	347	17	[286, 296]	287	286	14
150	[397, 421]	399	389	19	[318, 330]	319	306	15
185	[450, 482]	454	453	22	[339, 351]	340	327	16
240	[521, 550]	524	510	25	[402, 417]	404	388	18

Table 4 Maximum allowed n regarding voltage drops in normal operating conditions for all schemes

l m	q, mm^2 120	150	185	240
	n_{max}			
200	24	26	29	32
300	19	21	23	26
400	16	18	20	22

Table 6 Reliability constraints

l m	Scheme A		Schemes B, C, D, E	
	$\overline{SAIDI}_{0.9}$ hr./cust. yr	n_{max}	$\overline{SAIFI}_{0.9}$ int./cust.yr	n_{max}
200	2.984	22	0.997–1.000	44
300	2.946	15	0.984–0.999	31
400	2.911	11	0.971–0.999	24

The lowest $\overline{SAIFI}_{0.9}$ values within the intervals given in Table 6 are for scheme B and the highest, for scheme E.

4 Conclusions

The paper presents general models for assessing the costs and application limits of typical urban distribution feeder schemes, which could help in the planning of new and developing of existing networks. Such an approach was missing in the available literature. New feeders' solutions with reserve cable are also proposed, and analyzed in the paper. The mathematical model, used for the analysis of various solutions, takes into account the uncertainty of some inputs by modeling them as grey variables, which can have any value within the defined intervals the bounds of which can be determined from practical experience. As indicated, the application of the grey numbers theory makes it possible to quantify the obtained calculation results by a possibility grade.

For a numerical illustration, the developed mathematical models have been applied to the Belgrade 10 kV distribution network. The performed calculations have shown the following for the considered network:

- The cables with the highest conductor cross section appeared to be the cheapest solutions for all schemes.
- Scheme A is the cheapest one considering the total annual cost. The limits of this scheme are determined by the *SAIDImax* constraint. However, these limits will be rarely exceeded in practice, as the results of the performed analysis have shown. This means that scheme A could be considered as an adequate solution for urban distribution networks in many cases.
- -Schemes B and D are the second cheapest analyzed solutions satisfying all considered aspects. There are only some restrictions concerning the supply of the reserved feeder in case of the worst fault location because of voltage drops. The mentioned restrictions are more pronounced for smaller conductors' cross sections and longer cable sections between neighboring customers' TSs.
- Schemes C and E are the most expensive ones. These solutions behave much better than schemes B and D in the emergency situations. This advantage could justify their application in the cases when reliability very sensitive customers should be supplied.

Appendix

Grey numbers

If lower and upper limits of an information G can be estimated by real numbers, the information can be presented as an interval grey number

$$G = [\underline{G},\ \overline{G}], \tag{A1}$$

with $\underline{G}$ and $\overline{G}$ designating the lower and upper bounds of the grey number. The basic grey numbers operations are

$$G_1 + G_2 = [\underline{G}_1 + \underline{G}_2, \overline{G}_1 + \overline{G}_2] \tag{A2}$$

Table 5 Maximum allowed n regarding voltage drops in case of worst failure location

l m	Schemes B and D				Schemes C and E			
	q, mm^2				q, mm^2			
	120	150	185	240	120	150	185	240
	n_{max}				n_{max}			
200	9	10	11	13	13	15	16	18
300	6	6	7	9	11	12	13	15
400	4	4	5	6	9	10	11	12

$$G_1 - G_2 = [\underline{G}_1 - \overline{G}_2, \overline{G}_1 - \underline{G}_2] \tag{A3}$$

$$G_1 \cdot G_2 = [\min(\underline{G}_1 \cdot \underline{G}_2, \underline{G}_1 \cdot \overline{G}_2, \overline{G}_1 \cdot \underline{G}_2, \overline{G}_1 \cdot \overline{G}_2), \max(\underline{G}_1 \cdot \underline{G}_2, \underline{G}_1 \cdot \overline{G}_2, \overline{G}_1 \cdot \underline{G}_2, \overline{G}_1 \cdot \overline{G}_2)] \tag{A4}$$

$$G_1/G_2 = [\underline{G}_1, \overline{G}_1] \cdot \left[\frac{1}{\overline{G}_2}, \frac{1}{\underline{G}_2}\right] \tag{A5}$$

Possibility

$$P(G_1 \le G_2) = \frac{\max(0, L(G_1) + L(G_2) - \max(0, \overline{G}_1 - \underline{G}_2))}{L(G_1) + L(G_2)} \tag{A6}$$

where

$$L(G) = \overline{G} - \underline{G} \tag{A7}$$

Possibility function $P(G \le X)$, where X is a fixed number, equals,

$$P(G \le X) = \frac{\max(0, L(G)) - \max(0, \overline{G} - X))}{L(G)} \tag{A8}$$

If

$$\underline{G} \le X \le \overline{G} \tag{A9}$$

we obtain from (A8)

$$P(G \le X) = \frac{X - \underline{G}}{\overline{G} - \underline{G}} \tag{A10}$$

X is with possibility p greater than or equal to G if

$$P(G \le X) = p \tag{A11}$$

or, explicitly,

$$X \ge \overline{G}_p = p\overline{G} + (1 - p)\underline{G} \tag{A12}$$

$\overline{G}_p$ can be named the p possibility upper bound of G. As

$$P(X \le G) = \frac{\overline{G} - X}{\overline{G} - \underline{G}} \tag{A13}$$

X is with possibility p lower than or equal to $\underline{G}_p$, the p possibility lower bound of G, if

$$X \le \underline{G}_p = p\underline{G} + (1 - p)\overline{G} \tag{A14}$$

By bearing in mind (A12) and (A14), we can say that $\underline{G}$ and $\overline{G}$ are 1 possibility lower and upper bounds of G.

From (A12) and (A14) it follows

$$\underline{G}_{0.5} = \overline{G}_{0.5} = \left(\underline{G} + \overline{G}\right)/2 \tag{A15}$$

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