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ORIGINAL PAPER

Towards robust controller design using μ-synthesis approach for speed regulation of an uncertain wind turbine

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Abstract

Wind turbines are subjected to factors like fatigue, aerodynamics, structural flexibility and wind turbulence which lead to uncertain behaviour. This affects stability and deteriorates the performance of the large structured wind turbine. In order to reduce the effects of uncertainties on the system performance and its structure, a robust controller design is necessary. In this paper, it is proposed to design a μ-synthesis-based robust controller to overcome the effects due to uncertainties. A $\pm\,25\%$ variation in the values of the system matrix elements is considered for analysis in this paper. A 109th order of the proposed μ-controller is obtained, wherein it is reduced to a 7th order controller by using the balanced truncation method. The robust stability and robust performances are satisfactorily achieved with both these controllers. Furthermore, the worst-case performance of the uncertain wind turbine is also analysed.

Keywords Robust μ-controller · Robust stability · Robust performance · Uncertainty · Wind turbine · Model order reduction

1 Introduction

According to preliminary statistics announced by World Wind Energy Association [1] on 25 February 2019, the overall capacity of all wind turbines installed worldwide by the end of 2018 reached 597 GW. It says that 50,100 MW were added in the year 2018, slightly less than in 2017 when 52,552 MW were installed. The year 2018 was second in a row with a growing number of new installations but at a lower rate of 9.1%, after 10.8% growth in 2017. All wind turbines installed by the end of year 2018 can cover close to 6% of the global electricity demand. This scenario in wind power capture led to increase in the overall size of the wind turbine ranging from 1 kW to several MWs. These large structures are available as vertical-axis (VAWT) and horizontal-axis wind turbines (HAWT). The components of the upwind HAWT machine are shown in Fig. 1. The advantages of HAWT over VAWT are: (1) the entire rotor can be placed on top of a tall tower that implies proximity to larger wind speeds and hence increased power capture (2) blades with provision for a maximum pitch angle of 18°, and (3) no need for tensioned cables (guy wires) which are used to add structural stability. As the size of the wind turbine increases, the loads on its structure also increase [3,4].

With the increase in load and the stochastic nature of the wind input on a big structure like the wind turbine, its control becomes difficult. There are researchers who are dealing with various controllers for wind turbine like individual pitch control to limit power in high winds, particularly for large turbines [5], classical proportional integral differential (PID) controllers combined with fuzzy logic [6] for blade pitch regulation in region 3. The uncertainties in the wind turbine model cannot be considered under the effect of PID and Fuzzy logic controllers. The robust model reference adaptive controller design for wind turbine speed regulation simulated by using fatigue, aerodynamics, structural flexibility and wind turbulence (FAST) code is presented in [7] for a three-state model of the wind turbine. Here again, the uncertainties were not considered. In [8], the standard H_∞ controller design for a variable speed wind turbine is

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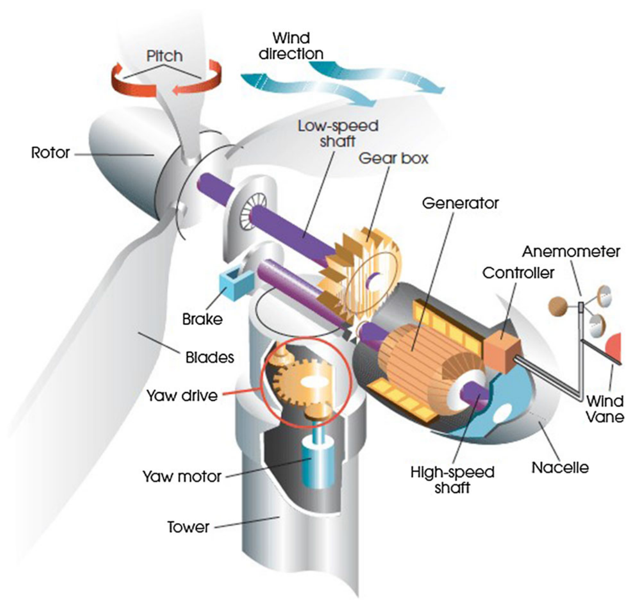


Fig. 1 Horizontal-axis wind turbine (HAWT)—components [2]

obtained. The robust control under parametric uncertainty is overviewed in [9]. A tutorial on robustness and fragility of high-order controllers is provided in [10], and a robust control approach for hydraulic excavators using μ-synthesis is presented in [11].

Intuitively, it can be said that wind turbines are vulnerable to external disturbances and measurement noise. The differences between the mathematical model used for design and the actual dynamics of the model lead to uncertainties. The potential threat to the stability and performance under the influence of uncertainties on the wind turbines gained prominence and is hence considered for analysis in this paper. The design of robust controller for an uncertain wind turbine must be able to deal with variations in the internal and external parameters of the wind turbine. Improper control of wind turbines subjected to uncertainties owing to its large size and cost may lead to loss in reliability and economic aspects. Therefore, the focus lays generally on the active control of larger flexible wind turbines subjected to uncertainties that lead to reduction in the losses incurred economically and structurally which sums up to costly compensation. In the robust stability analysis, generally the designed μ-controller has an order which is much greater than the order of the nominal turbine model. Hence, it is necessary to apply the model order reduction techniques to preserve or retain the most important properties of the original controller. In such a way, the complexity of the system reduces [12–14]. The balanced truncation model order reduction method [15,16] is used to reduce the original higher-order controller in order to reduce the complexity in realization of the physical higher-order controller.

The paper is organized as follows: Sect. 2 discusses the linearized wind turbine model of the CART2 wind turbine. In Sect. 3, the implementation of the robust stability concepts and the model order reduction technique are presented. Section 4 shows the simulation results for various scenarios encountered by the uncertain wind turbine model. Finally, the conclusions are made from the obtained results for the original and reduced order robust μ-controller designed for an uncertain wind turbine for speed regulation by controlling the rotor collective blade pitch angle.

2 A linearized uncertain modelling of the CART2 wind turbine

The wind turbine considered for study in this paper is a Controls Advanced Research Turbine (CART2) developed in the National Wind Technology Center (NWTC), a sub-centre of National Renewable Energy Laboratory [17], Colorado. The CART2 is a 600-kW and 2-bladed horizontal-axis wind turbine having the rotor collective blade pitch angle as the control input and the perturbation in the wind speed as the disturbance input. The measured output variable is considered as the generator speed. This machine is developed using the high-fidelity turbine simulator code known as FAST [17,18]. The obtained models are reliable since they consider the flexible bodies (blades, low-speed shaft and tower) and the rigid bodies (earth, base plate, nacelle, generator and hub) of the wind turbine. Similarly, the controllability and the observability of the uncertain wind turbine were preserved upon inclusion of the uncertainty. The uncertain wind turbine and the robust controller located in feedback are shown in Fig. 2.

2.1 A seven-state linear wind turbine model

The state space linearized model of a wind turbine with seven states, a control input u (perturbed rotor collective blade pitch angle, $\delta\beta$), a disturbance input u_d (perturbed wind speed, δw) and the measured control output y (generator speed, ω) is expressed as

$$\begin{aligned} M\dot{x} &= Ax + Bu + \Gamma u_d \\ y &= Cx \end{aligned} \tag{1}$$

where x is the state vector, $\dot{x}$ denotes the time derivative of x, u is the control input, u_d is the disturbance input, C is the output vector to be measured, M is the mass matrix, A is the system matrix, B is the control input gain matrix and Γ is the disturbance input gain matrix that provides the relationship between measured output and the turbine states. The seven-state linearized wind turbine model along with the matrices that is developed by FAST [18] is given in Eq. (2).

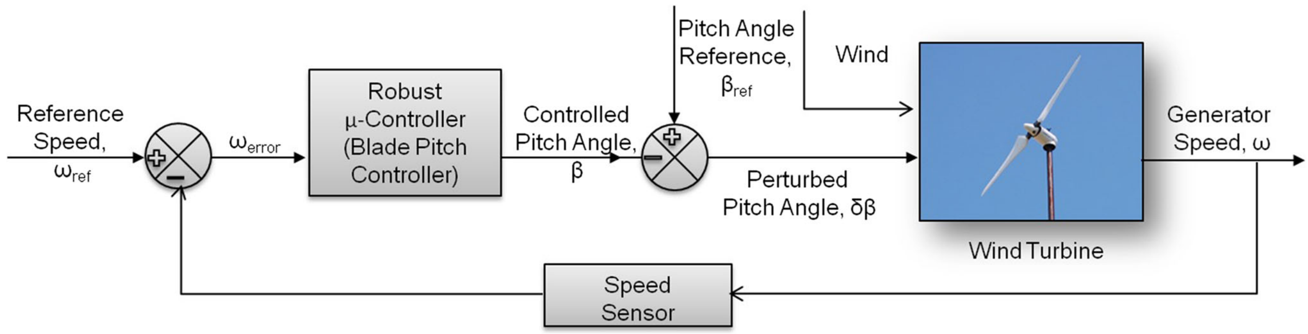


Fig. 2 Representation of the uncertain wind turbine along with the robust μ-controller for regulation of generator speed, ω, by controlling the pitch angle, β

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{11} & M_{14} & 0 & 0 & 0 & M_{17} \\ 0 & 2M_{14} & I_{\text{rot}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{\text{gen}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2M_{71} & 0 & 0 & 0 & 0 & M_{77} \end{bmatrix} \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \\ \dot{x_6} \\ \dot{x_7} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -K_{11} & -C_{11} & -C_{14} & 0 & 0 & -K_{17} & -C_{17} \\ -2K_{41} & -2C_{41} & \gamma - C_d & -1 & C_d & -K_{47} & -C_{47} \\ 0 & 0 & K_d & 0 & -K_d & 0 & 0 \\ 0 & 0 & C_d & 1 & -C_d & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -2K_{71} & -2C_{71} & -C_{74} & 0 & 0 & -K_{77} & -C_{77} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} + \begin{bmatrix} 0 \\ \zeta_b \\ \zeta \\ 0 \\ 0 \\ 0 \\ \zeta_t \end{bmatrix} \delta\beta + \begin{bmatrix} 0 \\ \alpha_b \\ \alpha \\ 0 \\ 0 \\ 0 \\ \alpha_t \end{bmatrix} \delta w$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{bmatrix}^{\text{T}} \tag{2}$$

The seven states of the system considered in this model are: x_1 is the perturbed rotor first symmetric flap mode displacement, x_2 is the perturbed rotor first symmetric flap mode velocity, x_3 is the perturbed rotor rotational speed, x_4 is the perturbed drive-train torsional spring force, x_5 is the perturbed generator rotational speed, x_6 is the perturbed tower first fore-aft mode displacement and x_7 is the perturbed tower first fore-aft mode velocity. In addition, $\delta\beta$ represents the perturbations in rotor collective pitch angle and is the basic control input considered in this paper, while δw represents the perturbations in the turbulent wind speed and is assumed uniform across the rotor disk. The constants M_{ij}, K_{ij} and C_{ij} represent the mass, stiffness and damping elements of the respective matrices (i, j = 1,2,…7), ζ represents the partial derivative of the rotor aerodynamic torque with respect to $\delta\beta$, α represents the partial derivative of the rotor aerodynamic torque with respect to δw and γ represents the partial derivative of the rotor aerodynamic torque with respect to ω.

I_{rot} and I_{gen} represent the moment of inertia of rotor and generator, respectively, and K_d and C_d represents the stiffness and damping factors [18]. The standard state space form of Eq. (1) with two inputs u and u_d is as follows:

$$\begin{aligned} \dot{x} &= (M^{-1}A)x + (M^{-1}B)u + (M^{-1}\Gamma)u_d \\ y &= Cx \end{aligned} \tag{3}$$

where $M^{-1}A$, $M^{-1}B$ and $M^{-1}\Gamma$ are the nominal matrices with respect to the state space model of the wind turbine. The block diagram representation of Eq. (3) is shown in Fig. 3.

2.2 Open-loop nominal wind turbine

The nominal values of the CART2's state matrices A, B and Γ are taken from [18]. The open-loop poles obtained from the eigen analysis of matrix A are: -0.039888 ± 22.574j; -4.4422 ± 13.508j; -0.11715 ± 5.8673j and -0.12094, which implies there are three pole pairs and an individual pole. The first and third pole pairs are lightly damped and represent the drive-train torsion mode and the tower first fore-aft mode, respectively, the second pole pair represents the rotor first symmetry flap mode which is highly damped, whereas the generator speed is represented by the last pole [18]. Speed regulation improves when the generator pole moves farther away to the left from its own open-loop value and when damping is increased to the lightly damped pole pairs. The real part of the first pole is very close to the origin and the slightest of uncertainties in the system parameters will further deteriorate the robust properties which are further discussed in Sect. 3. The general specifications of the wind turbine are given in Table 1.

2.3 Modelling of uncertain wind turbine

In practical systems such as wind turbine, uncertainties are unavoidable. The uncertain behaviour is due to many factors (i) disturbances due to wind speed variations, noise generated

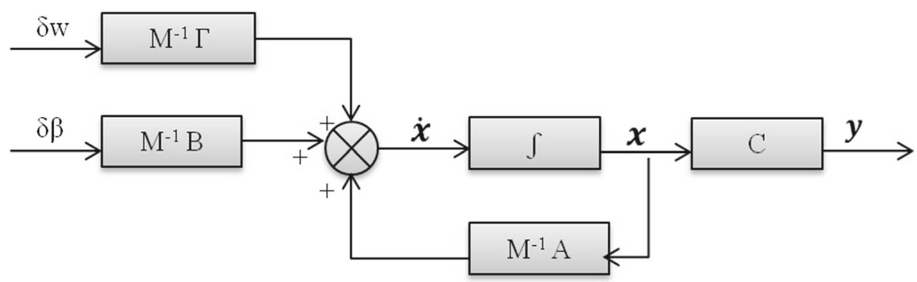


Fig. 3 Block diagram of a nominal wind turbine

Table 1 General specifications of the CART2 HAWT machine

Specifications	Type/numerical values
Turbine type	Horizontal axis
	Upwind rotor
	Teetering hub
Power regulation	Full span
	Blade pitch control
Number of blades	2
Rotor speed	42 rpm
Rotor diameter	43.3 m
Hub height	36.3 m

while measurement of the generator speed ω and (ii) dynamic perturbations that are incurred due to the differences in the mathematical model and the actual dynamics of the wind turbine in operation. Typically, the dynamic perturbations include the unmodelled high-frequency dynamics, neglected nonlinearities and variations in the system parameters that are due to changes in the environmental conditions, wear-and-tear factors of the wind turbine in operation. The stability and performance of any control system may be adversely affected by these factors. The variations due to the high wind speeds that affect the component loads, the moments of inertia acting on the rotor and generator, the stiffness coefficients, the mass coefficients, etc. reflect changes in the elements of the system matrix of its mathematical model of Eq. (2) dynamically. These dynamic perturbations which occur anywhere in the wind turbine can be lumped into a single block known as the perturbation block or the uncertainty block represented by Δ, as shown in Fig. 4. It is called as unknown unstructured uncertainty transfer function $\Delta(s)$ [19]. The actual dynamics of the wind turbine model with perturbations $G_{\text{pert}}(s)$ is given by

$$G_{\text{pert}}(s) = G_{WT}(s) + \Delta(s), \tag{4}$$

where $G_{WT}(s)$ is the packed transfer function of the nominal wind turbine and $\Delta(s)$ corresponds to the parameter variations and is a diagonal matrix having a specified structure, and therefore, it is called "structured uncertainty". It is represented in the block Δ given by

$$\Delta = \text{diag}[\delta_i a_i] \tag{5}$$

where, i = 1, 2, 3, …,15. In this analysis, there are 15 elements that were chosen to introduce uncertainty. These elements represent those which get affected by mass, damper, spring, moments of inertia of rotor and generator of the nominal system matrix of Eq. (3). They are subjected to ±25% uncertainties. Here, $a_i{}'s$ are considered as the system elements of Eq. (3) and δ_i represents the relative changes in these parameters and $\delta_i \leq 1$.

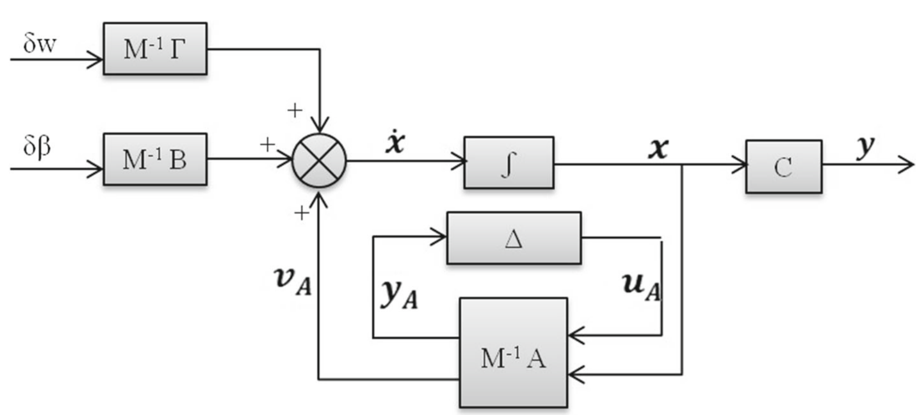


Fig. 4 Block diagram of a nominal wind turbine with uncertainty included

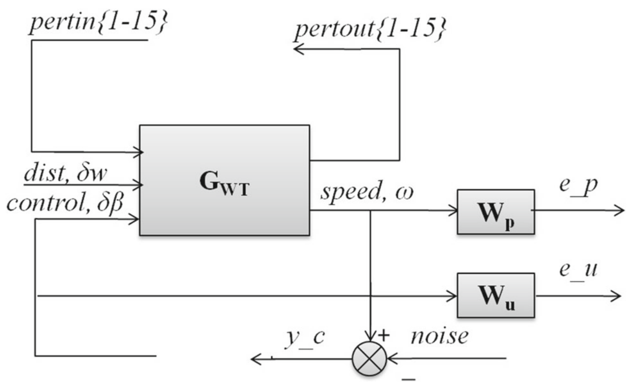


Fig. 5 Open-loop structure of the wind turbine with uncertainty shown in the form of exogenous variable vectors pertin and pertout with weighting functions

3 Robust control design

In real-time control system design, generally the actual model differs from its mathematical representation. One of the criteria to be met in a controller's design is to reduce such differences, if any. In addition, the following criteria should be followed in the design of the robust controller, and they are (i) the controller must stabilize the plant, if it is not originally stable, (ii) the controller must drive the system towards internal stability, and (iii) stabilizing controllers has to be designed. In other words, if the designed controller exhibits robust performance and robust stability while controlling an uncertain wind turbine, the designed controller is said to be a robust [19].

3.1 System interconnections

The open-loop model of the interconnected structure is shown in Fig. 5. The variables pertin and pertout have 15 elements and the variable e_p has three elements, while the variables control ($\delta\beta$), dist (δw), e_u and y_c has an element each. The open-loop structure of the wind turbine is created in MATLAB environment by using the command sysic. Hence, in this interconnected system there are 17 inputs and 20 outputs including 15 exogenous variables contained in pertin and pertout.

3.2 Robust design specifications

In the presence of possible uncertainties, if a controller can achieve certain specified performance criterion and remain stable, then the designed controller is said to be robust. In the design of a robust μ-controller for an uncertain wind turbine, the primary objective is to find a stabilizing controller out of a set of all stabilizing controllers such that its closed-loop system is robust.

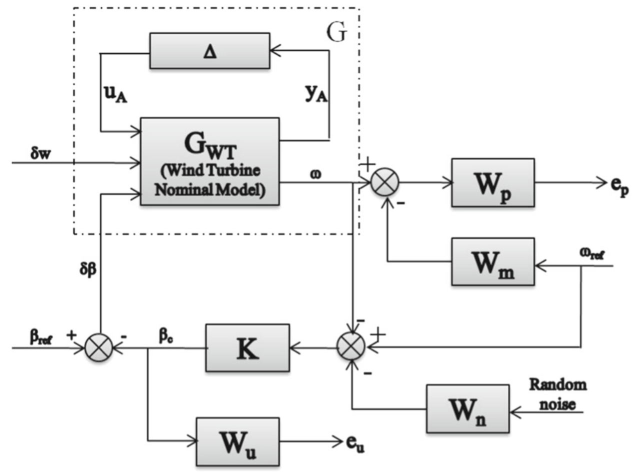


Fig. 6 Closed-loop configuration of the uncertain wind turbine and the μ-controller K with weighting functions

Figure 6 depicts the closed-loop structure of the uncertain wind turbine G, where K represents the μ-controller that has to be designed. The variable ω_{ref} represents the speed reference input, and ω represents the generator output speed. The output from the controller is taken as the controlled pitch angle β_c, while δw is wind disturbance input. The signals ω_{ref}, δw and noise are assumed to be energy bounded and hence are normalized. In order to maintain good performance specifications like tracking, disturbance attenuation and noise rejection for all values of ω_{ref}, δw and noise, their energy should not exceed 1, i.e. the value of μ should be minimized which is called as the gain of the corresponding transfer function matrices [19].

The weighting functions that are used in the μ-synthesis-based robust controller design are given by:

$$
\begin{aligned}
W_m(s) &= \frac{s+10}{s+500};\ W_u(s) = \frac{4*\mathrm{e}^{-30}*s+4*\mathrm{e}^{-13}}{10*s+1};\\
W_n(s) &= 2*\mathrm{e}^{-06}*\frac{s+1}{s+1000}\\
W_p(s) &= \begin{bmatrix} w_{p1}(s) & 0 & 0 \\ 0 & w_{p2}(s) & 0 \\ 0 & 0 & w_{p3}(s) \end{bmatrix}\\
w_{p1}(s) &= \frac{5*\mathrm{e}^{-18}}{10*s+20};\ w_{p2}(s) = \frac{0.08511*s+17.87}{100*s+20};\\
w_{p3}(s) &= \frac{1*\mathrm{e}^{-18}}{100*s+20}
\end{aligned}
\tag{6}
$$

where $W_p(s)$ enables to meet the tracking requirement and $W_u(s)$ compensates the disturbance, $W_m(s)$ is the model transfer function and $W_n(s)$ is the noise transfer function. These weighting functions are stable and of minimum phase.

Table 2 Summary of iterations obtained in μ-synthesis-based robust control

Iteration number	1	2	3
Controller order	15	27	109
γ value achieved	4915.86	4.796	0.884
Peak μ value	102.708	1.240	0.878

These weighting functions are used to meet the control objectives for μ-synthesis.

3.3 μ-synthesis controller design

The interconnected structure for μ-synthesis is shown in Fig. 6. The structured uncertainty block Δ corresponds to the uncertainties used in the modelling of the wind turbine system. Mathematically, the block structure Δ can be defined as follows:

$$\Delta_P = \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_F \end{bmatrix} : \Delta \in C^{15\times 15}, \Delta_F \in C^{5\times 1} \quad (7)$$

The first block Δ is described in Sect. 2.3, and the second block Δ_F is a fictitious block having uncertainty, wherein the robust performance objectives are included. The inputs to Δ_F are the weighted error signals e_p which has three elements and one element each of e_u and y_c, while the output is ω which is the generator speed. In order to meet the design objectives by the μ-synthesis approach, it has to achieve the singular value $\mu < 1$, given in Eq. (8), by finding a stable controller K for all frequency, $\omega \in[0,\infty]$.

$$\mu_{\Delta_{G_{WT}}}[F_L(G_{WT}, K)(j\omega)] < 1 \quad (8)$$

where $F_L(G_{WT}; K)(jw)$ is the lower fractional transformation of G_{WT} and K [19]. The controller $K(s)$ is designed by μ theory using D-K iteration. Fulfilment of the above condition guarantees robust performance of the closed-loop system. The μ-synthesis automates the procedure by using D-K iterations in the robust control toolbox which are shown in Table 2. The γ value achieved is 0.884, and the peak μ value is 0.878 in the third iteration. Hence, it can be stated that the designed controller using μ-synthesis approach is robustly stable for $\pm 25\%$ uncertain wind turbine.

3.4 Model order reduction in the μ-controller

The order of the μ-controller obtained in the design is 109 as shown in Table 2. The physical realizability of this high-order controller is quite tedious and expensive, and it is hence proposed to reduce it to 7th order using balanced truncation method. The reduction in its order significantly simplifies the

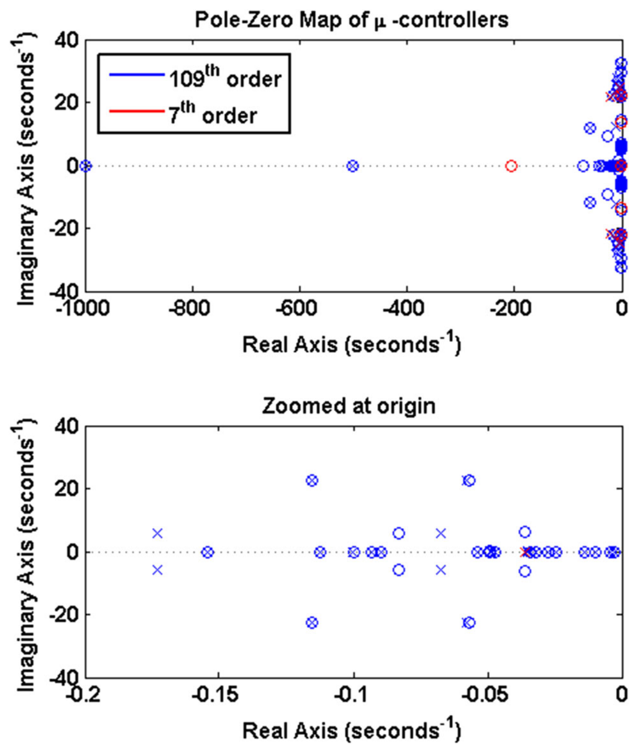


Fig. 7 Pole-zero map locations of the robust original 109th $order\mu$-controller. The inner frames replicate the pole- zero locations near origin in order to show that none of the poles and zeros are on the right half of the s-plane

realization of the controller physically. The physical realizability of the higher-order controller is complex. The key qualitative properties, viz. stability, realizability with good time and frequency response matching are retained in the simplified model [12–14]. The model order reduction is achieved based on the balanced truncation method [15,16]. The MATLAB command balancmr performs the balanced truncation model order reduction method and reduces to 7th order from the 109th order original controller. Figure 7 shows the pole- zero locations of the 109th order original μ-controller and that of the 7th order reduced μ-controller. The pole-zero locations at origin are also depicted in the same graph (heading "Zoomed at origin") to indicate that all the poles and zeros lie on the left half of s-plane. The speed is regulated to 42 rpm for the nominal wind turbine with both the original and reduced μ-controllers as shown in Fig. 8.

3.5 Robust stability

In order to achieve robust stability of the closed-loop system, it must be internally stable for each possible, uncertain plant dynamics $G = F_U(G_{WT},\Delta)$. The nature of the turbulent wind velocity is invariably uncertain, and hence, the stiffness and damping factors that are associated with the blades, the nacelle joints and the cantilevered tower to the earth get

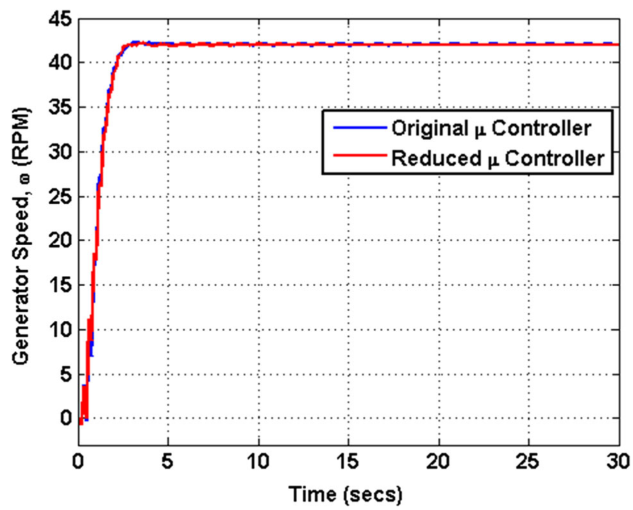


Fig. 8 Closed-loop transient response of the nominal wind turbine for speed regulation using original and reduced μ-controllers

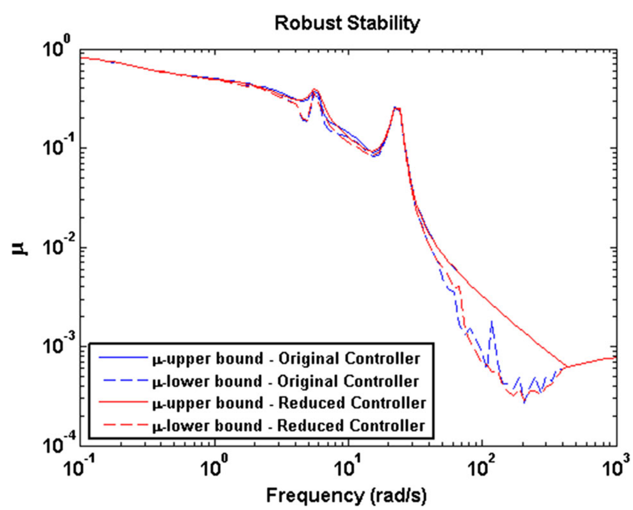


Fig. 9 Robust stability analysis (upper and lower bounds) for original and reduced μ-controllers

affected along with the moment of inertia of generator I_{gen} and rotor I_{rot}. The need for the design of robust controller for an uncertain wind turbine arises due to such deviations. From Fig. 9, it can be stated that, for stable uncertain Δ, the closed-loop system is robustly stable since the value of $\mu(0.878)$ achieved is < 1.

Hence, for an uncertainty level of $\pm 25\%$, a sample of ten possible uncertain plant models are obtained for which the value of μ is achieved to be < 1 by both the designed original 109th order and the reduced 7th order controllers as shown in Fig. 9. It means that the system retained its stability for all its values in the range from 75 to 125% of the nominal values.

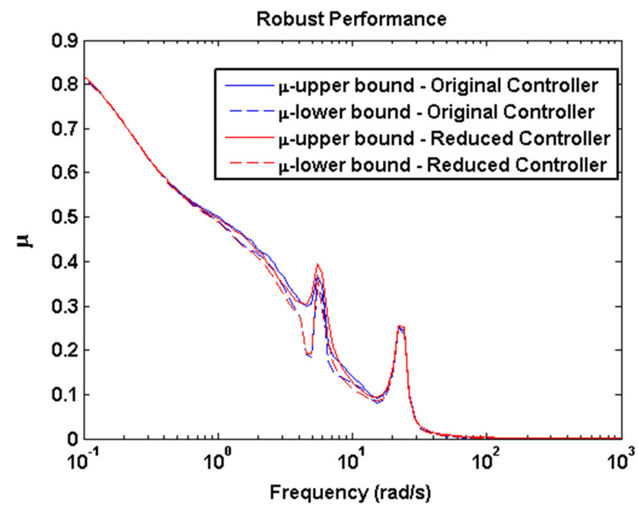


Fig. 10 Robust performance analysis (upper and lower bounds) for original and reduced μ-controllers

3.6 Robust performance

In order to achieve robust performance by the closed-loop system, the following are to be met: (i) it must remain stable internally for each $G = F_U(G_{WT}, \Delta)$, and (ii) the performance criterion should be satisfied for each G. The results of the structured singular value are calculated after repeating three D-K iterations [19]. These results are shown in Table 2, wherein the maximum value of γ obtained in the first iteration is 4915.86. The subsequent iterations are continued until the value of γ goes less than 1. In the final or third iteration for this design, the value of γ achieved is 0.884 and that of μ is 0.878. Since both the values are less than 1, it means that the robust performance of the closed-loop system is achieved. The designed μ-controller is of order 109, and it is achieved after three iterations. The performance objectives usually are to ensure good tracking, good disturbance attenuation and good noise rejection such that for any reference, disturbance and noise inputs, the energy does not exceed 1 ($\mu < 1$), as obtained in Fig. 10.

4 Simulation results

In order to test the effectiveness of the control strategy, extensive analyses are performed on the proposed robust μ-controllers (original and reduced) acting on an uncertain wind turbine. A $\pm 25\%$ uncertainty is included in the nominal model of the wind turbine to regulate the generator speed, ω along with the perturbed rotor first symmetric flap mode displacement, x_1 cm and the perturbed tower first fore-aft mode displacement, x_6 cm. The proposed robust μ-controller is 109th order, and hence, a balanced truncation method is applied to reduce it to a 7th order controller. Comparisons of

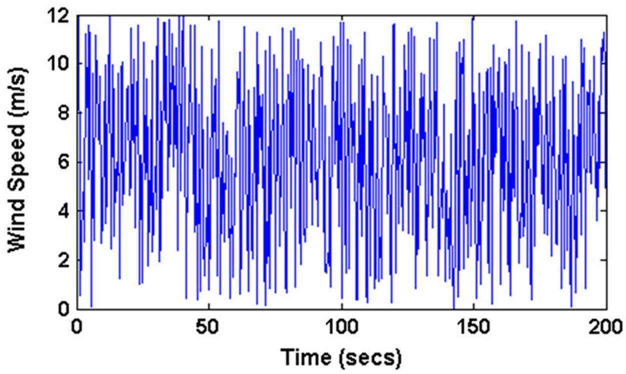


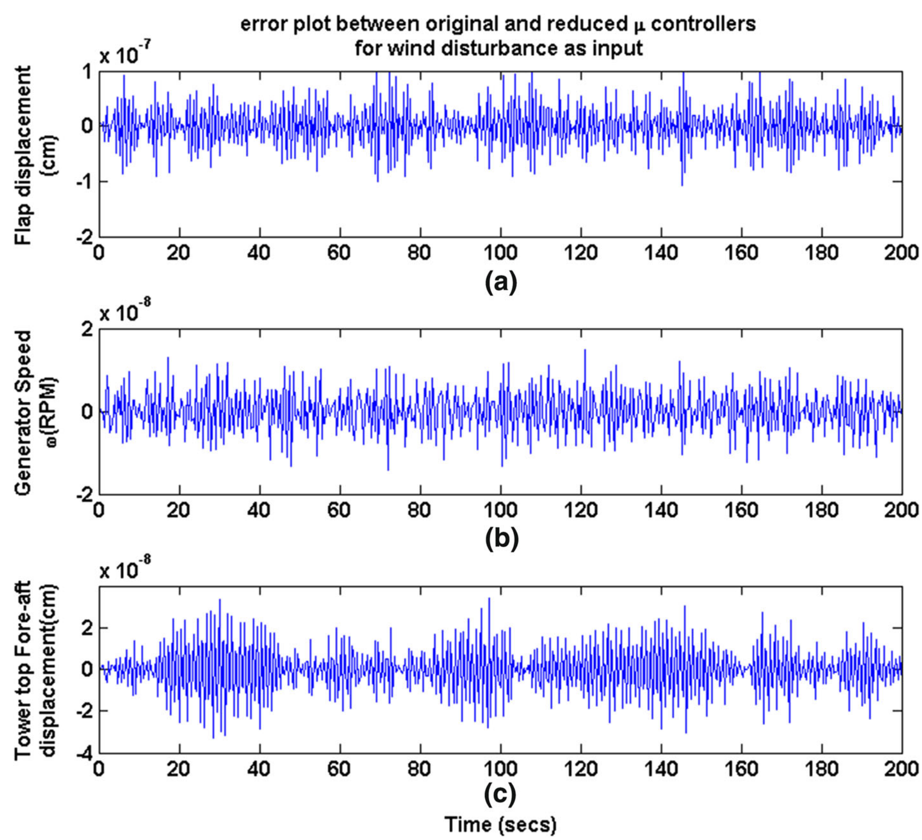
Fig. 11 Turbulent wind profile (maximum value of 12 m/s) applied as the disturbance input, δw

performances of the original proposed robust μ-controller and the reduced controller are shown in each case study. The simulation results are obtained from four case studies to verify the robustness of the proposed μ-controllers. They are: (i) generator speed variations when only turbulent wind conditions of (0–12) m/s are applied, (ii) regulation of generator speed by achieving step changes in the controlled pitch angle of the blade, (iii) regulation of generator speed for step changes in the controlled pitch angle along with wind disturbance input and (iv) worst-case performances of controllers for generator speed regulation by controlling the pitch angle under turbulent wind conditions of (0–12) m/s. The uncertain wind turbine's performance in the presence of robust μ-controller is analysed when all the ten samples of the uncertain wind turbine are following the specified performance criteria. These transient responses of the states of the system x_1 , x_5 and x_6 are shown in the following case studies.

4.1 Generator speed variations for disturbance wind input

The wind conditions generally are highly turbulent for tall towers, so random wind speed variations from 0 to 12 m/s as shown in Fig. 11 are considered as disturbance input for study in this paper. The blades of the wind turbine are affected early on for such external disturbances. This results in deviations in the specified states (x_1, x_5 and x_6). The performances of the original and reduced μ-controllers are shown in Fig. 12 for ten samples of the uncertain wind turbine that were taken into account within the ±25% uncertainty considered. This error plot shows the comparison between both the controllers for each state. The criterion in model order reduction is that the trajectory followed by the original controller must closely be followed by the reduced controller. The smaller the error indicates the close proximity of both the performances.

Fig. 12 Error plot between the original and reduced μ-controller for wind disturbance as the input for an uncertain wind turbine



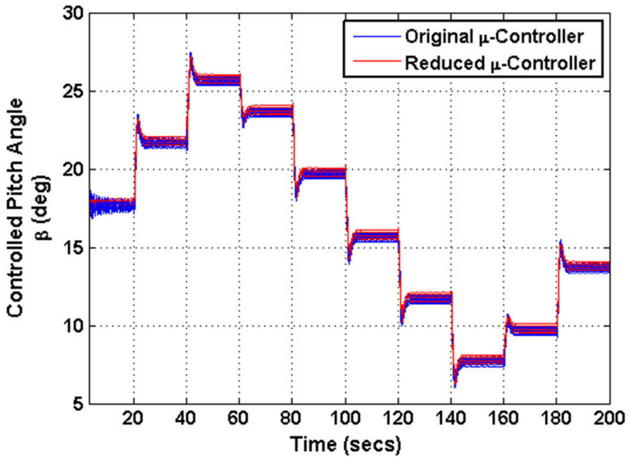


Fig. 13 Controlled pitch angle input to the wind turbine β_c which is an output from the robust μ-controller. The graph indicates ten samples of the uncertain wind turbine for both the original and reduced μ-controllers

It is evident from Fig. 12a that the rotor first symmetric flap mode displacement x_1 cm is shown with a maximum error value of only $1 * 10^{-7}$ cm during the entire operating time of 200 s. Similarly, in Fig. 12b it can be seen that for the rated speed of 42 rpm denoted as x_5 or ω, the maximum value of error lies between $\pm 2 * 10^{-8}$ RPM for both the controllers. Finally, the perturbed tower first fore-aft mode displacement, x_6 cm in Fig. 12c shows a maximum error of $3 * 10^{-8}$ cm. The error plots of the three states indicates that the difference in the performances of both the controllers is small which emphasizes the fact that the performance of the reduced controller is in tandem with that of the original μ-controller.

4.2 Regulation of generator speed by achieving step changes in the blade pitch angle considered as control input

In this section, the step changes in the control pitch angle of the blades are achieved that are shown in Fig. 13. The obtained values of control input are 18°, 22°, 26°, 24°, 20°, 16°, 12°, 8°, 10° and 14°. Each value of control input is considered for a period of 20 s. The total time period considered for study here is 200 s. For such step changed control input values of β_c, the transient behaviour of the uncertain wind turbine (ten samples) with the robust μ-controller and the reduced μ-controller is shown by the states x_1 , x_5 and x_6 in Fig. 14. The rotor first symmetric flap mode displacement x_1 cm shows a transient response of 4 cm initially, i.e. below 3 s and goes to zero displacement and stays there during the steady state, and it is depicted in Fig. 14a. From this figure, it is evident that at every step change in the control input, i.e. after every 20 s the displacement curve displayed transient state momentarily and reached steady state for the rest of the period.

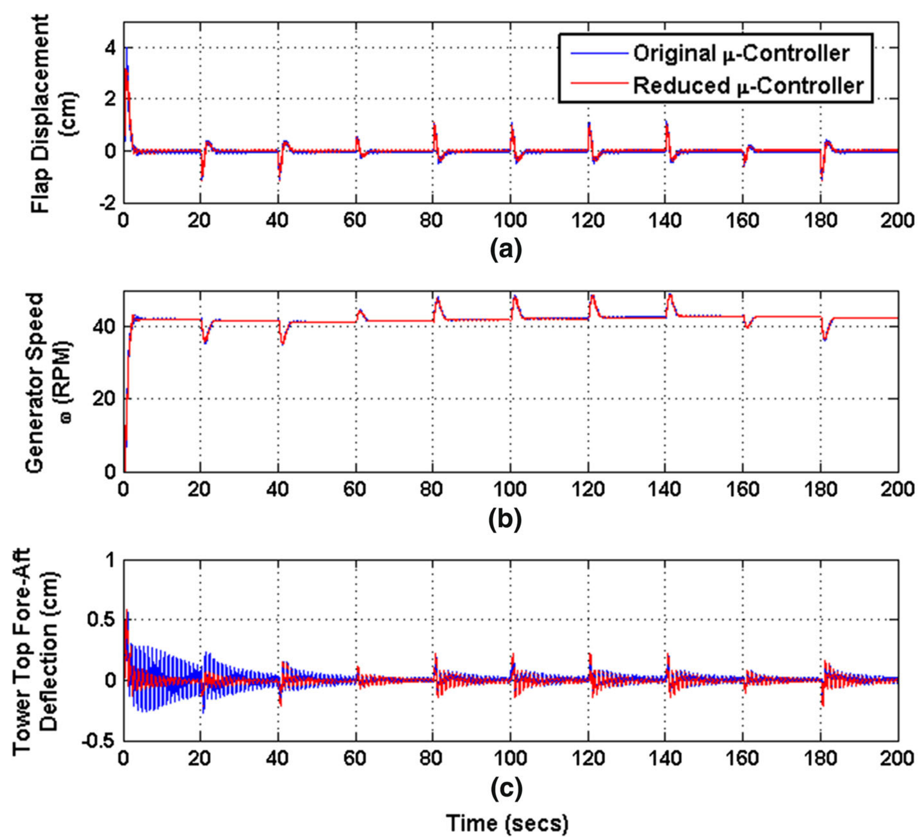


Fig. 14 Transient responses of the three states x_1, x_5 and x_6 for ten samples of the uncertain wind turbine by controlling the pitch angle 18°, 22°, 26°, 24°, 20°, 16°, 12°, 8°, 10° and 14° in steps and each value of β_c prevails for 20 s

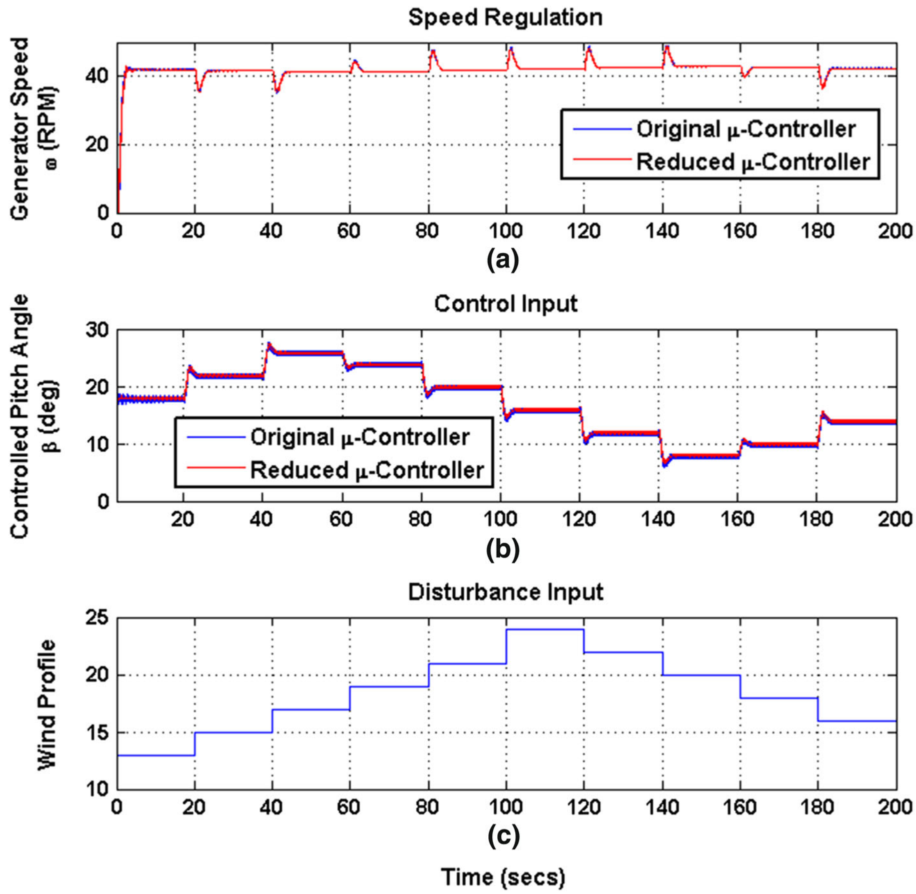


Fig. 15 Regulation of generator speed to its rated value of 42 rpm for two inputs (step changes in controlled pitch angle β_c and step changes in disturbance wind input) for ten samples of the uncertain wind turbine. The values of β_c varied from 8° to 26° in steps for 20 s duration. The values of wind disturbance input varied from 13 to 24 m/s for 20 s

Similarly, both the robust μ-controllers regulate the speed of the generator to 42 revolutions per minute (rated speed) for all the step changes achieved in the control pitch angle of the blades as shown in Fig. 14b. For every change in step of β_c, the generator speed either dropped or rised for less than 2 s and maintained its position in steady state for the rest of the period until the next step change r.

Finally, the perturbed tower first fore-aft mode displacement x_6 cm shows decaying oscillations between − 0.2 and 0.5 cm which is considerably negligible for the tower of a 600-kW CART2-sized machine. When such a heavy wind turbine is placed atop the tower, due to the rotation of the rotor, there are occurrences of vibrations in the associated components to the generator. But, the proposed controller reduces the vibrations in the tower due to rotation to a very small value of 0.5 cm. This is shown in Fig. 14c.

4.3 Regulation of generator speed for step changes in the controlled pitch angle input along with wind disturbance input

In addition to the step changes stated in Sect. 4.2 for the controlled pitch angle β_c of the blades, the application of the step changed wind disturbance input is also applied simultaneously on the uncertain wind turbine to regulate the generator speed x_5 RPM and the other two states x_1 cm and x_6 cm in this section. The disturbance input, wind, is varied in steps for a period of 20 s from a minimum value of 13 m/s to a maximum value of 24 m/s as shown in Fig. 15c. The controlled pitch angle β_c is shown in Fig. 15b. For both these inputs, the generator speed is regulated to 42 rpm as evident in Fig. 15a.

The transient responses of the three states (a) flap symmetric displacement x_1 in cm, (b) generator speed x_5 in rpm and (c) the tower first fore-aft mode displacement x_6 in cm for the two inputs mentioned in Fig. 15b, c for ten samples of the uncertain wind turbine are shown in Fig. 16a, b, c, respectively.

In Fig. 16a, the rotor first symmetric flap mode displacement x_1 cm shows a transient response of 4 cm below 3 s and goes to zero displacement and stays in the steady-state period for every 20 s. Similarly, in Fig. 16b, the rated speed (42 RPM) x_5 or ω is obtained by all the ten samples of the uncertain wind turbine for both the controllers in less than 3 s. Finally, it is shown in Fig. 16c the perturbed tower first fore-aft mode displacement x_6 cm is deviating between − 0.2 and 0.5 cm which is considerably negligible for a 600-kW CART2 machine. When such a heavy mass wind turbine is placed atop the tower, due to the rotation of the rotor, there

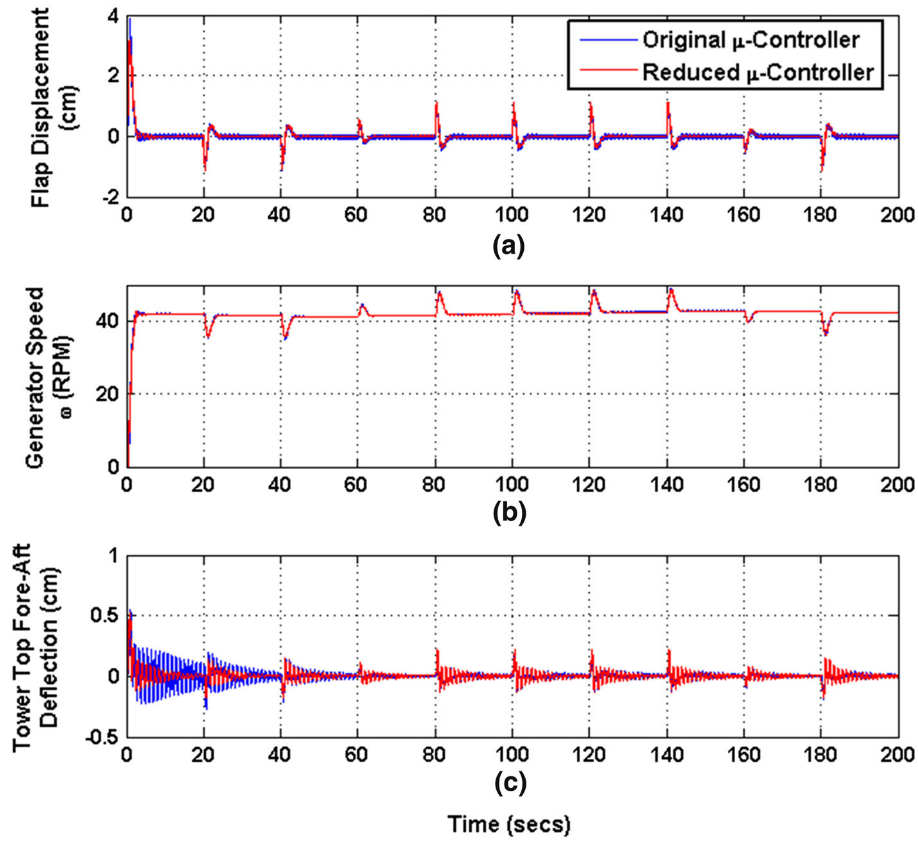


Fig. 16 Transient responses of the three states **a** flap symmetric displacement in cm, **b** generator speed in rpm and **c** the tower first fore-aft mode displacement in cm for two inputs (step changes in controlled pitch angle β_c and step changes in disturbance wind input) for ten samples of the uncertain wind turbine

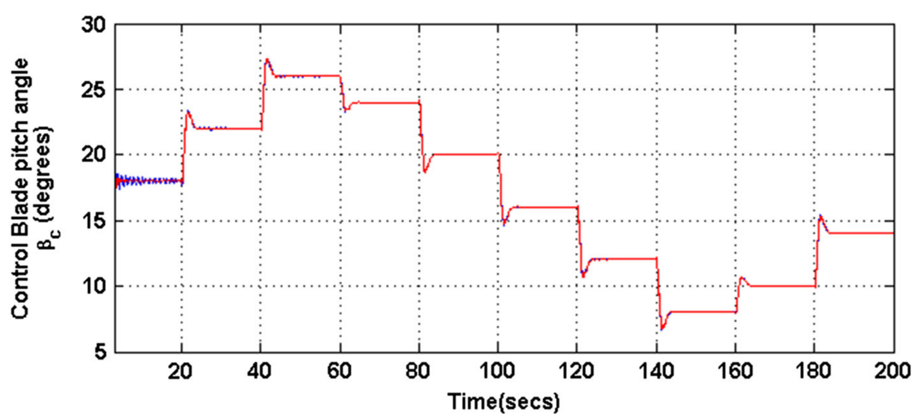


Fig. 17 Controlled pitch angle β_c in steps for the worst-case conditions under turbulent wind conditions of 12 m/s

are occurrences of vibrations in the associated components to the generator. But, the proposed controller reduces the vibrations in the tower due to rotation to a very small value of 0.5 cm.

4.4 Worst-case performances of controllers for generator speed regulation by controlling the pitch angle under turbulent wind conditions of 12 m/s

Worst-case performance [19] is a measure of the robustness of the controller that should meet the performance specifications and the system's stability in the presence of uncertainty. In this case, ten samples of the system parameters ranging from the upper and lower bounds, i.e. + 25% to − 25% of the nominal values, are considered. The powerful robust control toolbox in MATLAB helps in analysing the robustness by directly calculating the upper and lower bounds on worst-case performance. With uncertainties included into the nominal parameters of the stable system, it is likely that the performance gets degraded for specific values of its uncertain elements. Worst-case performance measure is one such value that indicates the level of degradation due to modelled uncertainty. The exact robust performance margin is obtained by the comparison of the nominal systems performance with that of the performance of the upper and lower bounds of

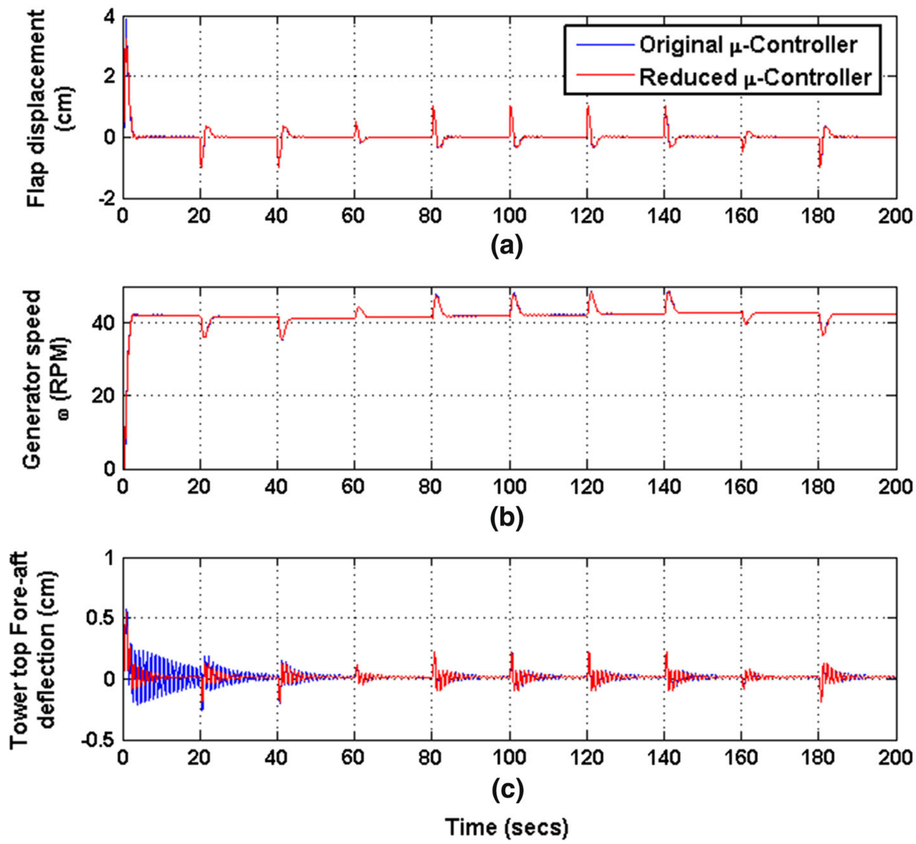


Fig. 18 Representation of the worst-case performances of the uncertain wind turbine shown as transient responses of the three states x_1, x_5 and x_6 for both the inputs (step changed control pitch angle and the wind disturbance input of 12 m/s.)

uncertainty being applied on the system. The controlled pitch angle β_c for the worst case is shown in Fig. 17, and the worst-case transient responses of state variables of wind turbine are shown in Fig. 18.

5 Conclusions

A μ-controller is designed for $\pm 25\%$ uncertain seven-state wind turbine model developed by FAST. A reduced order controller is further obtained by the implementation of the balanced truncation model order reduction method. Comparisons are made by implementing both the controllers in the closed loop of an uncertain wind turbine for the possible scenarios. First, the robust properties are satisfactorily achieved. The values of $\gamma = 0.884$ and $\mu = 0.878$ are less than 1 which indicates that the proposed controllers are exhibiting robust performance and robust stability when using μ-synthesis approach. MATLAB simulations are presented to test the effectiveness of the controllers first on the application of turbulent wind disturbance input ranging its magnitude from 0 to 12 m/s. The desired generator speed and the other prominent states (rotor first symmetric flap mode displacement, x_1 cm and the perturbed tower first fore-aft mode displacement, x_6 cm) were studied to obtain a minimal error in the performances of the original and reduced μ-controllers. The second case shows that the step changes in the controlled pitch angle β_c has considerably less effect on the three states of the ten samples of the uncertain wind turbine. The third case shows the simultaneous effect of step changed blade pitch angle input and step changed wind disturbance input on all the state variables to obtain a satisfactory generator speed regulation of 42 rpm. Finally, the worst-case performance is analysed, i.e. out of the ten uncertain samples considered within the uncertainty level of $\pm 25\%$ of the nominal values, the nominal performance is compared with that of the worst-case samples (i.e. $\pm 25\%$ of the nominal value). The robustness of the designed μ-controller and the reduced controller is evident with this measure.

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