

Solution of multi-objective optimal power flow using efficient meta-heuristic algorithm

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Abstract An efficient meta-heuristic algorithm-based multi-objective optimization (MOO) technique for solving the multi-objective optimal power flow (MO-OPF) problem using incremental power flow model based on sensitivities and some heuristics is proposed in this paper. This paper is aimed to overcome the drawback of traditional MOO approach, i.e., the computational burden. By using the proposed efficient approach, the number of power flows to be performed is reduced substantially, resulting the solution speed up. In this paper, the generation cost minimization and transmission loss minimization are considered as the objective functions. The effectiveness of the proposed approach is examined on IEEE 30 and 300 bus test systems. All the simulation studies indicate that the proposed efficient MOO approach is approximately 10 times faster than the evolutionary-based MOO algorithms. In this paper, some of the case studies are also performed considering the practical voltage-dependent load modeling. The simulation results obtained using the proposed efficient approach are also compared with the evolutionary-based Non-dominated Sorting Genetic Algorithm-2 (NSGA-II) and the classical weighted summation approach.

Keywords Evolutionary algorithms · Generation cost · Multi-objective optimal power flow · Pareto optimal solutions · Sensitivity · Transmission loss

1 Introduction

The role of optimal power flow (OPF) is more significant in modern power system operation and control. Engineers continue to find new uses for OPF programs. Thus, OPF may become more popular and easy to use as conventional power flows. The OPF problem can be solved for minimum generation cost which satisfies the power balance equations and system operating constraints. Normally, the classical optimization methods uses sensitivity analysis and gradient-based techniques. But, the OPF is a highly nonlinear, discrete and multi-modal optimization problem. Therefore, these conventional techniques are not suitable for solving this problem. The real-world problems naturally involve multiple and conflicting objectives to be optimized simultaneously. Defining multiple objectives often gives better idea of the problem.

Majority of the classical multi-objective optimization (MOO) algorithms convert the true multi-objective problem into a single-objective optimization problem by using some user defined functions. The challenges of multi-objective optimal power flow (MO-OPF) are generation of best solutions, generation of uniformly distributed Pareto set, maximizing the diversity of the developed Pareto set, computational efficiency, etc. In this paper, an attempt has been made to improve the computational efficiency of MO-OPF. Several MOO approaches have been developed in the literature. For example, classical weighted summation approach [1], penalty function approach [2], non-dominated sorting genetic algorithm (NSGA)-based approach [3], ϵ -constrained approach [4], etc. The main challenge in a multi-objective environment is to minimize the distance of the generated solutions to the Pareto set and maximize diversity of the developed Pareto set. A good Pareto set may be obtained by appropriate guiding of search process through careful design of reproduction operators and fitness assign-

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ment strategies. To obtain diversification in the Pareto set, special care has to be taken in the selection process.

Reference [5] proposes a multi-objective differential evolution (DE)-based technique to solve the OPF problem considering different objective functions and operational constraints with a clustering algorithm to manage the size of the Pareto set. The performance of population-based algorithms, i.e., particle swarm optimization (PSO), evolutionary programming (EP) and genetic algorithm (GA) for solving the MO-OPF problem, is proposed in [6]. A multi-objective Modified Imperialist Competitive algorithm for solving the MO-OPF problem considering the generation cost, voltage deviation, emission and transmission losses impacts are proposed in [7]. Reference [8] proposes a multi-objective-based DE algorithm based on forced initialization to solve the OPF problem which is formulated as a nonlinear MOO problem. An adaptive group search optimization algorithm for solving the OPF problem considering the accurate multi-objective model is proposed in [9]. In Reference [10], a non-dominated sorting multi-objective opposition-based gravitational search algorithm has been proposed to solve different single and MO-OPF problems. In Reference [11], an Artificial Intelligence techniques are used to solve the MO-OPF problem incorporating FACTS devices with valve point loading (VPL) effect considering transmission loss and voltage deviation minimizations objectives.

A MO-OPF technique using PSO considering two conflicting objectives, i.e., generation cost and environmental pollution, is proposed in [12]. An improved artificial bee colony algorithm to solve the OPF problem involving various objectives, i.e., minimization of the total generation cost, minimization of atmospheric pollutant emissions, minimization of active power losses and minimization of voltage deviations is proposed in [13]. A mathematical model for solving the MO-OPF problem considering uncertainties modeled by fuzzy numbers considering generation cost, total gas emission and voltage profile as the objective functions is proposed in [14]. The gray wolf optimizer and DE algorithms to solve the OPF problem considering the indicator of static line stability index are proposed in [15]. Reference [16] proposes a DE-based integrated approach in a flexible package-based GUI using MATLAB program and adapted to enhance the solution of MO-OPF under contingency situation considering multi shunt FACTS devices. An OPF formulation to enhance the overall transient stability of the power system in addition to the traditional fuel cost minimization is proposed in [17]. Reference [7] proposes various multi-objective variants based on a decomposition approach, where the MOO problem is decomposed into a number of scalar optimization sub-problems which are simultaneously optimized. A biogeography-based optimization algorithm to solve the constrained OPF problems in power systems, considering valve point nonlinearities of generators, is proposed

in [18]. A multi-objective differential evolution algorithm (DEA) which is formulated as a nonlinear MOO problem based on forced initialization for solving the OPF problem is proposed in [19].

Reference [20] proposes a stochastic weight trade-off chaotic non-dominated sorting PSO algorithm for solving MO-OPF considering generator fuel cost and transmission loss objectives. Reference [21] presents the applications of Wolf algorithm for determining the optimal settings on OPF control variables. Reference [22] presents the defects of existing models and algorithms for solving the transient stability constrained OPF. In reference [23], a non-dominated sorting MO gravitational search algorithm is proposed for optimal adjustments of the power system control variables which involve both continuous and discrete variables of OPF problem. A nonlinear optimal control problem as a multi-objective mathematical optimization problem is formulated in reference [24], and it is solved using the harmony search algorithm. Reference [25] proposes a multi-objective mathematical model and an adaptation of the Strength Pareto Evolutionary Algorithm II (SPEA2) for the mixed-model assembly line balancing and equipment selection problem. Reference [26] proposes an efficient MOO approach based on micro genetic algorithm (MGA) for solving the MOO problems. An efficient multi-objective ant colony optimization-I is proposed in [27] to exploits the good performance of ant colony optimization which is enhanced using the ideas borrowed from evolutionary optimization. A MOO of a boot-shaped rib in a cooling channel was conducted to assess the trade-off between two conflicting objectives, i.e., heat transfer performance and pressure drop has been proposed in [28]. A multi-objective energy-efficient task scheduling problem on a green data center partially powered by the renewable energy is proposed in [29].

From the above literature review, it can be observed that the main drawback of all the MOO approaches is the excessive execution time. Therefore, this paper is aimed to overcome this drawback. In this work, the differential evolution algorithm (DEA) is selected to implement the proposed efficient multi-objective optimal power flow (MO-OPF) approach. The proposed MOO is used to find the optimum values of both continuous and discrete control variables. Two standard IEEE test systems (i.e., IEEE 30 bus and 300 bus) are adopted, and the MO-OPF problem is solved considering the generation cost minimization and transmission loss minimization as the objective functions.

The rest of the paper is organized as follows. The mathematical problem formulation is described in Sect. 2. Section 3 presents the proposed efficient approach for the single- and multi-objective optimization problems. Simulation studies on IEEE 30 and 300 bus test systems are given in Sect. 4. The contributions with concluding remarks are drawn in Sect. 5.

2 Problem formulation

In this paper, the generator active power outputs, generator bus voltage magnitudes, transformer tap settings and bus shunt susceptances are considered as the control variables. The objective functions considered in this paper are described next:

2.1 Generation cost

For the active power optimization problem, the total generation cost minimization (F_1) is considered as the objective function while satisfying all the generating units and system’s operating constraints. The minimization function can be obtained as sum of the fuel costs of all the generating units [30].

minimize,

$$F_1 = \sum_{i=1}^{N_G} (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \quad \$/h \quad (1)$$

where N_G is the number of generating units, P_{Gi} is the active power generation of unit- i . a_i , b_i and c_i are the fuel cost coefficients of unit- i .

2.2 Transmission loss

For the reactive power optimization problem, the total system transmission losses minimization is selected as the objective function. The converged load flow solution gives the bus voltage magnitudes and phase angles. Using these, the active power flow through the lines can be evaluated. Net system power loss is the sum of power loss in each transmission line [31].

minimize,

$$F_2 = \sum_{k=1}^{N_{line}} g_k (V_i^2 - V_j^2 - 2V_i V_j \cos(\theta_{ij})) \quad MW \quad (2)$$

where N_{line} is the number of transmission lines. g_k is the conductance of k th line. θ_{ij} is the voltage angle difference between bus i and bus j .

2.3 Practical voltage-dependent load modeling

Generally, the load demands are modeled as the constant power loads. But, the practical aggregated load demands seen from Extra high voltage (EHV) buses are voltage-dependent. This is because of the effects of sub-transmission and distribution system reflected in the equivalent load representation. Some of the case studies presented in this paper considers this practical and realistic load model [32,33]. For the steady

state studies, ZIP (polynomial) or exponential load models are used. The exponential load model used in this work is expressed as,

$$P_{Di} = P_{Di}^0 \left(\frac{V_i}{V_i^0} \right)^{np} \quad (3)$$

$$Q_{Di} = Q_{Di}^0 \left(\frac{V_i}{V_i^0} \right)^{nq} \quad (4)$$

where P_{Di} is the active power load at bus i , Q_{Di} is the reactive power load at bus i . P_{Di}^0 , Q_{Di}^0 and V_i^0 are the nominal values of the active, reactive power loads and the voltage magnitude, respectively. np and nq are voltage exponents which depends on the type and composition of the load. Subscript ‘0’ indicates nominal values of respective variables.

2.4 Equality and inequality constraints

2.4.1 Equality constraints

These constraints are typical power flow equations.

$$0 = P_i = P_{Gi} - P_{Di} - V_i \sum_{j=1}^n V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \quad (5)$$

$$0 = Q_i = Q_{Gi} - Q_{Di} - V_i \sum_{j=1}^n V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \quad (6)$$

In the above equations, $i = 1, 2, \dots, n$. Where n is the number of buses in the system. P_{Gi} and Q_{Gi} are the active and reactive power generations at bus- i . P_{Di} and Q_{Di} are the corresponding active and reactive load demands, respectively. In this paper, fast decoupled load flow (FDLF) is used for the solution of equality constraints.

2.4.2 Inequality constraints

A. Generator constraints: Generator voltage magnitudes, active and reactive power outputs are restricted by their lower and upper limits, and they are represented as,

$$V_{Gi}^{\min} \leq V_{Gi} \leq V_{Gi}^{\max} \quad (7)$$

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad (8)$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max} \quad (9)$$

B. Transformer constraints: Transformer taps have lower and upper setting limits.

$$T_i^{\min} \leq T_i \leq T_i^{\max} \quad (10)$$

C. Switchable VAR sources: The switchable VAR sources have restrictions as follows,

$$Q_{Ci}^{\min} \leq Q_{Ci} \leq Q_{Ci}^{\max} \quad (11)$$

D. Security constraints: These include the limits on the line flow limits and load bus voltage magnitudes.

$$S_{Li} \leq S_{Li}^{\max} \quad (12)$$

$$V_{Li}^{\min} \leq V_{Li} \leq V_{Li}^{\max} \quad (13)$$

2.5 Multi-objective optimization (MOO) problem

A MOO problem has many objectives which are to be optimized simultaneously. The optimization problem is subjected to a number of equality and inequality constraints which the solution should satisfy. The MOO problem is defined by [34],

$$\text{minimize/maximize } F_i(x) \quad i = 1, 2, \dots, N_{\text{obj}} \quad (14)$$

$$\text{subjected to } G_j(x) = 0 \quad j = 1, 2, \dots, M_{\text{eq}} \quad (15)$$

$$H_k(x) \leq 0 \quad k = 1, 2, \dots, N_{\text{ineq}} \quad (16)$$

Eq. (14) represents the objective function vector. Eqs. (15) and (16) represents the set of equality and inequality constraints. ‘ x ’ is a vector of decision control variables. The principle of an ideal MOO procedure is to find multiple trade-off optimal solutions with a wide range of values for objectives. The constraints on reactive power generations, branch flows, slack bus active power generation and load bus voltage magnitudes are treated as the penalty terms in the fitness function.

3 Proposed efficient approach for solving the MO-OPF problem

The general MOO problem solves two or more conflicting objective functions simultaneously subjected to various equality and inequality constraints. In this paper, the generation cost and transmission loss minimization objectives are optimized simultaneously. The general MO-OPF problem optimizes two or more objective functions are optimized simultaneously, and it is expressed as,

$$\text{minimize } [J_1(x), J_2(x)] \quad (17)$$

$$\text{subjected to } g(x) = 0 \quad (18)$$

$$h^{\min} \leq h(x) \leq h^{\max} \quad (19)$$

where F_1 is the generation cost minimization objective function and F_2 is the transmission loss minimization objective function. From the literature review, it can be observed that

the best way to solve this problem is by using the any meta-heuristic-based MOO algorithm with Pareto optimal front. However, this approach is computationally expensive. Hence, this paper proposes an efficient approach to obtain the Pareto optimal front.

In the proposed efficient approach, the first half of the specified number of Pareto optimal solutions are obtained by optimizing the generation cost minimization (F_1) function considering the other objective function [i.e., transmission loss (F_2)] as constraint, while the second half is obtained in a vice versa manner. Therefore, in the first variant, the generation cost minimization function (F_1) is considered as the objective function and transmission loss (F_2) is considered as the constraint. It can be expressed as,

$$\text{minimize } F_1(x) \quad (20)$$

$$\text{subjected to } F_2(x) \leq F_2^{\text{specified}} \quad (21)$$

$$g(x) = 0 \quad (22)$$

$$h^{\min} \leq h(x) \leq h^{\max} \quad (23)$$

where $F_1^{\text{specified}}$ is some specified value of generation cost. In the second variant, transmission loss minimization function (F_2) is considered as the objective function and generation cost function (F_1) as a constraint. It can be expressed,

$$\text{minimize } F_2(x) \quad (24)$$

$$\text{subjected to } F_1(x) \leq F_1^{\text{specified}} \quad (25)$$

$$g(x) = 0 \quad (26)$$

$$h^{\min} \leq h(x) \leq h^{\max} \quad (27)$$

where $F_2^{\text{specified}}$ is some specified value of transmission loss.

Let us consider that the externally maintaining population (i.e., an external population to retain the non-dominated solutions) size is $2N$. At every generation, newly found, non-dominated solutions are compared with existing external population and resulting solutions are preserved. However, in the proposed efficient multi-objective optimization approach these $2N$ non-dominated Pareto optimal solutions are generated by executing the proposed efficient differential evolution (DE) algorithm for $2N$ times. The first and $2N$ th solutions are the two extreme points on the Pareto optimal front. These extreme solutions can be obtained by solving the Eqs. (20) and (24) by relaxing the constraints (21) and (25), respectively. Hence, for the first point on the Pareto optimal front, generation cost objective function (F_1) attains optimum value (F_1^{\min}), whereas the transmission loss (F_2) attains maximum value (F_2^{\max}). It can be expressed as,

$$\text{First point on the Pareto optimal front : } [F_1^{\min}, F_2^{\max}] \quad (28)$$

For the $2N$ th point on the Pareto optimal front, the system losses (F_2) attain optimum value (F_2^{\min}), whereas the generation cost (F_1) attains maximum value (F_1^{\max}). It can be expressed as,

$$2N\text{th point on the Pareto optimal front: } [F_1^{\max}, F_2^{\min}] \quad (29)$$

The rest ($2N - 2$) points on the Pareto optimal front can be obtained by solving the Eqs. (20) and (24), $N - 1$ times with different $F_2^{\text{specified}}$ and $F_1^{\text{specified}}$ values, respectively, in each run. For the $N - 1$ runs of Eq. (20), the $F_2^{\text{specified}}$ varies as,

$$F_2^{\text{specified}} = [F_2^{\max}] - \frac{t [(F_2^{\max}) - (F_2^{\min})]}{N} \quad t = 1, 2, 3, \dots, N - 1 \quad (30)$$

Similarly, for the $N - 1$ runs of Eq. (24), the $F_1^{\text{specified}}$ varies as,

$$F_1^{\text{specified}} = F_1^{\max} - \frac{t [F_1^{\max} - F_1^{\min}]}{N} \quad t = 1, 2, 3, \dots, N - 1 \quad (31)$$

After obtaining these $2N$ points/solutions, sort in the ascending order of F_1 value obtained for each solution leads to the Pareto optimal front.

3.1 Implementation of proposed efficient multi-objective optimization approach

The efficient MOO approach uses the incremental variables model based on the sensitivities. The quadratic generation cost function with incremental variables is expressed as [35],

$$F_1 = \sum_{i=1}^{N_G} [a_i + b_i(P_{Gi}^0 + \Delta P_{Gi}) + c_i(P_{Gi}^0 + \Delta P_{Gi})^2] \quad \$/\text{h} \quad (32)$$

The generator fuel cost objective function considering valve point loading (VPL) effect in terms of incremental variables is given by [35],

$$F_1 = \sum_{i=1}^{N_G} \left[a_i + b_i(P_{Gi}^0 + \Delta P_{Gi}) + c_i(P_{Gi}^0 + \Delta P_{Gi})^2 + \left| d_i \sin \left(e_i \left(P_{Gi}^{\min} - \left(P_{Gi}^0 + \Delta P_{Gi} \right) \right) \right) \right| \right] \quad \$/\text{h} \quad (33)$$

Generator with prohibited operating zones (POZs) has discontinuous input-output characteristics, as it is difficult to find actual POZ by the performance testing. Generally, the best economy is achieved by avoiding the operation in areas

that are in actual operation. The feasible operating zones of i^{th} generating unit is described as [35],

$$P_{Gi} \in \begin{cases} P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi,1}^l \\ P_{Gi,k-1}^u \leq P_{Gi} \leq P_{Gi,k}^l \\ P_{Gi,z_i}^u \leq P_{Gi} \leq P_{Gi}^{\max} \end{cases} \quad (k = 2, 3, \dots, z_i) \quad (34)$$

The system transmission loss minimization objective function in terms of incremental variables is represented as [36],

$$F_2 = G_{kk} (V_k^0 + \Delta V_k)^2 + \sum_{j \neq k}^n (V_k^0 + \Delta V_k) (V_j^0 + \Delta V_j) Y_{kj} \cos [\theta_{kj} + (\delta_j^0 + \Delta \delta_j) - (\delta_k^0 + \Delta \delta_k)] \quad (35)$$

The above Eqs. (33) and (35) are nonlinear, non-convex, and they can be solved simultaneously using the proposed efficient MOO algorithm.

3.2 Constraints

3.2.1 Constraints on control variables

The incremental changes of active power generation (ΔP_{Gi}), generator bus voltage magnitudes (ΔV_{Gi}), transformer tap settings (ΔT_i), and bus shunt susceptances ($\Delta B_{sh,i}$) are constrained by their minimum and maximum limits, and they are represented using [37],

$$(P_{Gi}^{\min} - P_{Gi}^0) \leq \Delta P_{Gi} \leq (P_{Gi}^{\max} - P_{Gi}^0) \quad (36)$$

$$(V_{Gi}^{\min} - V_{Gi}^0) \leq \Delta V_{Gi} \leq (V_{Gi}^{\max} - V_{Gi}^0) \quad (37)$$

$$(T_i^{\min} - T_i^0) \leq \Delta T_i \leq (T_i^{\max} - T_i^0) \quad (38)$$

$$(B_{sh,i}^{\min} - B_{sh,i}^0) \leq \Delta B_{sh,i} \leq (B_{sh,i}^{\max} - B_{sh,i}^0) \quad (39)$$

3.2.2 Constraint on reactive power generation

The constraints on changes in reactive power generation (ΔQ_{Gi}) is represented using,

$$(Q_{Gi}^{\min} - Q_{Gi}^0) \leq \Delta Q_{Gi} \leq (Q_{Gi}^{\max} - Q_{Gi}^0) \quad (40)$$

3.2.3 Line flow constraints

The power flow through each transmission line connected between bus i and bus j (P_{ij}) is restricted by,

$$(-P_{ij}^{\max} - P_{ij}^0) \leq \Delta P_{ij} \leq (P_{ij}^{\max} - P_{ij}^0) \quad (41)$$

3.2.4 Power balance constraint inside the DE optimization

The power balance constraint inside the DE in terms of incremental variables is represented using,

$$\sum_{i=1}^{N_G} \Delta P_{Gi} - \Delta P_{\text{loss}} = 0 \quad (42)$$

The detailed description of incremental function evaluation is represented in [35].

3.2.5 Updating the control variables

$$P_{Gi}^{t+1} = P_{Gi}^t + \Delta P_{Gi} \quad (43)$$

$$V_{Gi}^{t+1} = V_{Gi}^t + \Delta V_{Gi} \quad (44)$$

$$T_i^{t+1} = T_i^t + \Delta T_i \quad (45)$$

$$B_{\text{sh},i}^{t+1} = B_{\text{sh},i}^t + \Delta B_{\text{sh},i} \quad (46)$$

By using the above updated control variables, the generation cost, system losses, reactive power generation and power flow through lines can be determined.

3.3 Constraints handling

The handling of all the equality and inequality constraints are described next:

- The generator active power outputs (P_{Gi}), the generator bus voltage magnitudes (V_{Gi}), the transformer tap settings (T_i) and the switchable VAR sources (Q_{Ci}) are self-restricted between their lower and upper limits, and they are handled by the algorithm.
- The equality constraints are satisfied by the power flow solution.
- The constraints on slack generator power output ($P_{G, \text{Slack}}$), load bus voltage magnitudes (V_{Di}), reactive power generation output (Q_{Gi}) and branch flow (S_k) are restricted by adding a penalty to the objective function.

$$\begin{aligned} F_{\text{avg}} = & F + \rho_P (P_{G, \text{Slack}} - P_{G, \text{Slack}})^2 \\ & + \rho_Q \sum_{i=1}^{N_G} (Q_{Gi} - Q_{Gi}^{\text{limit}})^2 \\ & + \rho_V \sum_{i=1}^{N_D} (V_{Di} - V_{Di}^{\text{limit}})^2 \\ & + \rho_L \sum_{k=1}^{N_{\text{line}}} (S_{Lk} - S_{Lk}^{\text{limit}})^2 \end{aligned} \quad (47)$$

The limit values of above variables can be calculated using,

$$x^{\text{limit}} = \begin{cases} x^{\text{max}} & x > x^{\text{max}} \\ x^{\text{min}} & x < x^{\text{min}} \end{cases} \quad (48)$$

where x can be P_{Gi} , Q_{Gi} , V_{Di} and S_{ki} .

4 Results and discussion

The effectiveness of the proposed efficient multi-objective optimization algorithm is tested on standard IEEE 30 and 300 bus test systems [38]. The selected control parameters of DE are: mutation constant is 0.9, crossover constant is 0.5, population size is 50 and the maximum number of generations is 500. The number of non-dominated solutions, i.e., desired number of points on the Pareto optimal front selected, is 50 (i.e., for the proposed efficient MOO approach, $N=25$). The brief description of differential evolution algorithm (DEA) and the non-dominated sorting genetic algorithm-2 (NSGA-II) has been presented in Appendices 1 and 2, respectively.

4.1 Simulation results on IEEE 30 bus system

The IEEE 30 bus system data, minimum and maximum limits for the control variables and the initial operating points are taken from [35,36,39]. In this paper, the Pareto optimal front obtained with the proposed efficient MOO approach is compared with the classical weighted summation approach and the evolutionary-based Non-dominated Sorting Genetic Algorithm-2 (NSGA-II) algorithm. In the classical weighted summation approach, all the objective functions are linearly combined into a single objective using the appropriate weight factors [40].

$$F = W_1 F_1 + W_2 F_2 \quad (49)$$

where W_1 and W_2 are the weight factors such that $W_1 + W_2 = 1$. By varying these weight factors, different portions of the Pareto optimal front can be generated. The resulting single-objective optimization problem is solved using the proposed efficient approach applied to the DE algorithm.

4.1.1 Case 1: Solving the multi-objective optimal power flow (MO-OPF) problem with quadratic cost function

Figure 1 depicts the distribution of Pareto optimal solutions in the Pareto optimal front obtained using the proposed efficient MOO approach and the evolutionary-based NSGA-II algorithm. In the proposed efficient MOO approach, the proposed efficient single-objective optimization considering cost minimization objective with transmission loss constraint is run for 25 times and the loss minimization objective function with cost constraint is run for 25 times. These 50 runs of proposed efficient approach gives the entire Pareto optimal front. From

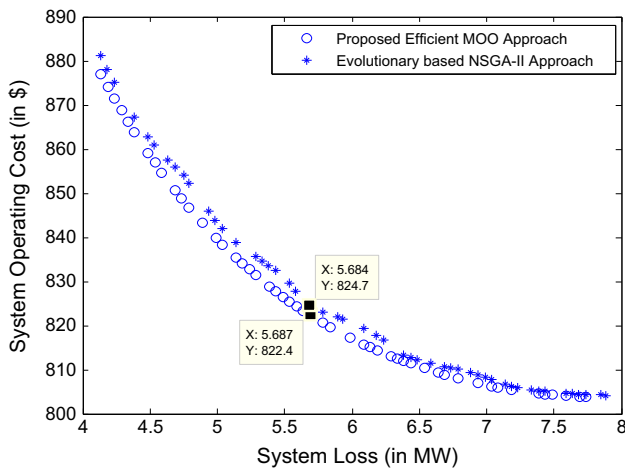


Fig. 1 Pareto optimal fronts for case 1 using proposed efficient MOO approach and NSGA-II algorithm

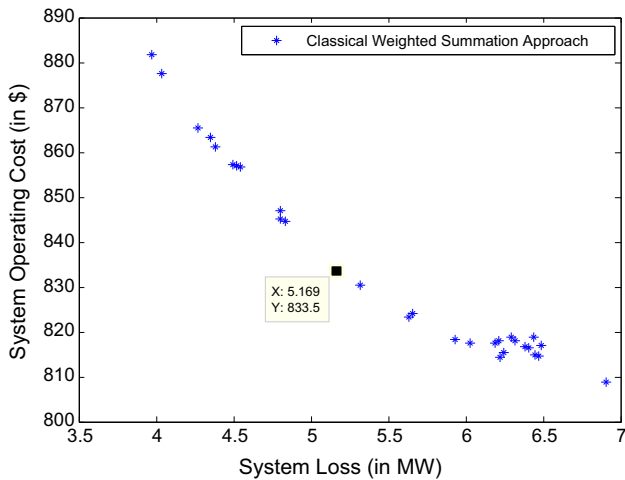


Fig. 2 Pareto optimal front for case 1 using the classical weighted summation approach

Fig. 1, it can be observed that the Pareto optimal solutions obtained with proposed efficient MOO approach are diverse and well distributed over the entire Pareto optimal front.

Whereas the evolutionary algorithms are population based, the reproduction operation causes new generations by recombination of old solutions. This enables finding several members of Pareto optimal set in a single run, instead of performing a series of separate runs. Figure 1 also shows the best Pareto optimal front obtained using the NSGA-II algorithm in a single run. However, as explained earlier the difficulty of this evolutionary-based NSGA-II algorithm is the excessive computational time.

Figure 2 depicts the Pareto optimal solutions obtained with the linear combination/weighted coefficients of generation cost and transmission loss minimization objective functions. To generate the trade-off surface by the linear combination of weighted summation approach, the DE algorithm is run

Table 1 Performance metrics for comparing the Pareto fronts of IEEE 30 bus system (case 1)

Performance metrics	Proposed efficient MOO approach	Evolutionary-based NSGA-II algorithm
Set coverage metric (C)	0.4632	0.0516
Extent metric (α)	166.1755	158.2591
Spacing metric (s)	1.4653	1.6082

for 50 times using various W_1 in the range between 0 and 1. From the Pareto optimal front obtained with this weighted summation approach, it is clear that by varying the weight factors in equal increments do not guarantee the uniformly distributed solutions in the Pareto optimal front. Further, all the non-dominated solutions cannot be obtained, and some of the solutions obtained are inferior.

The Pareto optimal front obtained with proposed efficient MOO approach has been compared with the best Pareto front obtained from the NSGA-II algorithm with respect to 3 performance metrics (i.e., set convergence, extent and spacing), and their values are given in Table 1. The more details about these performance metrics can be found in the Reference [41].

The second row in Table 1 presents the set coverage metrics (C) of proposed efficient MOO approach and evolutionary-based NSGA-II algorithm. The obtained C using the proposed efficient MOO is 0.4632, which shows that about 23 non-dominated Pareto optimal points obtained by the NSGA-II algorithm are dominated by the points on the Pareto optimal front obtained by the proposed efficient MOO approach. Whereas the C obtained using the NSGA-II is 0.0516, i.e., only two Pareto optimal points obtained by the proposed efficient MOO approach are dominated by the solutions on the Pareto optimal front obtained by the NSGA-II algorithm. This shows the convergence superiority of the proposed efficient approach over the NSGA-II approach. The obtained values of α and s using the proposed efficient MOO approach shows the better spread and spacing of Pareto optimal solutions as compared to the NSGA-II algorithm. This shows the diversity superiority of the proposed efficient approach over the NSGA-II algorithm.

After obtaining the Pareto optimal front, the best compromise solution can be found using the fuzzy min-max approach [32]. Table 2 presents the optimum control variable settings and the best compromise solution obtained by using the proposed efficient approach, evolutionary-based NSGA-II approach and the classical weighted summation approach. From this table, it can be observed that the minimum generation cost and transmission loss obtained by using the proposed efficient MOO approach are better than the other two approaches. The computational/execution time

Table 2 Control variables and the objective function values for case 1 using the proposed efficient MOO approach, evolutionary-based NSGA-II approach and the classical weighted summation approach

Control variables and objective function values	Case 1 using proposed efficient MOO approach	Case 1 using evolutionary-based NSGA-II approach	Case 1 using classical weighted summation approach
P_{G1} (MW)	130.24	128.87	115.51
P_{G2} (MW)	51.12	49.74	48.73
P_{G5} (MW)	29.85	28.80	37.69
P_{G8} (MW)	34.85	34.99	33.71
P_{G11} (MW)	23.64	28.39	29.48
P_{G13} (MW)	19.38	18.29	23.45
V_1 (p.u.)	1.1	1.0965	1.0917
V_2 (p.u.)	1.0912	1.0835	1.0794
V_5 (p.u.)	1.0671	1.0523	1.0576
V_8 (p.u.)	1.0718	1.0594	1.0729
V_{11} (p.u.)	1.0765	1.0870	1.0294
V_{13} (p.u.)	1.0959	1.0729	1.0929
$T_{6,9}$ (p.u.)	0.9625	0.95	0.95
$T_{6,10}$ (p.u.)	1.05	1.0375	1.05
$T_{4,12}$ (p.u.)	0.975	0.9625	1.0125
$T_{28,27}$ (p.u.)	1.0125	1.00	1.0125
$b_{sh,10}$ (p.u.)	0.05	0.05	0.05
$b_{sh,12}$ (p.u.)	0.04	0.02	0.01
$b_{sh,15}$ (p.u.)	0.02	0.04	0.02
$b_{sh,17}$ (p.u.)	0.05	0.02	0.05
$b_{sh,20}$ (p.u.)	0.04	0.02	0.01
$b_{sh,21}$ (p.u.)	0.05	0.03	0.02
$b_{sh,23}$ (p.u.)	0.02	0.05	0.00
$b_{sh,24}$ (p.u.)	0.01	0.05	0.02
$b_{sh,29}$ (p.u.)	0.05	0.05	0.02
Generation cost (\$/h)	822.43	824.72	833.53
System loss (MW)	5.687	5.684	5.169
Computational time (s)	17.0209	183.2058	69.8614

required for solving Case 1 using the proposed efficient MOO approach, NSGA-II approach and the classical weighted summation approach using the proposed efficient approach is 17.0209, 183.2058 and 69.8614s, respectively; i.e., the proposed efficient MOO approach is 10.76 times faster than the NSGA-II approach. Overall, it can be concluded that the proposed efficient MOO approach provides good quality Pareto optimal fronts as compared to evolutionary-based NSGA-II approach and the classical weighted summation approach in an extremely efficient manner.

4.1.2 Case 2: Solving the MO-OPF problem considering the cost minimization function with VPL and POZs effects

In this case also, the system operating cost minimization and loss minimization are considered as the two conflict-

ing objectives to be optimized. Figure 3 shows the Pareto optimal fronts obtained using the proposed efficient MOO approach and the NSGA-II algorithm for Case 2. From this figure, it can be observed that the Pareto optimal solutions obtained using the proposed efficient MOO approach are diverse and well distributed over the entire Pareto optimal front.

Figure 4 depicts the Pareto optimal front obtained using the classical weighted summation approach for Case 2. From this figure, it can be observed that by varying the weight factors in equal intervals do not guarantee the uniformly distributed solutions in the Pareto optimal front. As mentioned earlier, after obtaining the Pareto optimal fronts, the best compromise solution can be obtained using the fuzzy min-max approach. Table 3 shows the optimum control variables settings and the best compromise solution obtained by using

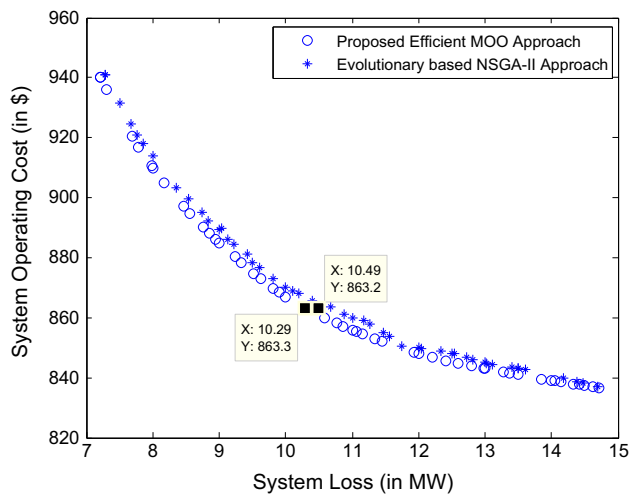


Fig. 3 Pareto optimal front for case 2 using the proposed efficient MOO Approach and the evolutionary-based NSGA-II approach

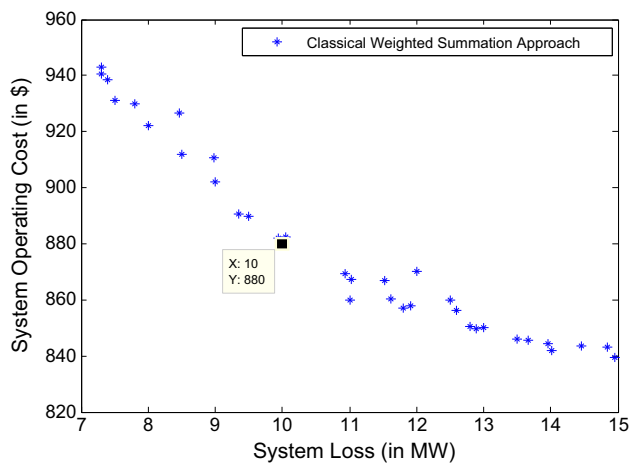


Fig. 4 Pareto optimal front for case 2 using the classical weighted summation approach

the proposed efficient MOO approach, NSGA-II algorithm and the classical weighted summation approach.

From Table 3, it can be observed that the best compromise solutions (i.e., system operating cost and transmission loss) obtained using the proposed efficient MOO approach, NSGA-II and classical weighted summation approach are (863.31 \$/h and 10.29 MW), (863.20 \$/h and 10.49 MW) and (880 \$/h and 10 MW), respectively. This shows that the best compromise solution obtained with the proposed efficient MOO approach is better than the other two approaches. The time required for the execution for solving Case 2 using the proposed efficient MOO approach, NSGA-II and weighted summation approach is 19.0856, 189.915 and 78.468 s, respectively; i.e., the proposed efficient MOO approach is approximately 10 times faster than the NSGA-II approach.

4.1.3 Case 3: Solving the MO-OPF problem with quadratic cost function considering the voltage-dependent load modeling

In this case study, the MO-OPF problem is solved considering the practical voltage-dependent load modeling. Figure 5 presents the Pareto optimal front obtained using the proposed efficient MOO algorithm and the NSGA-II algorithm. As mentioned earlier, the generation cost minimization objective considering the voltage-dependent load modeling and transmission loss constraint is executed for 25 times, and the transmission loss minimization objective considering the voltage-dependent load modeling and total cost as constraint is executed for 25 times. These 50 optimal points gives the complete Pareto optimal front. From Fig. 5, it can be observed that the Pareto optimal solution obtained with the proposed efficient MOO approach is diverse and well distributed over the entire Pareto optimal front.

The Pareto optimal front obtained using the proposed efficient MOO approach is also compared with the evolutionary-based NSGA-II algorithm. Figure 5 also depicts the Pareto optimal front of NSGA-II algorithm. However, the computational time required for the proposed efficient MOO is much lesser than the time required for the evolutionary-based NSGA-II algorithm.

After obtaining the Pareto optimal fronts, the best compromised solutions can be obtained using the fuzzy min-max approach. Table 4 presents the optimum control variables settings and objective function values for Case 3. The best compromise solution obtained with the proposed efficient MOO approach has the total generation cost of 801.2480 \$/h and the total transmission losses of 6.174 MW, whereas, by using, the evolutionary-based NSGA-II algorithm has the total generation cost of 812.6738 \$/h and the total transmission losses of 5.9362 MW.

From the above, it can be observed that the optimal Pareto front and the best compromise solution obtained using the proposed efficient MOO approach are almost same as NSGA-II approach with lesser computational time. From Table 4, it can be observed that the computational time required for the proposed efficient MOO approach is 17.5633 s and for NSGA-II approach is 184.1936 s, i.e., the proposed efficient MOO approach is 10.49 times faster than the evolutionary-based NSGA-II approach.

4.2 Simulation results on IEEE 300 bus system

IEEE 300 bus system [38] consists of 69 generators and 411 branches of which 62 branches are tap setting transformer branches and 12 buses have been selected as shunt compensation buses. The total load in the system is 23,246.86 MW. The case studies for IEEE 300 bus system are presented next:

Table 3 Control variables and the objective function values for case 2 using the proposed efficient MOO approach, evolutionary-based NSGA-II approach and the classical weighted summation approach

Control variables and objective function values	Case 2 using proposed efficient MOO approach	Case 2 using evolutionary-based NSGA-II approach	Case 2 using classical weighted summation approach
P_{G1} (MW)	123.35	126.19	121.11
P_{G2} (MW)	56.97	57.00	57.06
P_{G5} (MW)	37.54	38.25	38.89
P_{G8} (MW)	22.95	20.67	24.85
P_{G11} (MW)	29.57	29.62	29.47
P_{G13} (MW)	23.31	22.16	22.02
V_1 (p.u.)	1.0988	1.0965	1.0982
V_2 (p.u.)	1.0876	1.09	1.0871
V_5 (p.u.)	1.0588	1.0688	1.0658
V_8 (p.u.)	1.0812	1.0794	1.0753
V_{11} (p.u.)	1.0888	1.0553	1.0488
V_{13} (p.u.)	1.0929	1.0947	1.0982
$T_{6,9}$ (p.u.)	1.0125	1.1	1.025
$T_{6,10}$ (p.u.)	0.9875	0.975	0.9
$T_{4,12}$ (p.u.)	0.9875	1.0125	0.9875
$T_{28,27}$ (p.u.)	0.975	1.0	0.9875
$b_{sh,10}$ (p.u.)	0.05	0.05	0.05
$b_{sh,12}$ (p.u.)	0.02	0.05	0.02
$b_{sh,15}$ (p.u.)	0.02	0.03	0.05
$b_{sh,17}$ (p.u.)	0.02	0.05	0.02
$b_{sh,20}$ (p.u.)	0.04	0.04	0.05
$b_{sh,21}$ (p.u.)	0.01	0.01	0.05
$b_{sh,23}$ (p.u.)	0.03	0.00	0.03
$b_{sh,24}$ (p.u.)	0.00	0.00	0.02
$b_{sh,29}$ (p.u.)	0.05	0.02	0.01
Generation cost (\$/h)	863.31	863.20	880
System loss (MW)	10.29	10.49	10
Computational time (s)	19.0856	189.9152	78.4680

4.2.1 Case 1: Solving the MO-OPF problem with quadratic fuel cost function

Table 5 presents the optimum objective function values of MO-OPF problem considering the total generation cost and transmission losses as the objective functions to be solved simultaneously considering the constant load modeling (i.e., Case 1). The best compromise solution obtained for this case using the proposed efficient MOO approach has the generation cost of 857,442.6782 \$/h and the transmission losses of 678.2998 MW, whereas the solution obtained using the NSGA-II approach has the total generation cost of 857,623.0121 \$/h and the transmission losses of 681.4907 MW. From this table, it can be observed that both the techniques have almost same objective function values. But, the proposed efficient MOO approach is approximately 10.79 times faster than the evolutionary-based NSGA-II algorithm.

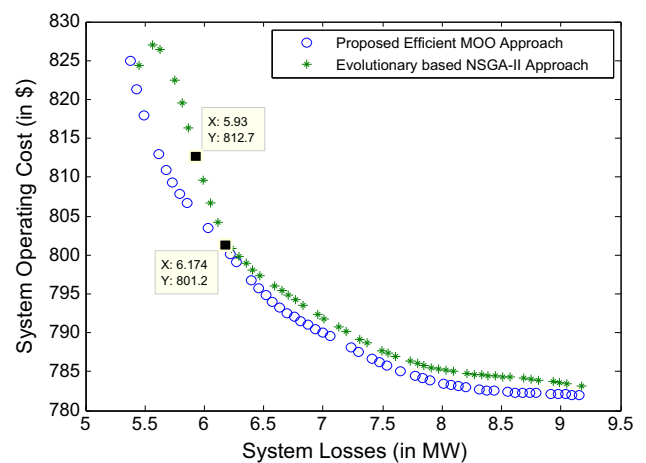


Fig. 5 Pareto optimal front for case 3 using the proposed efficient MOO approach and the evolutionary-based NSGA-II approach

Table 4 Control variables and the objective function values for Case 3 using the proposed efficient MOO approach and evolutionary-based NSGA-II approach

Control variables and objective function values	Case 3 using proposed efficient MOO approach	Case 3 using evolutionary-based NSGA-II approach
P_{G1} (MW)	148.4247	120.5049
P_{G2} (MW)	72.1520	53.5678
P_{G5} (MW)	20.9145	42.1325
P_{G8} (MW)	13.5165	30.2686
P_{G11} (MW)	16.5836	19.0794
P_{G13} (MW)	12.9983	12.7589
V_1 (p.u.)	1.0400	1.0529
V_2 (p.u.)	1.0400	1.0158
V_5 (p.u.)	1.0141	0.9665
V_8 (p.u.)	1.0100	0.9865
V_{11} (p.u.)	1.0082	0.9871
V_{13} (p.u.)	0.9529	0.9629
$T_{6,9}$ (p.u.)	1.0875	0.95
$T_{6,10}$ (p.u.)	1.0625	1.0750
$T_{4,12}$ (p.u.)	1.05	0.9625
$T_{28,27}$ (p.u.)	0.925	0.9875
$b_{sh,10}$ (p.u.)	0.02	0.02
$b_{sh,12}$ (p.u.)	0.03	0.00
$b_{sh,15}$ (p.u.)	0.04	0.04
$b_{sh,17}$ (p.u.)	0.01	0.02
$b_{sh,20}$ (p.u.)	0.04	0.05
$b_{sh,21}$ (p.u.)	0.05	0.04
$b_{sh,23}$ (p.u.)	0.02	0.02
$b_{sh,24}$ (p.u.)	0.04	0.01
$b_{sh,29}$ (p.u.)	0.02	0.03
Generation cost (\$/h)	801.2480	812.6738
System loss (MW)	6.174	5.9362
Total load demand (MW)	278.3546	272.3847
Computational time (s)	17.5633	184.936

Table 5 Optimum objective function values for Case 1 using the proposed efficient MOO and evolutionary-based NSGA-II approaches for IEEE 300 bus system

Objective function values	Case 1 using proposed efficient MOO approach	Case 1 using evolutionary-based NSGA-II approach
Generation cost (\$/h)	857,42.6782	857,23.0121
System loss (MW)	678.2998	681.4907
Total load demand (MW)	23,46.86	23,46.86
Computational time (min)	4.7504	51.2568

Bold values represent the optimum objective function values and corresponding computational time

Table 6 Optimum objective function values for Case 2 using the proposed efficient MOO and evolutionary-based NSGA-II approaches for IEEE 300 bus system

Objective function values	Case 2 using proposed efficient MOO approach	Case 2 using evolutionary-based NSGA-II approach
Generation cost (\$/h)	809,27.9451	801,63.0894
System loss (MW)	628.3249	630.8413
Total load demand (MW)	22,98.31	22,67.09
Computational time (min)	4.8076	52.0691

Bold values represent the optimum objective function values and corresponding computational time

4.2.2 Case 2: Solving the MO-OPF problem with quadratic cost function considering the voltage-dependent load modeling

Table 6 presents the optimum objective function values of MO-OPF problem considering the total generation cost and transmission losses as the objective functions to be solved simultaneously considering the practical voltage load modeling (i.e., Case 2). The best compromise solution obtained for this case using the proposed efficient MOO approach has the generation cost of 809,027.9451 \$/h and the transmission losses of 628.3249 MW, whereas the solution obtained using the NSGA-II approach has the total generation cost of 801,163.0894 \$/h and the transmission losses of 630.8413 MW. From this table, it can be observed that both the techniques have almost same objective function values. But, the proposed efficient MOO approach is approximately 10.83 times faster than the evolutionary-based NSGA-II algorithm.

From the above simulation studies, it can be observed that the proposed efficient MOO approach overcomes the main drawback, i.e., excessive execution time of the evolutionary-based MOO algorithms. The proposed efficient MOO approach is approximately 10 times faster than the evolutionary-based MOO approaches.

5 Conclusions

This paper proposes a new efficient multi-objective optimization algorithm to solve the MO-OPF problem. It uses the incremental power flow model and sensitivities; and the lower and upper bounds of the objective function values. In this paper, the MOO problem is solved considering the system operating cost, transmission losses and L-index as the objective functions while satisfying all the equality and inequality constraints. In this paper, the proposed efficient

MOO approach is implemented using the DE algorithm. The simulation studies are performed on IEEE 30 and IEEE 300 bus test systems to demonstrate the effectiveness of the proposed efficient MOO approach. The simulation results shows that the Pareto optimal solutions obtained with proposed efficient MOO approach are diverse and well distributed over the entire Pareto optimal front. All the simulation studies indicate that the proposed efficient MOO approach is approximately 10 times faster than the evolutionary-based MOO algorithms. Extending the proposed efficient MO-OPF approach to power system with renewable energy sources considering the security is a scope for future research.

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Appendix 1: Differential evolution algorithm (DEA)

DEA is an evolutionary computation technique developed by Storn and Price in 1995. The developed DEA is reliable and versatile function optimizer that is readily applicable to a wide range of optimization problems. DE is a simple population-based, stochastic search meta-heuristic optimization algorithm which is capable of handling nonlinear, non-differentiable and multi-model objective functions. DEA improves a population of candidate solution over several iterations using mutation, crossover and selection operators in order to reach an optimal solution. DE uses a population of floating point encoded individuals and mutation, crossover and selection operators to explore the solution space in search of global optima. DE is a novel evolution algorithm as it employs real-coded variables, instead of a binary or a gray representation. DE typically relies on mutation as the search operator and uses selection to direct the search toward the prospective regions in the feasible region [42]. For more details of DEA, the reader can refer Reference [39].

Appendix 2: Non-dominated sorting genetic algorithm-2 (NSGA-II)

In the MOO, the Pareto optimal solution (i.e., the non-dominated solution) improves at least one objective function without degrading the other objective functions. Using the MOO algorithm, Pareto optimal set/front, i.e., a set of non-dominated solutions, can be determined. In this paper, NSGA-II algorithm is used to find the Pareto optimal front. More details of NSGA-II can be found in the References [43,44]. After determining the Pareto optimal front, the fuzzy

min-max method is used to determine the best compromise solution.

References

- Rosehart WD, Canizares CA, Quintana VH (2003) Multiobjective optimal power flows to evaluate voltage security costs in power networks. *IEEE Trans Power Syst* 18(2):578–587
- Arya LD, Choube SC, Kothari DP (1997) Emission constrained secure economic dispatch. *Int J Electr Power Energy Syst* 19(4):279–285
- Abido MA (2003) A novel multiobjective evolutionary algorithm for environmental/economic power dispatch. *Electr Power Syst Res* 65(1):71–81
- Yokoyama R, Bae SH, Morita T, Sasaki H (1988) Multiobjective optimal generation dispatch based on probability security criteria. *IEEE Trans Power Syst* 3(1):317–324
- Abido MA, Al-Ali NA (2012) Multi-objective optimal power flow using differential evolution. *Arab J Sci Eng* 37(4):991–1005
- Kahourzade S, Mahmoudi A, Mokhlis HB (2015) A comparative study of multi-objective optimal power flow based on particle swarm, evolutionary programming, and genetic algorithm. *Electr Eng* 97(1):1–12
- Ghasemi M, Ghavidel S, Ghanbarian MM, Gharibzadeh M, Vahed AA (2014) Multi-objective optimal power flow considering the cost, emission, voltage deviation and power losses using multi-objective modified imperialist competitive algorithm. *Energy* 78:276–289
- Shaheen AM, El-Sehiemy RA, Farrag SM (2016) Solving multi-objective optimal power flow problem via forced initialized differential evolution algorithm. *IET Gener Transm Distrib* 10(7):1634–1647
- Daryani N, Hagh MT, Teimourzadeh S (2016) Adaptive group search optimization algorithm for multi-objective optimal power flow problem. *Appl Soft Comput* 38:1012–1024
- Bhowmik AR, Chakraborty AK (2015) Solution of optimal power flow using non dominated sorting multi objective opposition based gravitational search algorithm. *Int J Electr Power Energy Syst* 64:1237–1250
- Singh S, Verma KS (2015) “Artificial intelligence techniques for multi objective optimum power flow with valve point loading incorporating SVC”. *Int J Recent Dev Eng Technol* 4(6):41–48
- Hazra J, Sinha AK (2011) “A multi-objective optimal power flow using particle swarm optimization”. *Eur Trans Electr Power* 21(1):1028–1045
- He X, Wang W, Jiang J, Xu L (2015) An improved artificial bee colony algorithm and its application to multi-objective optimal power flow. *Energies* 8:2412–2437
- Salhi A, Naimi D, Bouktir T (2013) Fuzzy multi-objective optimal power flow using genetic algorithms applied to algerian electrical network. *Power Eng Electr Eng* 11(6):443–454
- El-Fergany AA, Hasanien HM (2015) Single and multi-objective optimal power flow using grey wolf optimizer and differential evolution algorithms. *Electr Power Compon Syst* 43(13):1548–1559
- Mahdad B, Srairi K (2013) A study on multi-objective optimal power flow under contingency using differential evolution. *J Electr Eng Technol* 8(1):53–63
- Abido MA, Ahmed MW (2016) Multi-objective optimal power flow considering the system transient stability. *IET Gener Transm Distrib* 10(16):4213–4221
- Medina MA, Das S, Coello CAC, Ramírez JM (2014) Decomposition-based modern metaheuristic algorithms for multi-

- objective optimal power flow—a comparative study. *Eng Appl Artif Intell* 32:10–20
19. Roy PK, Ghoshal SP, Thakur SS (2010) Biogeography based optimization for multi-constraint optimal power flow with emission and non-smooth cost function. *Expert Syst Appl* 37(12):8221–8228
 20. Shaheen AM, El-Sehiemy RA, Farrag SM (2016) Solving multi-objective optimal power flow problem via forced initialised differential evolution algorithm. *IET Gener Transm Distrib* 10(7):1634–1647
 21. Man-Im A, Ongsakul W, Singh JG, Boonchuay C (2015) “Multi-objective optimal power flow using stochastic weight trade-off chaotic NSPSO”. In: *IEEE innovative smart grid technologies—Asia, Bangkok*, 1–8
 22. Priyanto YTK, Hendarwin L (2015) “Multi objective optimal power flow to minimize losses and carbon emission using Wolf Algorithm”. In: *International seminar on intelligent technology and its applications, Surabaya*, 153–158
 23. Ye CJ, Huang MX (2015) Multi-objective optimal power flow considering transient stability based on parallel NSGA-II. *IEEE Trans Power Syst* 30(2):857–866
 24. Bhowmik AR, Chakraborty AK, Babu KN (2014) “Multi objective optimal power flow using NSMOGSA”. In: *International conference on circuits, power and computing technologies, Nagercoil*, 84–88
 25. Ren P, Li N (2014) “Multi-objective optimal power flow solution based on differential harmony search algorithm”. In: *10th International conference on natural computation, Xiamen*, 326–329
 26. Oesterle J, Amodeo L (2014) Efficient multi-objective optimization method for the mixed-model-line assembly line design problem. *Procedia CIRP* 17:82–87
 27. Liu GP, Han X, Jiang C (2012) An efficient multi-objective optimization approach based on the micro genetic algorithm and its application. *Int J Mech Mater Des* 8(1):37–49
 28. Mortazavi-Naeini M, Kuczera G, Cui L (2015) Efficient multi-objective optimization methods for computationally intensive urban water resources models. *J Hydroinform* 17(1):36–55
 29. Seo JW, Afzal A, Kim KY (2016) Efficient multi-objective optimization of a boot-shaped rib in a cooling channel. *Int J Therm Sci* 106:122–133
 30. Lei H, Wang R, Zhang T, Liu Y, Zha Y (2016) A multi-objective co-evolutionary algorithm for energy-efficient scheduling on a green data center. *Comput Oper Res* 75:103–117
 31. Sailaja Kumari M, Maheswarapu S (2010) Enhanced genetic algorithm based computation technique for multi-objective optimal power flow solution. *Int J Electr Power Energy Syst* 32(6):736–742
 32. Shaw B, Mukherjee V, Ghoshal SP (2014) Solution of reactive power dispatch of power systems by an opposition-based gravitational search algorithm. *Int J Electr Power Syst* 55:29–40
 33. Surender Reddy S, Abhyankar AR, Bijwe PR (2011) Reactive power price clearing using multi-objective optimization. *Energy* 36(5):3579–3589
 34. Surender Reddy S, Abhyankar AR, Bijwe PR (2011) Multi-objective day-ahead real power market clearing with voltage dependent load models. *Int J Emerg Electr Power Syst* 12(4):1–22
 35. Abido MA (2003) Environmental/economic power dispatch using multiobjective evolutionary algorithms. *IEEE Trans Power Syst* 18(4):1529–1537
 36. Surender Reddy S, Bijwe PR, Abhyankar AR (2014) Faster evolutionary algorithm based optimal power flow using incremental variables. *Int J Electr Power Energy Syst* 54:198–210
 37. Surender Reddy S, Bijwe PR (2016) Efficiency improvements in meta-heuristic algorithms to solve the optimal power flow problem. *Int J Electr Power Energy Syst* 82:288–302
 38. *IEEE tutorial course on optimal power flow: solution techniques, requirements and challenges* (1996)
 39. [Online]. Available: <http://www.ee.washington.edu/research/pstca>
 40. Abou El Ela AA, Abido MA, Spea SR (2010) Optimal power flow using differential evolution algorithm. *Electr Power Syst Res* 80(7):878–885
 41. Varadarajan M, Swarup KS (2008) Solving multi-objective optimal power flow using differential evolution. *IET Gener Transm Distrib* 2(5):720–730
 42. Shrivastava A, Siddiqui HM (2014) A simulation analysis of optimal power flow using differential evolution algorithm for IEEE-30 bus system. *Int J Recent Dev Eng Technol* 2(3):50–57
 43. Deb K, Pratap A, Agarwal A, Meyarivan T (2002) A fast and elitist multi-objective genetic algorithm: NSGA II. *IEEE Trans Evol Comput* 6(2):182–197
 44. Shukla PK, Deb K (2007) On finding multiple Pareto-optimal solutions using classical and evolutionary generating methods. *Eur J Oper Res* 181:1630–1652