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ORIGINAL PAPER

Electromechanical modeling of a contactor with dc coil

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Abstract The purpose of this article is to model a contactor with dc coil using simple electromechanical equations. These models do not need a detailed description of internal characteristics of the contactor, and the parameters are estimated from external measurements (current in main coil, and operation time of contacts). Four models are formulated to obtain the operation time of contacts and to describe the current in contactor coil. Two ways for modeling the contactor have a constant equivalent area for airgap (with and without considering friction). The other two ways for modeling the contactor have a variable equivalent area for airgap; these two ways only have different parameters for models. The results of these models are compared with experimental measurements. Although the proposed models include several simplifications, these models are very accurate for obtaining the operation time of contacts for eight different values of the dc source voltage. The three transient stages of the current in contactor coil are obtained with these models, with good accuracy for the time of occurrence of the local minimum of the current. These facts are important, because they imply a good representation of the behavior of the contactor. The obtaining of the three transient stages of current in contactor coil, using the electromechanical models, had not been shown in the literature. This article shows the feasibility of obtaining such computed results, which are in agreement with experimental measurements.

Keyword Contactor with dc coil

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1 Introduction

The basic electromechanical equations for the description of the transient behavior of contactors with dc coil can be found in some textbooks (e.g., [1]). However, some simulation details for contactors are only described in specialized literature, for example: (a) the non-linear effect due to the limits for armature displacement [2,3], (b) the discontinuities in the armature path, due to the changes in the springs and masses in motion [4,5], and (c) the effects of small airgaps between armature and core when the contactor is closed [5].

On the other hand, the application of correction factors for the equivalent lengths and the equivalent areas of the magnetic field paths is usual in the analysis of magnetic circuits of transformers [6,7] and inductors [8,9]. Although the application of similar factors is not usual for contactors, there are examples of their use for contactors with ac coil [10,11]. A correction factor for the equivalent area of the airgap is used in this article and its effect is analyzed.

The models based on the estimation of magnetic fields in each point of the contactor geometry (using the finite-element method or other methods) require a very detailed description of the internal parts of the contactor (geometry and characteristics of the materials). In contrast, the models of this article require an estimation of their parameters, and such estimation is based only on the external measurements: current in the main coil and operation time of contacts. The electromechanical models and the models based on the calculation of magnetic fields usually have different applications for the analysis of electrical devices, and both are useful within their own scopes.

The transient current in the contactor coil has three stages. Some research articles [12–19] show currents similar to the results obtained for this article, but the finite-element method is always necessary for their models. The purpose of this article is to model a contactor with dc coil, using lumped parameters (i.e., without the detailed calculation of magnetic fields). Therefore, the contactor is modeled here without considering its specific geometry but only a simplified schematic representation. The obtaining of the three transient stages of the current in the contactor coil, using the electromechanical models, had not been shown in the previous literature. This article shows the feasibility of obtaining such computed results, which are in agreement with the experimental measurements. In addition, computed results for the operation time of contacts are accurate, and they are also obtained here using these electromechanical models.

The electromechanical models with lumped parameters might be useful as simple methods for analyzing the contactor when the applied voltage is varying in time, for example: (a) due to events in the electrical system, as short circuits or starts of large motors; and (b) due to the search of different ways for energizing the contactor coil, using electronic control [20,21]. On the other hand, these models might be necessary for some users of contactors, to avoid the search of the internal characteristics of the devices, because this could imply an undesired disassembling of its internal pieces.

Four ways for modeling the contactor were developed in this article: two ways have a constant equivalent area for the airgap, and the other two ways have a variable equivalent area for the airgap. The main contribution of this article is to show that the simple electromechanical models can be applied for obtaining the contactor operation time and the transient behavior of the coil currents when the device is tested with different dc source voltage values.

An article about electromechanical modeling of a contactor with ac coil, using lumped parameters, was recently published [22]. Both cases (ac and dc coils) are based on simple electromechanical models, and the computed results for the contactor operation time are accurate in both cases. In case of ac coil [22], the model is conceptually more complex, due to the presence of shading rings. In case of dc coil (this article), the obtaining of the correct behavior of the coil current was more laborious (and the search of model parameters required many more attempts).

2 Basic simplifications

The following simplifications were used for the model:

(a) Only an equivalent spring was considered, whose force varies linearly with the armature movement.
(b) The contactors usually have different springs, related to the armature and the auxiliary contacts. The effect of these different springs was not considered.
(c) The effect of the small collisions, produced in the core during the armature travel, was not considered.
(d) Changes in the masses in motion, which could occur during the armature travel, were not considered.
(e) An equivalent mass was considered that relates the accelerating force with the armature acceleration.
(f) Only a linear variation of the friction force with respect to the armature velocity was considered.
(g) The non-linear characteristic of the ferromagnetic core of the contactor was not considered.
(h) The presence of the iron remanent flux at the energizing instant was not considered.
(i) Correction factors for the variables in the magnetic paths were used only in the equivalent area of the airgap.

This set of simplifications is for obtaining a simple model, without considering the specific details of the tested device. For example, the changes in the springs and the masses in motion [4,5] may imply that physically there is not any equivalent spring, or equivalent mass, but these simplifications are necessary to achieve a simple model.

3 Electromechanical models

Figure 1 shows a simplified drawing of a contactor with dc coil. x is the armature position, $x = 0$ at the position limit when the coil is de-energized, and $x = d$ at a theoretical position equal to the gap length. There is other position limit at the end of the motion and it occurs at a position (x) which is less than d; however, for simplicity, the model considers the that end of the motion is at a position equal to d.

The magnetic force was assumed to be proportional to the square of the current and to the derivative of the inductance with respect to the position. However, some authors recognize that the algebraic expression for the magnetic force is

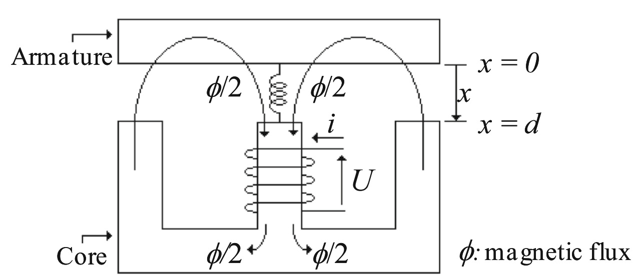

Fig. 1 Schematic representation of a contactor with dc coil

not simple [2,13], and the models based on finite-element method compute such force from the spatial distribution of the magnetic field [12–19]. The armature weight was considered, because the contactor was tested and modeled, as it is shown in Fig. 1 (x is in vertical direction).

The equations for describing the contactor dynamics are:

$$(i^2\,\mathrm{d}L/\mathrm{d}x)/2 + m\,g - K\,(x + x_0) - Bv = m\mathrm{d}v/\mathrm{d}t \tag{1}$$

$$U = R\,i + L\,\mathrm{d}i/\mathrm{d}t + i\,(\mathrm{d}L/\mathrm{d}x)\,v \tag{2}$$

$$v = \mathrm{d}x/\mathrm{d}t \tag{3}$$

$$L = N^2/(\Re_{\mathrm{FE}} + \Re_G) \tag{4}$$

where i: Electric current in the coil, L: Equivalent inductance of the coil, m: Equivalent mass of the mobile part of the contactor, g: Gravity acceleration, K: Equivalent spring constant, x_0: Equivalent spring position without elongation, B: Friction constant, v: Armature velocity, U: Voltage applied to the coil, R: Coil resistance, N: Number of turns of the coil, $\Re_{\mathrm{FE}}$: Iron equivalent reluctance, $\Re_G$: Gap equivalent reluctance.

The non-linear nature of this set of equations is due to the constraints given by the armature position limits ($0 \le x \le d$).

3.1 Model with constant equivalent area for the airgap

If the equivalent area of the airgap is considered to be constant and equal to the iron cross section, then the inductance is:

$$L = K_1/\{K_2 + (d - x)\} \tag{5}$$

$$K_1 = N^2\mu_0 A_{\mathrm{FE}} \tag{6}$$

$$K_2 = \Re_{FE}\mu_0 A_{\mathrm{FE}}. \tag{7}$$

A_{FE}: Iron equivalent cross section.
μ_0: Gap magnetic permeability.

3.2 Model with variable equivalent area for the airgap

The function $F(x)$ is the ratio between the equivalent gap area (A_G) and the equivalent iron area ($A_G = FA_{\mathrm{FE}}$). This function F is as follows:

$$F = 1 + (a - 1)(1 - x/d)^n. \tag{8}$$

where a: Value of $F(x)$ for $x = 0$, n: Exponent for the function $F(x)$.

With this function, the expression for the inductance is as follows:

$$L = K_1 F/\{K_2 F + (d - x)\}. \tag{9}$$

3.3 Implementation of these models

Equations (1–3) are a set of differential equations, where L can be obtained by Eqs. (5) or (9), as a function of x. These equations were programmed and solved using the Euler's numerical algorithm for differential equations. A very low integration step was selected (10^{-6} s), to avoid numerical errors. Other options for solving these differential equations are easily available (in software packages and/or in the literature). For these models, the electrical input is a step of dc voltage (U), the electrical output is the transient behavior of the current in the coil (i), and the mechanical outputs are the transient behaviors of armature position and armature speed (x, v). The search of the parameters for each model was performed to minimize the error of the model in comparison with the experimental results. Errors are simply computed by summing the absolute value of the difference between computed and measured results. The search of the parameters was performed with the help of two optimization tools ("solver" of Excel, and "fminsearch" of MATLAB). There are other optimization tools which can be applied in future research about this topic.

4 Results

A contactor with a 60 V dc coil was tested. The source voltage value for the tests was varied between 35 and 70 V, in steps of 5 V. Only the gap length and the coil resistance were assumed as known parameters ($d = 0.006$ m, $R = 476.5\,\Omega$). The graphical results show the measured current (α) and the current obtained through simulations (β).

The search of the parameters for the models was done to minimize the error for the time of occurrence of the local minimum of the current (main goal). On the other hand, the general minimization of the error for the evolution of the current was the second goal of this search procedure. This problem could be formulated as a multi-objective optimization problem, for which the solutions typically imply a compromise between the considered goals [23]. For this article, the parameters were found with the help of optimization tools by giving priority to the main goal (i.e., the second goal was searched after a satisfactory solution for the main goal was obtained, without allowing high increases of the main goal). Due to the non-linear nature of this problem, this procedure required many attempts of searching using the optimization tools. There were also attempts for simply using the minimization of the error for the evolution of the current as main goal; however, these attempts were not successful (some details about this topic are described at the last paragraph of Sect. 4.3).

4.1 Model with constant area for the equivalent airgap

Figures 2 and 3 show the results using the model with constant area, without and with considering the friction, respectively. Table 1 shows the values for the parameters of these models, in the International System of Units (SI).

The evolution of the current has three stages: in the first one, the current is increasing until its first local maximum; in the second one, the current is decreasing until its local minimum; and in the third one, the current is increasing until its steady value (these stages have been labeled for the case of 50 V in Fig. 2). The value of the local minimum, between the second stage and the third stage, occurs at the end of the armature movement (due to the action of the mechanical limit for the movement).

The closing of the normally open contacts is produced before the end of the movement, at $x = b$ $(b < d)$. A value for b was computed with each model, after the estimation of the model parameters. Each b value was calculated to minimize the average error in the closing time of the contacts. Using this procedure, the obtained values for b were 5.48 and 5.16 mm (without and with friction, respectively). The closing times of the normally open contacts are shown in Table 2. The results of Table 2 indicate that both models are accurate to predict the closing times, because the average errors are in the order of 1 ms.

4.2 Model with variable area for the equivalent airgap

Figures 4 and 5 show the results using the model with variable area, with two different sets of estimated parameters for the model (cases A and B). Table 3 shows the values for the parameters for both the cases, expressed in the International System of Units (SI).

Similar to the model with constant area, the optimal values for b were computed. The obtained results for b were 4.91 and

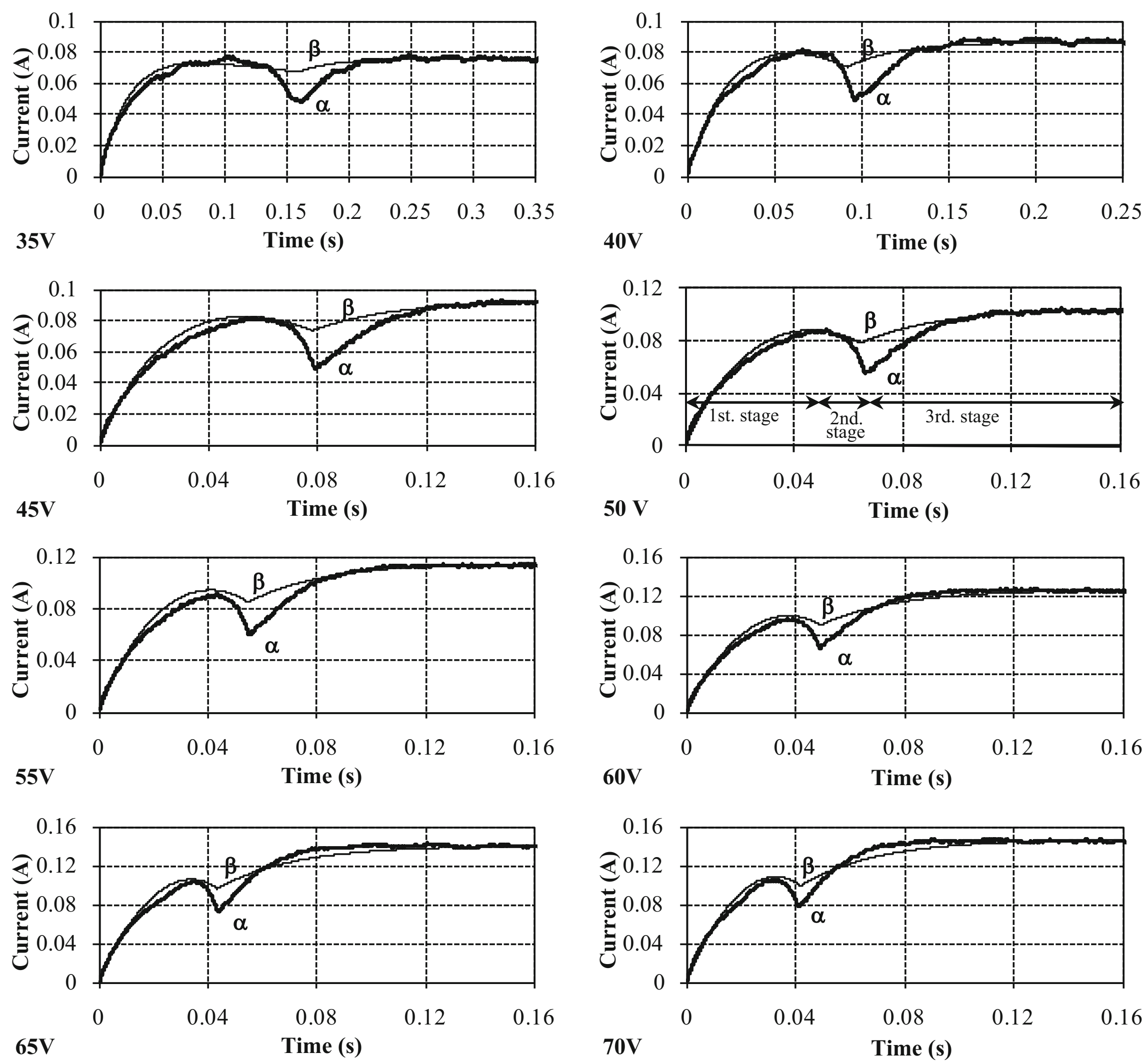

Fig. 2 Measured current (α) and result of the simulation (β), varying the source voltage. Model with constant area for the airgap, without friction

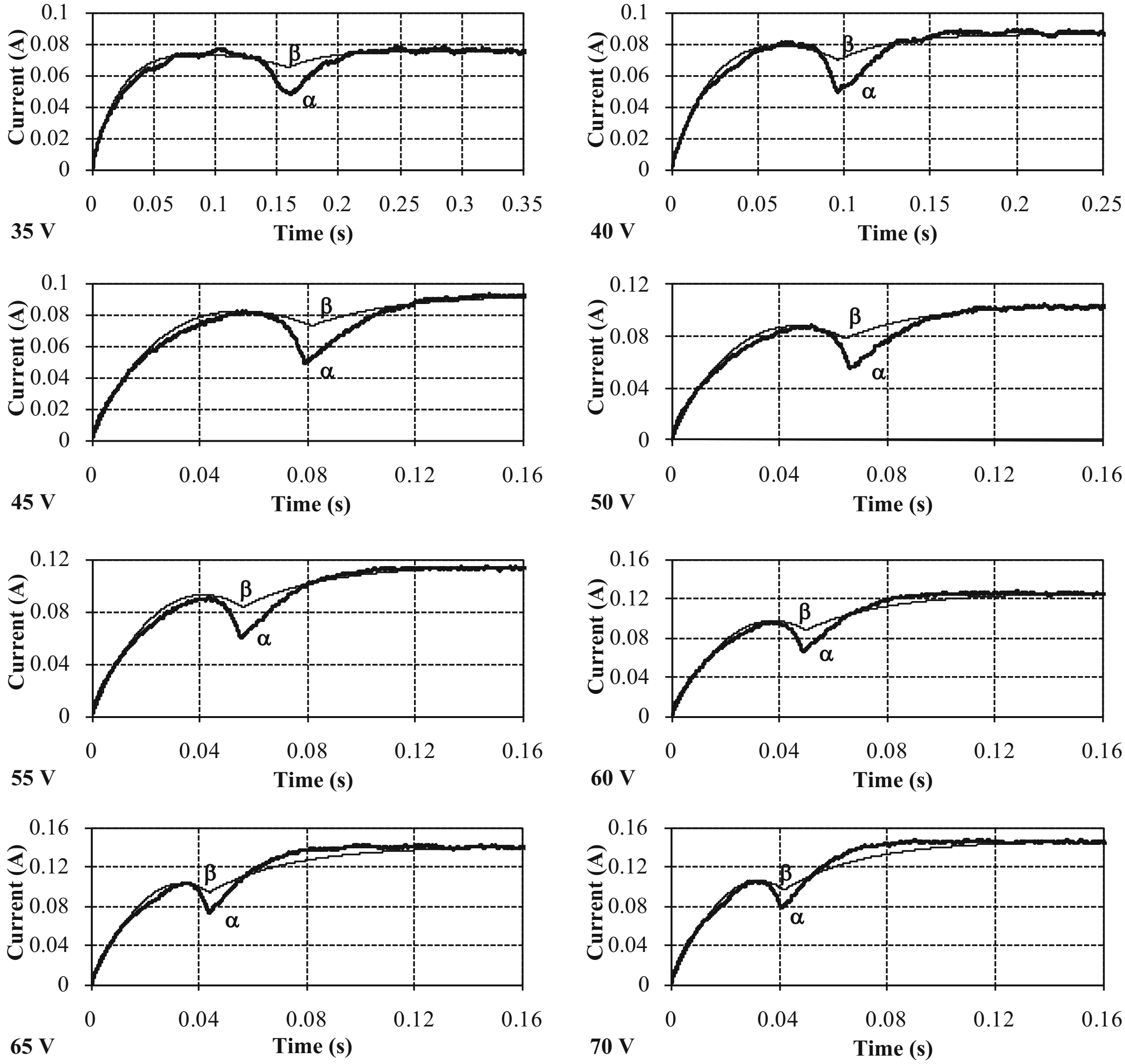

Fig. 3 Measured current (α) and result of the simulation (β), varying the source voltage. Model with constant area for the airgap, with friction

Table 1 Parameters for the model with constant area (SI units)

Parameters	Without friction	With friction
x_0	0.01277	0.03433
K_1	0.1993	0.2001
K_2	0.008888	0.008000
K	140	50
M	0.07	0.05
B	0	3

Table 2 Closing time (in ms) of the contacts. Model with constant area

Voltage (V)	Measured time	Results without friction		Results with friction	
		Time	Error	Time	Error
35	153.00	152.96	0.04	152.64	0.36
40	90.84	89.60	1.24	92.64	1.80
45	74.74	72.96	1.78	74.72	0.02
50	64.78	61.12	3.66	61.92	2.86
55	52.28	53.52	1.24	53.68	1.40
60	46.02	48.08	2.06	47.92	1.90
65	42.10	42.56	0.46	42.08	0.02
70	39.66	40.64	0.98	40.16	0.50

5.11 mm (cases A and B, respectively). The closing times of the normally open contacts are shown in Table 4. The results of Table 4 indicate that both models are accurate to predict the closing times, because the average errors are in the order of 1 ms.

The parameters for case A were selected to obtain a good approximation of the measured current at the first stage of its transient period (when the current is increasing, until its first local maximum). The parameters for case B were selected to obtain a good approximation of the measured current at the

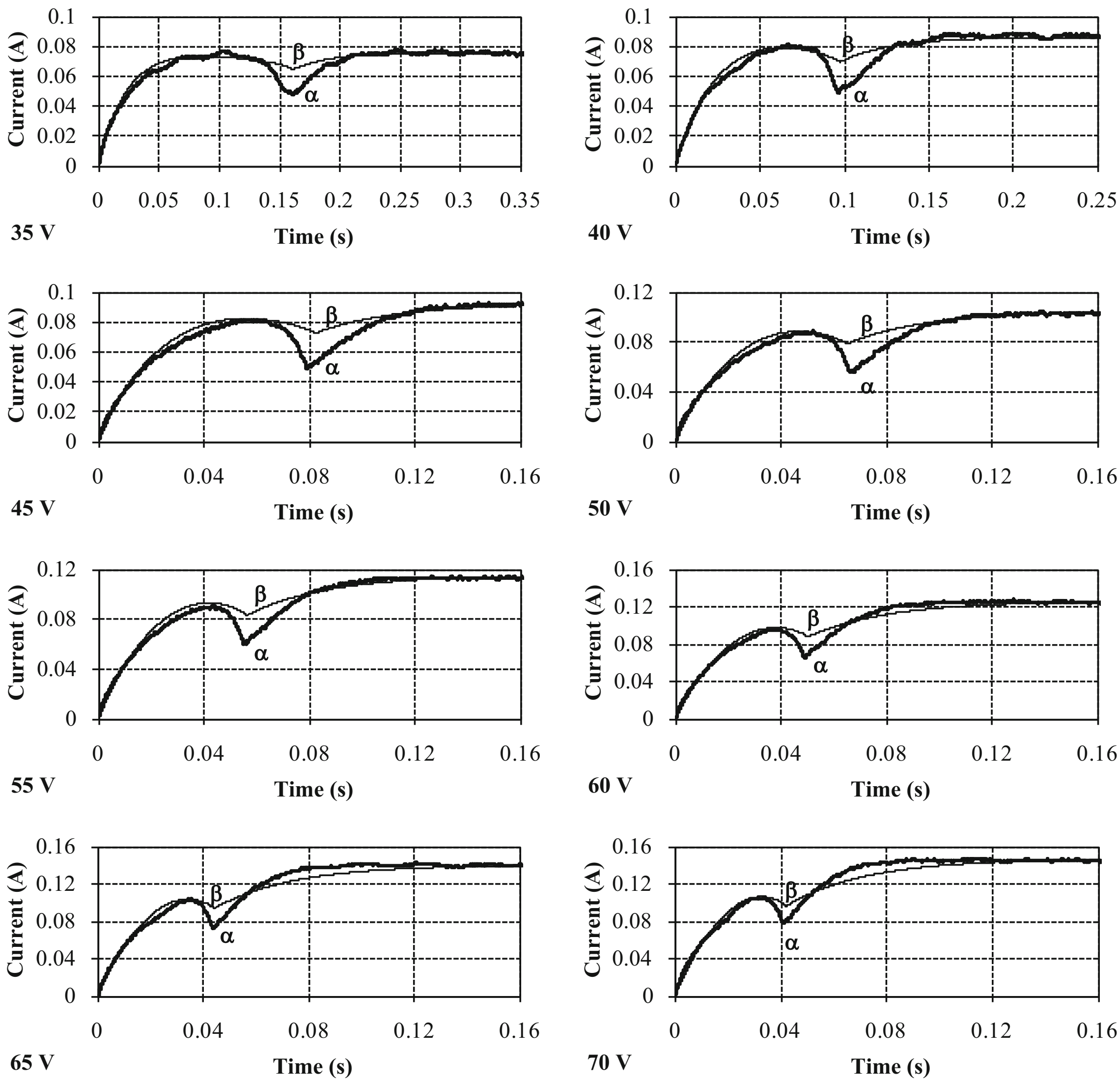

Fig. 4 Measured current (α) and result of the simulation (β), varying the source voltage. Model with variable area for the airgap (case A)

region adjacent to the local minimum (the region between the second stage and the third stage).

4.3 Analysis of the results

(a) The four ways used for modeling the contactor have a good precision for the time of occurrence of the local minimum of the current. This was the main goal for the search of the parameters of the models, because the errors on these values have the greatest influence in the qualitative comparison between the computed results and the experimental results. Consequently, the four ways used for modeling the contactor also have a good precision for the closing times of the normally open contacts.

(b) Experimental evaluation with a wide range of the dc source voltage was important to search the adequate parameters for the models. This allows a good certainty about the validity of the results.

(c) The two ways for modeling the contactor with constant gap area (with and without including the friction force in the simulation) had similar graphical results. The inclusion of the friction force effect implies the inclusion of an additional independent parameter (B), but this did not imply an important improvement in the graphical results of the model.

(d) For the graph zone near to the occurrence of the local minimum of the current, the case B of the model with variable gap area has the best results in comparison with the measured values. However, this case has the greatest errors for the first transient stage of the current (the increasing stage before the first local maximum). The simultaneous achievement of the lowest errors at both

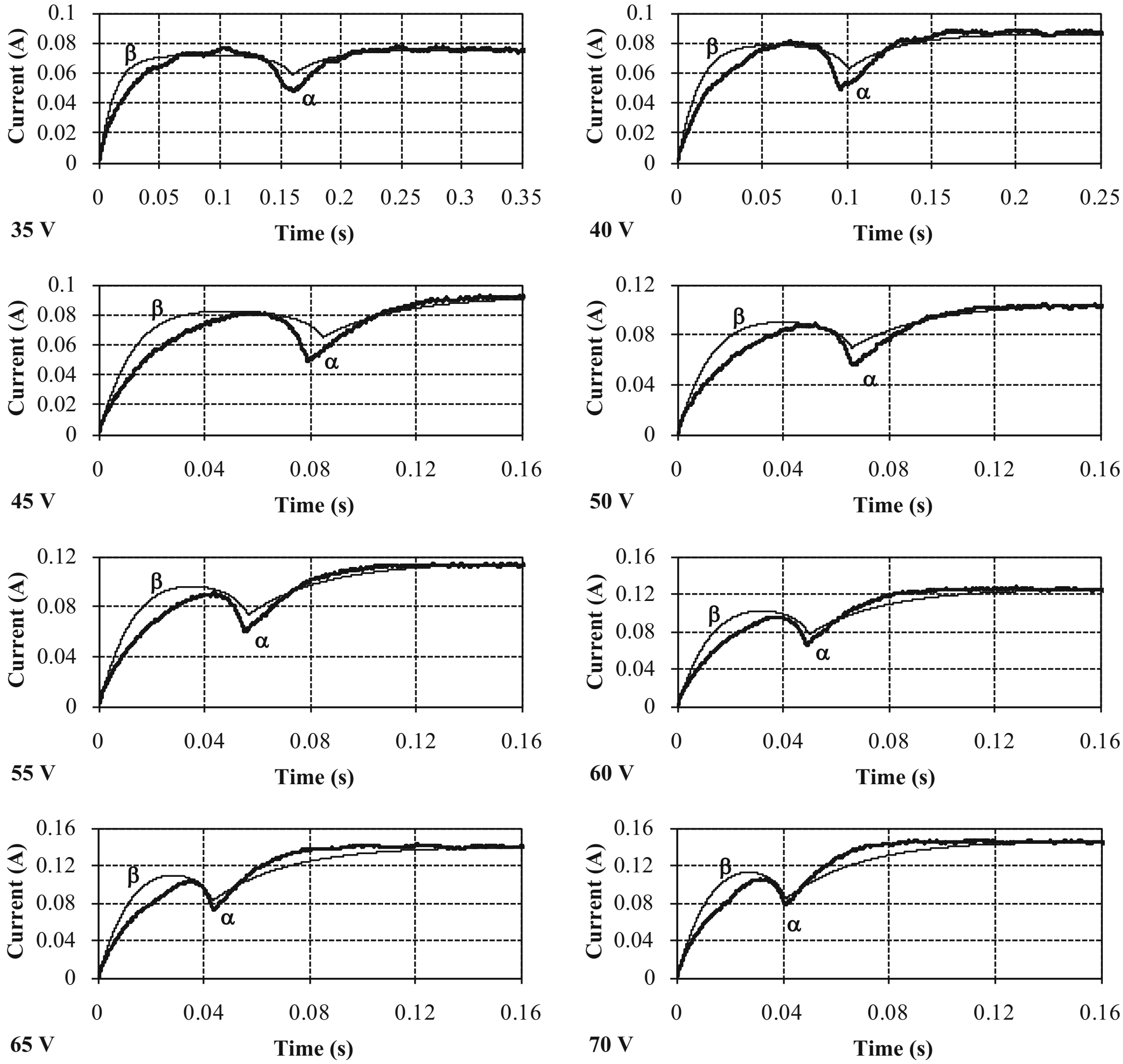

Fig. 5 Measured current (α) and result of the simulation (β), varying the source voltage. Model with variable area for the airgap (case B)

Table 3 Parameters for the model with variable area (SI units)

Parameters	Case A	Case B
n	0.1	0.7
a	1.500	1.424
x_0	0.027924	0.001205
K_1	0.13342	0.05150
K_2	0.009333	0.003792
K	60	630
m	0.05	0.07
B	3	13.5

Table 4 Closing time (in ms) of the contacts. Model with variable area

Voltage (V)	Measured time	Case A		Case B	
		Time	Error	Time	Error
35	153.00	152.96	0.04	149.76	3.24
40	90.84	92.16	1.32	94.08	3.24
45	74.74	74.32	0.42	75.28	0.54
50	64.78	61.52	3.26	61.52	3.26
55	52.28	53.44	1.16	52.80	0.52
60	46.02	47.68	1.66	46.72	0.70
65	42.10	41.92	0.18	40.72	1.38
70	39.66	39.92	0.26	38.72	0.94

stages of the transient behavior of the current was not possible in this work.

(e) For the graph zone near to the occurrence of the local minimum of the current, the model with constant gap area was not able of giving results similar to the case B of the model with variable gap area. This implies that the additional number of parameters (for modeling the correction factor for the air-gap area) was necessary to represent such graph zone. Further studies about the effect of the air-gap correction factors are recommended, because these factors modify the expressions for the magnetic force and equivalent inductance.

(f) Case A of the model with variable gap area has lower errors for the first transient stage of the current than case B. The graphical results for case A of the model with variable gap area are similar to the graphical results of the two models with constant gap area, but these three models and their parameters are different. Model with constant gap area without considering friction force has the lowest number of parameters; therefore, the additional number of parameters of the other two ways for modeling the contactor did not notoriously improve their graphical results. The models with additional number of parameters should give a more detailed physical description of the contactor; therefore, additional experiments should be done in the future, to take advantage of the additional number of parameters.

(g) Figure 6 shows an enlargement of the graphical results for the rated voltage, to observe more clearly some of the above-mentioned comparisons. Comparison between results of the models is similar for the other voltage values. The results in Figs. 2, 3, and 4 are very similar between themselves. The results in Figs. 3 and 4 are almost identical, although some common parameters for both models (K, m, x_0) are not identical. This highlights the possibility of obtaining similar results with different models (Figs. 2, 3, 4).

(h) Each set of parameters was considered the best found solution for each model. However, this does not imply certainty about the real value of physical variables related with these parameters. For example, the values for the equivalent spring constant (K) are very different for each model. Maybe, none of these values are near to the value that would be measured by a relationship between measured forces and distances, and such relationship might be different than the assumed linear model for the equivalent spring. The search of real physical values for the model parameters would require specific tests for such goal and it is out of the scope of this article. Directly measured physical parameters were assumed as known parameters (gap length and coil resistance), and the estimated parameters for each model are those which best represent the contactor measured behavior. The difference between real physical parameters and the parameters of simplified models is a typical problem in control theory, and it is almost philosophical. This problem has been highlighted here to avoid possible confusions: these models can represent the measured behavior of the contactor, but their parameters are not indirect measurements of the real physical variables.

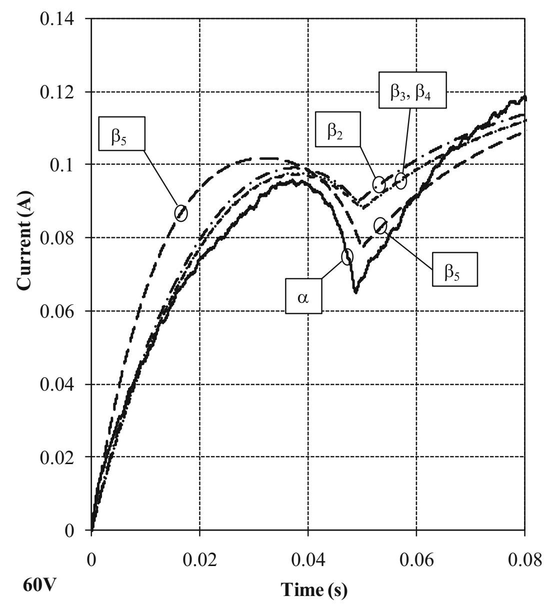

Fig. 6 Measured current (α) and result of the simulation (β) for the different models (β2: Fig. 2, β3: Fig. 3, β4: Fig. 4, β5: Fig. 5)

(i) The transient behavior of the current in the coil is in accordance with the results shown in some research articles [12–19], which are based on the finite-element method. However, obtaining similar results using simple electromechanical models was not found in the reviewed literature.

(j) The following examples of unsuccessful trials might be interesting for some readers. These trials were obtained by simply using the minimization of the error for the evolution of the current as main goal. Figure 7 shows a result for the model with constant area for the airgap, without friction, by defining the objective function as the average of the absolute errors for all the measurements (only the graphical result at one voltage is shown in Fig. 7, for the sake of simplicity). Figure 8 shows the corresponding result by including a weight factor equal to 10 for the measurements near to the local minimum of the current (i.e., near the end of the armature movement). In both the cases, the initial point for the search of optimal values was the result shown in Fig. 2 and Table 1. Several different definitions for the error of the approximation were tested, and the results shown in Figs. 7 and 8 are only two examples. The optimization tool minimized the selected objective functions, but the results obtained by the trial-and-error procedure are better for representing different details of the current in the contactor coil. This occurs, because the time instant for the local minimum of the current is very important, and the result for such value might be worse when the selected objective func-

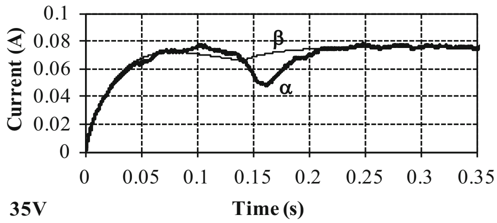

Fig. 7 Example of measured current (α) and result of the optimization (β) for the model with constant area for the airgap, without friction. For this case, the objective function is the average of absolute errors, and its minimum value is 0.0038 A (the value for the case of Fig. 3 is 0.0041 A)

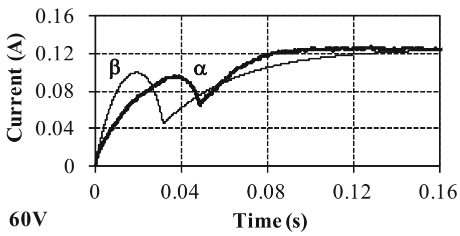

Fig. 8 Result is similar to Fig. 7, but the objective function has a weight factor for the points near to the local minimum of the measured current. For this case, the minimum value of the objective function is 0.012 A, and the value for the case of Fig. 3 is 0.018 A

tions are minimized. Of course, other attempts of using optimization tools for the estimation of parameters might be explored in future works.

5 Conclusion

A contactor with dc coil was modeled using lumped parameters, without considering its specific geometry but only its simplified schematic representation. The feasibility of obtaining the three transient stages of the contactor coil current with electromechanical models was demonstrated. Four ways of modeling the contactor were analyzed, and the results were compared with experimental measurements for different values of the dc source voltage. Although the proposed models include several simplifications, the four ways of modeling the contactor have a very good accuracy to represent the end of the armature movement and the closing time of the contacts.

Three of the four analyzed ways for modeling the contactor have the similar graphical results for the behavior of the coil current. The fourth one has the best result for the graph zone near to the occurrence of the local minimum of the current, but with the greatest errors for the first transient stage of the current. Future research might find the simultaneous achievement of the lowest errors at both the stages of the transient behavior of the current, but the results shown here can be already considered satisfactory for the simplicity of these models.

The models were experimentally evaluated for a wide range of dc source voltage values. This gave certainty about the validity of the results of the models, and this procedure is strongly recommended for testing any other model. Future research might find the minimum set of tests for obtaining certainty about the validity of the models. On the other hand, future research might also be directed toward: (a) experimental evaluation with voltages varying in time and (b) other techniques for electromechanical modeling and/or for parameter estimation.

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