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# **Sliding mode control with integral augmented sliding surface: design and experimental application to an electromechanical system**

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**Abstract** In this study, an integral augmented sliding mode control  $(SMC + I)$  has been proposed to improve control performance of systems. Stability of the closed-loop system is guaranteed in the sense of Lyapunov stability theorem. The effectiveness of the control solution is established by the stability analysis of the closed-loop system dynamics. The proposed controller is adopted to control speed of an electromechanical system. The experimental set-up reflects the emphasis on the practicability of the proposed sliding mode controller. The experimental results are presented and compared with the results obtained from conventional sliding mode control and Proportional + Integral + Derivative (PID) control. The experimental results verify that the proposed controller provides favorable tracking performance, faster and smoother speed regulation with regard to parameter variations and disturbances. The present study shows that the proposed controller, with its straightforward solution, is easily applicable to industrial problems and an alternative to conventional PID and sliding mode controllers.

**Keywords** Sliding mode control · Integral control · Experimental application

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# **1 Introduction**

Sliding mode control (SMC), first proposed in the early 1950s, is one of the effective nonlinear robust control methods since it provides the system dynamics with an invariance property to uncertainties once the system dynamics are controlled in the sliding mode [\[1\]](#page-7-0). It possesses many advantages including: (i) insensitivity to parameter variations and model uncertainties; (ii) external disturbance rejection; and (iii) fast dynamic responses and good transient performance [\[2](#page-7-1)[,3](#page-7-2)]. Moreover, the dynamic performance of a system under SMC method can be shaped according to the system specification by an appropriate choice of switching function [\[4\]](#page-7-3).

Sliding surface plays an important role in design of sliding mode controllers such that tracking errors and output deviations can be eliminated or reduced to a satisfactory level in practical applications [\[5](#page-7-4)]. Unfortunately, an ideal sliding mode controller has a discontinuous switching function and it is assumed that the control signal can be switched from one value to another infinitely fast [\[2](#page-7-1),[6\]](#page-7-5). Such a switching function produces chattering that is highly undesirable [\[2](#page-7-1)]. It appears as a high frequency oscillation near the desired equilibrium point and may excite the unmodeled high-frequency dynamics of the system. However, it is impossible to achieve infinitely fast switching control because of finite time delays for the control computation and limitations of physical actuators [\[7](#page-7-6)[,8](#page-7-7)]. One of the approaches to counteract the chattering phenomenon in SMC systems is the use of continuous approximation to smoothen the discontinuity [\[3](#page-7-2),[7](#page-7-6)[–10\]](#page-7-8). Error within some predetermined boundary layer is considered in these functions [\[11\]](#page-8-0).

Several research studies about SMC method have been performed in recent years and the method plays an important role in the application to practical problems [\[4](#page-7-3)[,10](#page-7-8),[12](#page-8-1)[–17](#page-8-2)]. Electrical machines such as direct current (DC) motor system



<span id="page-1-0"></span>**Fig. 1** Diagram of the electromechanical system

are very important in industrial applications and have a wide range of profile in their motions that is required to follow a predetermined speed or position under load [\[18](#page-8-3),[19\]](#page-8-4). The main advantages of these motors are of easy speed or position control and wide adjustable range [\[20](#page-8-5)]. The motors have been extensively used in several industrial applications [\[5](#page-7-4),[21](#page-8-6)[–23](#page-8-7)]. There have been considerable developments in nonlinear control schemes for the DC motors [\[21](#page-8-6),[24\]](#page-8-8), with simulation studies [\[4](#page-7-3)[,25](#page-8-9)] and experimental applications [\[5](#page-7-4)]. This has attracted extensive researches in the field of control engineering.

In the current study, conventional sliding mode control is improved such that an integral dynamic with an independent tuning parameter as a constant coefficient is added into conventional sliding surface to improve transient performance and steady-state accuracy, and to overcome drawbacks of the conventional SMC method. Solution is obtained on a surface in the proposed control mechanism, while it is obtained on a line in the conventional SMC. An experimental application is performed such that SMC + I controller is designed for the speed control of an electromechanical system, a DC motor connected to a load using belt mechanisms via shafts. The feasibility and effectiveness of the proposed sliding mode controller is experimentally demonstrated and the system is controlled using a computer. The results obtained from the present study are compared with the traditional PID

control system and conventional SMC system in dynamic responses of the closed-loop control.

## **2 Electromechanical system model**

Electromechanical system is given in Fig. [1](#page-1-0) and its block diagram is presented in Fig. [2.](#page-1-1) Electrical and mechanical equations of the system can be given as:

<span id="page-1-2"></span>
$$
v_a(t) = L_a \frac{d}{dt} i_a(t) + R_a i_a(t) + K_m \omega_m(t)
$$
\n(1)

$$
J_{\rm m}\left(\frac{\mathrm{d}\omega_{\rm m}(t)}{\mathrm{d}t}\right) = T_{\rm m}(t) - T_{\rm s1}(t) - R_{\rm m}\omega_{\rm m}(t) - T_{\rm f}(\omega_{\rm m})\tag{2}
$$

$$
J_1\left(\frac{d\omega_1(t)}{dt}\right) = T_{b1}(t) - T_{s2}(t) - R_1\omega_1(t) - T_f(\omega_1)
$$
 (3)

$$
J_{\rm L}\left(\frac{\mathrm{d}\omega_{\rm L}(t)}{\mathrm{d}t}\right) = T_{\rm b2}(t) - R_{\rm L}\omega_{\rm L}(t) - T_{\rm d}(t) - T_{\rm f}(\omega_{\rm L}) \tag{4}
$$

$$
T_{s1}(t) = k_{s1}(\theta_m(t) - K_{b1}\theta_1(t)) + B_{s1}(\omega_m(t) - K_{b1}\omega_1(t))
$$
 (5)

$$
T_{s2}(t) = k_{s2}(\theta_1(t) - K_{b2}\theta_L(t)) + B_{s2}(\omega_1(t) - K_{b2}\omega_L(t))
$$
 (6)

$$
\frac{d\theta_m(t)}{dt} = \omega_m(t), \quad \frac{d\theta_L(t)}{dt} = \omega_L(t), \quad \frac{d\theta_1(t)}{dt} = \omega_1(t) \tag{7}
$$

where  $v_a$  is the motor armature voltage,  $R_a$  and  $L_a$  are the armature coil resistance and inductance, respectively, *i*<sup>a</sup> is the armature current,  $K_{\rm m}$  is the torque coefficient,  $T_{\rm m}$  is the generated motor torque,  $\omega_{m}$ ,  $\omega_{1}$ ,  $\omega_{L}$ , are the rotational speeds of the motor,  $J_m$ ,  $J_1$ ,  $J_L$  are the moments of inertia,  $R_m$ ,  $R_1$ ,  $R_L$ are the coefficients of viscous-friction,  $T<sub>d</sub>$  is the external load disturbance,  $T_f$  is the nonlinear friction,  $T_{s1}$ ,  $T_{s2}$  are the transmitted shaft torques,  $T_{b1}$ ,  $T_{b2}$  are the transmitted torques from the belts, and  $K_{b1}$ ,  $K_{b2}$  are the belt constants.

The model for the nonlinear friction can be obtained by considering an asymmetrical characteristic as [\[19\]](#page-8-4):

$$
T_{\rm f}(\omega) = \left(\alpha_0 + \alpha_1 e^{-\alpha_2|\omega|}\right) \text{sgn1}(\omega) + \left(\alpha_3 + \alpha_4 e^{-\alpha_5|\omega|}\right) \text{sgn2}(\omega)
$$
 (8)



<span id="page-1-1"></span>**Fig. 2** Block diagram of the electromechanical system

where  $\alpha_i$  are constants,  $\alpha_i \in R$ ,  $\alpha_i > 0$ ,  $i = 0, \ldots, 5$ ,  $\alpha_0 \neq \alpha_3, \alpha_1 \neq \alpha_4, \alpha_2 \neq \alpha_5$ , and the functions sgn1 and sgn2 are defined as:

<span id="page-2-0"></span>
$$
sgn1(\omega) = \begin{cases} 1 & \omega \geq 0 \\ 0 & \omega < 0 \end{cases}, \quad \text{sgn2}(\omega) = \begin{cases} 0 & \omega \geq 0 \\ -1 & \omega < 0 \end{cases} \tag{9}
$$

The problem, however, with this system model given in Eq. [\(1\)](#page-1-2) to Eq. [\(9\)](#page-2-0) is that several parameters are needed and it is difficult to define all these parameters. To simplify the modeling process, model of the electromechanical system can be approximated using a second-order perturbed linear model including disturbances and uncertainties as [\[5](#page-7-4)]:

<span id="page-2-1"></span>
$$
\ddot{\omega}_{\mathcal{L}}(t) = -(A_{\mathbf{n}} + \Delta A)\dot{\omega}_{\mathcal{L}}(t) - (B_{\mathbf{n}} + \Delta B)\omega_{\mathcal{L}}(t) \n+ (C_{\mathbf{n}} + \Delta C)u(t) + d(t)
$$
\n(10)

where  $u(t)$  is the control input that denotes the motor armature voltage  $(u(t) = v_a(t))$ ,  $A_n$ ,  $B_n$  and  $C_n$  are the nominal system parameters,  $\Delta A$ ,  $\Delta B$  and  $\Delta C$  are the uncertainties introduced by system parameters, nonlinear friction, and unmodeled dynamics, and *d*(*t*) presents external disturbance. The second-order dynamic model, Eq. [\(10\)](#page-2-1), can be rearranged as

$$
\ddot{\omega}_{\mathcal{L}}(t) = -A_{\mathbf{n}}\dot{\omega}_{\mathcal{L}}(t) - B_{\mathbf{n}}\omega_{\mathcal{L}}(t) + C_{\mathbf{n}}u(t) + L \tag{11}
$$

where *L* denotes the lumped uncertainty that is bounded but unknown,  $|L| \le L_m$ ,  $L_m \in R^+$ ,  $R^+$  is space of the positive real constants. *L* is defined by

$$
L = -\Delta A \dot{\omega}_{\mathcal{L}}(t) - \Delta B \omega_{\mathcal{L}}(t) + \Delta C u(t) + d(t) \tag{12}
$$

# **3 Sliding mode control**

### 3.1 Conventional sliding mode control

Sliding surface,  $s(t)$  in the conventional SMC depends on the tracking error,  $e(t)$  and derivative(s) of the tracking error as [3]:

<span id="page-2-2"></span>
$$
s(t) = \left(\lambda + \frac{d}{dt}\right)^{n-1} e(t)
$$
\n(13)

where *n* denotes order of uncontrolled system,  $\lambda$  is a positive constant,  $\lambda \in R^+$ . If the system concerned is assumed to be of second-order  $(n = 2)$ , and then, in a two-dimensional phase plane related to Eq. [\(13\)](#page-2-2), the solution is a straight line passing through the origin. The tracking error starting from certain initial value will converge to the boundary layer and move inside the boundary layer towards the horizontal axis until it reaches  $de(t)/dt = 0$ . Under perturbation, the error will not coincide with the switching surface. Obviously, steady-state error in some level may occur, if the uncontrolled system does not include inherent integral action. On the other hand, the zero steady-state error is important in control systems such as servo and regulation problems in industrial applications [\[4](#page-7-3)].

#### 3.2 Integral augmented sliding mode control (SMC + I)

The sliding surface can be improved introducing an integral action into the sliding surface for steady-state accuracy defined as [\[26](#page-8-10)[,27](#page-8-11)]:

<span id="page-2-3"></span>
$$
s(t) = \left(\lambda + \frac{d}{dt}\right)^{n-1} e(t) + k_i \int\limits_0^t e(\tau)d\tau
$$
 (14)

where  $k_i$  is the integral gain,  $k_i \in R^+$ . The phase plane associated with the proposed switching surface, Eq. [\(14\)](#page-2-3), will be three-dimensional and the switching surface is a plane passing through the origin, if the order of uncontrolled system is assumed two,  $n = 2$ . Significance of the proposed control approach is that the solution is obtained on a plane, while the solution is obtained on a line in conventional SMC.

# 3.3 Stability

The control objective is to determine a control law, *u*(*t*) such that the tracking error,  $e(t)$  should converge to zero. The process of sliding mode control can be divided into two phases: the sliding phase with  $s(t) = 0$ ,  $\dot{s}(t) = 0$ , and the reaching phase with  $s(t) \neq 0$ . Control action corresponding to these two phases is performed in two parts: equivalent control and switching control [\[2,](#page-7-1)[28\]](#page-8-12). These can be derived separately. Conceptually, in sliding mode the equivalent control is described when  $s(t) = 0$  or  $\dot{s}(t) = 0$ , while the switching control is determined in the case of  $s(t) \neq 0$  [\[1\]](#page-7-0).

Taking derivative of the sliding surface given in Eq. [\(14\)](#page-2-3) with respect to time, for  $n = 2$  one has:

<span id="page-2-5"></span>
$$
\dot{s}(t) = \ddot{e}(t) + \lambda \dot{e}(t) + k_i e(t)
$$
\n(15)

A necessary condition for the tracking error to remain on the sliding surface  $s(t)$  is  $\dot{s}(t) = 0$  [\[2](#page-7-1)[,4](#page-7-3)[,7](#page-7-6)]:

<span id="page-2-4"></span>
$$
\ddot{e}(t) + \lambda \dot{e}(t) + k_i e(t) = 0 \tag{16}
$$

If the control gains,  $\lambda$  and  $k_i$ , are properly chosen such that the characteristic polynomial in Eq. [\(16\)](#page-2-4) is strictly stable, that is, roots of the polynomial are in the open left-half of the complex plane, it implies that  $\lim_{t \to \infty} e(t) = 0$  meaning that the closed-loop system is globally asymptotically stable [\[5](#page-7-4)[,15](#page-8-13)]. If the nominal parameters of the electromechanical system are inserted in to Eq. [\(15\)](#page-2-5), it is obtained

$$
\dot{s}(t) = \ddot{\omega}_{r}(t) + A_{n}\dot{\omega}_{L}(t) + B_{n}\omega_{L}(t) - L - C_{n}u(t) + \lambda\dot{e}(t) + k_{i}e(t)
$$
\n(17)

where  $\omega_{r}(t)$  is the set-point speed in the present electromechanical system problem. A necessary condition for the output,  $\omega_L(t)$ , to remain on the sliding surface is  $\dot{s}(t) = 0$ [\[2](#page-7-1),[4,](#page-7-3)[7\]](#page-7-6). The equivalent control law is obtained when  $\dot{s}(t) = 0$ for the unknown lumped uncertainty,  $L = 0$ :

<span id="page-3-0"></span>
$$
u_{\text{eq}}(t) = \frac{1}{C_{\text{n}}} (\ddot{\omega}_{\text{r}}(t) + A_{\text{n}} \dot{\omega}_{\text{L}}(t) + B_{\text{n}} \omega_{\text{L}}(t) + \lambda \dot{e}(t) + k_i e(t))
$$
\n
$$
(18)
$$

If the initial output trajectory is not on the sliding surface  $s(t)$ , or there is a deviation of the representative point from *s*(*t*) due to parameter variations and disturbances, the controller must be designed such that it can drive the output trajectory to the sliding mode  $s(t) = 0$ . The output trajectory under the condition that will move toward and reach the sliding surface is said to be on the reaching phase. For this purpose, the Lyapunov function can be chosen as

$$
V(t) = \frac{1}{2}s^2(t)
$$
 (19)

with  $V(0) = 0$  and  $V(t) > 0$  for  $s(t) \neq 0$ . A sufficient condition to guarantee that the trajectory of the error will translate from reaching phase to sliding phase is to select the control strategy, also known as reaching condition [\[2](#page-7-1)]:

<span id="page-3-1"></span>
$$
\dot{V}(t) = s(t)\dot{s}(t) < 0, \quad s(t) \neq 0. \tag{20}
$$

To satisfy the reaching condition, the equivalent control  $u_{eq}(t)$  given in Eq. [\(18\)](#page-3-0) is augmented by a switching control term,  $u_{sw}(t)$ , to be determined. Consider the system given in Eq. [\(10\)](#page-2-1) with uncertainties and disturbances and the sliding mode controller is designed such that

$$
u(t) = u_{\text{eq}}(t) + u_{\text{sw}}(t) \tag{21}
$$

If the system parameters and variables are inserted into the reaching condition given in Eq. [\(20\)](#page-3-1), it is obtained as

<span id="page-3-2"></span>
$$
s(t)\dot{s}(t) = s \left[ \ddot{\omega}_{\rm r}(t) + A_{\rm n} \dot{\omega}_{\rm L}(t) + B_{\rm n} \omega_{\rm L}(t) - C_{\rm n} \left( u_{\rm eq}(t) + u_{\rm sw}(t) \right) + \lambda \dot{e}(t) + k_{\rm i} e(t) \right]
$$
(22)

The switching control  $u_{sw}(t)$  in Eq. [\(22\)](#page-3-2) can be chosen as [\[2](#page-7-1)]

$$
u_{sw}(t) = k_s sgn(s) \tag{23}
$$

where  $k<sub>s</sub>$  is a positive constant and means upper bound of uncertainty,  $k_s \in R^+(k_s = L_m)$  [\[29](#page-8-14)], and sgn(.) denotes signum function defined as [\[2](#page-7-1)]

$$
sgn(s(t)) = \begin{cases} +1, & \text{if } s(t) > 0\\ 0, & \text{if } s(t) = 0\\ -1 & \text{if } s(t) < 0 \end{cases}
$$
(24)

Taking the derivative of the Lyapunov function with respect to time, one has

$$
\dot{V}(s(t)) = s(s)\dot{s}(t)
$$
\n
$$
= -s(t)k_sC_nsgn(s(t))
$$
\n
$$
= -k_sC_n|s(t)| \leq 0
$$
\n(25)

This implies that  $\dot{V}(s(t))$  is a negative semi-definite function. Define the following term:

$$
X(t) = k_{\rm s} C_{\rm n} |s(t)| \tag{26}
$$

therefore

$$
X(t) \leqslant -\dot{V}(s(t))\tag{27}
$$

then *t*

$$
\int_{0}^{1} X(\tau)d\tau \leqslant V(s(0)) - V(s(t))
$$
\n(28)

Since  $V(s(0))$  is bounded and  $V(s(t))$  is non-increasing and bounded, the following result can be concluded

$$
\lim_{t \to \infty} \int_{0}^{t} X(\tau) d\tau < \infty
$$
\n(29)

Also,  $\dot{X}(t)$  is bounded, and it can be shown that  $\lim_{t\to\infty}$  $X(t) = 0$  by the Barbalat's Lemma [\[2](#page-7-1)]. That is,  $s(t) \rightarrow 0$  as  $t \rightarrow \infty$ . By applying this switching control law, the sliding mode control system can be guaranteed to be stable in the Lyapunov sense.

# 3.4 Switching

It is known that the discontinuous switching functions can be approximated by their continuous switching functions to avoid the chattering of the control force and to achieve the exponential stability [\[7\]](#page-7-6). Instead of signum function, a saturation function has been used via introducing a thin boundary layer around the sliding surface to avoid chattering [\[2,](#page-7-1)[3\]](#page-7-2). For a more smooth change of the switching signal, a hyperbolic tangent function has also been used to improve the switching control effort as [\[5](#page-7-4),[30\]](#page-8-15):

$$
u_{sw}(t) = k_s \tanh\left(s(t)/\Omega\right) \tag{30}
$$

where  $\Omega$  is a positive constant,  $\Omega \in R^+$ , and it defines the thickness of the boundary layer that affects the steady-state accuracy and robustness.

#### **4 Experimental set-up and test results**

Diagram of the experimental set-up is shown in Fig. [3.](#page-4-0) A computer (Pentium II MMX, 300 MHz, 256 MB RAM) is used to implement proposed sliding mode controller. The output shaft speed is measured from an optical sensor (as revolutions per minute, rpm) and a tacho-generator (as volts) connected to the motor shaft. A series of pulses is generated in the optical sensor when the slotted disk, which is mounted on the motor shaft, is rotated. When the shaft of the dc motor is turned, a voltage is induced at the tacho-generator terminals and is

<span id="page-4-0"></span>



directly proportional to shaft speed. The motor to be studied operates with a maximum output shaft speed of 1,500 rpm. The motor drives a shaft that carries disks, which operate various transducers, and a tacho-generator. Motor speed is reduced by 9:1. The motor speeds at different input armature voltages are measured to obtain the tacho-generator characteristics and it is calculated to be 0.0017 V/rpm. The measured output data are transferred to the computer by a data acquisition card (Advantech, Model: PCL-1800, 330 kHz in speed, a conversion time of  $2.5 \mu s$ ., and  $0.01\%$  accuracy). The sampling period is taken to be 5 ms for all cases. Control calculations are performed in Matlab environment, in Simulink of Matlab [\[31](#page-8-16)]. The experimental application is performed in such a way that the measured signal is transferred to the Simulink via the data acquisition card, and is compared with the reference signal. The calculations are performed to produce the switching signal and the equivalent signal. The overall control signal (switching signal + equivalent signal) is sent to the power amplifier using the data acquisition card to control the real system.

As a preliminary work, the electromechanical system should be tested to define the gain coefficients,  $A_n$ ,  $B_n$  and *C*n. This process is needed to obtain the equivalent control input,  $u_{eq}(t)$  and should be performed before the closed-loop operation is allowed. In open-loop conditions, a step input signal, amplitude of 2.55 V, that corresponds to 1,500 rpm is applied to the electromechanical system from the computer over the data acquisition card. First-order plus dead-time model is one of the effective model types to approximate real systems, if response of the open-loop systems to an applied deterministic signal such as step input change do not possesses any overshoot, or the response looks like an overdamped response type. The modeling process is based on the process reaction curve method as [\[5](#page-7-4)[,32](#page-8-17)[–34](#page-8-18)]:

$$
G(s) \cong \frac{Ke^{-T_{\rm d}s}}{Ts + 1} \cong \frac{K}{(1 + T_{\rm d}s)(1 + Ts)}
$$
(31)

Therefore, the nominal mathematical model of the system is approximated to be:

$$
G(s) \cong \frac{0.786}{(1 + 0.01s)(1 + 0.27s)}
$$

Responses of the actual system and approximated system are illustrated in Fig. [4](#page-4-1) for the model validation such that solid line presents the approximated model response and the dotted line denotes the real system output, and the speed error is illustrated in Fig. [5.](#page-5-0) The output speed settles down after 1.3 s and the error is about  $\pm$  35 rpm and it is meaning that the modeling error is  $\pm 35/1$ , 200 =  $\pm$  2.9% in the approximated model. Since the mean error is zero, it satisfies the fact that the steady-state modeling error is zero. Using the approximate system model, the model parameters are calculated to be  $A_n = 103.74$ ,  $B_n = 370.37$  and  $C_n = 291.11$ . Conventional PID controller parameters are determined using process reaction curve method and the gain coefficients are calculated to be  $K_p = 21$ ,  $T_i = 0.25$ and  $T_D = 0.0717$ . Heuristic method is used in general to choose the sliding mode controller parameters. As the design



<span id="page-4-1"></span>**Fig. 4** Response of the actaul system and approximated system



**Fig. 5** Modeling error

<span id="page-5-0"></span>

<span id="page-5-1"></span>**Fig. 6** Responses of PID, SMC and SMC+I control system to 0–1200 rpm step setpoint speed change

guidelines, first, the sliding mode controller parameters must be all positive real. Second, the polynomial, given in Eq. [\(16\)](#page-2-4), must be stable. Third, the switching signal should be minimized not to hurt the actuator. The overshoot is not desired. From all these, to satisfy the requirements the parameters are selected as  $\lambda = 12$ ,  $\Omega = 30$  and  $k_s = 4$  for the conventional SMC system, and  $\lambda = 12$ ,  $k_i = 1$ ,  $\Omega = 30$  and  $k_s = 5$  for the SMC + I system.

Responses of the closed-loop control system to 0–1200 rpm set-point changes are illustrated in Fig. [6](#page-5-1) for PID, SMC and SMC + I control systems respectively. The performance of the system with the proposed sliding mode controller ( $SMC + I$ ) is much better than the system with the conventional PID controller and conventional sliding mode controller such that no overshoot, smaller rise time, and smaller settling time in magnitude were obtained from the proposed controller. On the other hand, conventional PID controllers did not meet the needs of precise control, since these resulted in some overshoots and larger settling time in magnitude. The variations of the control efforts were shown in Fig. [7](#page-5-2) for PID control, and in Fig. [8](#page-5-3) for SMC and  $SMC + I$ control such that while the control effort of SMC+I system is a little bit larger in transient conditions from that of SMC system, it is smaller in steady-state conditions in magnitude and settles down in 0.18 s, while the control effort of SMC system settles down in 0.22 s. The control effort of PID control, in Fig. [7,](#page-5-2) is very large in magnitude and oscillating in transient conditions, and settles down in 0.5 s. Such a large variation in the control effort is not desired in most of the control applications, since it may be harmful for the actuators. Some quantitative performance indicators of the speed tracking quality are presented in Table [1](#page-6-0) for PID, SMC and SMC  $+$  I controllers.



**Fig. 7** Control effort with PID control

<span id="page-5-2"></span>

<span id="page-5-3"></span>Fig. 8 Control efforts with SMC and SMC + I control

**Table 1** Time domain specifications

<span id="page-6-0"></span>

Rise	Settling	Overshoot	Output
time (ms)	time	(%)	deviations in
	$(ms)$ (5%)		steady-state (rpm)
			$\pm 10.3$
172	208	0.0	$\pm$ 9.3
133	166	0.0	$\pm 8.9$
	190	260	4.6

Some comments can be given about the tuning parameters,  $\lambda$  and  $k_i$  to affect operation of the closed-loop system. If the switching control signal  $u_{sw}$  in Fig. [3,](#page-4-0) is considered,  $\lambda$ behaves basically as a typical proportional gain. In addition,  $\lambda$  affects the equivalent control, Eq. [\(18\)](#page-3-0), as a derivative gain. The integral gain *ki* in Fig. [3](#page-4-0) adjusts rates of error integration. Also, the switching control is limited and the overall control input ( $u_{sw} + u_{eq}$ ) should be reasonable not to hurt the actuator and to satisfy the design requirements.

Variations of the switching (hitting) control and equivalent control, for SMC system and SMC + I system are illustrated in Fig. [9,](#page-6-1) [10,](#page-6-2) [11,](#page-6-3) and [12,](#page-6-4) respectively. It is clearly seen that magnitudes of the variations in  $SMC + I$  control system are smaller than that of the variations in SMC control system for both the switching and equivalent control efforts. The variations of the sliding signal  $s(t)$  during the control are shown in Fig. [13](#page-7-9) for both SMC system and SMC+I system, respectively. It can be noted that the sliding signal is  $s(t) \neq 0$  when the error signal is not zero. This means that the sliding mode is in the reaching surface up to about 0.2 s and then arrives sliding surface in SMC + I system. When it reaches to sliding surface, theoretically it is expected the sliding function to be zero,  $s(t) = 0$ . In practical applications, there are always some small deviations and fluctuations at



<span id="page-6-1"></span>**Fig. 9** Switching control effort with SMC control



<span id="page-6-2"></span>Fig. 10 Switching control effort with SMC + I control



<span id="page-6-3"></span>**Fig. 11** Equivalent control effort with SMC control



<span id="page-6-4"></span>**Fig. 12** Equivalent control effort with SMC + I control

the output measured variable because of uncertainties and disturbances. Here, the average value of the sliding function is zero,  $s(t) = 0$ .



<span id="page-7-9"></span>**Fig. 13** Sliding signal,  $s(t)$  in SMC and SMC + I system

## **5 Conclusions**

In this study, a sliding mode control with an integral augmented sliding surface  $(SMC + I)$  has been proposed to improve control performance of systems. Present algorithm was adopted to control the speed of a computer-controlled electromechanical system while the nominal system is assumed known. Approximated second-order-system model is used in the present design, since many of the industrial systems can be modeled using a second-order model. The practical application is associated with the sliding mode controller as a computational-intelligence approach to the engineering problems. Experimental application was carried out to test the effectiveness of the present controller, SMC + I. From the experimental results the proposed SMC + I controller is more effective to be applied for the speed control of the electromechanical system due to the uncertainty handling capabilities and disturbance rejection of the design method. In order to avoid the chattering phenomena, a hyperbolic function has been used. The closed-loop system is in the sliding mode at all times and the tracking error converges to zero exponentially under the existence of parameter uncertainties and disturbances. The closed-loop system has been proved and shown to be globally exponentially stable in the sense of Lyapunov theorem.

Based on the experimental results and the time domain specifications presented in Table [1,](#page-6-0) it can be concluded that the control performance of the electromechanical system was significantly improved with SMC + I control system compared with the conventional PID and SMC control systems. Experimental results also confirm the fact that the sliding mode controllers are reasonable candidates to use in industrial applications and these can be considered to be alternative to usual PID controllers, since the present design algorithm is simple to use and easy to understand with its straightforward solution, and the computational task is not a problem

any more because of high-speed computers and application tools to use in industrial applications.

The merit of integral augmented control structure has been verified by experimental testing of an electromechanical system. Three controllers are tested: [\(1\)](#page-1-2) conventional PID; [\(2\)](#page-1-2) conventional sliding mode and [\(3\)](#page-1-2) integral augmented sliding mode. The experimental results show that the proposed controller has much better steady-state and transient performance.

The current theoretical study and experimental application showed that there are several advantages of the present design method such that: (i) conventional sliding surface is improved to include integral action, (ii) solution is simple and easy to understand including straightforward design process and control calculations are not complex, (iii) experimentalbased model is used in design procedure, while theoretical model-based model obtained using differential equations includes many parameters which are difficult to define, (iv) tuning is easy since the guidelines for the design give clues to designer how to select the tuning parameters, and (v) the design method is experimentally applied to an electromechanical system, and this supports the fact that the current controller is applicable to industrial problems.

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# <span id="page-7-0"></span>**References**

- 1. Utkin VI (1977) Variable structure systems with sliding modes. IEEE Trans Automa Control 22:212–222
- <span id="page-7-1"></span>2. Slotine JJ, Li W (1991) Applied nonlinear control. Prentice Hall, Englewood Cliffs
- <span id="page-7-2"></span>3. Boiko I, Fridman L, Iriarte R, Pisano A, Usai E (2006) Parameter tuning of second-order sliding mode controllers for linear plants with dynamic actuators. Automatica 42:833–839
- <span id="page-7-3"></span>4. Utkin VI (1993) Sliding mode control design principles and applications to electric drives. IEEE Trans Ind Electron 40:23–26
- <span id="page-7-4"></span>5. Eker ˙I (2006) Sliding mode control with PID sliding surface and experimental application to an electromechanical plant. ISA Trans 45:109–118
- <span id="page-7-5"></span>6. Jang MJ, Chen CL, Chen CK (2002) Sliding mode control of hyperchaos in Rössler systems. Chaos Solitons and Fractals 13:1465–1476
- <span id="page-7-6"></span>7. Khalil HK (1996) Nonlinear systems. Prectice Hall, Upper Saddle River
- <span id="page-7-7"></span>8. Hung JV, Gco W, Hung JC (1993) Variable structure control: a survey. IEEE Trans Ind Electron 40:2–22
- 9. Zhou F, Fisher DG (1992) Continuous sliding mode control. Int J Control 55:313–327
- <span id="page-7-8"></span>10. Park KB, Lee JJ (1997) Sliding mode controller with filtered signal for robot manipulators using virtual plat/controller. Mechatronics 7:277–286
- <span id="page-8-0"></span>11. Slotine JJ (1984) Sliding controller design for nonlinear systems. Int J Control 40:421–434
- <span id="page-8-1"></span>12. Lim CL, Jones NB, Spurgeon SK, Scott JJA (2003) Reconstruction of human neuromuscular control signals using a sliding mode control technique. Simul Model Pract Theory 11:222–235
- 13. Lin FJ, Wai RJ, Kuo RH, Liu DC (1998) A comparative study of sliding mode and model reference adaptive observers for induction motor drive. Electr Power Syst Res 44:163–174
- 14. Walchko KI, Novick D, Nechyba MC (2003) Development of a sliding mode control system with extended Kalman filter estimation for subjugator. Florida Conference on Recent Advances in Robotics FCRAR, Dania Beach
- <span id="page-8-13"></span>15. Wai RJ, Lin CM, Hsu CF (2004) Adaptive fuzzy sliding mode control for electrical servo drive. Fuzzy Sets Syst 143:295–310
- 16. Sha D, Bajic VB, Yang H (2002) New model and sliding mode control of hydraulic elevator velocity tracking system. Simul Pract Theory 9:365–385
- <span id="page-8-2"></span>17. Khan MK, Spurgeon SK (2006) Robust MIMO water level control in interconnected twin-tanks using second order sliding mode control. Control Eng Pract 14:375–386
- <span id="page-8-3"></span>18. Lyshevski SE (1999) Nonlinear control of mechatronic systems with permanent-magnet DC motors. Mechatronics 9:539–552
- <span id="page-8-4"></span>19. Jang JO, Jeon GJ (2000) A parallel neuro-controller for DC motors containing nonlinear friction. Neurocomputing 30:233–248
- <span id="page-8-5"></span>20. Eker I (2004) Experimental on-line identification of an electromechanical system. ISA Transactions 43:13–22
- <span id="page-8-6"></span>21. Horng JH (1999) Neural adaptive tracking control of a DC motor. Inf Sci 118:1–13
- 22. Yavin Y, Kemp PD (2000) Modelling and control of the motion of a rolling disk: effect of the motor dynamics on the dynamical model. Comput Methods Appl Mech Eng 188:613–624
- <span id="page-8-7"></span>23. Mummadi VC (2000) Steady-state and dynamic performance analysis of PV supplied DC motors fed from intermediate power converter. Solar Energy Mater Solar Cells 61:365–381
- <span id="page-8-8"></span>24. Kara T, Eker ˙I (2004) Nonlinear closed-loop direct identification of a dc motor with load for low speed two-directional operation. Electr Eng 86(2):87–96
- <span id="page-8-9"></span>25. Chern TL, Wu YC (1993) Design of brushless DC position servo systems using integral variable structure approach. IEE Proc Electr Power Appl 140:27–34
- <span id="page-8-10"></span>26. Eker İ, Akınal ŞA (2005) Sliding mode control with integral action and experimental application to an electromechanical system. In: CIMA'05 International Computing Sciences Conferences(ICSC)– 1st International ICSC Symposium on Industrial Application Of Soft Computing, 15–17 December, Istanbul, TURKEY
- <span id="page-8-11"></span>27. Seshagiri S, Khalil KH (2002) On introducing integral action in sliding mode control. In: Proceedings of 41st conference on decision and control, Nevada, pp 1473–1478
- <span id="page-8-12"></span>28. Chan PT, Rad AA, Wang J (2001) Indirect adaptive fuzzy sliding mode control: Part II: parameter projection and supervisory control. Fuzzy Sets Syst 122:31–43
- <span id="page-8-14"></span>29. Chu WH, Tung PC (2005) Development of an automatic arc welding system using a sliding mode control. Int J Mach Tools Manuf 45:933–939
- <span id="page-8-15"></span>30. Ha QP, Nguyen QH, Rye DC, Whyte HFD (2001) Fuzzy sliding mode controllers with applications. IEEE Trans Ind Electron 30:  $2 - 21$
- <span id="page-8-16"></span>31. Eva PE (1996) The MATLAB Handbook. Addison-Wesley, **Harlow**
- <span id="page-8-17"></span>32. Chen CT, Peng ST (2006) Design of a sliding mode control system for chemical processes. J Process Control 15:515–530
- 33. Camacho O, Smith CA (2000) Sliding mode control: an approach to regulate nonlinear chemical processes. ISA Trans 39:205–218
- <span id="page-8-18"></span>34. Seborg DE, Edgar TF, Mellichamp DA (1989) Process Dynamics and Control. Wiley, New York