Economic Theory **14**, 705–715 (1999)

Economic Theory

Long run equilibria in an asymmetric oligopoly*

Yasuhito Tanaka

Faculty of Law, Chuo University, 742-1, Higashinakano, Hachioji, Tokyo, 192-03, JAPAN (e-mail: yasuhito@tamacc.chuo-u.ac.jp)

received: September 19, 1997; revised: June 18, 1998

Summary. Consider an oligopolistic industry composed of two groups (or populations) of firms, the low cost firms and the high cost firms. The firms produce a homogeneous good. I study the finite population evolutionarily stable strategy defined by Schaffer (1988), and the long run equilibrium in the stochastic evolutionary dynamics based on imitation and experimentation of strategies by firms in each group. I will show the following results. 1) The finite population evolutionarily stable strategy (ESS) output is equal to the competitive (or Walrasian) output in each group of the firms. 2) Under the assumption that the marginal cost is increasing, the ESS state is the long run equilibrium in the stochastic evolutionary dynamics in the limit as the output grid step, which will be defined in the paper, approaches to zero.

Keywords and Phrases: Asymmetric oligopoly, Finite population evolutionarily stable strategy, Long run equilibrium.

JEL Classification Numbers: C72, L13.

1 Introduction

Recently Vega-Redondo (1997) studied the long run evolutionary equilibrium in a symmetric oligopoly with a homogeneous good, and showed that the Walrasian behavior, that is, profit maximization given the market clearing price is the long run equilibrium strategy. I consider an oligopolistic industry composed of two groups (or populations) of firms, the low cost firms and the high cost firms, producing a homogeneous good, and study the finite population evolutionarily stable strategy defined by Schaffer (1988), and the long run equilibrium in

* I wish to thank an anonymous referee for very helpful comments.

the stochastic evolutionary dynamics based on imitation and experimentation of strategies by firms in each group. The long run equilibrium is the state where it spends most of the time in the long run when the probability of experimentation (mutation) becomes very small. I will show the following results. 1) The finite population evolutionarily stable strategy (ESS) output is equal to the competitive (or Walrasian) output in each group of the firms. 2) Under the assumption that the marginal cost is increasing, the ESS state is the long run equilibrium in the stochastic evolutionary dynamics in the limit as the output grid step, which will be defined below, approaches to zero.

Schaffer (1988) proposed a concept of an evolutionarily stable strategy (ESS) for a finite (or small) population as a generalization of the standard ESS concept for an infinite (or large) population by Maynard Smith (1982). It is called *the finite population ESS*. He showed that the finite population ESS is not generally a Nash equilibrium strategy. In Schaffer (1989) he applied this concept to an economic game, and showed that the strategy which survives in economic natural selection is the relative, not absolute, payoff maximizing strategy. He considered the following survival rule in economic natural selection. Firms are born with strategies and cannot change their strategies in response to changing circumstances. At the end of each period, if the payoff of Firm i is larger than the payoff of Firm j, the probability that Firm i survives in the next period is larger than the probability that Firm j survives in the next period.

Alternatively we can consider that the survival rule operates on strategies, not firms, and the population proportion of successful strategies grows by firms' imitation of strategies. In this paper I consider the following model. Firms can observe decisions of other firms, but do not know the exact form of the demand function, and can not compute their best responses to other firms' strategies. On the other hand, each firm knows that the cost functions for all firms in the group to which it belongs are the same and the game is symmetric. When all firms in each group choose the same strategy, denoting it by s_1, in a symmetric oligopoly their profits are equal, and they do not have incentive to change their strategies. Now, suppose that one firm (the mutant firm) experiments a different strategy, s_2. If this firm makes higher profit than the rest of the firms, they will wish to imitate the mutant firm's success. On the other hand, if the mutant firm makes lower profit than the rest of the firms, they will not wish to imitate the mutant firm's failure, and in fact the mutant firm will wish to switch s_2 to s_1. If, starting from s_1, experimenting always leads to lower profit for the mutant firm than for the rest of the firms, then s_1 is an ESS[1].

Hansen and Samuelson(1988) also presented analyses about evolution in economic games. They showed that with small number of firms the surviving strategy in economic natural selection, they called such a strategy a *universal survival strategy*, is not a Nash equilibrium strategy. Their *universal survival strategy* is essentially equivalent to Schaffer's finite population evolutionarily stable strategy. They said, "In real-world competition, firms will be uncertain about the

[1] For a more detailed analysis of an imitation behavior, see Schlag (1998).

profit outcomes of alternative strategies. This presents an obvious obstacle to instantaneous optimization. Instead, firms must search for and learn about more profitable strategies. As Alchian (1950) emphasizes, an important mechanism for such a search depends on a comparison of observed profitability across the strategies used by market participants. That is, search for better strategies is based on *relative* profit comparisons.".

The authors of some recent papers studying evolutionary dynamics such as Robson and Vega-Redondo (1996) and Vega-Redondo (1997) consider a model of stochastic evolutionary dynamics assuming imitation dynamics of players' strategies. On the other hand, in other some papers such as Kandori and Rob (1995) and Kandori and Rob (1998) best response dynamics are assumed. In best response dynamics each player chooses a strategy in period $t + 1$ which is a best response to other players' strategies in period t. Thus players must know the whole payoff structure of the game, and must be able to compute their best responses. While in imitation dynamics, players simply mimic successful other players' strategies. For an infinite population, imitation dynamics coincide with best response dynamics. I think that imitation dynamics is more appropriate than best response dynamics for an economic game with boundedly rational players, and a finite population evolutionary game is more interesting than an infinite population one. There are experimental studies about the role of imitation in an economic decision making such as Pingle and Day (1996) and Offerman and Sonnemans (1998). Pingle and Day (1996) argued that imitation plays an important role in real-world decision making because it is one of the procedures that allows the decision maker to economize on decision costs.

In the next Sect. I consider the finite population evolutionarily stable strategy in an oligopoly with two groups of firms producing a homogeneous good, and show that the ESS output is equal to the competitive output in each group. In Sect. 3 I will show that the ESS state is the long run equilibrium in the limit as the output grid step approaches to zero.

2 The finite population evolutionarily stable strategy

Consider an oligopolistic industry composed of N firms producing a homogeneous good. There are two groups (populations) of firms, the low cost firms and the high cost firms. The number of the low cost firms is n_1, and the number of the high cost firms is n_2. n_1 and n_2 are integer numbers which are not smaller than 3, and $n_1 + n_2 = N$. The output of the i-th low cost firm is denoted by x_i, and the output of the i-th high cost firm is denoted by y_i. The price of the good is denoted by p.

The inverse demand function is

$$p(X + Y) \text{ where } X = \sum_{i=1}^{n_1} x_i \text{ and } Y = \sum_{i=1}^{n_2} y_i .$$

X is the total output of the low cost firms, and Y is the total output of the high cost firms. $p(\cdot)$ is decreasing in $X + Y$.

The cost function for the low cost firms is

$$c_l(x_i) = c(x_i),$$

and the cost function for the high cost firms is

$$c_h(y_i) = \lambda c(y_i) \text{ where } \lambda > 1.$$

The cost for the high cost firms is proportionally higher than the cost for the low cost firms. $c(\cdot)$ is increasing and twice differentiable. Further I assume

Assumption 1 $c(x_i)$ is convex, that is, the marginal costs, $c'(x_i)$ and $\lambda c'(y_i)$, are increasing.

The profits of the low cost firms are written as follows,

$$\pi_i^l(x_i, X+Y) = p(X+Y)x_i - c(x_i), \; i = 1, 2, \cdots, n_1.$$

Similarly, the profits of the high cost firms are written as follows,

$$\pi_i^h(y_i, X+Y) = p(X+Y)y_i - \lambda c(y_i), \; i = 1, 2, \cdots, n_2.$$

I consider an evolutionary game in which N firms repeatedly play an N firms oligopoly stage game. In this evolutionary game the population is N, and the stage game is also an N players game. Thus it is a so called *playing the fields model* of evolutionary game. Strategies for the firms are their outputs. The firms repeatedly play the stage game in each period, and may change their strategies between one period and the next period. This dynamic problem is treated in the next section. In this section I consider the finite population evolutionarily stable strategy of the stage game in each group of firms.

Consider the state in which all low cost firms choose x^*. If, when one low cost firm (mutant firm) chooses a different strategy x', the profits of the firms who choose x^* are larger than the profit of the mutant firm given the outputs of the high cost firms, and this relation holds for all $x' \neq x^*$, then x^* is the finite population evolutionarily stable strategy (ESS) for the low cost firms[2]. Formally, x^* is the finite population ESS if

$$\pi_i^l(x^*, X+Y) > \pi_j^l(x', X+Y), \; \forall x' \neq x^* \text{ and all } i \neq j \quad (1)$$
$$\text{where } x_j = x', \text{ and } x_i = x^* \text{ for all } i \neq j \text{ in } X.$$

In this expression it is assumed that the mutant firm is the j-th low cost firm. According to Schaffer (1988) we can find the finite population ESS as the solution of the following problem,

$$x^* = \arg\max_{x'} \varphi^l, \quad (2)$$

where

[2] Schaffer's definition is weaker. He defines x^* as the finite population ESS if Eq. (1) is satisfied with weak inequality. I adopt the definition with strong inequality. About the definitions of the finite population ESS, see Crawford (1991).

$$\varphi^l = \pi_j^l(x', X+Y) - \pi_i^l(x^*, X+Y) = p(X+Y)x' - c(x') - p(X+Y)x^* + c(x^*). \quad (3)$$

If there is a unique value of x' which maximizes Eq. (2) given x^* and the total output of the high cost firms $Y = \sum_{i=1}^{n_2} y_i$, then x^* satisfies Eq. (1) since φ^l has the maximum value, which is zero, only when $x' = x^*$.

Differentiation of φ^l with respect to x' yields

$$p(X+Y) + p'(X+Y)(x' - x^*) - c'(x') = 0.$$

$p'(X+Y)$ is the derivative of the inverse demand function. Then, the condition for maximization of φ^l is written as

$$p(X+Y) - c'(x^*) = 0 \text{ where } X = n_1 x^*. \quad (4)$$

The value of x^* obtained from Eq. (4) depends on $Y = \sum_{i=1}^{n_2} y_i$. Denote such x^* when all high cost firms choose y by $x^*(y)$. Call $x^*(y)$ *the ESS output for the low cost firms relative to* y. It is obtained from the following equation.

$$p(n_1 x^*(y) + n_2 y) - c'(x^*(y)) = 0. \quad (5)$$

The finite population ESS for the high cost firms is similarly defined, and is obtained as the solution of the following problem,

$$y^* = \arg\max_{y'} \varphi^h,$$

where

$$\begin{aligned} \varphi^h &= \pi_j^h(y', X+Y) - \pi_i^h(y^*, X+Y) \\ &= p(X+Y)y' - \lambda c(y') - p(X+Y)y^* + \lambda c(y^*). \end{aligned}$$

By similar procedures the condition for maximization of φ^h is obtained as follows,

$$p(X+Y) - \lambda c'(y^*) = 0 \text{ where } Y = n_2 y^*. \quad (6)$$

Similarly to $x^*(y)$, from Eq. (6) we obtain *the ESS output for the high cost firms relative to* x. It is denoted by $y^*(x)$, and is obtained from the following equation.

$$p(n_1 x + n_2 y^*(x)) - \lambda c'(y^*(x)) = 0. \quad (7)$$

The conditions for the finite population ESS for both groups are written as

$$p(n_1 x^* + n_2 y^*) - c'(x^*) = 0, \quad (8)$$

and

$$p(n_1 x^* + n_2 y^*) - \lambda c'(y^*) = 0, \quad (9)$$

Eq. (8) and Eq. (9) mean that the price of the good is equal to the marginal cost for both low cost and high cost firms. Subsequently, we obtain the following proposition.

Proposition 1. *The finite population ESS output of the low cost firms and that of the high cost firms are equal to the competitive outputs.*

Denote the pair of ESS outputs of the low cost and the high cost firms obtained from Eq. (8) and Eq. (9) by (x^*, y^*). Since the marginal cost is increasing, we find $x^* > y^*$, that is, the ESS output of the low cost firms is larger than that of the high cost firms.

3 The long run equilibrium

In this Sect. I will show that the finite population ESS output for each group obtained in the previous section is the long run equilibrium strategy in the stochastic evolutionary dynamics based on imitation and experimentation of strategies by the firms.

Kandori et al. (1993), Kandori and Rob (1995), Vega-Redondo (1996) and Vega-Redondo (1997) presented analyses of long run equilibria of dynamic and stochastic evolutionary games. In our model, N players (firms) play an oligopoly game in each period. According to Robson and Vega-Redondo (1996) and Vega-Redondo (1997) I consider the following imitation dynamics of the firms' strategies. In period $t+1$ each low cost (or high cost) firm has a chance with positive probability less than one to change its strategy to the strategy which achieved the highest profit among the strategies chosen by some low cost (or high cost) firms in period t. If the strategy of one firm in period t achieved the strictly highest profit in its group, this firm does not change its strategy. If in period t the highest profit was attained by two or more firms in one of the groups even when they chose different strategies, in period $t+1$ each firm in this group may choose either strategy among the strategies which attained the highest profit in period t. If all firms in one of the groups chose the same strategy in period t, since in such a state this strategy achieved the strictly highest profit, the firms in this group do not change their strategies.

As in Vega-Redondo (1997) I assume that the firms in each group must choose their outputs from a finite grid $\Gamma = \{0, \delta, 2\delta, \cdots, v\delta\}$ where $\delta > 0$ and $v \in \mathbb{N}$ are arbitrary. δ is called the grid step. It is required that the finite population ESS outputs x^* and y^* belong to this grid. δ can be arbitrarily small to make the grid sufficiently fine. The state of the imitation dynamics is identified with the output profile. The state space is denoted by Ω which is equal to Γ^N. Denote the transition matrix of this dynamics by $T(\omega, \omega')$, and by $T^{(m)}(\omega, \omega')$ the corresponding m-step transition matrix, where $\omega, \omega' \in \Omega$.

In addition to this dynamic adjustment, there is a random mutation. In each period, each firm switches (mutates) its strategy with probability ε. Mutation may be interpreted as experimentation of a new strategy by the firms. All strategies may be chosen with positive probability. Thus the complete dynamic is an ergodic Markov chain, and it has the unique stationary distribution. Consider the limit of the stationary distribution of the Markov chain as $\varepsilon \to 0$. The long run equilibria are the states which are assigned positive probability in the limit. This adjustment process is the same as that in Robson and Vega-Redondo (1996) and Vega-Redondo (1997). It has the stochastic nature even without mutation since each firm has a chance to change its strategy independently with some positive probability, and the number of firms in each group who change their strategies in period $t+1$ to the best strategy in period t is the stochastic variable without mutation. In period $t+1$ all firms may choose the best strategy in period t with strictly positive probability.

I define a *limit set* of the dynamics without mutation. A set A is a limit set if this set is closed under the finite chains of positive probability transitions. That is,

(1) $\forall \omega \in A, \forall \omega' \notin A, T(\omega, \omega') = 0.$

(2) $\forall \omega \in A, \omega' \in A, \exists m \in \mathbb{N}$ such that $T^{(m)}(\omega, \omega') > 0.$

If in period t some firms choose different strategies in one of the groups, at least one firm has a chance to change its strategy with positive probability without mutation, and all firms in each group may choose the same strategy in period $t+1$. Thus such a state can not be included in any limit set, and in any state included in some limit set all firms in each group must choose the same strategy. On the other hand, in any state in which all firms in each group choose the same strategy no firm has incentive to change its strategy except for mutation. Therefore a limit set is identified as a set which include a single state in which all firms in each group choose the same strategy[3]. We need no mutation to move from any state, which is not included in a limit set, to a state in some limit set. Thus a long run equilibrium must be in some limit set. According to Kandori and Rob (1995) I consider a reduced Markov chain defined on the limit sets. Denote the state in which all low cost firms choose x and all high cost firms choose y by $\omega(x, y)$. The number of the states (including the states where $x = 0$ or $y = 0$) is $(v + 1)^2$.

Now construct a directed graph which contain the directed paths from $\omega(x, y)$ to $\omega(x^*, y^*)$ for all $(x, y) \neq (x^*, y^*)$, and the path from $\omega(x^*, y^*)$ to some state $\omega(x', y')$, $(x', y') \neq (x^*, y^*)$. Denote this graph by G.

Eliminating the path from $\omega(x, y)$ to $\omega(x^*, y^*)$ in G we get an $\omega(x, y)$-tree. Similarly, eliminating the path from $\omega(x^*, y^*)$ to $\omega(x', y')$ in G we get a $\omega(x^*, y^*)$-tree. An $\omega(x, y)$-tree is a collection of directed branches $(\omega(x_1, y_1), \omega(x_2, y_2))$, $\omega(x_2, y_2)$ is the successor of $\omega(x_1, y_1)$, which satisfy the following conditions[4],

(1) Except for $\omega(x, y)$, each state has a unique successor.
(2) There is no closed loop.

Denote the total number of mutations in an $\omega(x, y)$-tree (or $\omega(x^*, y^*)$-tree) by $C(x, y)$ (or $C(x^*, y^*)$). Based on the results in Freidlin and Wentzel (1984), in their Proposition 4 Kandori and Rob (1995) showed that the long run equilibria comprise the states having minimum $C(x, y)$.

From the arguments in the previous section we see that, since x^* and y^* are the finite population ESS outputs, one mutation is not sufficient and we need at least two mutations to move from the state $\omega(x^*, y^*)$ to any other state. Therefore $C(x, y) \geqq (v + 1)^2$ for $\omega(x, y) \neq \omega(x^*, y^*)$.

Next we examine how many mutations we need to move from a state $\omega(x, y)$, $(x, y) \neq (x^*, y^*)$, to the state $\omega(x^*, y^*)$. Consider a state in which all low cost firms choose x', all high cost firms choose y', and $x' \neq x^*(y')$. $x^*(y')$ is the

[3] This result is similar to Proposition 1 in Vega-Redondo (1997).

[4] For more details about a tree see Kandori et al. (1993) and Vega-Redondo (1996, 1997).

ESS output for the low cost firms relative to y' defined in the previous section. If one low cost firm (mutant firm) chooses $x^*(y')$, the profit of this mutant firm is given by

$$\pi_j^l(x^*(y'), X+Y) = p(X+Y)x^*(y') - c(x^*(y')). \tag{10}$$

And the profits of the other low cost firms are

$$\pi_i^l(x', X+Y) = p(X+Y)x' - c(x') \text{ for } i \neq j, \tag{11}$$

where

$$x_j = x^*(y'), \text{ and } x_i = x' \neq x^*(y') \text{ for all } i \neq j \text{ and } Y = n_2 y'.$$

Comparing Eq. (10) and Eq. (11), we obtain

$$\begin{aligned}&\pi_j^l(x^*(y'), X+Y) - \pi_i^l(x', X+Y)\\ &= p(x^*(y') + (n_1-1)x' + n_2 y')[x^*(y') - x'] - c(x^*(y')) + c(x').\end{aligned}$$

Now we can show

Lemma 1.

$$\pi_j^l(x^*(y'), X+Y) - \pi_i^l(x', X+Y) > 0. \tag{12}$$

Proof. $x^*(y')$ maximizes φ^l in Eq. (3) given Y. It means

$$p((n_1-1)x^*(y') + x' + n_2 y')[x^*(y') - x'] - c(x^*(y')) + c(x') > 0 \tag{13}$$

Consider two cases.

1. If $x^*(y') > x'$, since p is decreasing we have
$$p((n_1-1)x^*(y') + x' + n_2 y') < p(x^*(y') + (n_1-1)x' + n_2 y').$$
Then, we find that Eq. (13) implies Eq. (12) because $x^*(y') - x' > 0$.
2. If $x^*(y') < x'$, since p is decreasing we have
$$p((n_1-1)x^*(y') + x' + n_2 y') > p(x^*(y') + (n_1-1)x' + n_2 y').$$
Then, we find that Eq. (13) implies Eq. (12) because $x^*(y') - x' < 0$.

(Q.E.D.)

This lemma means that we can move from a state $\omega(x', y')$, $x' \neq x^*(y')$, to the state $\omega(x^*(y'), y')$ by one mutation.

Similarly, consider a state $\omega(x', y')$, $y' \neq y^*(x')$. If one high cost firm chooses $y^*(x')$, its profit is

$$\pi_j^h(y^*(x'), X+Y) = p(X+Y)y^*(x') - \lambda c(y^*(x')). \tag{14}$$

And the profits of the other high cost firms are

$$\pi_i^h(y', X+Y) = p(X+Y)y' - \lambda c(y') \text{ for } i \neq j, \tag{15}$$

where

$$y_j = y^*(x'), \text{ and } y_i = y' \neq y^*(x') \text{ for all } i \neq j \text{ and } X = n_1 x'.$$

Comparing Eq. (14) and Eq. (15), we obtain

$$\pi_j^h(y^*(x'), X+Y) - \pi_i^h(y', X+Y)$$
$$= p(n_1 x' + y^*(x') + (n_2 - 1)y')[y^*(x') - y'] - \lambda c(y^*(x')) + \lambda c(y'). \quad (16)$$

Now we can show

Lemma 2.

$$\pi_j^h(y^*(x'), X+Y) - \pi_i^h(y', X+Y) > 0. \quad (17)$$

Proof. Similar to Lemma 1.

This lemma means that we can move from a state $\omega(x', y')$, $y' \neq y^*(x')$, to the state $\omega(x', y^*(x'))$ by one mutation.

(x^*, y^*) is obtained from Eq. (8) and Eq. (9). $x^*(y')$ and $y^*(x')$ are obtained from Eq. (5) with $y = y'$ and Eq. (7) with $x = x'$. Now we can show

Lemma 3.

$$|x^*(y') - x^*| < \frac{n_2}{n_1}|y' - y^*|, \quad (18)$$

and

$$|y^*(x') - y^*| < \frac{n_1}{n_2}|x' - x^*|. \quad (19)$$

Proof. See Appendix.

Eq. (18) implies

$$|x^*(y^*(x')) - x^*| < \frac{n_2}{n_1}|y^*(x') - y^*|. \quad (20)$$

Combining Eq. (19) and Eq. (20), we find

$$|x^*(y^*(x')) - x^*| < |x' - x^*|.$$

Similarly, we obtain

$$|y^*(x^*(y')) - y^*| < |y' - y^*|.$$

Thus, there is a sequence of the states which starts from $\omega(x', y')$, passes through $\omega(x^*(y'), y')$, through $\omega(x^*(y'), y^*(x^*(y')))$, through $\omega(x^*(y^*(x^*(y'))), y^*(x^*(y')))$ and so on. The sequence approaches to $\omega(x^*, y^*)$. If the grid of the firms' outputs is sufficiently fine, this sequence of the states converges to some limit state which is in some neighbourhood of $\omega(x^*, y^*)$. We can move from one state to the next state by only one mutation. Then we obtain the following proposition.

Proposition 2. *For every $\eta > 0$, there is some $\delta' > 0$ such that if the grid step δ is no larger than δ', the long run state of the corresponding finite-state process is in some η-neighborhood of the ESS state $\omega(x^*, y^*)$.*

Denote a limit state of some sequence by $\omega(x^0, y^0)$, which may be different for different starting states. As $\delta \to 0$, any $\omega(x^0, y^0)$ converges to $\omega(x^*, y^*)$. Then we find $C(x^*, y^*) = (v+1)^2 - 1$, which is the number of the states other than $\omega(x^*, y^*)$. It is smaller than $C(x, y)$ for $(x, y) \neq (x^*, y^*)$.

From Proposition 1 and these results we obtain

Proposition 3. *The long run equilibrium output of the low cost firms and that of the high cost firms are equal to the competitive output for each group in the limit as the output grid step approaches to zero.*

4 Conclusion

In this paper I have considered an evolutionary game theoretic model of asymmetric oligopoly with two groups (populations) of firms in which the low cost firms and the high cost firms choose their outputs. I have shown that the finite population evolutionarily stable strategies (ESS) of the low cost and the high cost firms are equal to the competitive outputs. And I have shown that, under the assumption that the marginal cost is increasing, the ESS output in each group of firms is the long run equilibrium strategy in the stochastic evolutionary dynamics.

Appendix: Proof of Lemma 3

Rewriting Eq. (8), Eq. (9), Eq. (5) with $y = y'$ and Eq. (7) with $x = x'$,

$$p(n_1 x^* + n_2 y^*) - c'(x^*) = 0, \tag{21}$$

$$p(n_1 x^* + n_2 y^*) - \lambda c'(y^*) = 0, \tag{22}$$

$$p(n_1 x^*(y') + n_2 y') - c'(x^*(y')) = 0, \tag{23}$$

and

$$p(n_1 x' + n_2 y^*(x')) - \lambda c'(y^*(x')) = 0. \tag{24}$$

Consider two cases.

1. If $y' > y^*$, since $p(\cdot)$ is decreasing and $c'(\cdot)$ is increasing, from Eq. (21), Eq. (23) we have $x^*(y') < x^*$ and $c'(x^*(y')) < c'(x^*)$. Then we find

$$p(n_1 x^*(y') + n_2 y') < p(n_1 x^* + n_2 y^*),$$

and

$$n_1 x^*(y') + n_2 y' > n_1 x^* + n_2 y^*.$$

Arranging this expression, we obtain

$$x^*(y') - x^* > -\frac{n_2}{n_1}(y' - y^*). \tag{25}$$

2. If $y' < y^*$, similarly we have $x^*(y') > x^*$ and $c'(x^*(y')) > c'(x^*)$. Then we find

$$p(n_1 x^*(y') + n_2 y') > p(n_1 x^* + n_2 y^*),$$

and

$$n_1 x^*(y') + n_2 y' < n_1 x^* + n_2 y^*.$$

Arranging this expression, we obtain

$$x^*(y') - x^* < -\frac{n_2}{n_1}(y' - y^*). \tag{26}$$

Since Eq. (25) holds when $y' > y^*$ and $x^*(y') < x^*$, and Eq. (26) holds when $y' < y^*$ and $x^*(y') > x^*$, we obtain

$$|x^*(y') - x^*| < \frac{n_2}{n_1}|y' - y^*|.$$

By similar procedures, using Eq. (22) and Eq. (24), we obtain

$$|y^*(x') - y^*| < \frac{n_1}{n_2}|x' - x^*|.$$

(Q.E.D.)

References

Alchian, A.: Uncertainty, evolution and economic theory. J. Political Econ. **58**, 211–221 (1950)

Crawford, V. P.: An 'evolutionary' interpretation of Van Huyck, Battalio, and Beil's experimental results on coordination. Games and Econ. Behav. **3**, 83–94 (1991)

Freidlin, M. I., Wentzel, A. D.: Random perturbations of dynamical systems. Berlin Heidelberg New York: Springer 1984

Hansen, R. G., Samuelson, W.: Evolution in economic games. J. Econ. Behav. Org. **10**, 315–338 (1988)

Kandori, M., Mailath, G., Rob, R.: Learning, mutation, and long run equilibria in games. Econometrica **61**, 29–56 (1993)

Kandori, M.,Rob, R.: Evolution of equilibria in the long run: a general theory and applications. J. Econ. Theory **65**, 383–414 (1995)

Kandori, M.,Rob, R.: Bandwagon effects and long run technology choice. Games and Econ. Behav. **22**, 30–60 (1998)

Maynard Smith, J.: Evolution and the theory of games. Cambridge: Cambridge University Press 1982

Offerman, T., Sonnemans, J.: Learning by experience and learning by imitating successful others. J. Econ. Behav. Org. **34**, 559–575 (1998)

Pingle, M.. Day, R. H.: Models of economizing behavior: Experimental evidence. J. Econ. Behav. Org. **29**, 191–209 (1996)

Vega-Redondo, F.: Evolution, games and economic behaviour. Oxford University Press 1996

Vega-Redondo, F.: The evolution of Walrasian behavior. Econometrica **65**, 375–384 (1997)

Robson, A., Vega-Redondo, F.: Efficient equilibrium selection in evolutionary games with random matching. J. Econ. Theory **70**, 65–92 (1996)

Schaffer, M. E.: Evolutionarily stable strategies for a finite population and a variable contest size. J. Theoretical Biology **32**, 469–478 (1988)

Schaffer, M. E.: Are profit-maximisers the best survivors? A Darwinian model of economic natural selection. J. Econ. Behav. Org. **12**, 29–45 (1989)

Schlag, K. H.: Why imitate, and if so, how? J. Econ. Theory **78**, 130–156 (1998)