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Speculative securities*

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Summary. A speculative security is an asset whose payoff depends in part on a random shock uncorrelated with economic fundamentals (a sunspot) about which some traders have superior information. In this paper we show that agents may find it desirable to trade such a security in spite of the fact that it is a poorer hedge against their endowment risks at the time of trade, and has an associated adverse selection cost. In the specific institutional setting of innovation of futures contracts, we show that a futures exchange may not have an incentive to introduce a speculative security even when all traders favor it.

Keywords and Phrases: Information revelation, Sunspots, Security design, Futures contracts, Trading volume.

JEL Classification Numbers: D82, G14.

1 Introduction

This paper studies the problem of security design in an asset market with asymmetrically informed traders. It shows that an optimal asset may be a "speculative security," an asset whose payoff depends on a random shock unrelated to endowments and preferences about which some agents have private information. Such a "private information sunspot" introduces noise in the price system that

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reduces the amount of information transmitted to uninformed traders. Under certain conditions, this results in better risk sharing, making all agents better off. In other words, agents prefer to trade a speculative security instead of a nonspeculative one,[1] even though the sunspot dependence by itself makes the asset a worse hedging instrument, and less information revelation imposes an adverse selection cost on traders.

One of the central issues in the literature on security design (for an overview, see Allen and Gale, 1994; Duffie and Rahi, 1995) in the presence of informational asymmetries among agents is to determine the optimal degree of dependence of security payoffs on the private information of agents. One line of research emphasizes the adverse selection that insiders face when the private information component is included in the asset payoff. With asymmetric information about security returns, uninformed traders are reluctant to trade with informed traders and, as a result, the gains from trade are not fully exploited. Rahi (1996) and DeMarzo and Duffie (1995) are papers that fall in this category. In the first paper a risk averse entrepreneur finds it optimal to issue an asset whose payoff does not depend on the entrepreneur's private information. In the second paper, in a risk neutral world, the authors show that in a wide range of cases an intermediary optimally issues an information-free security as a way to minimize the adverse selection problem.

In Marín and Rahi (1995), we point out a countervailing effect of information revelation on security design. We show that, in some cases, it is welfare-improving to restrict the set of tradable securities to reduce the incorporation of private information into the price system. Information has negative value insofar as it restricts risk sharing – risks that have already been resolved can no longer be shared in the market. We label this effect the Hirshleifer effect, since it was Jack Hirshleifer (Hirshleifer, 1971) who first pointed out this aspect of information revelation. In general, when risk sharing is an issue, both effects, the adverse selection and the Hirshleifer effect, will be present in any security design problem in which financial structures differ in the amount of information revealed by prices.

In this paper the incorporation of an asymmetric information component in the asset payoff plays a role that is similar to the reduction of the number of tradable securities in Marín and Rahi (1995). Both are devices that inject noise in the equilibrium price system in a way that less information is revealed to market participants in equilibrium. However, we find the result in the present paper even more striking since the private information component is a lottery completely unrelated to economic fundamentals as opposed to Marín and Rahi (1995) where the private information is correlated with agents' endowments. Furthermore, we generalize Rahi (1996) and show that under certain conditions

[1] In our setting, an asset that does not depend on the sunspot is traded for hedging reasons alone, while a sunspot-dependent asset is also an instrument for speculation in the sense that some agents can trade to exploit their superior information. Hence our use of the terms "speculative" and "nonspeculative."

the main conclusion of that paper – that an optimally designed security minimizes adverse selection – is reversed.

In the traditional view of financial markets, both partial revelation of information (informational inefficiency) and higher volatility of asset prices are welfare-reducing. This is clearly the view that motivates financial regulations such as disclosure rules, margin requirements, and circuit-breakers. We show that, in a fairly standard framework, there are circumstances in which this view is untenable. In our model, a nonspeculative security leads to informational efficiency and minimizes price volatility but may nevertheless be Pareto dominated by a speculative security.

After demonstrating that a speculative security may be Pareto-preferred, we investigate the incentives of a futures exchange to introduce such a security. Maximization of trading volume is widely perceived to be an appropriate objective for a futures exchange, both in practice and in the theoretical modeling of futures innovation. However, we find that a volume-maximizing exchange may or may not have an incentive to innovate a speculative contract depending on the way the contract size is normalized, which is essentially arbitrary. A more satisfactory objective is maximization of transaction fee revenue, wherein the exchange designs a futures contract and chooses the fees traders have to pay per transaction. It turns out that in this case the exchange always chooses not to include the speculative component in the asset payoff, even when it makes all the traders in the economy better off.

The paper is organized as follows. In the next section we describe the model, which is a variant of the exponential-normal framework for studying security design that is outlined in Duffie and Rahi (1995). We derive the rational expectations equilibrium for any given asset that is made available for trade. In Section 3, we analyze the welfare impact of security design, and provide conditions under which a speculative asset Pareto dominates a nonspeculative asset. Section 4 looks at the security design problem from the perspective of a futures exchange. We explore the link between the choice of futures contract and trading volume, and show that a revenue-maximizing contract is necessarily nonspeculative. Section 5 concludes. Proofs and technical results are in the Appendix.

2 The model

We consider a static one-good economy with a single risky asset and a riskfree bond whose interest rate is normalized to zero.[2] Both assets are in zero net supply. There are two agents with CARA utility. Agent 1 has risk aversion coefficient r_1 and initial endowment $e_1 := k_1 x z$, where x and z are independent normal random variables[3] and k_1 is a scalar. The random variable x is privately known to agent 1 at the time of trading and can be thought of as the size of his hedging needs. Agent 1 also observes a private signal s. This signal is normally distributed,

[2] We do not need a riskless asset when the risky security is a futures contract, as in Section 4.

[3] All random variables are defined on a fixed probability space.

independent of x and z. By a judicious choice of units we can normalize the variances of x, z, and s to be one. Agent 2 has risk aversion r_2 and endowment $e_2 := k_2 z$. He has no private information. We refer to agent 1 as "informed" and agent 2 as "uninformed."[4] We assume that these agents behave competitively; hence they should be thought of as representing a continuum of each type.

The aggregate risk in the economy is given by $(xk_1 + k_2)z$, which is independent of s. Indeed s is a pure "sunspot," extraneous uncertainty which is independent of agents' preferences and endowments. The parameters k_1 and k_2 determine the size of the aggregate risk, as well as the degree of heterogeneity among traders (and hence the gains from trade). We make the following technical assumption:

Assumption 1. $\quad r_1^2 k_1^2 < 1.$

This is a necessary and sufficient condition for the *ex ante* expected utility of the informed agent to be well-defined (as will be clear from the proof of Lemma A.3 in the Appendix). On the other hand, the uninformed agent's expected utility is well-defined for all r_2 and k_2. We also take k_1 to be nonzero (for otherwise, equilibrium will necessarily be fully revealing).

After agent 1 has observed his private signals, he trades the available securities with agent 2 in a competitive rational expectations equilibrium. Subsequently, all uncertainty is resolved, the assets pay off, and consumption takes place.

We now parameterize the risky asset as follows. The payoff of this asset, denoted f, is linear in the endowment risk z and the signal s:

$$f = az + bs, \tag{1}$$

with a nonzero. Our goal is to analyze the effect on agents' welfare of the choice of the security design parameters, a and b. If b is zero, we refer to the asset as "nonspeculative." There is no asymmetric information about the payoff of a nonspeculative asset and it is traded for hedging reasons alone. In fact it is a perfect hedge for the z risk. For nonzero b we term the asset "speculative." As mentioned earlier, the speculative component s is pure sunspot uncertainty independent of agents' endowments. Furthermore, as will become clear shortly, it matters precisely because there is asymmetric information about it.[5]

A position θ_i in the risky asset leaves agent i with end-of-period wealth

$$w_i := e_i + \theta_i(f - p), \tag{2}$$

where p is the asset price. Agents have rational expectations and learn from prices, *i.e.* they know what the random variable p is and condition on it. The

[4] For concreteness, one can think of the agents as farmers, where k_i is the size of farmer i's farm, x is the productivity per acre for the first farmer (while it is one for the second farmer), and z is the (exogenous) price of the farm output at the time of harvest.

[5] The conventional usage of the term "sunspot" is for a random variable that is extraneous with respect to the primitives of the economy – preferences, endowments, and asset payoffs. However, in this paper, the asset is not exogenously given. Hence we refer to s as a sunspot even though the asset payoff may depend on it.

information of agent 1 is $\mathcal{F}_1 := (s, x, p)$, and that of agent 2 is $\mathcal{F}_2 := p$. Agent i solves the following maximization problem:

$$\max_{\theta_i \in \mathcal{M}_i} \quad E[-\exp(-r_i w_i)], \tag{3}$$

where w_i is given by (2), and $\mathcal{M}_i$ is the space of $\mathcal{F}_i$-measurable random variables. Assuming for the moment that p, f, and e_i are joint normal (which will be the case in equilibrium), any choice of θ_i leaves net wealth w_i normally distributed, conditional on $\mathcal{F}_i$. Therefore, agent i's expected utility is

$$\begin{aligned} E[-\exp(-r_i w_i)] &= -E\left[E[\exp(-r_i w_i)|\mathcal{F}_i]\right] \\ &= -E\left[\exp\left(-r_i\left[E(w_i|\mathcal{F}_i) - \tfrac{r_i}{2}\mathrm{Var}(w_i|\mathcal{F}_i)\right]\right)\right]. \end{aligned} \tag{4}$$

Let

$$\mathcal{E}_i := E(w_i|\mathcal{F}_i) - \frac{r_i}{2}\mathrm{Var}(w_i|\mathcal{F}_i). \tag{5}$$

The problem (3) is equivalent to choosing a position θ_i to maximize $\mathcal{E}_i$ pointwise for each realization of $\mathcal{F}_i$. From (2):

$$\mathcal{E}_i = E(e_i|\mathcal{F}_i) + \theta_i\left[E(f|\mathcal{F}_i) - p\right] - \frac{r_i}{2}\left[\mathrm{Var}(e_i|\mathcal{F}_i) + \theta_i^2\mathrm{Var}(f|\mathcal{F}_i) + 2\theta_i\mathrm{cov}(f, e_i|\mathcal{F}_i)\right]. \tag{6}$$

The solution to (3) can readily be calculated:

$$\theta_i = \frac{E(f|\mathcal{F}_i) - p - r_i\mathrm{cov}(f, e_i|\mathcal{F}_i)}{r_i\mathrm{Var}(f|\mathcal{F}_i)}, \tag{7}$$

so that

$$\theta_1 = \frac{bs - p - r_1 k_1 a x}{r_1 a^2} \tag{8}$$

and

$$\theta_2 = \frac{bE(s|p) - p - r_2 k_2 a}{r_2[a^2 + b^2\mathrm{Var}(s|p)]} \tag{9}$$

For reasons of tractability, we limit our attention to linear equilibria.

Definition 1. *A linear rational expectations equilibrium is a 3-tuple of random variables* (θ_1, θ_2, p)*, such that* p *is of the form:*

$$p = \overline{p} + \alpha x + \beta s, \qquad (\overline{p}, \alpha, \beta) \in \mathbb{R}^3; \tag{10}$$

given this price function, θ_i *solves the utility maximization problem (3) for* $i = 1, 2$*; and markets clear for every realization of private information:*

$$\theta_1 + \theta_2 = 0. \tag{11}$$

Given an asset of the form (1), there exists a unique equilibrium in the linear class.

Lemma 2.1. *There exists a unique linear rational expectations equilibrium. The price function is*

$$p = \frac{1}{D}\left[[(r_1 + r_2)b^2 + r_1^2 r_2 k_1^2 (a^2 + b^2)](bs - r_1 k_1 ax) - r_1 r_2 k_2 a (r_1^2 k_1^2 a^2 + b^2)\right],$$

and the equilibrium asset position of agent 1 is

$$\theta_1 = \frac{1}{aD}\left[r_1^2 k_1^2 a(bs - r_1 k_1 ax) + r_2 k_2 (r_1^2 k_1^2 a^2 + b^2)\right],$$

where

$$D := (r_1 + r_2)(r_1^2 k_1^2 a^2 + b^2) + r_1^2 r_2 k_1^2 b^2.$$

The equilibrium asset position of agent 2 is, of course, just the negative of that of agent 1. By observing the equilibrium price (and knowing the price function), the uninformed agent learns the random variable $(bs - r_1 k_1 ax)$, which is a linear combination of the informed agent's private signals. Thus the equilibrium is partially revealing – the informed agent knows the realizations of s and x but the uninformed agent does not. If b is zero, the information s becomes irrelevant; the equilibrium is then fully revealing with respect to the relevant information, x.

3 Welfare

In this section we analyze the impact of security design on agents' welfare. In particular, we are interested in identifying conditions under which a speculative asset Pareto dominates a nonspeculative asset. We measure agent i's welfare by his certainty-equivalent wealth in equilibrium. Any given asset of the form (1) gives rise to a unique linear equilibrium (Lemma 2.1), with associated terminal wealth $w_i^*(a, b)$ for agent i. The certainty-equivalent of w_i^* is given by

$$\mathscr{U}_i := -\frac{1}{r_i}\ln\left[E[\exp(-r_i w_i^*)]\right]. \tag{12}$$

Hence $\mathscr{U}_i$ is agent i's certainty-equivalent wealth in equilibrium, or equilibrium utility for short.

Lemma A.3 in the Appendix gives closed-form expressions for agents' equilibrium utilities. These depend on the asset payoff parameters, a and b, only through μ^2, where $\mu := b/a$. Differentiating, and using Assumption 1, we obtain the following comparative statics results:

Lemma 3.1. *For the informed agent,*

$$\left(\frac{\partial \mathscr{U}_1}{\partial(\mu^2)}\right)_{\mu=0} > 0$$

if and only if

$$r_2^2k_2^2(r_1+r_2)(1-r_1^2k_1^2)\left[r_1^2(1+r_1^2k_1^2)-2r_2(r_1+r_2)(1-r_1^2k_1^2)\right]$$
$$> \left[(r_1+r_2)^2(1-r_1^2k_1^2)+r_1^4k_1^2\right]\cdot\left[(r_1+r_2)(1+r_1^2k_1^2)+2r_1^2r_2k_1^2\right].$$

For the uninformed agent,

$$\left(\frac{\partial \mathscr{U}_2}{\partial(\mu^2)}\right)_{\mu=0} > 0$$

if and only if

$$r_1^2r_2^2k_2^2(r_1+r_2) > \left[(r_1+r_2)(1-r_1^2k_1^2)+2r_1^2r_2k_1^2\right]\cdot\left[(r_1+r_2)^2+r_1^2r_2^2k_1^2\right].$$

The lemma provides necessary and sufficient conditions for a local Pareto improvement with a speculative asset relative to a nonspeculative asset. In order to understand these conditions we need to analyze the interplay between the various effects of introducing the speculative component into the asset payoff. First, we have the spanning effect: an asset with a speculative component is a worse hedging instrument. Since the equilibrium is partially revealing for nonzero b, the uninformed agent can no longer get a perfect hedge and reduces his trading activity. Second, there is the adverse selection effect: with partial revelation the uninformed agent is less willing to trade, again reducing the set of (information-constrained) feasible allocations. Both these effects operate in the same direction and the result would be to make both agents worse off. The incidence of the welfare loss depends on prices.

Counteracting these forces is the Hirshleifer effect. Less accurate information on aggregate risk at the time of trade can improve agents' welfare. To see more clearly how this can happen, let us think of a simple Edgeworth box economy with a single good, two states of the world, and two agents. Suppose there is symmetric information and asset markets are complete. Then agents can trade to a point on the contract curve. If, however, the true state of the world is revealed before trade, no trade takes place. This is clearly inefficient unless the initial endowment is on the contract curve.

In our economy, the Hirshleifer effect is somewhat more subtle. Introducing dependence on extraneous private information results in partial revelation of endowment-related information (x). Even though the asset payoff is independent of x, this risk can be hedged to some extent (from the *ex ante* point of view) since portfolios can depend on x. As in the Edgeworth box example, partial revelation of endowment risk can improve risk sharing.

To get a Pareto improvement, the Hirshleifer effect must outweigh the spanning and adverse selection effects. Lemma 3.1 provides (necessary and sufficient) conditions on the parameters of the model for which this is the case. A necessary condition for the utility of the informed agent to be higher with a speculative asset is that the risk aversion of the uninformed agent, r_2, is small. This can be interpreted as saying that the negative spanning effect should be weak, since the extra noise in a speculative asset hurts the uninformed agent (and through prices, the informed agent)[6] more the higher is his risk aversion. If r_2 is small, we can obtain a Pareto improvement provided the size of the uninformed agent's endowment, k_2, is large. The idea here is that for the Hirshleifer effect to dominate, the aggregate initial risk must be large, otherwise less revelation does not have enough scope for improving risk sharing. The only question that remains is why we cannot generate the same effect through a large k_1. The reason is that prices become less informative with respect to s as k_1 increases (recall that prices reveal the random variable $(bs - r_1k_1x)$), exacerbating both the adverse selection and spanning effects. Indeed as $r_1^2k_1^2$ approaches one (which it cannot exceed due to Assumption 1), the utility improvement condition for the informed agent is necessarily violated. We can circumvent this by reducing r_1 as we increase k_1 (keeping r_1k_1 constant, say), but this again strengthens the adverse selection effect as the informed agent becomes almost a pure speculator while prices do not reveal any more information. In this case we see that the utility improvement condition for the uninformed agent becomes difficult to satisfy. To summarize:

Proposition 3.2. *If the uninformed agent is not too risk averse and his risk exposure is sufficiently large, both agents prefer a speculative to a nonspeculative asset.*

If k_2 is zero, we are in the setting of Rahi (1996). Lemma 3.1 confirms that in this case the informed agent prefers a nonspeculative asset, as was shown in Rahi (1996). In fact it goes further: a speculative asset is Pareto dominated. As we have seen, however, these results hold only because the Hirshleifer effect is weak.

Note that adding a speculative component to the asset payoff can be Pareto improving, even though it necessarily results in higher volatility of the asset price, as can be verified from the equilibrium price function (Lemma 2.1):

Proposition 3.3. *The variance of the price of an asset with payoff* $(z + bs)$ *is increasing in* $|b|$.

Propositions 3.2 and 3.3 show that market efficiency, as commonly understood in finance, and low volatility may be in conflict with Pareto efficiency. A nonspeculative security has a fully revealing price. Introducing a speculative component causes the price to be partially revealing and more volatile. Nevertheless, under the conditions of Proposition 3.2, all agents prefer to trade a speculative security.

4 Futures innovation

We now study security design by a futures exchange. If the exchange can levy lumpsum fees on agents who wish to trade its contracts, and is thus able to extract some of the surplus that agents get from trading, it will issue a speculative

[6] Since the informed agent knows s he does not face a negative spanning effect directly. Hence a restriction on his risk aversion coefficient r_1 is not needed.

contract under the conditions of Proposition 3.2. In actual practice, although lumpsum fees are charged in the form of seat prices, the seats derive their value from commissions that exchange members can charge nonmembers who wish to trade the exchange's contracts. Exchanges also seem to be concerned about the volume of trade in their contracts. For a discussion, see Duffie and Rahi (1995).

The theoretical literature on futures innovation has, by and large, taken trading volume as the objective function of a futures exchange (see, for example, Duffie and Jackson, 1989; Marín and Rahi, 1995; Rahi, 1995). We adopt this perspective in Section 4.1. We find, however, that trading volume is a problematic criterion. A more satisfactory formulation, presented in Section 4.2, is one in which the exchange charges commissions and maximizes revenue. This type of objective function has been considered previously by Hara (1995) and $\bar{\text{O}}$hashi (1992). However, these authors do not characterize the optimal contract when the exchange chooses both the fee and the security payoff.

4.1 Trading volume

We first analyze optimal contract design by an exchange that maximizes the expected volume of trade, $\mathscr{V} := E(|\theta_1| + |\theta_2|)$. The equilibrium asset position θ_1 (see Lemma 2.1) is homogeneous of degree -1 in the standard deviation of the asset payoff. Some kind of normalization is clearly needed, since trading volume can be increased arbitrarily by scaling down the contract. In previous work it has typically been assumed that the standard deviation of the contract is one, which in our case means setting $a^2 + b^2 = 1$.

Proposition 4.1. *With the normalization $a^2+b^2 = 1$, if the informed agent is more risk averse than the uninformed, and the risk exposure of the uninformed agent is large enough, a volume-maximizing futures exchange will prefer a speculative contract to a nonspeculative one.*

It appears that a volume-maximizing exchange will innovate a speculative contract under conditions similar to those in which all agents find a speculative asset desirable. On closer examination, however, the "normalization" we have used is not that innocuous. What, after all, is the rationale behind taking the standard deviation of the contract to be one, especially in an asymmetric information setting in which the standard deviation is different across agents at the time of trade? For instance, the informed agent in this model knows the realization of the signal so that the conditional variance of the asset payoff for him depends only on the coefficient a. With the normalization $a^2+b^2 = 1$, the exchange can choose an asset with an arbitrarily large speculative component, and a correspondingly small weight on the risk factor z, thus inducing the informed agent to trade an arbitrarily large quantity. Indeed, it is straightforward to check that, under the conditions of Proposition 4.1, a volume-maximizing contract does not exist. Volume approaches infinity as a approaches zero (and b goes to infinity), but there is no trade when a is zero.

An alternative normalization one might use is $a = 1$. Here the interpretation is that the exchange starts from a benchmark contract with payoff equal to z and contemplates adding on a speculative component bs. We get the following result for this case:

Proposition 4.2. *With the normalization $a = 1$, a volume-maximizing futures exchange will always choose a nonspeculative contract.*

Once again one wonders if this result is not driven simply by the fact that introducing a speculative component in the asset payoff increases the variance of the payoff for the uninformed agent. Our conclusion is that it is problematic to measure the volume of trade. Any normalization of the size of the contract is arbitrary.

4.2 Transactions fee revenue

We now provide a model of a revenue-maximizing futures exchange that chooses both the contract and a fee for trading the contract. This construction requires no normalization, as we shall see. The exchange designs a contract with payoff of the form (1). The contract is traded by two (groups of) agents as in Section 2. The utility functions, endowments and information of these agents are as in Section 2. In this case, however, transactions incur a fee that must be paid to the exchange. For the sake of analytical convenience we choose a quadratic fee structure. Specifically, if an agent's asset position is θ, he pays $\frac{T}{2}\theta^2$. The exchange chooses the security design parameters a and b and the transactions fee T to maximize expected revenue, *i.e.* it solves the following optimization problem:

$$\max_{a,b,T} E(T\theta_1^2). \tag{13}$$

Note that in equilibrium the squared asset position is the same for both agents. To solve this problem we need to compute the rational expectations equilibrium for any given a, b, and T. The terminal wealth of agent i is

$$w_i = e_i + \theta_i(f - p) - \frac{T}{2}\theta_i^2.$$

Analogous to (7), the optimal position is

$$\theta_i = \frac{E(f|\mathscr{F}_i) - p - r_i \operatorname{cov}(f, e_i|\mathscr{F}_i)}{r_i \operatorname{Var}(f|\mathscr{F}_i) + T}.$$

The definition of equilibrium is the same as Definition 1, supplemented with (13). We refer to the economy described in this subsection as the "transactions fee economy" to differentiate it from the economy that we have studied heretofore. We first derive the rational expectations equilibrium:

Lemma 4.3. *There exists a unique linear rational expectations equilibrium in the transactions fee economy. The price function is*

$$p = \frac{1}{\overline{D}}\Big[\big([(r_1+r_2)a^2+T]b^2+r_1^2k_1^2a^2[r_2(a^2+b^2)+T]\big)$$
$$\times(bs-r_1k_1ax)-r_2k_2a(r_1a^2+T)(r_1^2k_1^2a^2+b^2)\Big],$$

and the equilibrium asset position of agent 1 is

$$\theta_1 = \frac{a}{\overline{D}}\Big[r_1^2k_1^2a(bs-r_1k_1ax)+r_2k_2(r_1^2k_1^2a^2+b^2)\Big],$$

where

$$\overline{D} := [(r_1+r_2)a^2+2T](r_1^2k_1^2a^2+b^2)+r_1^2r_2k_1^2a^2b^2.$$

The proof is analogous to that of Lemma 2.1. It turns out that the exchange's fee revenue can be expressed as a function of $\tau := \frac{T}{a^2}$ and μ. See Lemma A.4, which also gives the equilibrium utility (certainty-equivalent wealth) of the traders for any choice (τ,μ) of the exchange.[7]

Proposition 4.4. *The revenue-maximizing contract for the exchange is the non-speculative contract ($\mu = 0$). The optimal transactions fee τ is $\frac{r_1+r_2}{2}$.*

In the previous section we saw that under some conditions (Proposition 3.2) traders unanimously prefer a speculative asset. The above proposition, on the other hand, asserts that a revenue-maximizing exchange would never introduce a speculative asset. One might ask if a speculative asset would be preferred by all traders if the exchange can levy transaction fees. In other words, if the traders could dictate the choice of contract knowing that the exchange would charge fees to maximize its revenue given the contract, is it possible that they would still want a speculative contract? The following lemma gives the requisite conditions:

Lemma 4.5. *In the transactions fee economy, for the informed agent,*

$$\left(\frac{\partial\mathscr{U}_1}{\partial(\mu^2)}\right)_{\mu=0} > 0$$

if and only if

$$r_2^2k_2^2(r_1+r_2)(1-r_1^2k_1^2)\big[r_1(3r_1+r_2)^2(1+r_1^2k_1^2)$$
$$-8r_2(r_1+r_2)(1-r_1^2k_1^2)(3r_1+r_2+2r_1^5k_1^4)\big]$$
$$> \big[8(r_1+r_2)^2(1-r_1^2k_1^2)+r_1^3k_1^2(3r_1+r_2)\big]$$
$$\cdot\big[(3r_1+r_2)[r_1+r_2+r_1^2k_1^2(r_1+2r_2)]+r_1^7r_2k_1^6\big].$$

For the uninformed agent,

$$\left(\frac{\partial\mathscr{U}_2}{\partial(\mu^2)}\right)_{\mu=0} > 0$$

[7] It is clear that the security design parameter a has no substantive effect on the exchange's revenues or on agent's utilities. It merely scales the transaction fee and the weight on the speculative component of the asset.

if and only if

$$
\begin{aligned}
&r_2^3 k_2^2 (r_1 + r_2)(9r_1^2 - 15r_2^2 - 10r_1 r_2) \\
&\quad > \left[(r_1 + r_2)(r_1 + 2r_2) + r_2(r_1 + r_2)(1 - r_1^2 k_1^2) + 4r_1^2 r_2^2 k_1^2\right] \\
&\qquad \cdot \left[8(r_1 + r_2)^2 + r_1^2 r_2 k_1^2 (r_1 + 3r_2)\right].
\end{aligned}
$$

Proof. From Lemma A.4, the revenue-maximizing τ for a given μ is

$$
\tau = \frac{r_1 + r_2}{2} + \frac{r_1^2 r_2 k_1^2 \mu^2}{2(r_1^2 k_1^2 + \mu^2)}.
$$

Substituting this in the utility expressions in Lemma A.4, and differentiating, we obtain the result. □

From the proof one can see that the exchange charges a higher transactions fee for a speculative contract than for a nonspeculative one. Nevertheless the traders prefer a speculative contract under conditions similar to those required for Proposition 3.2.

Proposition 4.6. *If the uninformed agent is not too risk averse and his risk exposure is sufficiently large, both agents prefer a speculative to a nonspeculative asset, even if the futures exchange optimally charges a transactions fee.*

This proposition is an immediate corollary of Lemma 4.5. The intuition behind the result is the same one as explained in the previous section. The only additional effect in this case is that a speculative asset is associated with a higher transactions fee, making it more expensive for agents to hedge their risk. This results in a greater utility loss the more risk averse the agent is. That is why a necessary condition for the uninformed agent's utility to be higher with a speculative contract is that his risk aversion be small. This condition was not needed in the absence of transaction fees (see Lemma 3.1).

5 Conclusion

We have shown in this paper that a sunspot-dependent speculative security can lead to a better allocation of risk than a nonspeculative security. The sunspot introduces noise in the price system which may be desirable. We have also considered a particular institutional mechanism for the design of securities: innovation of futures contracts. Introducing a speculative component into the asset payoff may increase or decrease trading activity depending on how this activity is measured. But the effect on the commission revenue of a futures exchange is unambiguous – revenue is always lower with a speculative security. Hence the incentives of an futures exchange may be in conflict with the interests of hedgers.

While we make strong parametric assumptions to demonstrate these results, our analysis raises issues of a more general nature. Security innovation not only affects spanning but also alters the information that agents have when they trade. The informational effect is two-fold: adverse selection considerations point to the desirability of more revelation, while the Hirshleifer effect works in the opposite direction. These effects need to be traded off when securities are designed. Developing general criteria for determining which effect is stronger in any given instance remains an open problem.

The fact that it is difficult to find examples of real-world speculative securities suggests that spanning and adverse selection may be more important than the Hirshleifer effect. On the other hand, it could be a problem of misalligned incentives, as highlighted by our analysis of futures innovation. It is in this latter instance that there may be a case for regulatory intervention.

Appendix

Lemma A.1. *Suppose* $\mathbf{A}$ *is a symmetric* $n \times n$ *matrix,* $\mathbf{b}$ *is an* n*-vector,* c *is a scalar, and* $\mathbf{w}$ *is an* n*-dimensional normal variate:* $\mathbf{w} \sim N(\mathbf{0}, \Sigma)$, Σ *positive definite. Then* $E[\exp(\mathbf{w}^\top \mathbf{A}\mathbf{w} + \mathbf{b}^\top \mathbf{w} + c)$ *is well-defined if and only if* $(\mathbf{I} - 2\Sigma\mathbf{A})$ *is positive definite, and*

$$E[\exp(\mathbf{w}^\top \mathbf{A}\mathbf{w} + \mathbf{b}^\top \mathbf{w} + c) = |\mathbf{I} - 2\Sigma\mathbf{A}|^{-\frac{1}{2}} \exp[\frac{1}{2}\mathbf{b}^\top (\mathbf{I} - 2\Sigma\mathbf{A})^{-1}\Sigma\mathbf{b} + c].$$

Proof.

$$\begin{aligned}
&E[\exp(\mathbf{w}^\top \mathbf{A}\mathbf{w} + \mathbf{b}^\top \mathbf{w} + c)] \\
&= \int_{\mathbb{R}^m} \exp(\mathbf{w}^\top \mathbf{A}\mathbf{w} + \mathbf{b}^\top \mathbf{w} + c)(2\pi)^{-\frac{m}{2}}|\Sigma|^{-\frac{1}{2}} \exp(-\frac{1}{2}\mathbf{w}^\top \Sigma^{-1}\mathbf{w})\, d\mathbf{w} \\
&= \int_{\mathbb{R}^m} (2\pi)^{-\frac{m}{2}}|\Sigma|^{-\frac{1}{2}} \exp[-\frac{1}{2}\mathbf{w}^\top(\Sigma^{-1} - 2\mathbf{A})\mathbf{w} + \mathbf{b}^\top \mathbf{w} + c]\, d\mathbf{w} \\
&= \int_{\mathbb{R}^m} (2\pi)^{-\frac{m}{2}}|\Sigma|^{-\frac{1}{2}} \exp[-\frac{1}{2}(\mathbf{w} - \overline{\mathbf{w}})^\top(\Sigma^{-1} - 2\mathbf{A})(\mathbf{w} - \overline{\mathbf{w}}) \\
&\quad + \frac{1}{2}\mathbf{b}^\top(\Sigma^{-1} - 2\mathbf{A})^{-1}\mathbf{b} + c]\, d\mathbf{w} \\
&= |\Sigma|^{-\frac{1}{2}}|(\Sigma^{-1} - 2\mathbf{A})^{-1}|^{\frac{1}{2}} \exp[\frac{1}{2}\mathbf{b}^\top(\Sigma^{-1} - 2\mathbf{A})^{-1}\mathbf{b} + c],
\end{aligned}$$

where $\overline{\mathbf{w}} = (\Sigma^{-1} - 2\mathbf{A})^{-1}\mathbf{b}$. The result follows immediately. □

We will need this lemma to calculate *ex ante* utilities. The following lemma, which is straightforward to show, is useful for analyzing the volume of trade:

Lemma A.2. *Let* $X \sim N(m, \sigma^2)$. *Then*

$$E(|X|) = \frac{1}{\sqrt{2\pi}}\left[2\sigma e^{-\frac{1}{2}(\frac{m}{\sigma})^2} + m\int_{-\frac{m}{\sigma}}^{\frac{m}{\sigma}} e^{-\frac{1}{2}y^2} dy\right].$$

In particular,

$$\frac{\partial E(|X|)}{\partial m} = \frac{1}{\sqrt{2\pi}} \int_{-\frac{m}{\sigma}}^{\frac{m}{\sigma}} e^{-\frac{1}{2}y^2} dy$$

and

$$\frac{\partial E(|X|)}{\partial(\sigma^2)} = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{1}{2}(\frac{m}{\sigma})^2}.$$

Proof of Lemma 2.1. Using (10) and the standard theory of the multivariate normal distribution (see, for example, Anderson (1984), Ch. 1):

$$E(s|p) = \frac{\beta(p-\overline{p})}{\alpha^2+\beta^2}, \qquad \mathrm{Var}(s|p) = \frac{\alpha^2}{\alpha^2+\beta^2}.$$

Substituting in (9), and using the market clearing condition (11), we obtain the equilibrium price function in terms of $\overline{p}, \alpha,$ and β:

$$\begin{aligned} p \quad = \quad & \frac{1}{Q}\Big(r_1a^2\left[r_2k_2a(\alpha^2+\beta^2)+b\beta\overline{p}\right]+r_1r_2k_1a\left[a^2(\alpha^2+\beta^2)+b^2\alpha^2\right]x \\ & -r_2b\left[a^2(\alpha^2+\beta^2)+b^2\alpha^2\right]s\Big), \end{aligned}$$

where

$$Q := r_1a^2b\beta - r_2b^2\alpha^2 - a^2(r_1+r_2)(\alpha^2+\beta^2).$$

Comparing coefficients with (10), we can solve for $\frac{\alpha}{\beta}$, and subsequently $\overline{p}, \alpha$, and β. The asset position θ_1 can now be derived by substituting the price function in (8). □

Lemma A.3. *The equilibrium utility of agents is given by*

$$\mathscr{U}_1 = \frac{1}{2r_1}\left[\ln M + \frac{r_1^2r_2^2k_2^2(1-r_1^2k_1^2)(r_1^2k_1^2+\mu^2)^2}{F^2M}\right]$$

and

$$\mathscr{U}_2 = \frac{1}{2r_2}\left[-r_2^2k_2^2 + \ln(1+r_1^4r_2^2k_1^4N) + \frac{r_2^4k_2^2(r_1^2k_1^2+\mu^2)N}{1+r_1^4r_2^2k_1^4N}\right],$$

where

$$\begin{aligned} F &:= \quad (r_1+r_2)(r_1^2k_1^2+\mu^2)+r_1^2r_2k_1^2\mu^2, \\ M &:= \quad (1-r_1^2k_1^2)+r_1^6k_1^4F^{-2}[r_1^2k_1^2+(1-r_1^2k_1^2)\mu^2], \qquad (14) \\ N &:= \quad F^{-2}[r_1^2k_1^2+\mu^2+r_1^2k_1^2\mu^2]. \end{aligned}$$

Proof. Using (6) and (7), in equilibrium,

$$\begin{aligned}\mathscr{E}_i \quad &= E(e_i|\mathscr{I}_i) - \tfrac{r_i}{2}\mathrm{Var}(e_i|\mathscr{I}_i) + \theta_i\left[E(f|\mathscr{I}_i) - p - r_i\mathrm{cov}(f, e_i|\mathscr{I}_i)\right]\\ &\quad -\tfrac{r_i}{2}\theta_i^2\mathrm{Var}(f|\mathscr{I}_i)\\ &= E(e_i|\mathscr{I}_i) - \tfrac{r_i}{2}\mathrm{Var}(e_i|\mathscr{I}_i) + \tfrac{r_i}{2}\theta_i^2\mathrm{Var}(f|\mathscr{I}_i).\end{aligned}$$

Now, from (4), (5) and (12),

$$\mathscr{U}_i = -\frac{1}{r_i}\ln\left[E[\exp(-r_i\mathscr{E}_i)]\right],$$

and the desired expressions follow from an application of Lemma A.1. The positive definiteness condition in Lemma A.1 is equivalent to Assumption 1. □

Lemma A.4. *For any given choice (τ, μ) of the exchange, the equilibrium utility of agents in the transactions fee economy is given by*

$$\mathscr{U}_1 = \frac{1}{2r_1}\left[\ln\overline{M} + \frac{r_1r_2^2k_2^2(r_1+\tau)(1-r_1^2k_1^2)(r_1^2k_1^2+\mu^2)^2}{\overline{F}^2\overline{M}}\right]$$

and

$$\mathscr{U}_2 = \frac{1}{2r_2}\left[-r_2^2k_2^2 + \ln(1 + r_1^4r_2^2k_1^4\overline{N}) + \frac{r_2^4k_2^2(r_1^2k_1^2+\mu^2)\overline{N}}{1 + r_1^4r_2^2k_1^4\overline{N}}\right],$$

where

$$\begin{aligned}\overline{F} &:= (r_1 + r_2 + 2\tau)(r_1^2k_1^2 + \mu^2) + r_1^2r_2k_1^2\mu^2,\\ \overline{M} &:= (1 - r_1^2k_1^2) + r_1^5k_1^4(r_1+\tau)\overline{F}^{-2}[r_1^2k_1^2 + (1 - r_1^2k_1^2)\mu^2],\\ \overline{N} &:= \overline{F}^{-2}r_2^{-1}[(r_2+\tau)(r_1^2k_1^2+\mu^2) + r_1^2r_2k_1^2\mu^2].\end{aligned}$$

The revenue of the exchange is

$$\tau\overline{F}^{-2}(r_1^2k_1^2+\mu^2)[r_1^2k_1^2(r_1^2k_1^2+r_2^2k_2^2) + r_2^2k_2^2\mu^2].$$

Proof of Proposition 4.1. We denote the mean and standard deviation of θ_1 by m_θ and σ_θ respectively. Using the expression for the equilibrium holding θ_1 from Lemma 2.1 and the normalization $a^2 + b^2 = 1$, we get

$$m_\theta = \frac{(1+\mu^2)^{\frac{1}{2}}}{F}\cdot r_2k_2(r_1^2k_1^2+\mu^2) \tag{15}$$

and

$$\sigma_\theta^2 = \frac{1+\mu^2}{F^2}\cdot r_1^4k_1^4(r_1^2k_1^2+\mu^2), \tag{16}$$

where F is given by (14). The expected volume of trade is

$$\mathscr{V} = 2E(|\theta_1|).$$

Now

$$\frac{\partial \mathscr{V}}{\partial(\mu^2)} = 2\left[\frac{\partial E(|\theta_1|)}{\partial m_\theta} \cdot \frac{\partial m_\theta}{\partial(\mu^2)} + \frac{\partial E(|\theta_1|)}{\partial \sigma_\theta^2} \cdot \frac{\partial \sigma_\theta^2}{\partial(\mu^2)}\right].$$

Using Lemma A.2, (15) and (16),

$$\left(\frac{\partial \mathscr{V}}{\partial(\mu^2)}\right)_{\mu=0} = \sqrt{\frac{2}{\pi}}\left[\frac{r_2k_2(r_1-r_2)}{2(r_1+r_2)^2} \cdot \int_{-\frac{m_\theta}{\sigma_\theta}}^{\frac{m_\theta}{\sigma_\theta}} e^{-\frac{1}{2}y^2}dy - \frac{(r_1+r_2)(1-r_1^2k_1^2)+2r_1^2r_2k_1^2}{\sigma_\theta(r_1+r_2)^3} \cdot e^{-\frac{1}{2}\left(\frac{m_\theta}{\sigma_\theta}\right)^2}\right],$$

which is positive if $r_1 > r_2$ and k_2 is sufficiently large. □

Proof of Proposition 4.2. If $a = 1$, the mean and standard deviation of the equilibrium holding θ_1 are

$$m_\theta = \frac{1}{F} \cdot r_2k_2(r_1^2k_1^2 + \mu^2)$$

and

$$\sigma_\theta^2 = \frac{1}{F^2} \cdot r_1^4k_1^4(r_1^2k_1^2 + \mu^2),$$

respectively, where F is given by (14). From Lemma A.2, the volume of trade $\mathscr{V}$ is increasing in $|m_\theta|$ and σ_θ, both of which are decreasing in μ^2. Hence the result. □

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