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Environmental sustainability, nonlinear dynamics and chaos*

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Summary. This paper studies the possibility of nonlinear dynamics in a simple overlapping generations model with the environment – the John-Pecchenino (1994) model. We show that if people's concerns towards greener preferences and the maintenance efficiency relative to degradation are not sufficiently high, cyclically or chaotically fluctuating equilibria are more likely to exist; moreover, under a specific condition, a complicated topological structure might emerge. Our short-run analysis complements John and Pecchenino's long-run analysis and our findings suggest that the associated transition towards an environmentally sustainable state is not trivial.

Keywords and Phrases: Environmental-growth models, Sustainability, Complex dynamics.

JEL Classification Numbers: Q20, O41, C62.

1 Introduction

Recently, economists have shown considerable interest in studying the interplay between the environment and economic development; see, among others, Beltratti et al. (1993), Tahvonen and Kuuluvainen (1993), John and Pecchenino (1994), John et al. (1995), and Bovenberg and Smulders (1995, 1996). These authors introduce environmental variables into growth models, of which some display endogenous growth while others do not, intending to characterize the properties associated with a state known as an

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environmentally sustainable state and to study what and how environmental policy may affect this state. In the literature, such a state is usually measured by a constant stock of natural capital, also referred to as environmental quality.

Most of these models dealing with environmental issues restrict the analysis to steady states, thereby ignoring the transitional dynamics towards the long-run positions. The exception is Bovenberg and Smulders (1996), who find sharp differences between short-run and long-run effects of environmental policy. However, since transitional paths in the Bovenberg-Smulders model eventually converge to a balanced growth path, the dynamics can still be regarded as simple dynamics. There are two more reasons to believe that transitions in environmental-growth models, in particular in those models which follow the tradition of neoclassical growth models, not only are important but also can be rather complex. First, although environmental-growth models resemble the otherwise standard growth models, where the concept of an environmentally sustainable state corresponds to that of a steady state, one fundamental difference between the two types remains. In the former, there exist two opposing forces that balance each other towards the sustainable level: on one hand, economic agents' consumption activities hurt the process; whereas, on the other hand, their commitments to environmental preservation aid the process. Hence, it is conceivable that the interplay of these two effects, which is generally absent in traditional growth models, may give rise to nontrivial dynamics.

Second, the steady-state analysis is based on a presumption that the sustainable level is achievable. Thus, in these models, convergence eventually prevails. However, studies in nonlinear dynamics suggest that given the above two effects the convergence prediction appears oversimplified. In particular, it is well understood by now that such a system may give birth to bounded equilibrium paths that never converge to the steady state, a possibility that has not been exploited in environmental-growth models.

To investigate the possibility of complex dynamics in environmental-growth models, we choose an influential but admittedly highly stylized model in the literature – the John-Pecchenino (1994) model. In the model, individuals live for two periods, working while young and consuming while old. The young divide their wages between investment in physical capital and investment in the environment, a public good. We take the renewable resource to be the environmental resource, such as a rain forest, the climate, species diversity, etc. This resource serves as a source of utility and may serve as a complement to consumption, which attempt to capture the popular belief that environmental resources are an important determinant of the quality of life, not only for the present generation but also for our long-term successors. The latter is channelled through two ways. One is that the consumption by the current generation degrades the environment bequeathed to future generations, while the other is that their investment in environmental quality improves the environment bequeathed to future generations.

Under plausible assumptions on the functional forms of the utility and production, it is found that this model can produce perfect foresight equilibria, in which the level of environmental quality and other endogenous variables fluctuate forever. When people's concerns towards greener preferences and the maintenance efficiency relative to degradation are not sufficiently high, cyclically or chaotically fluctuating equilibria are more likely to exist. Moreover, under a specific condition, a complicated topological structure might emerge. These findings suggest that the dynamics towards the sustainable state may be complex and government policy can be implemented to induce a smooth convergence.

2 A simple model of growth with the environment

The economic environment

As in John and Pecchenino (1994), the model economy is populated by a large number of identical consumers and perfectly competitive firms. At each period t there are two overlapping generations of individuals, the young and the old. Abstracting from the issue of population growth, the size of each generation is constant and normalized to unity. A representative individual in each generation derives utility from consumption in old age, c_{t+1}, and an index of the quality of the environment when consumes, E_{t+1}.[1] The utility function, $U(\cdot,\cdot)$, is assumed to be increasing, strictly concave, and twice continuously differentiable, satisfying $U_c > 0, U_E > 0, U_{cc} < 0, U_{EE} < 0, U_{cE} \geqslant 0$,[2] and the Inada conditions.

An individual works only in the first period of life, supplying inelastically one unit of labor and earning a real wage of w_t. After optimally allocating w_t for saving for old-age consumption, s_t, she invests the remaining for improvement in environmental quality, m_t. When old, she supplies her savings inelastically to firms and earns the gross return $(1 + r_{t+1} - \delta)$, where r and δ represent the real rate of return on and the depreciation rate of capital, respectively. The life-cycle budget constraints she faces are

$$w_t = s_t + m_t \quad \text{and} \quad c_{t+1} = (1 + r_{t+1} - \delta)s_t. \tag{1}$$

The quality of the environment evolves according to (see John and Pecchenino, 1994)

$$E_{t+1} = (1 - b)E_t - \beta c_t + \gamma m_t, \tag{2}$$

[1] It is well known that this assumption abstracts from the consumption-saving decisions of agents. Although this is a standard practice in overlapping generations models, its economic limitation should be acknowledged. I thank an anonymous referee for clarifying this point.

[2] This proviso implies that consumption and environmental quality may be complements. Two variables are said to be Edgeworth complements (substitutes) if the marginal utility of one increases (decreases) as the quantity of the other increases.

where $b \in (0,1)$ measures the autonomous evolution of environmental quality, the term βc_t is the consumption degradation of the environment by the old, and γm_t measures environmental improvement. The coefficients β and γ, which measure the degree of consumption externality and the maintenance efficiency, respectively, are assumed to be time invariant.

Firms act competitively and use constant returns technology. Output per worker is given by the production function, $f(k_t) : \mathbf{R}_+ \to \mathbf{R}_+$, where k_t is the capital-labor ratio. The production function is assumed to be twice continuously differentiable and strictly concave in k_t. Each firm maximizes profits, hiring labor to the point where the marginal product of labor equals the wage and renting capital to the point where the marginal product of capital equals its rental rate:

$$w_t = f(k_t) - k_t f'(k_t) \quad \text{and} \quad r_t = f'(k_t). \tag{3}$$

Perfect foresight competitive equilibrium

The representative consumer behaves competitively, taking all prices as given. She maximizes U subject to (1) and (2). The first-order condition is given by

$$(1 + r_{t+1} - \delta)U_c(c_{t+1}, E_{t+1}) - \gamma U_{\mathrm{E}}(c_{t+1}, E_{t+1}) = 0. \tag{4}$$

Intuitively, Eq. (4) states that the marginal rate of substitution between consumption and environmental quality must equal the marginal rate of transformation. Hence, a *perfect foresight competitive equilibrium* of this model is characterized by the first-order condition (4) and the pricing equations (3); moreover, the goods market clears, which requires the demand for good in each period to equal its supply, or aggregate investment to equal aggregate saving:

$$k_{t+1} = s_t. \tag{5}$$

Our strategy next is to reduce the dimensionality of the system to a commandable one so that the model becomes simpler. To this end, for convenience, a simplistic technical hypothesis is put forward with regard to the functional form of the utility:

Assumption. *Define* $\eta_{\mathrm{E}} \equiv (E/c)U_{\mathrm{E}}/U_c > 0$. *This elasticity parameter is assumed to be constant.*

This proviso encompasses a number of functional forms, including logarithmic, Cobb-Douglas and constant elasticity of substitution (CES). Hence, eq. (5) now becomes

$$c_{t+1} = E_{t+1}[1 + f'(k_{t+1}) - \delta]/(\eta_{\mathrm{E}}\gamma). \tag{6}$$

From (1), we obtain an expression tying the capital stock to the environmental quality:

$$k_{t+1} = E_{t+1}/(\eta_{\mathrm{E}}\gamma), \tag{7}$$

which implies that the representative agent chooses her optimal mix of saving and maintenance in such a way that higher capital stock is associated with higher environmental quality. Then, suppressing m_t from (1), (3) and (5) and substituting it along with (6) and (7) into (2) yield a dynamic equation for E_t:

$$E_{t+1} = \frac{\eta_E}{1+\eta_E}\left\{[1 - b - \frac{\beta(1-\delta)}{\eta_E\gamma}]E_t + [\gamma(1-\alpha(k_t)) - \beta\alpha(k_t)]f(k_t)\right\}. \quad (8)$$

where $\alpha(k) \equiv kf'(k)/f(k)$ is capital's share of output. To further simplify the analysis, we assume that the parameter α is constant. Evidently, some popular forms of production function satisfy this assumption, e.g., the Cobb-Douglas function. As a consequence, the dynamic equilibrium is described by the following first-order nonlinear difference equation:[3]

$$\begin{aligned} E_{t+1} &= \frac{\eta_E}{1+\eta_E}\left\{\left[1 - b - \frac{\beta(1-\delta)}{\eta_E\gamma}\right]E_t + A[\gamma(1-\alpha) - \beta\alpha]\left(\frac{E_t}{\eta_E\gamma}\right)^{\alpha}\right\} \\ &= a_0 E_t + a_1 (E_t)^{\alpha} \equiv G(E_t; \eta_E, \beta, \gamma), \end{aligned} \quad (9)$$

where the two constant coefficients are defined as

$$a_0 = \frac{(1-b)\eta_E\gamma - \beta(1-\delta)}{\gamma(1+\eta_E)}, \text{ and } a_1 = \frac{A\eta_E^{1-\alpha}[\gamma(1-\alpha) - \beta\alpha]}{\gamma^{\alpha}(1+\eta_E)}.$$

Inspection of both parameters, a_0 and a_1, reveals that they can be of either sign; moreover, it can be readily verified that $a_0 < 1$. Indeed, as shown below, these properties open the possibility of a rich dynamic behavior for map G. To establish this point, we find that the existence of a non-trivial steady state crucially depends on the sign of a_1. It is then straightforward to show that a necessary condition is: $\gamma(1-\alpha) - \beta\alpha > 0$, or $a_1 > 0$, which is being imposed in what follows. Note that for this to hold, it requires either better maintenance technologies (higher γ) or lower environmental degradation by consumption (lower β), apparently consistent with the widely accepted belief that public intervention is needed regarding environmental issues. Without proper maintenance, sustainable development is not possible. To sign a_0, three fundamental parameters prove to be essential: people's attitude towards preserving the environment, η_E, maintenance technologies, γ, and the depletion rate of the environment by consumption, β. From the expression for a_0, it is evident that a_0 is positive if both η_E and γ are sufficiently large while β is sufficiently small, and vice versa. The subsequent analysis considers the dynamics for all possible cases of a_0. Interestingly, this parameter, jointly determined by η_E, γ and β, is 'tunable', and as it varies, the system generates a whole spectrum of rich transitional dynamics.

[3] The complete arguments of the function $G(\cdot)$ should be $G(E_t; \eta_E, \beta, \gamma, b, \delta, \alpha)$. However, since parameters (b, δ, α) are generally independent of individuals' activities towards the environment, we drop them from $G(\cdot)$ and skip the discussions of their impacts on the equilibrium dynamics. The remaining parameters (η_E, β, γ) are therefore referred to as the fundamental parameters of the model.

Before leaving this section, we look at the simple dynamics for the case when $a_0 \geq 0$. The unique steady state equilibrium is now given by $E^* = [a_1/(1-a_0)]^{1/(1-\alpha)}$. The following proposition highlights the kind of dynamics:

Proposition 1. *Suppose $a_1 > 0$ is satisfied. Then, if $a_0 \geq 0$, map G is monotonically increasing; for all $E_0 \in (0, \infty)$, $\lim_{t\to\infty} G^t(E_0) = E^*$. That is, there exists a unique and asymptotically stable (or attracting) positive steady state.*

It is worthwhile to note that $a_0 \in (0, 1)$ is equivalent to $\beta/\gamma < [(1-b)/(1-\delta)]\eta_{\mathrm{E}}$. This proposition implies that when the relevant parameters fall in the above domain, given any initial level of the environmental quality, there exists a path that converges to the steady state. This steady-state level, in general, corresponds to the *sustainable development* level in the literature. Thus, under these reasonable assumptions, sustainable development will be achievable in the long run, as long as people's concerns on the environment and the maintenance efficiency are not sufficiently low or the degradation is not sufficiently high. However, one immediate question arises as to what happens when these model parameters fail to satisfy the above conditions. To this the attention turns in the following section.

3 The emergence of complex deterministic dynamics

This section considers the case when $a_0 < 0$, or equivalently $\beta/\gamma > [(1-b)/(1-\delta)]\eta_{\mathrm{E}}$. Recall that $a_1 > 0$ remains valid. To aid the exposition, define $a \equiv -a_0 > 0$. The equilibrium condition (9) can thus be rewritten as

$$E_{t+1} = a_1 E_t^{\alpha} - aE_t \equiv G(E_t; \eta_{\mathrm{E}}, \beta, \gamma). \tag{10}$$

Interestingly, despite the seemingly simplistic form implied by (10), it is highly nonlinear, as seen below. It is straightforward to show that the G map satisfies the following properties:

(P.1) $G(0) = G(\hat{E}) = 0$, where $\hat{E} \equiv (a_1/a)^{1/(1-\alpha)}$ is the upper bound for the quality E;

(P.2) G is C^1-*unimodal*: G is once continuously differentiable and there exists $\bar{E} = (\alpha a_1/a)^{1/(1-\alpha)} \in (0, \hat{E})$ such that G is strictly increasing on $[0, \bar{E})$ and strictly decreasing on $(\bar{E}, \hat{E}]$;

(P.3) $\lim_{\mathrm{E}\to 0} G'(0) = +\infty$;

(P.4) the unique positive steady state equilibrium is given by $E^* \equiv [a_1/(1+a)]^{1/(1-\alpha)}$;

(P.5) $G'(E^*) = \alpha - (1-\alpha)a < 1$.

(P.6) If $a >=< \alpha^{-\alpha/(1-\alpha)}/(1-\alpha)$, $G(\bar{E}) >=< \hat{E}$; G maps $[0, \hat{E}]$ into itself if $a \leq \alpha^{-\alpha/(1-\alpha)}/(1-\alpha)$.

Brief mention should be made here on (P.6). It implies that there exist circumstances in which G does not map $[0, \hat{E}]$ into itself, thereby admitting more complicated dynamic structure. To ensure this, the capital share α must satisfy that $(1-\alpha)/\alpha > \alpha^{-\alpha/(1-\alpha)}/(1-\alpha)$.[4] Under empirically plausible values, this condition can be met in generic cases.[5]

The above properties are conveniently depicted in Fig. 1. From (P.6), two cases of a need to be distinguished, since conceivably they yield disparate dynamic structures. We start by analyzing the first case.

Case 1: $a \le a^{-\alpha/(1-\alpha)}/(1-\alpha)$

In this case, G maps $[0, \hat{E}]$ into itself. That is, any positive iterates of the set of an initial quality level will remain in the interval. Even though a falls in the interval, it is found that the dynamical system (10) may still undergo a series of bifurcations, including the appearances of cycles and the transition to aperiodic or chaotic behavior. First, when a is not sufficiently high, the dynamics are, broadly speaking, qualitatively similar to those described in Proposition 1.

Proposition 2. *(i) If* $a \in (0, \frac{\alpha}{1-\alpha}]$*, then, for all* $E_0 \in (0, \hat{E}), \lim_{t\to\infty} G^t(E_0) = E^*$*.* E^* *is a stable node; and (ii) If* $a \in (\frac{\alpha}{1-\alpha}, \frac{1+\alpha}{1-\alpha}]$*, then, for all* $E_0 \in (0, \hat{E})$*,* $\lim_{t\to\infty} G^t(E_0) = E^*$*.* E^* *is a stable spiral.*

The relevant parameter ranges for a correspond to the two scenarios in which the slopes of G evaluated at the fixed point E^* belong to

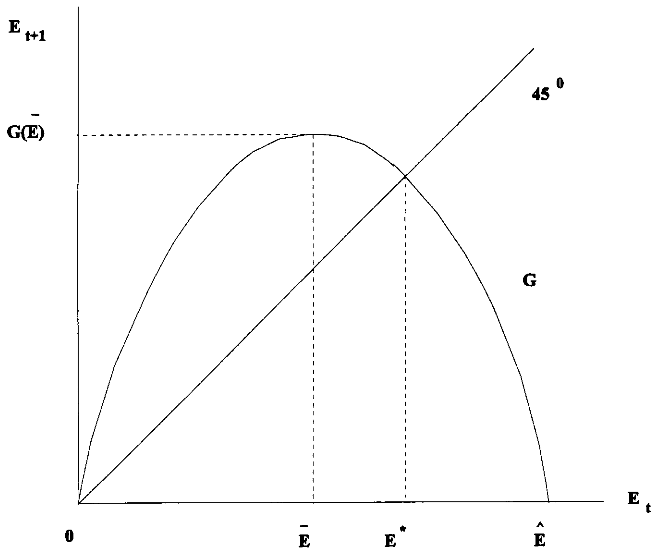

Figure 1. The structure of map G

[4] This inequality is briefly derived as follows. From the definition of a_0, we have $a = -a_0 < \beta/\gamma$. On the other hand, the assumption of $a_1 > 0$ requires that $\beta/\gamma < (1-\alpha)/\alpha$. Thus, $a < (1-\alpha)/\alpha$. Using (P.6), the inequality follows.

[5] To satisfy the inequality, a value of α slightly smaller than 0.3 is sufficient. In economic studies, $\alpha = 0.25$ is often utilized.

$G'(E^*) \in (0, \alpha)$ and $G'(E^*) \in (-1, 0)$, respectively. These ranges can be expressed in terms of the fundamental parameters (η_E, γ, β).

Next, we show that the model displays endogenous fluctuations when the quality level approaches the sustainable equilibrium. Since this sort of one-dimensional discrete functions has been thoroughly studied in the existing literature on nonlinear dynamics, we only summarize the main results. These are given in the following proposition.

Proposition 3. *(i) If* $a \in \left(\frac{1+\alpha}{1-\alpha}, \frac{\alpha^{-\alpha/(1-\alpha)}}{1-\alpha}\right)$, *map G generates a two-period cycle, and the set of* $E_0 \in (0, \hat{E})$ *such that* $G^t(E_0)$ *converges to the steady state,* E^*, *is at most countable; (ii) As a rises, the dynamical system* (10) *experiences period-doubling or flip bifurcations. That is, there exists a value* $a^* \in \left(\frac{1+\alpha}{1-\alpha}, \frac{\alpha^{-\alpha/(1-\alpha)}}{1-\alpha}\right)$ *such that attracting cycles of period* 2^n, for $n \geq 2$, *emerge; and (iii) As a further rises, there exists a value* $a^{**} \in \left(\frac{1+\alpha}{1-\alpha}, \frac{\alpha^{-\alpha/(1-\alpha)}}{1-\alpha}\right)$ *such that a period* 3 *cycle of map G emerges if* $a > a^{**}$.

Four comments are in order. First, by Sarkovskii's theorem (see Devaney (1989) p. 62), period 3 implies the existence of all other periods.[6] Moreover, by Li and Yorke's (1975) theorem, period 3 implies *topological chaos*; that is, there is an uncountable set $S \subset (0, \hat{E})$ such that no orbits that start in S will converge to one another or to any period orbits. Second, as recognized by Grandmont (1986), the Li-Yorke theorem says nothing about the size of the set of chaotic equilibria. S may be just a collection of disjoint points with zero Lebesgue measure, i.e., the probability of starting in S is zero, while any initial condition outside S results in an orbit converging to a cycle of finite period. In other words, the existence of topological chaos cannot be regarded as a priori for the existence of *ergodic chaos*, meaning that the asymptotic properties of solution sequences can be summed up by an absolutely continuous distribution that serves as an asymptotic sufficient statistic for the dynamical system (10). Grandmont (1986) shows that, in this case, $\{G^t(E^*)\}$ is either mapped into weakly stable cycles with measure zero for S or to unstable cycles with positive measures for S.[7]

Third, since a stable cycle must surround an unstable equilibrium, we notice that once the system is attracted by a stable cycle, the steady state or sustainable equilibrium is stable but not asymptotically stable. Clearly, without examining the stability condition, we are unable to determine how some economically interesting and observable variables behave along such a transition, which is why it is essential to study transitional dynamics.

Finally, Proposition 3 suggests that as a is progressively raised from moderate values to large ones, the system equilibria span from endogenously

[6] To prove the existence of a non-degenerated 3-period cycle, one key condition requires that $G^3(\bar{E}) < \bar{E} < G(\bar{E})$; see Grandmont (1985) for details.

[7] It is widely agreed that a unimodal map with a negative *Schwartzian derivative*, defined as $SG \equiv G'''/G' - (3/2)(G''/G')^2$, could have at most one weakly stable cycle. However, one could easily verify that for this map the sign of the Schwartzian derivative is far less straightforward: it at least alters the sign once.

fluctuating to periodic to chaotic ones, if any. It can be easily shown that these are equivalent to small values of η_E and large ratios of β/γ.[8] Thus, this proposition says that when individuals' environmental concerns are not great and the degradation is rapid relative to maintenance, chaotic equilibria are more common than others. These results accord reasonably well with intuition. For instance, when people attach low amenities towards green preferences, (6) and (7) indicate that they tempt to choose high optimal ratios of consumption to environmental quality. As a result, this exerts large negative external effects across generations on the evolution of the quality level, as manifested in (2). To put it differently, high consumption levels by the previous generation serve as a bottleneck on the preservation of the environment enjoyed by the present generation, thereby distorting the dynamic paths approaching the sustainable equilibrium.

Case 2: $a > \alpha^{-\alpha/(1-\alpha)}/(1-\alpha)$

When a is successively raised to some high values (provided that the set for (η_E, γ, β) remains nonempty), the maximum value of G, $G(\bar{E})$, exceeds the maximum value of E, $\hat{E}$, and thus there are $2^{n+1}-1$ disjoint closed intervals that escape from $(0, \hat{E})$ after $n+1$ iterates of G, i.e., $G^{n+1}, n \geq 0$. Or equivalently, there are 2^{n+1} disjoint open intervals that remain in $(0, \hat{E})$; see, e.g., Devaney (1989). Note that the non-negativity constraint of E requires that the level of environmental quality for those which have escaped becomes zero. Clearly, the topological structure under this case is much more complicated than under the previous case. The following proposition summarizes the result:[9]

Proposition 4. *If* $\alpha^{\alpha}[(1-\alpha)a-1]^{(1-\alpha)} > (a-1)/a$, *the set of an initial environmental quality that never escapes from* $(0, \hat{E})$ *is a Cantor set; that is, it is a closed, totally disconnected, and perfect subset of* $(0, \hat{E})$. *As a result, given any initial value, the subsequent transitional dynamics for E become very complex.*

This proposition provides a sufficient condition under which the aforementioned set of an initial quality has zero Lebesgue measure. It implies that the topological complexity or symbolic dynamics is more likely, when the public's environmental awareness are low or the degradation relative to the maintenance efficiency is high.

One interesting characteristic of this model is that the dynamic properties presented in this section apply not only to the level of environmental quality, but also to other model variables as well, namely, capital stock, output,

[8] The equivalent ranges for (η_E, γ, β) are shown to be $\{[(1-b)+(1+\alpha)/(1-\alpha)]/(1-\delta)\}\eta_E + (1+\alpha)/[(1-\alpha)(1-\delta)] < \beta/\gamma < \{[(1-b)+\alpha^{-\alpha/(1-\alpha)}/(1-\alpha)]/(1-\delta)\}\eta_E + \alpha^{-\alpha/(1-\alpha)}/[(1-\alpha)(1-\delta)]$.

[9] The simple proof is as follows: From Devaney (1986), two conditions suffice for such a set being Cantor: $G'(E) = -1$ and $G(E) > 1$. Specifically, solving for E from $G'(E) \equiv \alpha a_1 E^{\alpha-1} - a = -1$ and substituting it into the second inequality, we obtain the sufficient condition.

consumption, and investment. Take output as an example. Recall that it is defined as $f(k) = f[E/(\eta_E\gamma)]$, and we say that function f is a *homeomorphism* that maps $[0, \hat{k}]$ into $[0, f(\hat{k})]$, where $\hat{k} \equiv \hat{E}/(\eta_E\gamma)$ and $f(\hat{k}) \equiv A[\hat{E}/(\eta_E\gamma)]^\alpha$; that is, f is one to one and onto, continuous and f^{-1} is also continuous. Therefore, f and E describe the same dynamics.

Figure 2 shows diagrammatically the relevant parameter ranges. As we move northwest, we raise β/γ relative to η_E. The steady state, E^*, is at first a stable node, becomes a stable spiral, and then loses stability along the line marked with $a = (1+\alpha)/(1-\alpha)$. Once across that line, the system undergoes a series of period-doubling bifurcations while spiral equilibrium orbits, repelled by E^*, are mapped into periodic cycles of 2^n. Eventually, sufficiently low η_E and high β/γ may result in cycles of period 3, which might lead to the emergence of chaotic trajectories, and at last it gives rise to the topological complexity. One apparent policy implication for filtering away or avoiding the sort of complicated dynamic structures when the economy approaches the sustainable equilibrium is to induce a shift towards greener preferences (increasing η_E) and improve the maintenance technologies (reducing β/γ). This happens to be one of the main themes of the Earth Summit in Rio de Janeiro in June 1992.

4 Concluding remarks

Although periodic and aperiodic phenomena are known to be common in ecological systems, they have not been well understood in general equilibrium economic models with the environment. The present paper has attempted to fill this lacuna. In a simple overlapping generations model, we

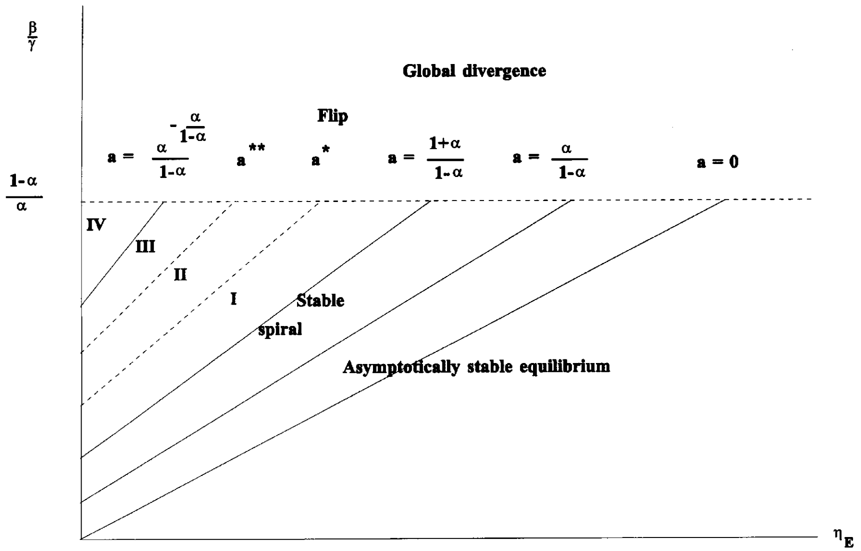

Figure 2. The relevant parameter ranges and the resulting dynamics; I cycle of period 2, II cycles of period 2^n, III cycles of period 3, IV topological complexity

have demonstrated, by making use of the notion of the bifurcation of a dynamical system, the possibility of nonlinear dynamics, including complex periodic orbits, nonexplosive chaotic deterministic trajectories, and a complicated topological structure. This study, hopefully to some extent, helps gain a better understanding of cyclical and chaotic behavior in environment-related economic systems. On the other hand, we also hope to shed some light on the analysis and discussion of sustainable development, suggesting that the associated transition is not too trivial to ignore.

This paper also contributes to the literature on economic dynamics and chaos. In the existing literature, it is well known that a simple overlapping generations model is capable of generating cyclical or chaotic equilibria, see, e.g., Benhabib and Day (1982) and Grandmont (1985). These studies seem to imply that cycles can exist only if individuals show rates of preferences for the present that are high enough (or equivalently, the discount rate is sufficiently close to zero), which often lead to unrealistic values. A recent paper by Balasko and Ghiglino (1995) argues that these models depend crucially on an assumption that preferences are homothetic. In a pure exchange economy, they establish a sufficient condition for the existence of cycles in the case when the homotheticity requirement is dropped. By contrast, in this paper, we take the other extreme assumption that rates of preference for the present are zero (recall that individuals only consume when old); but, the homotheticity requirement is retained for preferences over old-age consumption and environmental quality. What is relatively unknown in the literature is that whether or not chaotic equilibria may appear in this case, namely, when future utilities are discounted not so strongly. Interestingly, we show that once environmental variables are taken into account, the standard overlapping generations model can generate cycles with less stringent conditions.

Another strand of the literature departs from the above tradition of studying low discount rate by looking at other possibilities. For example, Majumdar and Mitra (1994) incorporate the wealth effect, where the utility function also depends on capital stock, and find that this modified model can exhibit cyclical and chaotic equilibria. Benhabib and Nishimura (1993) and Raut and Srinivasan (1994) study and confirm the existence of nonlinear dynamics in growth models with endogenous fertility.[10] In this regard, the alternative possibility in a model with the environment presented here enriches the existing literature on complex dynamics.

Finally, we would like to acknowledge that our analysis is subject to several qualifications which call for further research. For the sake of brevity, we just outline two of them. First, this model abstracts from environmental externalities from other sources, such as from production. It would be interesting to examine the issues considered here in a more general model with other linkages. Second, in this study, we focus on the case in which optimal

[10] I am grateful to an anonymous co-editor for bringing Majumdar and Mitra's and Raut and Srinivasan's papers to my attention.

environmental quality can be expressed by a one-dimensional difference equation. It may be possible to extend the model to the case of a higher dimension.

References

Balasko, Y., Ghiglino, C.: On the existence of endogenous cycles. Journal of Economic Theory **67**, 566–577 (1995)

Beltratti, A., Chichilnisky, G., Heal, G.: Sustainable growth and the green golden rule. NBER Working Paper No. 4430 (1993)

Benhabib, J., Day, R.: A characterization of erratic dynamics in the overlapping generations model. Journal of Economic Dynamics and Control **4**, 37–55 (1982)

Benhabib, J., Nishimura, K.: Endogenous fertility and growth. In: Becker, R. (ed.) Equilibrium, growth, and trade, vol. 2, pp. 237–247. New York: Academic Press

Bovenberg, A., Smulders, S.: Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model. Journal of Public Economics **57**, 369–391 (1995)

Bovenberg, A., Smulders, S.: Transitional impacts of environmental policy in endogenous growth model. International Economic Review **37**, 861–893 (1996)

Devaney, R.: An introduction to chaotic dynamical systems 2nd edn. New York: Addison-Wesley (1989)

Grandmont, J.: On endogenous competitive business cycles. Econometrica **53**, 995–1045 (1985)

Grandmont, J.: Periodic and aperiodic behavior in discrete one-dimensional dynamical systems, Chapter 13. In: Hildenbrand, W., Mes-Colell, A. (eds.) Contributions to Mathematical Economics. In Honor of Gerard Debreu. Amsterdam: North Holland 1986

John, A., Pecchenino, R.: An overlapping generations model of growth and the environment. Economic Journal **104**, 1393–1410 (1994)

John, A., Pecchenino, R., Schimmelpfennig, D., Schreft, S.: Short-lived agents and the long-lived environment. Journal of Public Economics **58**, 127–141 (1995)

Li, T., Yorke, J.: Periods three implies chaos. American Mathematical Monthly **82**, 985–992 (1975)

Majumdar, M., Mitra, T.: Periodic and chaotic programs of optimal intertemporal allocation in an aggregative model with wealth effects. Economic Theory **4**, 649–676 (1994)

Raut, L., Srinivasan, T.: Dynamics of endogenous growth. Economic Theory **4**, 777–790 (1994)

Tahvonen, O., Kuuluvainen, J.: Economic growth, pollution and renewable resources. Journal of Environmental Economics and Management **24**, 101–118 (1993)