Economic Theory **15**, 389–419 (2000)

Economic Theory

An experimental study of decisions in dynamic optimization problems*

Charles Noussair[1] **and Kenneth Matheny**[2]

[1] Department of Economics, Krannert School of Management, Purdue University, West Lafayette, IN 47907, USA (e-mail: noussair@mgmt.purdue.edu)
[2] Macroeconomic Advisers LLC., St Louis, MO 63105, USA (e-mail: matheny@macroadvisers.com)

Received: December 7, 1998; revised version: March 12, 1999

Summary. We study decisions of subjects who are given an incentive to solve dynamic optimization problems with the structure of a single-agent, one-sector, closed economy macroeconomic model. The decision task involves a sequence of choices of consumption and investment levels. Treatment variables consist of the initial endowment of capital stock, the production technology available to the economy, and the method of creating the structure of an infinite-horizon model. The study includes and contrasts data from both American and Japanese participants. We find that whether over- or underinvestment relative to the optimum occurs depends on the production technology, but not on the initial endowment of capital stock, nor the subject pool used, nor the method of implementing the infinite horizon. Sudden episodes of maximal consumption called *binges*, which are always suboptimal, are widely observed.

Keywords and Phrases: Dynamic optimization, Consumption, Investment, Experiment.

JEL Classification Number: C91.

* We thank Barry Sopher, Ernst Fehr, an anonymous referee, as well as participants at the 1997 Economic Science Association Meetings, at the April 1998 Midwest Macroeconomics Meetings, at the Purdue University Theory Workshop, at the Institute for Empirical Economics of the University of Zurich, and at the Laboratory of Economic Theory and Analysis at Lyon seminar series, for helpful comments. We thank Steve Tucker, Byung-Uk Chong, and Oksana Kikhtenko for research assistance. We are especially grateful to Professor Fumihiko Hiruma for help in conducting the experiments in Japan and to Mark Olson for many detailed discussions and helpful insights. We thank the Japan Society for the Promotion of Science, the Krannert School of Management at Purdue University, and the Center for International Business, Education and Research at Purdue University, for financial support.

Correspondence to: C. Noussair

1 Introduction

The concept of dynamic decision making is fundamental to much of modern economics. An understanding of the relationship between current decisions and future outcomes is crucial for the analysis of economic growth, the role of saving and investment, the properties of asset markets, and other important topics. Many macroeconomic models focus on the outcomes of optimal behavior of agents and therefore assume that agents are able to solve dynamic decision problems which may be very complex. Economists have devoted much attention to mathematical techniques for solving such problems and the research has yielded many important insights into the nature of economic activity. The rapid development of optimization techniques to solve theoretical models has outpaced the empirical study of the actual decisions made by human agents in the setting of the models.

In this paper we use an experimental approach to study the decisions of human subjects who are given a monetary incentive to solve a particular representative-agent dynamic model widely studied in macroeconomics. In a representative-agent dynamic model, economic actions are modeled as being consistent with the choices of a single decision maker, who maximizes the discounted utility of consumption over the appropriate time horizon. The particular problem we consider was first studied by Ramsey (1928). In the model, there is a single agent in a one-sector, closed economy with concave production and utility functions. The agent maximizes utility over an infinite horizon, starting with an initial level of savings in the form of physical capital. Capital is used to produce output, which is either consumed or invested in augmenting the capital stock used in future production. If the agent follows the optimal decision path, the economy's capital stock converges asymptotically to an optimal steady-state level.

The model, which is described in detail in Section 2, is a prototypical infinite horizon model with consumption and investment, discounting, and a production function exhibiting decreasing returns to capital input, and is at the core of modern macroeconomics and related fields. For example, most of equilibrium business cycle research, including both real business cycle modeling (Kydland and Prescott, 1982) and equilibrium models with increasing returns or other frictions (Benhabib and Farmer, 1996), is based on more complex variations of the framework we study. Variations on this model are widely used in such diverse fields as monetary economics (Cooley and Hansen, 1989), growth theory (Cass, 1965; Koopmans, 1965), and international economics (Backus, Kehoe, and Kydland, 1992). Related models are also used in the valuation of risky assets (Lucas, 1978). However, despite the central importance of dynamic optimization problems with consumption and investment decisions to both theoretical and applied research, there have been only a few laboratory studies of dynamic decision-making and even fewer with direct relevance to macroeconomics, even if the latter is broadly defined.

We present the data from an experiment that reproduces the structure of the theoretical model, and uses a cash payment structure to create the incentives that exist in the model. We design the experiment to conform as closely to the model as possible, taking a literal interpretation of the decision problem. We present our subjects with an individual choice problem, in which each agent has the role of a social planner in a one-agent economy. The decision situation is chosen to be computationally demanding, in order that the optimal policy not be obvious from introspection.

The experiment is not designed to assess whether the Ramsey growth model is a good description of how economies outside the laboratory grow. The design of the experiment removes, as the model removes, complications resulting from the existence of multiple agents, such as inefficiencies resulting from strategic behavior or externalities, which are features of naturally-occurring economies. The experiment does not address the question of whether economies behave as if such complications are important, nor the question of which institutions might cause the economy to behave like a representative-agent model. The experimental design also does not consider the political processes by which a social planner might be chosen. Rather, we consider whether individual human agents, when faced with the exact decision situation embodied in the social planner's problem, make decisions in accordance with the model's qualitative as well as quantitative predictions. We vary the parameters to evaluate the comparative dynamic predictions of the model, and to identify whether particular parameter configurations are more likely to lead to suboptimal decisions than others.

The experimental design is organized into a *basic design* which has two factors and two levels of each factor, and two extensions of the basic design. In the basic design, there are two levels of initial endowment of capital stock, one higher than the optimal steady state level of capital and one lower. Along the optimal decision path convergence to the optimal steady state is from above in the high endowment treatment and from below in the low endowment treatment. Two different production functions are used. Varying the production function changes the speed of convergence of the capital stock to the optimal steady-state level along the optimal trajectory.

In addition, we add two treatments to the basic design to address two important methodological issues. The first issue, which arises in studies like this one, is how to implement an infinite horizon model in the laboratory. We take two different approaches. In the basic design we impose an exogenous constant probability of terminating the economy at each time t, which, under appropriate assumptions, is equivalent, from the point of view of the agent, to an infinite horizon with discounting. In the other approach, we terminate the economy at a fixed time T, which is known to the subjects in advance, discount the payoffs from time 1 until time T, and award the subjects the discounted value of the capital stock remaining after time T assuming they made optimal decisions from that point on. The optimal decision is exactly the same for the two implementations of the infinite horizon. We compare the two methods for one of the treatment cells of the basic design.

A second methodological issue which sometimes arises in experimental research is concern about the use of only one subject pool. The data from the basic design was generated by undergraduate students at Purdue University, in Indiana, located in the United States. As a check on the robustness of our results, for two of the treatment cells, we replicate our experiment using undergraduate students from Waseda University, located in Tokyo, Japan. A finding of no subject-pool effect would strengthen our results in light of the considerable cultural difference between the two groups and popular views about differing intertemporal choice behavior between inhabitants of the two countries.

Although the theoretical framework we implement in our laboratory setting is relatively simple from the standpoint of modern empirical and theoretical research, the decision problems are much more complex than those sometimes associated with laboratory research in economics. Optimal decisions require solving a key first-order condition that equates marginal utilities of consumption at different dates. This is a standard and noncontroversial result in economics, but it does require difficult computations on the part of decision-makers, such as those in our experiment. Because optimal solutions are relatively complex, we would not be inclined to reject the theory's general applicability if laboratory subjects are unable to precisely replicate theoretical predictions. Instead, we look for broad qualitative coherence or incoherence between the model's key static and dynamic predictions, and those outcomes observed in the laboratory.

There is considerable evidence from other research in behavioral economics that suggests that dynamic optimization problems are difficult for subjects. For example, Hey (1988) studied a dynamic optimization problem in which subjects chose consumption and savings over a time horizon of random length. He observed consumption behavior that differed from optimal behavior, but also found that the comparative statics of consumption decisions were identical to those under optimal decision making. Fehr and Zych (1996) studied a dynamic decision problem in which subjects were given incentives to intertemporally optimize the consumption of a fictitious addictive good. Consumption at any point in time lowered the marginal utility of consumption in future rounds, in a similar manner to the building up of a tolerance to an addictive substance. They found a tendency toward excess consumption. Noussair and Olson (1997) studied decisions over a ten-round horizon in a setting in which at most twelve discrete choices were available in each round. They found that decisions were generally suboptimal at first, but improving with repetition, with some tendency toward overdepletion of capital stock near the end of the time horizon. Cox and Oaxaca (1992) studied behavior in search experiments and observed early termination of search compared to the optimal decision of a risk-neutral agent, but consistent with the presence of risk aversion on the part of subjects.[1] Our experiment was not designed to provide a direct comparison with any previous experimental studies. Rather, we chose our particular design to enable a direct comparison between our data and the Ramsey growth model. The previous experiments in dynamic decision-making do suggest that substantial departures from optimal de-

[1] An interesting literature has been concerned with the study of dynamic decision making with a focus on how agents discount the future. See for example Albrecht and Weber (1997), Benzion, Rapoport, and Yagil (1989), Gigliotti and Sopher (1997), Loewenstein (1987, 1988) and Thaler (1981). These studies tend to find strong departures from standard theoretical models of intertemporal choice.

cision making might be observed in our experiment, but because of differences in experimental designs, do not suggest what might be the specific nature of such departures.

The data from our experiments show that in some treatments, overconsumption relative to the optimum consistently occurs, and in the other treatments there is some tendency toward underconsumption. Whether or not subjects over- or underconsume depends on the production technology. Moreover, the direction of deviations in consumption and capital stock from the optimum is not affected by the ending rule nor by the subject pool employed. We also find a tendency under all treatments toward sudden episodes of great overconsumption and depletion of capital stock, a phenomenon to which we refer as a *binge*, rather than the pattern of consumption and investment smoothing over time suggested by the theoretical model. The incidence of binges declines as subjects acquire more experience with the decision situation.

The next section describes the theoretical model we are testing, Section 3 describes the procedures of the experiment, Section 4 lists the hypotheses tested, and Sections 5 and 6 present the results of the study and our final thoughts.

2 Description of theoretical model

2.1 Model

In the theoretical model corresponding to our experiment, each agent is assumed to maximize the present discounted value of current and future utility given in equation (1), subject to a sequence of resource constraints as in equation (2) and a given strictly positive initial capital stock, k_0.

$$\max \sum_{t=0}^{\infty} (1+\rho)^{-t} u(c_t) \tag{1}$$

$$c_t + k_{t+1} \leq f(k_t) + (1-\delta)k_t, \qquad \forall t \geq 0. \tag{2}$$

Depreciation of the capital stock occurs at the rate $\delta \in (0, 1]$. Utility and production functions u and f are strictly increasing, concave, and differentiable. The subjective rate of time preference ρ is positive. These assumptions guarantee that (2) will hold with equality in every round t.

Necessary and sufficient conditions for optimal choices of consumptions and capital stocks include the Euler equation in (3), the transversality condition in (4), and (5), which is equivalent to (2) under the assumption of non-satiation.

$$u'(c_t) = (1+\rho)^{-1} \left[1 - \delta + f'(k_{t+1})\right] u'(c_{t+1}), \qquad \forall t \geq 0 \tag{3}$$

$$\lim_{t\to\infty} (1+\rho)^{-t} u'(c_t) k_{t+1} = 0 \tag{4}$$

$$k_{t+1} = f(k_t) + (1-\delta)k_t - c_t, \qquad \forall t \geq 0. \tag{5}$$

Equations (3)–(5) constitute a well-known nonlinear planar dynamical system in capital stock and consumption.[2] The steady-state solution is a time-invariant one where $c_t = \bar{c}$ and $k_{t+1} = \bar{k}$, $\forall t \geq 0$, satisfying

$$\bar{c} = f(\bar{k}) - \delta\bar{k} \tag{6}$$

$$f'(\bar{k}) = \rho + \delta. \tag{7}$$

The properties of the utility function (other than non-satiation) have no bearing on the single positive steady state. Assumptions on f guarantee that there is exactly one steady state with strictly positive capital and consumption.[3] The functions $G(k_t)$ and $H(k_t)$ in (8) and (9) are the phaselines for capital and consumption.

$$k_{t+1} \geq k_t \iff c_t \leq f(k_t) - \delta k_t \equiv G(k_t) \tag{8}$$

$$c_{t+1} \geq c_t \iff c_t \geq f(k_t) - \delta k_t + (k_t - \bar{k}) \equiv H(k_t). \tag{9}$$

The steady state $(\bar{k}, \bar{c})$ occurs at the intersection of these two phaselines. Away from the positive steady state this model exhibits a familiar saddle-point property: for any given initial capital stock there is a unique optimal sequence of consumption and savings decisions from the initial period onward. That is, there is a unique solution satisfying expressions (1) and (2), or alternatively, (3)-(5). The solution has the property that the levels of capital stock and consumption converge to the optimal steady state. Some properties of optimal solutions are summarized in Section 2.2 below.

Although it is well-known that optimal solutions to models of this type are unique, it is difficult to compute analytic solutions, except in special cases. We use a shooting algorithm to compute optimal sequences of capital and consumption to 6 significant digits.[4] Figures 1 and 2 in Section 3 display optimal sequences of capital stock and consumption for the four different specifications of the model in our experiment.

2.2 Dynamical properties

The solution to (3)-(5) has four well-known dynamical properties:

1. For any given initial capital stock, $k_0 \geq 0$, optimal sequences of consumption and capital are unique.

2. Convergence to the steady state is strictly monotonic whether $k_0 > \bar{k}$ or $k_0 < \bar{k}$. If $k_0 = \bar{k}$, then $(c_t, k_t) = (\bar{c}, \bar{k})$ every round beginning with round 0.

3. Changes in the capital stock (net investment) are larger the further k_t is from the steady state.

[2] See for example Azariadis (1993, chapters 7 and 14).

[3] There is another steady state at the origin; however, that solution represents a permanent absence of activity.

[4] The algorithm is similar to one used by King and Rebelo (1989). *GAUSS* code is available on request.

4. The speed of convergence to the steady state and other dynamical properties are determined by the parameters, including α, δ, and ρ. Changes in ϕ have no effect on convergence in proportional terms. ϕ matters mostly by shifting the scale of capital and consumption but is otherwise unimportant.

3 The experiment

3.1 The basic design

The parameters for the basic design are given in Table 1. The design is a two-by-two design with two levels of initial capital stock, 3 and 50, and two different production functions, given in (10) and (11).

$$f^F(k_t) = 25.23 * k_t^{.2} \tag{10}$$

$$f^S(k_t) = 0.884 * k_t^{.9} \tag{11}$$

The parameters are chosen so that all treatments have the same optimal steady state capital stock, $\bar{k} = 14$. Convergence to $\bar{k}$ is from above when $k_0 = 50$ and from below when $k_0 = 3$. We refer to the $k_0 = 3$ and $k_0 = 50$ treatments as the *Low* and *High* treatments respectively. k_t^H and k_t^L denote the capital stock holdings in round t under High endowment and Low endowment respectively. Convergence is predicted to be faster when $\alpha = .2$ than when $\alpha = .9$. Therefore, we refer to the $\alpha = .2$ and the $\alpha = .9$ treatments as the *Fast* and the *Slow* treatments respectively. $k_t^F, k_t^S, f^F(k_t)$ and $f^S(k_t)$ will refer to the capital stock holdings in round t and the production functions under Fast and Slow convergence. The depreciation rate was set to $\delta = .5$ and the discount rate to $\rho = 1/9$ in all treatments. The utility function used in all treatment cells is:

$$U(c_t) = 100 * ln(1 + c_t) \tag{12}$$

The optimal sequences of capital stock holdings and consumption for the four treatment cells are shown in Figures 1 and 2.

Table 1. Parameters for the basic design

						Steady state	
Treatment	k_0	α	ϕ	δ	ρ	$\bar{k}$	$\bar{c}$
Fast/Low	3	0.2	25.23	0.5	1/9	14	35.78
Slow/Low	3	0.9	0.88	0.5	1/9	14	2.51
Fast/High	50	0.2	25.23	0.5	1/9	14	35.78
Slow/High	50	0.9	0.88	0.5	1/9	14	2.51

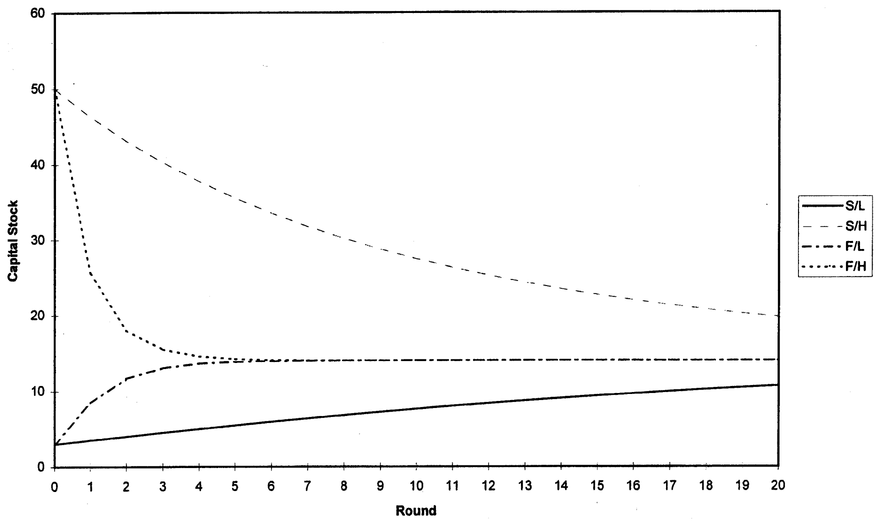

Figure 1. Predicted capital stock holdings over time

3.2 Implementation

The data were gathered in seven sessions. One of the sessions was conducted at Waseda University, Tokyo, Japan, and the rest of the sessions were conducted at Purdue University, Indiana, USA. As soon as subjects arrived for their session, they went through the instructions, which were computerized. The instructions are available from the authors upon request. After he completed the instructions, each subject solved one of the decision problems described above 23 times. We will refer to each of the attempts to solve the problem as a *period*, so that the experiment consisted of 23 periods, where each period corresponded to an "infinite" horizon. Each subject solved the same decision problem repeatedly for the entire session in which she participated.

In each of the treatments of the basic design, the *Random Ending Rule* was in effect. Under the Random Ending Rule, each period consisted of an uncertain number of rounds,[5] and the probability of the period terminating in the current round was 10 percent in every round. Each round corresponded to a time t in equations (10)-(12). The 10 percent probability of each round being the final round in a period induced a rate of time preference $\rho = 1/9$. The round at which a period would end was drawn randomly in advance from the appropriate distribution. The same random draws of period lengths were used for every subject to facilitate comparisons of data from different subjects and treatments. For example, period 18 for each subject in any of the *Random Ending Rule* treatments consisted of an identical number of rounds.

[5] The use of the terms *rounds* and *periods* in this manner may seem somewhat unusual to some readers. However, it seems more natural to us to think of each "infinite horizon" in the experiment as the relevant unit of time, and therefore we call each of these units a period.

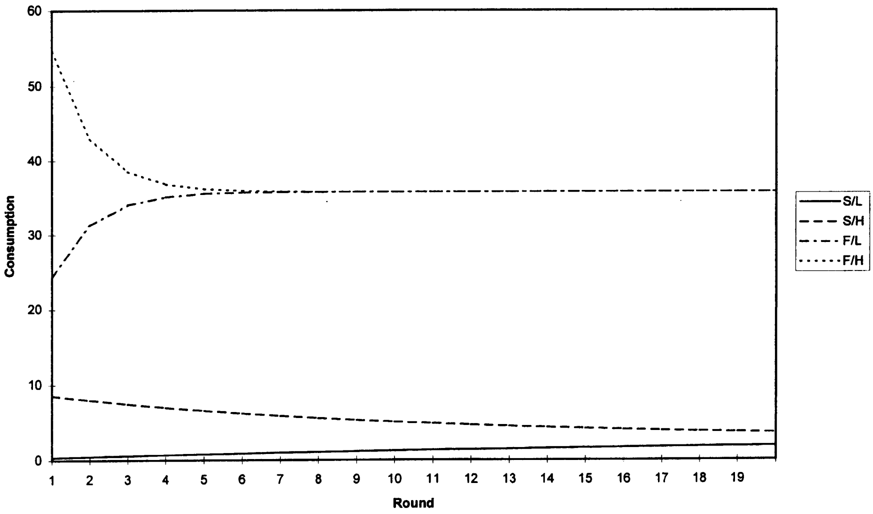

Figure 2. Predicted consumption levels over time

Since the model under investigation has a representative agent, the decision situation was presented as an individual choice problem. Each agent was her own economy, and could observe the decisions of any of the other participants at any time. The experiment was completely computerized except that subjects were provided with two sheets of paper: a *Production Schedule* and a *Token Value Sheet*. The Production Schedule described the production function and the Token Value Sheet described the utility function.

There are three variables described in the Production Schedule. The first variable is the current level of capital stock k_t, labeled as *Units of A Used in Production*. If the agent used all of his capital for production at time t, this provided $f(k_t) + .5k_t$ to divide between consumption for the current round c_t and capital for the next round k_{t+1}. The term $.5k_t$ is the undepreciated capital stock in round $t + 1$. The amount $f(k_t) + .5k_t$ is indicated by *Total A + X Produced* on the Production Schedule. If the subject consumed the maximum amount possible, his remaining capital stock would equal $.5k_t$, the amount that results by allowing the capital stock in k_t to depreciate and making zero gross investment in the round. This amount ($.5k_t$) is given in the column entitled *Minimum A*. If the subject consumed zero in round t, his capital stock in round $t + 1$ would be $k_{t+1} = f(k_t) + .5k_t$.

In each round t each subject chose a level of capital stock $k_{t+1} \in [.5k_t, f(k_t) + .5k_t]$. Before committing himself to a specific choice, a subject could type in any value for k_{t+1} and the computer calculated the implied value of c_t (given by (2) when (2) holds with equality) as well as $u(c_t)$. The consumption good was consumed immediately and could not be stored for future rounds. Of course, however, undepreciated capital stock did carry over to future rounds. The entire past history of the individual's *own* choices was accessible on the computer screen at all times to help in making decisions.

Subjects were awarded $u(c_t)$ tokens in each round, based on how much c_t they produced, according to (12). $u(c_t)$ was expressed in terms of tokens earned for producing units of c_t in a round and was indicated to each subject on his *Token Value Sheet*. The same utility function was used each round. In each round the tokens received were added to the total earned. The tokens earned could be converted to US dollars or Japanese yen at the end of the experimental sessions at a rate known in advance to subjects. Thus, maximizing cash earnings in a period of the experiment was equivalent to maximizing (1) and subjects' cash earnings in a period were proportional to the value of (1) attained. The first three periods did not count toward subjects' earnings. The next twenty periods did count toward final earnings. There were no participation fees or other non-salient rewards given in the experiment.

Subjects were required to spend at least 75 minutes on the instructions and making decisions in the 23 periods. They were informed that they would not be able to receive their actual cash payments until the 75 minutes had elapsed. They were not allowed to engage in any activity other than the experiment during the 75 minutes. These requirements were intended to reduce the incentive to make decisions as quickly as possible. 60 out of 65 subjects finished the experiment within five minutes after the 75 minute minimum. Earnings varied between $9 to $20 in the American sessions and between 900 and 1800 Yen (1 $US = 125 Yen) in the Japanese session.

3.3 Random versus fixed ending

In the basic design, the infinite time horizon was implemented by using a probabilistic ending rule, which we call the *Random Ending* rule. The use of a probabilistic ending rule to represent an infinite-horizon model yields the same optimal solution as the deterministic infinite horizon model only under the assumption of risk neutrality in the final monetary payoff on the part of subjects. To see this, let m_i^j equal the money subject i earns in period j and $E(m_i^j)$ the expected earnings of subject i in period j.

$$E(m_i^j) = \sum_{t=0}^{\infty} p_t u(c_t) \tag{13}$$

where p_t is the probability that the period continues until at least round t. If p_t is chosen so that

$$p_t = (\frac{1}{1+\rho})^t \tag{14}$$

then equation (13) is the same as the maximand in equation (1). The correct p_t can be induced by specifying a probability $\frac{\rho}{1+\rho}$ of the period ending after the current round. Let $V_i(\sum_{j=1}^{20} m_i^j)$ be the subject's utility for the final monetary payoff in the experimental session, which has 20 periods with monetary payoffs.

If the agent is risk neutral in the final monetary payoff, then choosing c_t to maximize the expected value of (13), a problem with exogenous uncertainty, is equivalent to (1), a problem with no uncertainty. However, if the agent is risk-averse, the two problems are no longer equivalent. Risk aversion affects the optimal solution in maximizing (13), in which uncertainty is present, but does not affect the maximization of (1) in which there is no uncertainty. Under the Random Ending rule, risk aversion would lead the agent to consume a greater fraction of her resources than under risk-neutrality, in order to smooth out payoffs for differing realizations of period length, even if it involves a lower expected monetary payment for the period.

Thus, if a lower than predicted level of investment were observed in the basic design, one possible explanation would be the presence of risk aversion. We therefore added the *Fixed Ending Rule* treatment in which there was no uncertainty about the final round of the period. Under the Fixed Ending Rule, each period consisted of ten rounds. In addition to receiving tokens based on their consumption in each round, subjects earned tokens based on the level of capital stock they held after round 10.[6] In order to make the optimal solution exactly the same as in the Random Ending treatment, the number of tokens awarded for terminal capital stock was the (discounted) quantity of tokens which they would receive if they were to make the optimal decisions beginning in round 11 over an infinite horizon.

Because the period had zero probability of ending before round 10 under the Fixed Ending Rule, and in order to make the decision problem identical under the two ending rules, the payoffs from consumption in rounds 1-10 were discounted by 10 percent from round to round. Instead of an identical token value sheet in each round, as in the Random Ending treatments, subjects received ten different sheets, one for each round, which reflected the discounting that occurred round by round. There was also a Token Value Sheet for A, which indicated the final buyout values of capital stock after round 10. Any given subject used the same eleven sheets in every period. The Fixed Ending Rule was only used for the Slow/Low parameters. This choice was made because initial experimentation with the Random Ending Rule indicated an underinvestment in capital stock under Slow/Low, and we conjectured that risk aversion in the final monetary payoff might have been the cause.

3.4 Cross-cultural differences

We had an opportunity to test the robustness of our results with a second subject pool, undergraduates at Waseda University in Tokyo, Japan. The use of Japanese subjects is of particular interest in light of the different patterns of saving and

[6] Though the infinite horizon in the theoretical model in Section 2 is stated as beginning in round zero, in the experimental sessions the initial round was presented to subjects as round 1, because we thought the label of round 0 might suggest to subjects that the round was a practice round that did not count toward their earnings. In Sections 3-6 we refer to the initial round in a period as round 1, as we did in the experiment.

consumption between residents of Japan and of the United States. In the session run at Waseda, the .9 (Slow) production function was used. Of the eleven subjects from Waseda, six had an initial endowment of 3 (Low) and five had an initial endowment of 50 (High).

3.5 Data available

Table 2 below summarizes the available data, by initial endowment, production function, ending rule and location.

Table 2. Summary of data gathered

Treatment	Num.subj.	Endowment	Prod.func.	Ending rule	Location
Slow/Low	11	3	$.884k_t^{.9}$	Random	Purdue
Slow/High	10	50	$.884k_t^{.9}$	Random	Purdue
Fast/Low	10	3	$25.23k_t^{.2}$	Random	Purdue
Fast/High	11	50	$25.23k_t^{.2}$	Random	Purdue
Slow/Low/Fixed	12	3	$.884k_t^{.9}$	Fixed	Purdue
Slow/Low/Japan	6	3	$.884k_t^{.9}$	Random	Waseda
Slow/High/Japan	5	3	$.884k_t^{.9}$	Random	Waseda

The basic design consisted of the four treatment cells, Slow/Low, Slow/High, Fast/Low and Fast/High. All of the data in the basic design used the Purdue subject pool and the Random Ending Rule. The basic design therefore allows for comparisons between the two levels of initial endowment and the two production functions.

4 Hypotheses

The eight hypotheses listed in this section are, with the exception of Hypothesis 8, implications of the theoretical model outlined in Section 2. The optimal sequences of capital and consumption in all four cells of the basic design are given in Figures 1 and 2. Since the steady state capital stocks ($\bar{k}$) are equal in the 4 cases, comparisons of speed of convergence are straightforward. Increases in the production parameter α slow convergence to the steady state both in absolute value and in proportional terms. Convergence to the steady state is monotonic, and at a decreasing rate in absolute value. Capital and consumption are either both greater or both less than their steady state values.[7]

The first hypothesis is derived directly from the data used to generate Figures 1 and 2. We do not expect the exact point predictions of the theoretical model

[7] The model has other comparative statics and dynamics predictions, which are not directly examined in the experiments reported here. In particular, increases in discounting (ρ) and depreciation (δ) produce faster convergence, and changes in ϕ have no bearing on speed of convergence in proportional terms. King and Rebelo (1993) demonstrate that changes in substitution rates, between factor inputs, and between consumptions at different dates, can affect transition paths.

to be observed, because we recognize that the required calculations are very demanding for subjects. However, we do hypothesize that there is no systematic tendency for capital stocks to be higher or lower than those along the optimal trajectory.

Hypothesis 1 *Median capital stock holdings are no different from the capital stock holdings along the optimal trajectory.*

Hypothesis 1 postulates that there is no general bias toward over- or under-consumption. The second hypothesis concerns a more general implication of the theoretical model, which makes clear predictions about increases and decreases over time in capital stock levels. We state Hypothesis 2 in both a strong and in a weak version. The strong version, which is a restatement of Property 2 of Section 2.2, says that when initial endowment is Low (High), subjects should increase (decrease) capital stock holdings monotonically over the course of a period, but not overshoot the optimal steady state level of capital stock. The weak version of the hypothesis takes into account the difficulty of determining the optimal steady state level of capital stock and therefore merely requires monotonicity of the capital stock holdings.

Hypothesis 2 *Strong Version:* $\bar{k} > k_{t+1}^L > k_t^L, \forall t$ *and* $\bar{k} < k_{t+1}^H < k_t^H, \forall t$. *Capital Stock Holdings are Moving Monotonically Over Time Toward the Optimal Steady State Level.*

Weak Version: $k_{t+1}^L > k_t^L, \forall t$ *and* $k_{t+1}^H < k_t^H, \forall t$ *Capital Stock Holdings are Moving Monotonically Over Time but Possibly Overshoot the Optimal Steady State Level.*

While Hypothesis 2 is concerned with the direction of convergence, Hypothesis 3 deals with the speed of convergence, and is also stated in a strong as well as a weak version.

Hypothesis 3 *Strong Version: For all t,* $|k_t^F - \bar{k}| < |k_t^S - \bar{k}|$ *and the strong version of Hypothesis 2 holds. The Speed of Convergence to the Optimum is Greater in the Fast Treatments than in the Slow Treatments.*

Weak Version: For all t, $|k_t^F - \bar{k}| < |k_t^S - \bar{k}|$. *Capital Stock Holdings are Closer to the Optimal Steady State Level in the Fast Treatments than in the Slow Treatments.*

Both versions of Hypothesis 3 are versions of Property 4 of Section 2.2, and both the strong and the weak version can be evaluated in the High and Low endowment data separately. The strong version requires that convergence of the capital stock toward the optimal steady state level take place and that convergence take place more quickly in Fast treatments than in Slow treatments, for a given initial endowment. The weaker version merely states that, controlling for t, the capital stock in the Fast convergence treatments is closer to the optimal steady state level than in the Slow convergence treatments.

Hypotheses 4-6 are statements about differences between decisions and earnings in different treatments, and all are implications of the theoretical model, which predicts a failure to reject all three null hypotheses.

Hypothesis 4 *There are no differences in the earnings realized, relative to the maximum possible earnings, between the four treatment cells of the basic design.*

Hypothesis 5 *There are no differences in decisions and earnings realized, relative to the maximum possible earnings, between the American and the Japanese subjects.*

Hypothesis 6 *There are no differences in decisions between the Fixed Ending Rule and the Random Ending Rule treatments.*

Hypothesis 4 asserts that subjects can solve all of the problems in the basic design equally well. The theoretical model predicts that decisions follow the optimal path in all treatments and therefore that there would be no difference in earnings across treatments relative to the optimum. All of the subjects in the basic design are drawn from the same subject pool and use the same ending rule so that any differences in earnings would be due to some aspect of the actual parameters of the decision problems which might create a tendency to make more costly errors in decision making in some of the treatments relative to others.

Hypotheses 5 and 6 are methodological diagnostics which, if supported, could strengthen the results obtained from the data for the basic design. Hypothesis 5 asserts that the decisions of subjects do not differ between the two subject pools. Both groups of subjects were undergraduates at large universities, with no previous experience in economic experiments. If subjects at both universities generate similar patterns in the data, we would interpret this as support for our main results, especially considering the cultural differences between the two groups.

Hypothesis 6 postulates that subjects' risk aversion in the final monetary payoff is not strong enough to induce significantly different behavior under the two different ending rules. The hypothesis also rules out other causes of any differences in decisions under the two ending rules, and supporting the hypothesis would indicate that decisions do not depend on the manner in which we induced the payoff structure of the infinite horizon.[8] Hypothesis 7 considers behavior in the final round under the Fixed Ending Rule, the only situation in the study when the period ends with probability one immediately after the current decision.

Hypothesis 7 *Strong version: Optimal decisions, conditional on current capital stock holdings, are made in round 10 under the Fixed Ending Rule.*
Weak version: $|k_{10} - \bar{k}| < |k_{11} - \bar{k}|$.

This is a special decision situation because there is no uncertainty and there are no dynamic considerations, because the period ends with certainty after the current round. The strong version states that subjects take the optimal decision, given the capital stock in round 10. The weak version is that the capital stock is moving in the correct direction in round 11 relative to round 10, that is, that

[8] The hypothesis covers only decisions and not earnings because comparisons of earnings between the two ending rules are difficult. Under the Fixed Ending Rule, the terminal value of capital stock awarded to subjects assumes that optimal decisions would be made after round 10.

the net investment chosen in round 10 moves capital stock in the direction of the optimal steady state. Hypothesis 8, unlike the previous seven hypotheses, is not an implication of the theoretical model, but rather is suggested by previous experimental studies.

Hypothesis 8 *The earnings of subjects relative to the optimum are greater in the later periods of a session than in the earlier periods.*

Hypothesis 8 is a well-documented pattern in experimental economics. In many experiments, the performance of subjects, as measured by their cash earnings, tends to improve as they repeat the decision situation. Because of this effect, much of the analysis that follows in this paper focuses on behavior in the later periods of the sessions.

5 Results

5.1 Overview of patterns in the data

5.1.1 Capital stock holdings

Figures 3–9 show the level of capital stock holdings in period 19 for each of the seven treatments. Period 19 was chosen for display for two reasons. The first reason is that it took place relatively late in the session so that decisions reflect 18 periods of practice with the decision problem. The second reason is because of its relative length, 15 rounds under the Random Ending Rule.

Figure 3 shows the data for the Slow/Low treatment for period 19. The final level of capital stock was less than the optimal level for all but one of the subjects. In period 19, no subject monotonically increases his capital stock as predicted by the model, but five of the eleven subjects monotonically decrease it. In Figure 3, several occurrences of a phenomenon we will refer to here as a “binge”, can be observed. We say that a subject *binges* during a period if he consumes the maximum feasible amount, that is, he sets $k_{t+1} = .5k_t$, in some round of the period. In period 19 of Slow/Low, at least one binge occurs on the part of five of the eleven subjects.

Figure 4 shows the data from period 19 of the Slow/High treatment. As theory predicts, many subjects tended to monotonically reduce their level of capital stock over the course of the period. However, the reduction in capital stock is usually at greater than the optimal rate. At the end of period 19, eight of the ten subjects have less than the predicted capital stock holding. Six out of ten subjects binge at least once in period 19. It is apparent from the figures that there is a tendency to underinvest in the Slow/High treatment, as there is in Slow/Low.

Figure 5 displays data from Fast/Low. In period 19, seven out of ten subjects binge at least once. Unlike in the Slow/Low and Slow/High treatments, however, the bingeing in Fast/Low is often preceded by a large investment in capital. Seven of the ten subjects possess a quantity of capital stock greater than the optimal

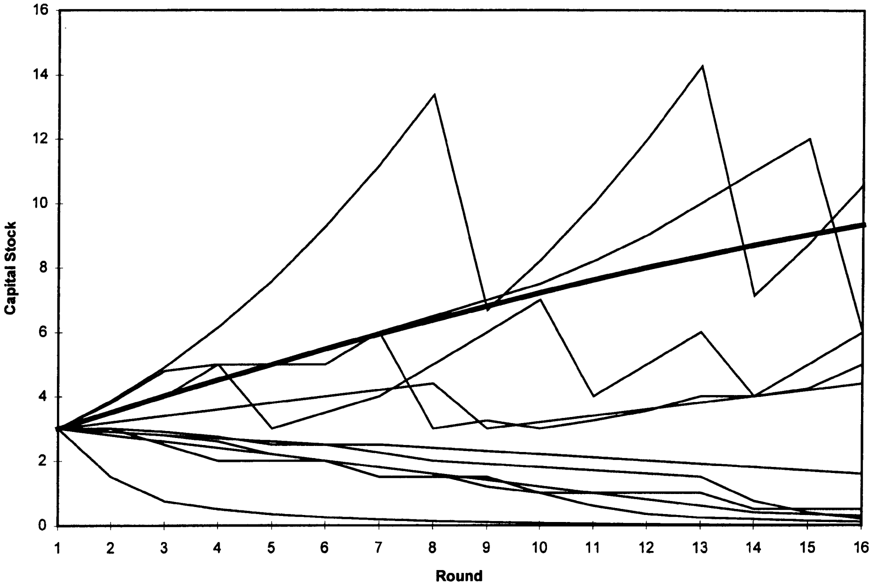

Figure 3. Capital stock holdings in the slow/low treatment: period 19: all subjects

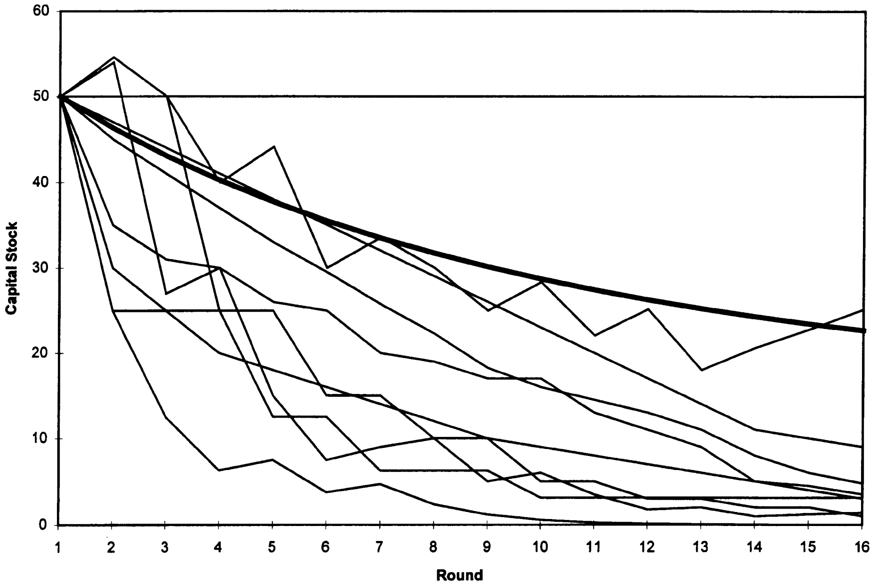

Figure 4. Capital stock holdings in the slow/high treatment: period 19: all subjects

level at the end of the period. Figure 6 corresponds to treatment Fast/High. The capital stock of six of the eleven subjects exceeds the level along the optimal trajectory in period 19.

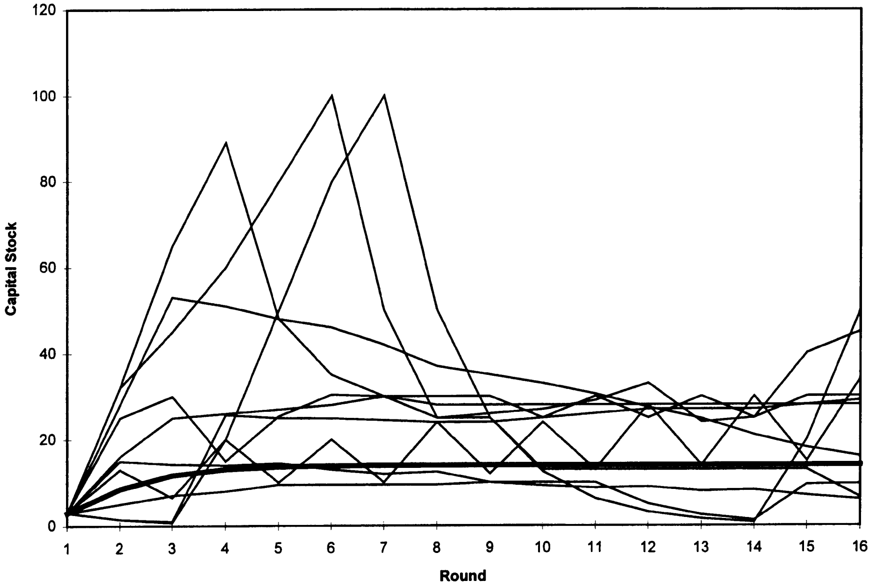

Figure 5. Capital stock holdings in the fast/low treatment: period 19: all subjects

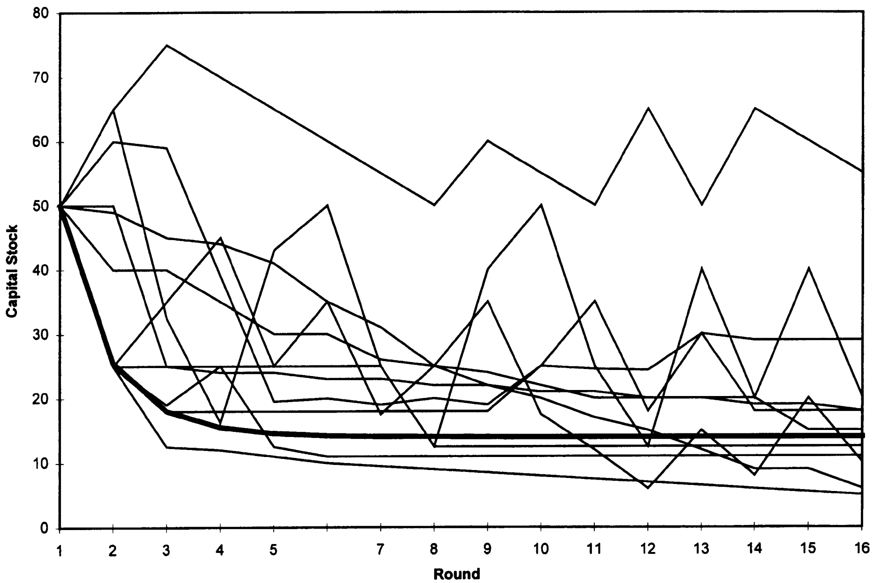

Figure 6. Capital stock holdings in the fast/high treatment: period 19: all subjects

Figure 7 shows the data from the Slow/Low/Fixed treatment. There is widespread monotonicity, though usually in the opposite direction as predicted. One subject is monotonically increasing, while five are monotonically decreasing, and six are not monotonic. The incidence of bingeing is lower under the Fixed Ending Rule than under the Random Ending Rule.

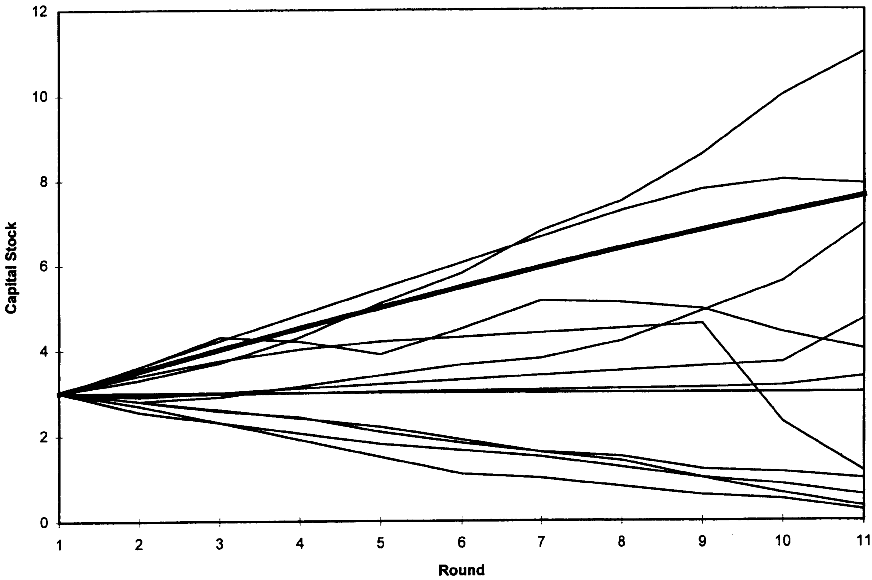

Figure 7. Capital stock holdings in the slow/low/fixed treatment: period 19: all subjects

The data from Slow/Low/Waseda are displayed in Figure 8. All 6 subjects have a capital stock below the optimal level in round 15 of period 19. The Slow/High/Waseda data are in Figure 9. All subjects monotonically decrease, and with the exception of one subject, all decrease capital stock faster than is optimal.

5.1.2 Consumption patterns

Figure 10 illustrates the consumption patterns of all subjects in period 19 of the Slow/Low treatment. In the early rounds of the period there is overconsumption. In round 1, ten of the eleven subjects consume more than the optimal quantity. By the end of the period, ten of eleven subjects are consuming less than along the optimal trajectory, a consequence of their earlier overconsumption and depletion of capital stock. Consumption binges can be seen as "spikes" on the graph. A similar pattern occurs in period 19 of Slow/High (not shown here). Nine of ten subjects in Slow/High have too little capital stock by round 15. As in Slow/Low, consumption by round is not nearly as smooth and regular as the theoretical prediction. Instead, frequent bingeing is observed.

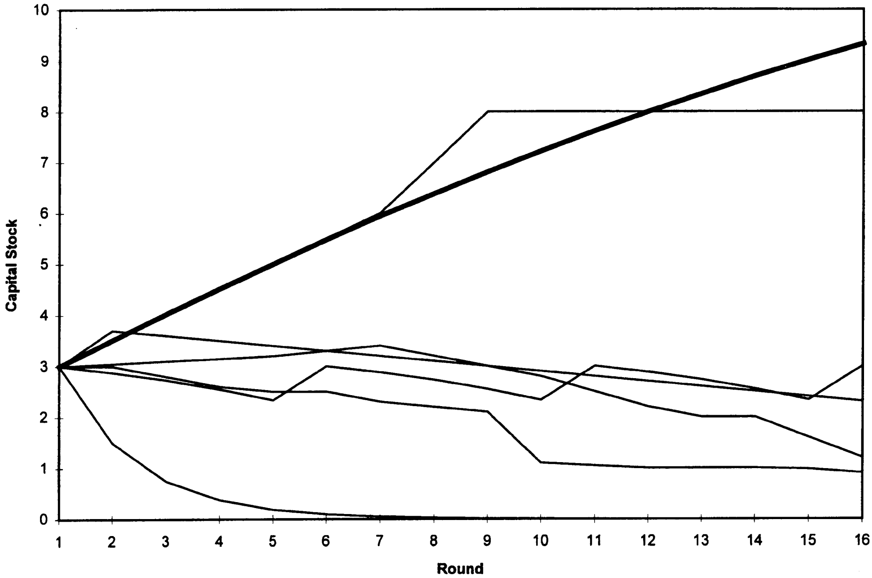

Figure 8. Capital stock holdings in the slow/low/waseda treatment: period 19: all subjects

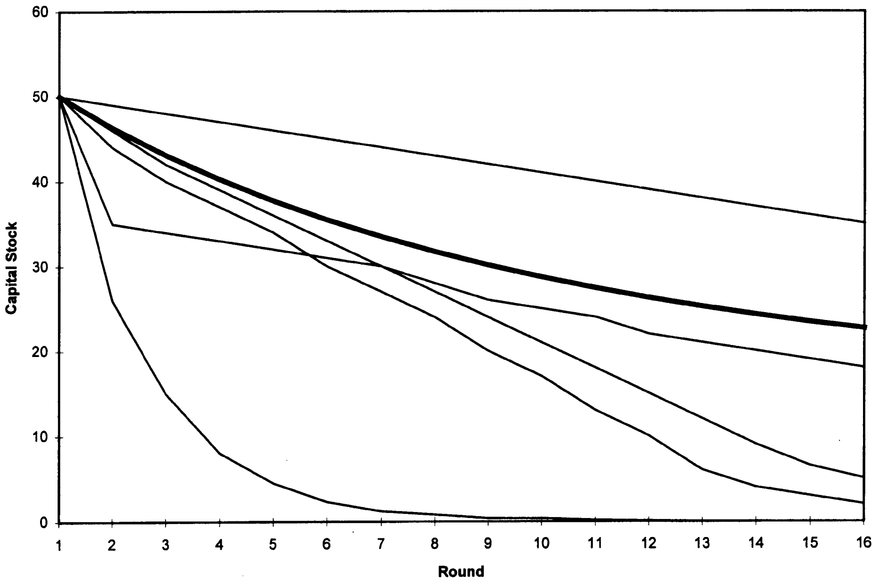

Figure 9. Capital stock holdings in the slow/high/waseda treatment: period 19: all subjects

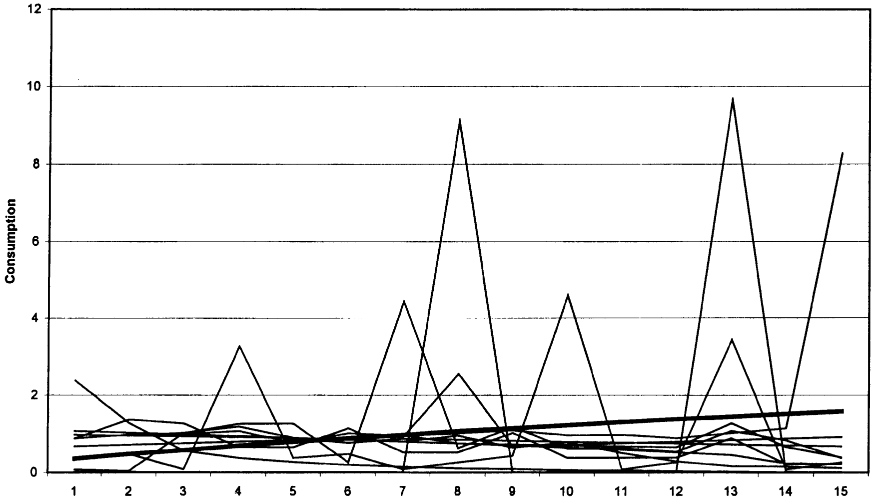

Figure 10. Consumption in the slow/low treatment: period 19: all subjects

In Figure 11, which graphs the data from the Fast/High treatment, underconsumption is usually observed in the early periods, but consumption is on average close to the theoretical prediction in later rounds, though some subjects exhibit an oscillating pattern of consumption behavior. The consumption patterns in Fast/Low have similar properties. In the two Fast treatments, unlike in the Slow treatments, some of the subjects (3 subjects in Fast/Low, and 4 subjects in Fast/High) consume nearly constant amounts which are very close to the optimal steady state level. Most of the subjects who oscillate between high and low consumption have an average consumption close to the optimal level.

In period 19 of the Slow/Low/Fixed treatment, as in the Slow/Low and Slow/High treatments, most of the subjects (10 out of 12) consume more than the optimal amount in the early rounds and, because they excessively deplete their capital stock, 9 of 12 consume less than the optimum in the late rounds. In the Slow/Low/Waseda treatment, the data also indicate overconsumption in early rounds followed by underconsumption in later rounds, due to insufficient remaining capital stock. In Slow/High/Waseda, the data show that the consumption of four of the five subjects closely tracked the optimal trajectory in the later rounds, while one subject binged in every round. In round 1, four of the five subjects consumed more than the optimal amount.

5.1.3 General patterns in the data

An overall picture emerges from the figures in sections 5.1.1 and 5.1.2. Under the Fast production function, there is great variation across individuals, but by period 19, a majority hold more capital stock than along the optimal path. Average consumption is close to the optimal steady state level, though with considerable

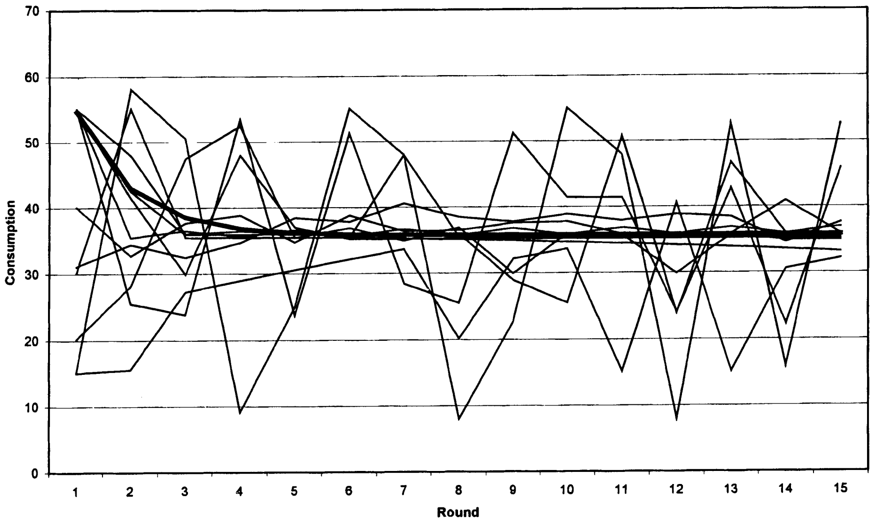

Figure 11. Consumption in the fast/high treatment: period 19: all subjects

variation round by round. Under the Slow production function, however, there is a strong general tendency to underinvest, regardless of initial endowment, subject pool, or ending rule. Under the Slow production function, consumption is greater than the optimum in the early rounds of a period, but since capital stock is depleted during the course of the period, the consumption in late rounds is lower than along the optimal trajectory. In all of the treatments, there is a tendency to binge and it is often the case that the same subject binges more than once per period. The next subsection details the results of statistical tests of the hypotheses listed in Section 4.

5.2 Tests of hypotheses

In this section we discuss the degree to which the data support the hypotheses listed in Section 4. Results 1-8 address Hypotheses 1-8, in order. The first result considers deviations of capital stock from the theoretical prediction, confirming the observations of Section 5.1.

Result 1: Whether overinvestment or underinvestment relative to the optimum occurs, depends on the production technology. Under Slow, there is underinvestment and under Fast, there is a tendency toward overinvestment.

Support for Result 1: Figure 12 compares the end-of-period capital stock holdings to the predicted level for all seven treatments, in the last five periods, for all subjects.[9] In Slow/Low, Slow/High and Slow/Low/Waseda, two-thirds or more of end-of-period capital stock holdings are less than 75% of the optimal level. We can reject the hypothesis that the capital stock holdings are equally likely to be greater than or less than the predicted value at the $p < .05$ level of significance for Slow/Low, Slow/High, Slow/Low/Fixed, and Slow/Low/Waseda. In both of the Fast treatment cells the majority of end-of-period capital stock levels are greater than 125% of the optimal level. For Fast/High we can reject the hypothesis that the holdings are equally likely to be higher or lower than the optimal value at the $p < .05$ level.[10] □

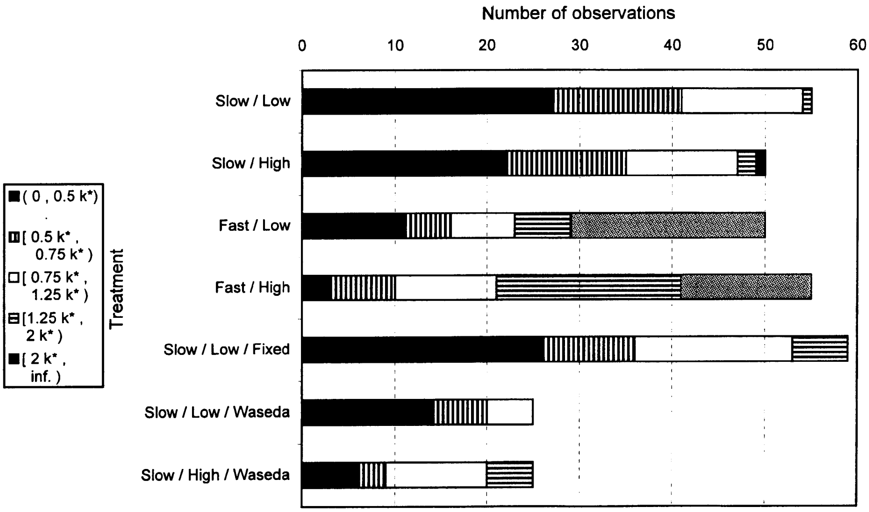

Figure 12. Ending capital stocks relative to optimum (k^*)

The next result documents the widespread nature of the monotonicity of capital stock in the data. The result also shows, however, that the monotonicity is not always consistent with the theoretical prediction. Under Slow/High and Slow/High/Waseda, the monotonic sequences are often observed to overshoot the optimal steady state level of capital stock and, in the Slow/Low, Slow/Low/Fixed and Slow/Low Waseda treatments, to usually move in the opposite direction as predicted.

Result 2: Most observed sequences of capital stock are monotonic, as predicted. However, under Slow/High, capital stock is depleted too quickly, and under

[9] We use the last five periods in evaluating Hypotheses 1-3, because decisions taken during these periods can be based on experience with previous periods of different lengths, which allows subjects to understand the rule for ending each period. The lengths of the last five periods were 15, 4, 2, 8, and 7 rounds. In Figure 12 there are five observations for each subject, one for each of the last five periods.

[10] Consider the variable x_i where $x_i = 1$ if the end of period capital stock is greater than along the optimal trajectory and $x_i = 0$ if it is less than along the optimal trajectory. We evaluate the hypothesis that x_i is drawn from a distribution with $P(x_i = 1) = P(x_i = 0) = .5$.

Slow/Low, capital stock is usually monotonically decreasing, contrary to the theoretical prediction of monotonic increase.

Support for Result 2: Table 3 lists the number of instances of monotonically increasing capital stock, monotonically decreasing capital stock, and bingeing, for each treatment cell. Each observation represents one entire period and for an observation to be classified as a monotonic increase (decrease) the subject must increase (decrease) capital stock holdings in every round of the period. The table contains all of the data from the first and the last five periods for all subjects. The data from the first five periods that counted toward subjects' earnings (periods 4-8) are in parentheses. The number before the comma is the number of observations of weak monotonicity ($k_{t+1} \geq$ or $\leq k_t, \forall t$) and the numbers after the commas is the number of observations of strict monotonicity ($k_{t+1} >$ or $< k_t, \forall t$).

Table 3. Instances of monotone increase, monotone decrease, and bingeing behavior: last five periods (first five periods in parentheses): all treatments

Treatment	Total obs.	Mon. inc.	Mon. dec.	Binge	Theo. pred.
Slow/Low	55	11,9(1,0)	40,20(40,20)	12(39)	Mon. Inc.
Slow/High	50	0,0(0,0)	31,11(40,20)	19(33)	Mon. Dec.
Fast/Low	50	11,8(9,7)	5,2(0,0)	19(31)	Mon. Inc.
Fast/High	55	1,1(0,0)	33,22(33,19)	34(40)	Mon. Dec.
Slow/Low/Fixed	60	13,13(4,2)	24,20(32,29)	10(15)	Mon. Inc.
Slow/Low/Waseda	30	8,3(0,0)	13,8(18,12)	5(13)	Mon. Inc.
Slow/High/Waseda	25	0,0(0,0)	25,25(23,18)	3(10)	Mon. Dec.

Note: An observation may be classified as both a monotonic decrease and as a binge.

Under all of the High Endowment treatments the large majority of subjects, in both subject pools and for both production functions, are monotonically reducing their capital stock in each round in the last five periods, as predicted. However, as can be seen in Figures 4, 6 and 9, this reduction in capital stock frequently goes beyond the optimal level of 14, especially for the Slow production function. In the late periods of the Slow/Low treatments, 73 percent of the Purdue subjects and 43 percent of the Waseda subjects are monotonically decreasing their capital stock holdings, which is counter to the theoretical prediction, whereas only 20 percent of the Purdue subjects and 27 of the Waseda subjects are monotonically increasing, as predicted. Under the Fixed Ending Rule and Low Endowment, 40 percent are monotonically decreasing and 22 percent are monotonically increasing. Thus, under the Slow production function and Low Endowment, regardless of subject pool and ending rule, *about twice as many subjects are monotonic in the wrong direction as in the correct direction.* Under Fast/Low only 32 percent of the subjects choose a monotonic sequence of capital stock holdings, a far lower percentage than in any other treatment cell, though most sequences are in the predicted direction. Thus, the strong version of Hypothesis 2 is not supported, neither under High nor under Low Endowment, and the weak version of Hypothesis 2 is supported for High Endowment only. □.

Table 3 also gives a comparison of the number of binges in the last five periods with the number of binges in the first five periods that counted toward subjects' earnings. The incidence of binges is lower in the last five periods in all of treatments, suggesting that some of the binges may be due to subjects sampling extreme strategies in the early periods to study their implications. The next result evaluates Hypothesis 3, which concerns the speed of convergence.

Result 3: The theoretical predictions regarding differences in speed of convergence between treatments are not supported.

Support for Result 3: The proposition is evaluated for the last five periods of data from the basic design. The strong version of Hypothesis 3 is not supported because the strong version of Hypothesis 2 is not supported. As for the weak version of Hypothesis 3, in each of the last five periods, the average end-of-period deviation from 14 is greater under Fast/Low than under Slow/Low, the opposite of the theoretical prediction.[11]. The average end-of-period deviation is greater under Fast/High than under Slow/High in two of the five periods. □

The next result focuses on the costliness of the departures from the optimal trajectory documented in Results 1-3, by comparing observed earnings to earnings along the optimal decision path. We use a measure called *efficiency*, a widely used measure of welfare in experimental economics, to compare earnings in different treatments.

Define the *efficiency* of subject i's decision in period j, E_i^j, as:

$$E_i^j = \text{(Earnings obtained by subject)/(Earnings along optimal path)}$$

where earnings refer to monetary payments, which are proportional to the realized value of the maximand in (1). Thus, efficiency represents the percentage of the payoff actually realized by the subject compared to the payoff he would have received by following the optimal policy.[12] As suggested by the earlier results, we can identify differences in earnings in the different treatments. The differences are described in the statement of Result 4.

[11] For each of the last five periods we calculate the average (across subjects) deviation in capital stock from the optimal level after the last round for each treatment (five different averages for each treatment, one for each of the last five periods) We then perform a pairwise comparison of the averages between Slow/Low and Fast/Low as well as between Slow/High and Fast/High.

[12] Note that when the Random Ending Rule is in effect, the earnings resulting from decisions along the optimal path are optimal in expectation given the distribution of period lengths, but are likely to be suboptimal for the actual realization of period lengths (an agent could improve earnings by consuming as much as possible immediately before the period ends). Therefore it is possible for efficiency to take on values greater than one. Of course, under the Random Ending Rule, the subject is unaware of when the period will end at the time he makes his decisions.

Result 4: Observed efficiency of decisions differs between treatments of the basic design. Efficiency is greater under the Fast than under the Slow production function. Efficiency is greater when the initial endowment is High than when it is Low.

Support for Result 4: Table 4 shows the results of the estimation of the following regression equation:

$$E_i = \beta_1 + \beta_2 High + \beta_3 Fast + \beta_4 Fast/High + \beta_5 Fixed + \beta_6 Waseda \tag{15}$$

Each unit of observation in the data described in Table 4 is the overall efficiency attained by one subject for the entire session so that there were a total of 65 observations. The coefficient β_1 indicates the average efficiency for the Slow/Low treatment, in which $\beta_2, \ldots, \beta_6$ all equal 0, which is 71.3 percent. The variable *High* takes on a value of 1 for the High Endowment treatment and 0 for Low Endowment. The variable *Fast* takes on a value of 1 for the Fast treatment and 0 for the Slow treatment. The variable *Fast/High* is an interaction term between Fast and High. The variable *Fixed* equals 1 if the Fixed Ending Rule is in effect and 0 otherwise. *Waseda* equals 1 if the data are generated by a Waseda University subject and 0 otherwise. The effect of the High endowment and Fast production function are both positive and significant at the 5 percent level, and the interaction term between High and Fast is not significantly different from zero. The actual average efficiencies in each treatment can be found in the last column of Table 5. *High* increases the level of efficiency of payoffs by about 8 percent of the maximum possible earnings level over *Low*, and *Fast* increases efficiency by about 18 percent of the maximum possible earnings level over *Slow*. □

Table 4. Estimated treatment effects: all periods

	Treatment					
	Slow/Low	High	Fast	Fast/High	Fixed	Waseda
Effect	.713	.083	.182	-.038	.095	.091
	(.031)	(.040)	(.047)	(.063)	(.046)	(.042)

The data in Table 4 allow us to address Hypothesis 5 and to state Result 5.

Result 5: There is a significant subject pool difference. The Waseda subjects receive higher earnings than the Purdue subjects.

Support for Result 5: The earnings result is seen from the data in Table 4. The coefficient for Waseda is positive and significant at the 5 percent level. The difference in the earnings of the two subject pools was about 9 percent of the optimal level of earnings. □

The Waseda subjects appear on average to be more sophisticated decision makers than the Purdue subjects. However, they are subject to the same types of bias. Under both the Slow/Low and the Slow/High parameters, both groups underinvest and overconsume, as can be seen in the figures in Section 5.1.

Result 6: Holdings of capital stock in the Slow/Low treatment are not different under the Fixed Ending rule than under the Random Ending Rule.

Support for Result 6: Comparing capital stock holdings after round 10 of the Slow/Low and Slow/Low/Fixed treatments in period 19 (the only period in the last five that lasted at least ten rounds under the random ending rule) using a rank-sum test, we find no significant difference at the five percent level of significance. The capital stock is lower than the optimal level under the Fixed Ending Rule, as it is under the Random Ending Rule. □

Result 6 is important because it indicates that the tendency to overconsume in the Slow/Low treatment is not a consequence of the Random Ending Rule. Result 7 considers the easiest decision situation in round 10 under the Fixed Ending Rule, for which there are no dynamic considerations.

Result 7: Decisions are not optimal in round 10 in the Fixed Ending Rule treatment. Capital stock holdings are moving toward the optimal steady state from rounds 10 to 11 in only 1/3 of the observations.

Support for Result 7: Overall, capital stock is moving in the correct direction over time in 80 of 240 observations. In period 23, the final period, when subjects have the most experience, capital stock moves in the correct direction for 5 of the 12 subjects. □

In S/L/F, as in S/L and S/L/W, the capital stock tends not to move in the correct direction over time. However, consider the variable $z = (k_{11} - k_{10}) * (k_{10} - k_9)$. A value of $z > 0$ means that capital stock is moving in the same direction from round 10 to 11 as it did from round 9 to 10. In the data from the last five periods under the Fixed Ending Rule, $z > 0$ in 49 out of 60 observations, while $z = 0$ and $z < 0$ for 10 and 1 observations respectively. This indicates that capital stock movements between round 10 and 11 are usually part of a strategy of reducing or increasing capital stock over two or more rounds. The next result considers changes in earnings over the course of the session.

Result 8: Earnings relative to the optimum are improving over the course of the sessions in two of the seven treatments. In the other five treatments earnings relative to the optimum are stable over the course of the sessions. Bingeing decreases over time in all of the treatments.

Support for Result 8: Consider the following regression equation:

$$E_i^j = \beta_0 + \beta_1 Period + \beta_2 Rounds \tag{16}$$

Where E_i^j = observed efficiency, Period = period number (the first period that counts toward earnings is coded as period 1 in the estimation, though it was actually the fourth period in the experiment, because it was preceded by three practice periods), and Rounds = number of rounds in the period. The equation is estimated for each treatment separately. The results are given in Table 5.

Table 5. Observed efficiency as a function of period number and length and overall average

Treatment	Constant	Periods	Rounds	Overall avg.
S/L	1.7*	-.004	−.058*	.713
	(.122)	(.006)	(.005)	
S/H	.917*	.004	−.011*	.796
	(.062)	(.003)	(.002)	
F/L	.769*	.003	.007*	.895
	(.049)	(.002)	(.001)	
F/H	.916*	.003*	0	.940
	(.018)	(.001)	(.001)	.
S/L/F	.73*	.007*	-	.808
	(.046)	(.001)		
S/L/W	1.69*	.002	−.056*	.805
	(.137)	(.009)	(.006)	
S/H/W	1.03*	.002	−.011*	.885
	(.056)	(.003)	(.002)	

Note: An asterisk indicates significance at less than the 5 percent level.

The results are mixed. The effect of the variable Period is significantly positive for Fast/High and Slow/Low/Fixed but insignificant for the other treatments. The fact that there is less bingeing in the late periods than in the early periods is described in the discussion of Table 3. □

Table 5 also shows the effect of the number of rounds on earnings relative to the optimum. The coefficient of *rounds* is negative for all four of the Slow treatments, Slow/Low, Slow/High, Slow/Low/Waseda, and Slow/High/Waseda. The intercept is greater than 1 for the Slow/Low and Slow/Low/Waseda treatments.[13] This is further evidence of overconsumption in Slow/Low, which raises earnings in short periods and lowers earnings in longer periods. When the period ended after a small number of rounds, earnings were on average higher than the theoretical prediction. In Fast/Low, in which there was overinvestment, efficiency was greater in longer periods.

5.3 Individual decision rules

As shown in Figures 3-11, there was considerable variation in the decision rules among subjects. In this subsection we list the specific decision rules which particular individual subjects used during each of the final five periods. Use of the same rule for the final five periods indicates that the subject has settled on

[13] Though payoffs are greater than along the optimal trajectory in Slow/Low for periods consisting of relatively few rounds, they are less than along the optimal trajectory during longer periods. The average earnings calculations in Table 5 reflect the fact that longer periods have greater weight in final earnings.

that particular rule. In the Slow/Low treatment, two of the subjects followed a strategy of *constant depletion* of capital stock. These subjects set $k_{t+1} = k_t - \alpha$, where $\alpha = .2$ for one subject and $\alpha = .1$ for the other, in each round, throughout the final five periods. One subject followed an *extreme* investment rule, setting $k_{t+1} \in \{.5k_t, f(k_t) + .5(k_t)\}$, that is either the minimum or the maximum feasible amount of investment, during every round of the last five periods. Two subjects followed a *near-exponential depletion* rule. They monotonically decreased capital stock in a manner such that $k_{t+1} = k_t - \alpha(k_t)$, where $\alpha(k_t)$ is a weakly monotonic increasing function of k_t.

At the individual level in Slow/High, there were also a few consistent patterns over the last 5 periods. One subject chose a *constant capital stock or binge* strategy. He set $k_{t+1} \in \{k_t, .5k_t\}$ either keeping his capital stock constant or bingeing in each round. Another subject used a *consumption targeting* strategy by always choosing $c_t \in \{1, 10\}$. One subject used a near-exponential depletion rule.

Some of the individual behaviors observed in the last five periods of Fast/Low were the following. Two subjects used a *multiple buildup and binge* strategy, bingeing at regular intervals and increasing capital stock in every period in which they were not bingeing. One participant used a *constant capital stock* rule for the last three periods, always setting $k_{t+1} = 3$. Three subjects built up capital early in the periods and then monotonically decreased (one subject weakly monotonically decreased) capital stock in every round after round three. One subject appeared to be targeting $k_{t+1} = c_t$.

Several behaviors are observed by individuals over the final five periods in Fast/High. One subject used the multiple buildup and binge strategy. Another subject followed the extreme rule. Two subjects used near-exponential depletion. Two subjects reduced capital stock over the first few rounds, and held a constant capital stock (of 11 units for one subject, of 18 units for the other) for the remainder of the period. One subject, during the last four periods, followed a strategy of repeatedly alternating bingeing for two rounds and building up capital stock for two rounds.

During the last five periods of Slow/Low/Fixed, one subject monotonically increased capital stock by an amount which was increasing in each round. Another subject used the constant capital stock rule during the last period and another subject approximated the constant capital stock rule by setting $k_{t+1} \in (2.85, 3.15)$. Two subjects monotonically increased capital stock over the course of the period and then invested the maximum feasible amount in round 10. On the other hand, one subject increased capital stock through round 9 and then binged in round 10. One subject slowly increased capital stock to about 6 units in round 6 and then slowly reduced it to 4 by round 10.

During each of the last five periods of Slow/Low/Waseda, one subject monotonically increased capital stock to a level of either 6 or 8 units, and then held a constant level of k. One subject approximated the constant capital stock rule for the last five periods and another subject approximated it for the last three periods. One subject binged in every round of every period. In Slow/High/Waseda, all five subjects decreased capital stock monotonically in each of the last five periods. One subject followed the constant depletion rule, lowering capital stock by 1 in each round.

Thus, inspection of the individual level decision rules used by subjects reveals wide heterogeneity, even after considerable experience with the decision problem. This heterogeneity is related to the fact that the problems are very difficult ones, so that subjects do not converge to common, near-optimal choices, as might be the case with an easier decision situation. The fact that subjects could not observe decisions or outcomes of decisions other than their own also limited the sources of information that they could use, to the outcomes of their own past decisions.

6 Discussion

The question of whether agents underinvest or overinvest relative to the optimum is a question without a straightforward yes or no answer. Rather, the answer depends on the parameters of the particular economy of interest. Among the economies studied here, we are able to identify economies in which underinvestment consistently occurs as well as economies in which there is overinvestment on average, and whether or not underinvestment or overinvestment takes place depends on the production technology available in the economy. The Fast production function leads to moderate overinvestment while the Slow production function leads to costly underinvestment.

Sudden episodes called binges, in which a subject consumes as much as possible in a given round, were widely observed in all of the treatments. Smoothing out consumption over time seems to be a difficult concept for subjects. The bingeing does not appear to be a manifestation of confusion or random behavior, but rather it seems that many subjects have determined that their best strategy is to suddenly consume as much as possible (often after building up capital stock) in the apparent belief that the optimal decision is to concentrate consumption in one or a few rounds rather than smoothing it out over all rounds. The incidence of bingeing does decline with experience, indicating that some subjects come to realize that concentrating consumption is suboptimal.

At first glance, a plausible explanation for binges under the Random Ending Rule is that subjects consume as much as possible in anticipation of the end of a period (rather like an investor attempting to “time the market” when making changes to one’s stock portfolio). A correctly timed binge can raise ex-post earnings. In fact, the incidence of binges is about 17% lower under the Fixed Ending Rule than under the Random Ending Rule. However, 83% of the binges cannot be accounted for by the ending rule. More generally, behavior did not differ substantially between Slow/Low and Slow/Low/Fixed. There was overconsumption in both treatments. Thus, we have no evidence that the two ways of representing the infinite horizon generated behavior substantively different from each other. Of course, we do not know (and we may never know) whether behavior in an actual infinite horizon would be different from under our ending rules.

Decisions are better under the Fast than under the Slow production function, in the sense that they lead to higher values of the objective function, as a percentage of the maximum feasible value. Under the Fast production function, the marginal product of capital is more elastic, which means that if the capital stock is below the optimal level, it can be quickly increased at low opportunity cost, and as it increases beyond the optimal level, the marginal product decreases rapidly, making further positive net investment more and more costly as capital stock increases. These properties tend to keep consumption at close to the optimal level. The adverse impact of bingeing is more severe in the Slow than in the Fast treatments, because it is more costly to rebuild capital stock under Slow than under Fast.

Decisions are better under High Endowment than under Low Endowment. Under High Endowment, in which it is optimal to reduce capital stock holdings over time, binges tend not to be as costly as they are under Low Endowment. The most difficult decision problem was Slow/Low, in which instead of monotonically increasing capital stock holdings over time, as predicted, subjects tended to monotonically *decrease* them. This behavior is not due to risk aversion, nor to an incorrect assessment of the probability of the period ending, since it also occurs under the Fixed Ending Rule.

Our results are strengthened by the use of two subject pools, Japanese as well as American subjects, and the observation of similar data in the two groups. Though the Japanese subjects received higher earnings than the American subjects, they exhibited the same type of departures from the theoretical prediction. Under the Slow production function, underinvestment is observed by the members of both subject pools. Binges are also observed among the Japanese subjects though they are less common than among the American subjects.

In our view, the data have individual-level as well as aggregate-level implications. At the individual level, the data indicate that biases exist in the way that agents make sequences of interdependent decisions. That such biases exist is also corroborated by previous experimental studies. In our design, the biases induce an individual to either over- or under-invest relative to the optimum, depending on the particular technology whereby investment is translated into future consumption in the economy. At the aggregate level, the results underscore the need for markets, institutions, and political processes to overcome the biases that exist at the individual level, and to minimize the impact that violations from optimal intertemporal choice have on the economy.

The results obtained here suggest to us three further lines of research. The first line of research is to focus on the manner of presentation of the data and decision-support systems that might improve individual decision making for this type of dynamic optimization problem. The second is to study the manner in which markets or other allocation mechanisms might be employed to lead an economy to evolve along the optimal trajectory. The third is to study political processes for choosing the decision makers, and whether certain types of processes lead to better decisions. Each of these topics is conducive to laboratory research.

References

Ahlbrecht, M., Weber, M.: An empirical study of intertemporal decision making under risk. Management Science **43**, 813–826 (1997)

Azariadis, C.: Intertemporal macroeconomics.Cambridge, MA: Blackwell 1993

Backus, D., Kehoe, P., Kydland, F.: International real business cycles. Journal of Political Economy **101**, 745–775 (1992)

Benhabib J., Farmer, R.: Indeterminacies and sector-specific externalities. Journal of Monetary Economics **37**, 421–443 (1996)

Benzion, U., Rapoport, A., Yagil, J.: Discount rates inferred from decisions: an experimental study. Management Science **35**, 270–284 (1989)

Cass, D.: Optimum growth in an aggregative model of capital accumulation. Review of Economic Studies **32**, 233–240 (1965)

Cooley, T., Hansen, G.: The inflation tax in a real business cycle model. American Economic Review **79**, 733–748 (1989)

Cox, J., Oaxaca, R.: Direct tests of the reservation wage property. Economic Journal **102**, 1423–1432 (1992)

Fehr, E., Zych, P.: The power of temptation: irrationally myopic excess consumption in an addiction experiment. Mimeo, University of Zurich (1995)

Gigliotti, G., Sopher, B.: Violations of present value maximization in income choice. Theory and Decision **43**, 45–69 (1997)

Hey., J., Dardanoni, V.: Optimal consumption under uncertainty: an experimental investigation The Economic Journal **98**, 105–116 (1988)

King, R., Rebelo, S.: Transitional dynamics and economic growth in the neoclassical model. NBER working paper #3185 (1989)

King, R., Rebelo, S.: Transitional dynamics and economic growth in the neoclassical model. American Economic Review **83**, 908–931 (1993)

Koopmans, T.: On the concept of optimal growth. In: The econometric approach to development planning. Skokie, IL: Rand McNally Press 1965

Kydland, F., Prescott, E.: Time to build and aggregate fluctuations. Econometrica **50**, 1345–1370 (1982)

Loewenstein, G.: Anticipation and the value of delayed consumption. Economic Journal **97**, 666–684 (1987)

Loewenstein, G.: Frames of mind in intertemporal choice. Management Science **34**(2), 200–214 (1988)

Lucas, R.: Asset prices in an exchange economy. Econometrica **46**, 1429–1445 (1978)

Noussair, C., Olson, M.: Dynamic decisions in a laboratory setting. Southern Economic Journal **64**, 979–992 (1997)

Ramsey, F.: A mathematical theory of saving. Economic Journal **38**, 543–559 (1928)

Thaler, R.H.: Some empirical evidence on dynamic inconsistency. Economics Letters **8**(3), 201–207 (1981)