



# Introduction to the special issue on mathematical economic epidemiology models

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The effects of the Covid-19 pandemic on global health and well-being, as well as on the global economy, have been devastating. Yet, Covid-19 was not the first, and will not be the last, zoonotic disease to affect humanity. Indeed, there is evidence that increased human-wildlife interface, together with ever expanding mobility around the globe, are contributing to an increased frequency of such events. Although the policy response to Covid-19 resulted in some successes, notably in the speed at which effective tests and, especially, vaccines against the SARS-CoV-2 virus were developed, the pandemic also exposed significant weaknesses in the ability of governments and institutions at all levels to respond in consistent, coordinated, and effective ways. These shortcomings point to the need for guidance from interdisciplinary approaches that incorporate social and economic factors, in addition to traditional epidemiological considerations.

In terms of modeling, it has become apparent that standard epidemiological models, which are designed to describe the evolution of infections and the effect of policy under various intervention scenarios, are ill equipped to evaluate important tradeoffs, especially those related to the effects on economic wellbeing resulting from various mitigation policies, such as prolonged lockdowns. Hybrid epidemiological-economic (often termed *epi-econ*) models were developed in order to better inform policy makers about effective mitigation that is in line with efficiency, as well as with individual incentives. Methodologically, this implies a change of modeling focus from dynamical systems, the study (mainly through simulations) of SEIR-like systems of differential equations, to *optimal control* theory. The latter typically involves the explicit intertemporal optimization of a functional describing combined epidemiological and economic objectives, subject to a set of dynamic equations describing the joint evolution of economic and epidemiological variables of interest. While this approach has already been providing a host of new insights that will help policy address future pandemics, important open questions remain.

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The papers in this volume can be grouped into two distinct categories. The first includes technical contributions addressing conceptual aspects not fully considered in the first generation of epi-econ models. These range from the lack of analytical approaches needed to characterize optimality conditions, to a variety of issues related to epidemic control policies. Arguably, testing and vaccination policies have not received as much attention as lockdowns in the existing literature, especially within models that incorporate spatial and demographic dimensions. The second category of papers are devoted to the development of a new generation of epi-econ models. These studies depart from the normative “social planner” approach and introduce behavior, through incorporating the decisions taken by individual agents in the economy. These may include whether to participate in remote versus in-person interactions, choosing testing frequency, deciding whether to take protective measures, such as wearing a mask, etc. Such decisions are strategic in nature, as they depend on the decisions taken by other agents, as well as on government policy. In a large economy, individual decisions can be assumed to have a negligible effect on aggregate variables. In that case, the theory of *mean-field games* provides a promising tool for identifying and characterizing the dynamics of the evolution of the pathogen in a large population. In recent years, mean-field games have seen considerable success in modeling distributional issues in macroeconomics, and we believe they will prove a similarly useful tool in future epi-econ research.

Perhaps the biggest challenge researchers in the field are currently facing is developing epi-econ models that are mathematically rigorous, tractable, and, at the same time, incorporate institutional and epidemiological detail that is necessary to provide useful guidance to public health policy practitioners. In their current state, mathematical models tend to be too stylized. Models that are more “realistic” tend to be hard to analyze and fully solve analytically, or even numerically. This volume contains fifteen recent papers that make important contributions to the mathematical approach and analysis of epi-econ models. Below we summarize the articles by topic, pointing to connections between current findings, as well as to some open questions that could lead to fruitful future research.

## 1 Dynamic epi-econ modeling and optimal control applications

Garibaldi, Moen, and Pissarides (2024) develop an elegant search-theoretic framework around a three-state discrete-time SIR. Agents in their model derive utility from social interactions, which can also lead to infections when they bring together susceptible and infected hosts through a labor-search-like matching function. The paper considers the effects of four types of externalities. A static externality occurs when individuals ignore the fact that their participation in consumption can lead to increased probability of infection from others. The intertemporal externalities can be divided into three groups: (1) a contagion externality generalizes the static externality, leading to subsequent dynamics of the disease spread; (2) a medical congestion externality; and (3) an immunity externality. The paper compares decentralized Nash outcomes with those chosen by a social planner. Externalities (1) and (3) move in opposite directions, which can lead to too much or too little shielding from economic activity in the

decentralized equilibrium. In the absence of vaccines, herd immunity is the only way to eradicate the disease. If the disease costs are sufficiently high, simulations show that the immunity externality can dominate the contagion externality, and agents in the decentralized solution may shield more than in the planner's solution. In contrast, when vaccines are available, a weaker immunity externality implies more shielding at the optimum than in the decentralized solution. Introducing a vaccine that is expected to arrive before herd immunity is reached implies that private agents shield more and the immunity externality is less severe. Thus, the prospect of a vaccine reduces the optimal market participation more than in the decentralized solution.

Federico, Ferrari, and Torrente (2024) investigate optimal vaccination policies when immunized individuals can still become infected. They employ techniques from viscosity theory and from the theory of regular Lagrangian flows to demonstrate several key theoretical results, among them a verification theorem for non-smooth (viscosity) solutions to the resulting Hamilton–Jacobi–Bellman equation. This is one of the first studies to combine regular Lagrangian flows with an optimal control problem. They then provide a numerical implementation of their model under quadratic costs and find that the optimal vaccination policy, which involves positive vaccination rates in the long run, is effective in mitigating infections.

Prieur, Ruan, and Zou (2024) use a stochastic epi-econ framework to study discrete optimal lockdowns accompanied by continuous vaccination policies. Their framework extends early epi-econ models, which have been overwhelmingly deterministic. The paper contains two additional salient features. The first involves a more realistic, discrete lockdown decision, which is modeled as an *impulse control*, as opposed to a continuous-time control. Second, they incorporate the risk of virus mutations, affecting the pathogen transmission rate. In simplified versions of their model, when the virus can mutate only once, and policy can alternate between two lockdown levels, the authors characterize the optimality conditions for impulse control and study the effect of the probability of a mutation on optimal policy. They find that, combined with vaccinations, lockdowns are effective prior to the occurrence of a mutation. In addition, the presence of uncertainty about a future mutation expedites lockdown interventions.

La Torre, Marsiglio, Mendivil, and Privileggi (2024b) also use a stochastic epi-econ model to study optimal treatment policies in the presence of random shocks associated with the diffusion of a new strain of a disease. New virus strains are modeled as random shocks affecting both the growth rate and the number of infected individuals. The probability of realization of these shocks is endogenous and depends on the number of infected, leading to a model with state-dependent probabilities. This novel functional dependence is rich enough to allow for a variety of disease characteristics, which in turn enhance the set of possible optimal treatment policies. Their model is sufficiently tractable to obtain closed-form solutions. Stochastic steady state dynamics are characterized by an invariant measure with a strictly positive prevalence support, implying that complete eradication of the disease is not possible even in the long run. However, optimal treatment can shift downward the support of the invariant measure, reducing the possible endemic prevalence levels associated with the steady state outcomes.

Calvia, Gozzi, Lippi, and Zanco (2024) use dynamic programming techniques to investigate the implications of non-convexities in epidemiological models for optimization. Such non-convexities are inherent, as they are imposed by the SIR dynamics, but they can also arise naturally due to cost structures. They derive continuity properties for the value function and optimality conditions for the associated optimization problem. The paper points to the need for incorporating general existence and uniqueness results, as well as for characterizing optimal trajectories. These could form a basis for the design of effective numerical methods to simulate the evolution of epidemics under various policies.

Goenka, Liu, and Nguyen (2024) use a combined dynamic general equilibrium-epidemiological model to study optimal lockdowns in the long run, when a disease like Covid-19 becomes endemic and immunity can eventually fail. Their analysis incorporates losses from increased mortality. They show that incorporating mortality costs is necessary for even partial lockdowns to be optimal in the endemic state of the disease. Higher infectivity leads to lower steady state lockdowns in their model, as the fraction of susceptible individuals in the population decreases. In addition, when non-compliance with quarantine policies is possible, a tradeoff emerges between higher compliance and lower quarantine levels.

SEIR-like epidemiological assumptions, which form the base for virtually all epi-econ models, do not typically give rise to realistic disease transmission patterns. This observation motivates Hritonenko and Yatsenko (2024) to propose an alternative approach, based on Volterra integral equation modeling, in order to study optimal lockdown policies under both finite and infinite horizons. Their analysis makes use of clinical data collected during the Covid-19 pandemic. Using adapted variational techniques, they develop a maximum principle leading to a characterization of optimal policies. They demonstrate that lockdowns can be effective in slowing down infections, but large numbers of infections can reemerge when the lockdown is lifted. Thus, optimal lockdowns can be useful tools in mitigating the impact of an epidemic until vaccines eventually become available.

The work by d'Albis, Augeraud-Véron, Coulibaly, and Desbordes (2024) investigates the relationship between disease propagation and mobility. They demonstrate theoretically and empirically that mobility and COVID-19 infections are jointly determined, and that this relationship may be subject to lags. An exogenous epidemic shock has an immediate effect on mobility, while an exogenous mobility shock influences epidemic variables with a delay. In the long run, disease contagiousness and mobility jointly shape epidemiological outcomes. Consistent with qualitative properties from the recent Covid-19 pandemic in France, they find that hospitalizations are highly sensitive to mobility, whereas mobility is less sensitive to hospitalizations. Intriguingly, it appears that unlike in the US, where voluntary reductions in mobility played an important role during the Covid-19 crisis, government policies on mobility were more likely to be necessary in France.

Kogan, El Ouardighi, and Herbon (2024) build a parsimonious finite horizon epi-econ model where dynamics is driven by three non-linear state equations describing, respectively, the law of motion of infections and the dynamics of labor and capital resources in a production economy with an  $Ak$  production technology. There are three control variables: consumption, mobility restrictions, and the treatment level for

infected individuals, subject to an upper bound on treatment capacity. In line with the Covid-19-related operations research literature, the authors impose a linear-quadratic objective function, which allows them to characterize analytically some important properties of the optimal policy paths. Fear of infection alone is shown to be highly harmful to capital in their model, without significantly protecting labor. Given an incompressible consumption level, they find that if the initial capital stock falls below a certain threshold, the capital stock would decrease over time and could eventually become depleted.

We end this section by summarizing two spatial epi-econ models. While spatial variables often play an important role in the epidemiology literature, they have not received a corresponding level of attention in epi-econ modeling. One of the reasons for this is that epidemic spreading over a (continuous) space typically requires the use of partial differential equation (PDE) modeling. The resulting infinite-dimensional model requires an additional level of mathematical sophistication, especially in setups that also involve optimal control. Camacho, Desbordes, and La Torre (2024) develop a spatial epi-econ model using a novel diffusion integro-differential PDE formulation. The policy maker aims to control the number of infections and, at the same time, minimize the cost of treatment. Using a linear-quadratic cost specification, the paper provides a closed-form expression for the optimal policy in the finite horizon case. They find that the disease is never eradicated and, under infinite horizon, everyone in the economy is eventually infected. La Torre, Liuzzi, and Marsiglio (2024a) also introduce a spatial dimension and study optimal regional mitigation policies to contain the spread of a pathogen. The policy authority in their model chooses how to optimally divide the spatial economy into regions employing different combinations of lockdown and treatment policies. The authors characterize the optimal policy in early and in advanced stages of the epidemic. Optimal lockdown intensity depends on several factors, including the initial distribution of disease prevalence, the amount of resources that can be diverted across regions, and the spatial–temporal evolution of the disease.

## 2 Heterogeneity and mean-field game epi-econ approaches

The efficient allocation of scarce tests was an important practical problem in many countries during the COVID-19 pandemic. Group testing has been proposed as a way to expand testing capacity. Bobkova, Chen, and Eraslan (2024) study optimal group testing, taking into account that individuals in the population are heterogeneous in terms of the risk of being infected. They develop an algorithm that reduces the number of tests needed compared to existing techniques. The paper shows that, when both low-risk and high-risk samples have sufficiently low infection probabilities, it is optimal to form heterogeneous groups, with exactly one high-risk sample per group. In a second stage, after a positive group test, when samples are heterogeneous, it is better to test the low-risk sample first. Heterogeneity then results in an advantage for their algorithm. Furthermore, the paper finds that for a range of parameters that include the COVID-19 positivity rate in the United States, the optimal size of a group test is four.

Makris (2024) incorporates private decisions on social distancing in a SIR model with featuring heterogeneous agents. The model is calibrated using UK COVID-19

data and is then used to investigate how mitigating policies depend on the induced responses across various population segments. They find that the predicted evolution of the epidemic is significantly different when social distancing is endogenous from when the mean contact rate is assumed to be an exogenous constant. They also find that policies that shut down essential sectors in the economy have a stronger impact on the death toll than on infections and on herd immunity, compared to policies that shut down non-essential sectors. Restrictions on social distancing can also generate welfare gains in their model.

Fabbri, Federico, Fiaschi, and Gozzi (2024) study a model where a continuum of agents participate in a discrete time, finite state, infinite horizon mean-field game. Their model incorporates an epidemiological component and an economic equilibrium component, as well as individual mobility decisions. Mobility affects agents' income, but also their probability of being infected and of infecting others. Strategic complementarities among individual mobility choices drive the evolution of aggregate economic activity, while infection externalities affect the disease diffusion. The paper demonstrates existence and provides a recursive method for computing an equilibrium. The model is then calibrated using data from the recent COVID-19 epidemic in Italy. Numerical investigations show that under no restrictions on mobility, peak disease prevalence in Italy would have been too high for the resulting hospital bed demand to be met. Thus, mobility restrictions might be part of an efficient regime.

Ghilli, Ricci, and Zanco (2024) develop a game-theoretic model that involves economic decision making in the form of "human capital accumulation" during the spread of an epidemic. Agents maximize their intertemporal utility in a mean-field model that involves a joint choice of the spatial distribution of agents, along with a human capital accumulation decision. The paper provides a numerical solution to the stationary mean-field game system. Existence proofs for solutions to associated systems remain challenging and pose an interesting topic for future research.

In conclusion, the contributions in this symposium volume represent a frontier in mathematical epi-econ modeling research. The articles range from optimal control applications to mean-field game approaches applied to epi-econ models. This remains an active field with many open questions. We are confident that future research will stimulate the advancement of new mathematical and numerical approaches that will provide us with a deeper understanding of the interplay between epidemics, social interactions, and policy.

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