Economic Theory
https://doi.org/10.1007/s00199-022-01424-6

RESEARCH ARTICLE

Wealth inequality, systemic financial fragility and government intervention

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Received: 3 August 2020 / Accepted: 8 March 2022

Abstract

Does wealth inequality make financial crises more likely? If so, how can a government intervene, and how does this affect the distribution of resources in the economy? To answer these questions, we study a banking model where strategic complementarities among wealth-heterogeneous depositors trigger systemic self-fulfilling runs. In equilibrium, higher wealth inequality increases directly the incentives to run of the poor, and indirectly those of the rich via higher bank liquidity insurance, thus increasing the probability of a systemic self-fulfilling run overall. A government intervention on illiquid but solvent banks redistributes resources towards the poor and makes systemic self-fulfilling runs less likely.

Keywords Heterogeneity · Financial intermediation · Bank runs · Government intervention

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This paper previously circulated as "A Theory of Government Bailouts in a Heterogeneous Banking System". We would like to thank Mostafa Beshkar, Diana Bonfim, Stephanie Chan, Isabel Correia, Luca Deidda, Robert Deyoung, Itay Goldstein, Hubert Kempf, Agnese Leonello, Luca David Opromolla, Pedro Teles, Harald Uhlig and the seminar participants, among others, at the 8th Lisbon Meetings in Game Theory and Applications, Banco de Portugal, Indiana University, Tilburg University, Maastricht University, ESSEC Business School and University of Bologna for their useful comments. We gratefully acknowledge the financial support from Fundação para a Ciência e Tecnologia, national funding through research Grant UIDB/05069/2020 and Grant PTDC/EGE-IND/31081/2017.

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Published online: 11 April 2022

JEL Classification D81 · G01 · G21 · G28

1 Introduction

Does wealth inequality make financial crises more likely? If so, how can a government intervene, and how does this government intervention affect the distribution of resources in the economy? The motivation for these questions comes from the debate over the role of wealth inequality in causing large financial crises. Kumhof et al. (2015) show that increasing wealth inequality preceded both the Great Depression and the Great Recession. Two of the most famous arguments to explain this observation (particularly regarding the latter episode) focus on the role played by government intervention. Stiglitz (2012) argues that higher wealth inequality depressed aggregate demand, forcing monetary authorities to lower interest rates too much for too long, thus fuelling a credit bubble and the following crisis. Rajan (2010) instead maintains that higher wealth inequality called for some form of redistribution, and politicians promoted it by allowing households to collateralize their real-estate wealth, thus again fuelling a credit bubble and a crisis. Our paper contributes to this debate by developing an alternative theory directly connecting wealth heterogeneity among banks' depositors to systemic financial fragility, and in particular to the probability of systemic self-fulfilling bank runs. Bank runs have always been considered a crucial trigger of financial crises since the seminal work of Friedman and Schwartz (1963) on the US National Banking Era. Moreover, they have also played a critical role in more recent episodes, like in Argentina in 2001, Uruguay in 2002, Greece in 2015 and the Great Recession (Gorton 2010). We show that increasing wealth inequality has different effects on the incentives to run of the rich and of the poor, and brings about higher systemic financial fragility. Moreover, a government intervention that redistributes from rich to poor brings about the "trickle up" effect of lower systemic financial fragility for the whole economy.

To formalize our arguments, our starting point is the seminal work by Goldstein and Pauzner (2005). In this framework, banks provide liquidity insurance to their depositors against idiosyncratic shocks that force them to consume in an interim date, i.e. before their investments mature. To provide liquidity insurance, banks engage in maturity transformation: They issue short-term liabilities (i.e. deposits) backed by long-term assets. We extend the Goldstein and Pauzner (2005) framework by assuming that the depositors are divided into groups of different wealth levels, that are homogeneous within themselves. The depositors fully deposit their wealth in the banks. We assume that wealth maps one-to-one into deposits to reflect the stylized fact of Table 1: Excluding the lowest 25th percentile (for which the median net worth in 2007 is less than US$1500) the ratio of transaction accounts (e.g. deposits and deposit-like instruments) to total financial assets is basically constant across percentiles of US households' net worth.[1] As such, deposit inequality is almost an exact mirror of wealth inequality.

[1] Transaction accounts as a percentage of total assets are also basically constant, between 0.39 and 0.56 percent. These stylized facts refer to 2007, but are quite stable across time, too. In the last twenty years, the Gini indices of transaction accounts as a percentage of either total financial assets or total assets have been

Table 1 US households' deposit heterogeneity, median values for percentiles of net worth

	Percentiles of household net worth, 2007				
	Less than 25	25–49.9	50–74.9	75–89.9	90–100
Transaction accounts (% of total financial assets)	6.25	2.84	2.62	2.74	2.32
Transaction accounts (% of total assets)	0.56	0.39	0.44	0.52	0.51

The economy is populated by a large number of banks. Because wealth is observable, banks create separate accounts for each wealth group and offer group-specific (or equivalently wealth-specific) deposit contracts. In the real world, these banks would correspond to bank conglomerates or "universal" banks that offer different independent services depending on the amounts deposited, for example through separate commercial and investment banking units that are "ring-fenced" one from the other.[2] In the model, the rationale for this assumption is that banks exist because they are a mechanism to implement the constrained efficient allocation in a decentralized economy in the Diamond and Dybvig (1983) tradition. Hence, they must behave in the same way as a constrained social planner. In our framework, a constrained social planner—i.e. operating all banks and subject to runs—would find it optimal to pool deposits within wealth groups to provide insurance against idiosyncratic shocks, and take into account the investment externality. However, offering the same deposit contract to all depositors independently of their wealth would entail an ex-ante redistribution from rich to poor. That would be equivalent to an insurance against states of nature, that is arguably not Pareto efficient.

To repay the depositors, the banks invest the deposits into a productive asset. This yields a positive return with some probability, which represents the aggregate state of the economy. The return also negatively depends on the total fraction of depositors who withdraw in the interim date in the whole economy. In this way, we introduce an investment externality, which is similar to Morris and Shin (2000) and is in the spirit of the production externality of Romer (1990). Another way to justify this assumption is in terms of aggregate demand externalities (Bebchuk and Goldstein 2011): If one bank is forced to liquidate projects prematurely, that negatively affects aggregate demand, thus reducing overall profits in the economy (Cooper and John 1988) and eventually impacting other banks.

In such an environment, depositors' decisions to withdraw in the interim period are subject to within-group strategic complementarities: The more depositors expect the other depositors in their own wealth group to withdraw in the interim date, the higher their incentives to withdraw in the interim date are, too.[3] Because of this, if the realization of the aggregate state is common knowledge, the economy exhibits two equilibria: one in which only the depositors who are hit by the idiosyncratic shocks

Footnote 1 continued
almost constant, and equal to around 20 and 12%, respectively. Calculations are available upon request. Source: US Survey of Consumer Finance.

[2] In the context of mutual funds for example, regulation prevents cross-subsidization across funds, in order to rule out Ponzi schemes.

[3] For a survey on games with strategic complementarities see Amir (2005).

withdraw in the interim date, and one in which all depositors withdraw because they expect everybody else to do the same, and are afraid that banks have insufficient resources to pay them all. In this latter case, we say that depositors' expectations trigger a banking crisis in the form of a self-fulfilling "run". Put differently, banks can become insolvent—i.e. not able to repay depositors according to the deposit contract—for two reasons: either because their investments turn out unproductive or because of a run in the interim period.

To characterize a unique equilibrium, we follow the "global game" literature (Carlsson and van Damme 1993; Morris and Shin 1998) and assume that each depositor observes a heterogeneous private noisy signal about the realization of the aggregate state—the probability of success of the long term asset. Based on the noisy signal, a depositor forms posterior beliefs about the true aggregate state and the running behavior of the other depositors in their wealth group, and ultimately decides whether to run on their bank. This happens if the signal that they receive is lower than a certain wealth-specific threshold, i.e. the critical type, which therefore is a measure of the financial fragility of each wealth group.

The presence of the investment externality implies that the strategic complementarities operate both within and between wealth groups. As such, all depositors must form posterior beliefs also about the running behavior of the depositors in the whole economy. This represents a theoretical challenge, that we solve by adapting to this framework the concept of "Belief Constraint" (Sakovics and Steiner 2012). We start by defining the critical belief of a group as the belief of the group's critical type about the aggregate number of individuals running in the economy. Then, the "Belief Constraint" states that the average of the critical beliefs, i.e. the beliefs regarding the aggregate number of individuals running, has a constant probability distribution. We can then employ this property to identify the unique running threshold in the economy. Another consequence of the investment externality is that the wealth-specific thresholds that trigger a run in each group are all functions one of the others, making the identification of each group's critical belief a daunting task. Yet, as the volatility of the noisy signals goes to zero all thresholds cluster around a unique value. In other words, a self-fulfilling run becomes "systemic": Depositors with different levels of wealth run together, following a common threshold strategy. The Belief Constraint allows us characterize the unique threshold by averaging out the indifference conditions between running and not running of depositors of the different wealth levels. In this way, the common threshold strategy turns out to be increasing in the endogenous bank liquidity insurance provided to each wealth group. This means that this economy features financial contagion through expectation formation. High liquidity insurance for a wealth group increases the likelihood of a systemic self-fulfilling run in the whole economy, as it increases the self-fulfilling incentives to run of that wealth group, and as a consequence of all the others, too. Moreover, the unique threshold turns out to be a decreasing function of depositors' wealth. This happens because, when the wealth of a group increases, the incentives to wait and not join a run of a depositor belonging to that group increase. Importantly, both effects are convex as depositors' utility is concave.

The two previous mechanisms are at the core of the positive relation between wealth inequality and systemic financial fragility that emerges in our environment. To see that, we run a comparative statics exercise. We increase the wealth of a rich group and lower the wealth of a poor group in the same amount, so that the aggregate endowment of the economy stays constant. This has the direct effect of increasing the incentives to wait of the rich, but decreasing more the incentives to wait of the poor. Moreover, as we prove that in equilibrium liquidity insurance is increasing in wealth, higher wealth inequality lowers the liquidity insurance for the poor, and increases it for the rich. This brings about the indirect effect of lowering the incentives to run of the first, but increasing more the incentives to run of the second. In other words, by increasing the incentives to run of the poor directly and of the rich indirectly, higher wealth inequality leads to higher systemic financial fragility.

The presence of systemic financial fragility justifies a government intervention against self-fulfilling runs, which are inefficient because are not due to bad fundamentals but only to coordination failures (Allen and Gale 2004). We assume the existence of an economy-wide government that, in order to maximize total welfare, employs resources outside of the banking system to provide subsidies and affect the depositors' incentives to run. We focus our attention on a bank liquidity assistance in the spirit of the lender of last resort (LOLR): The government subsidizes banks, but only when they are illiquid but solvent. Since the classical argument by Bagehot (1873), this has been considered the standard instrument with which public authorities have resolved financial fragility due to bank runs, and has some sound economic rationale (Diamond and Dybvig 1983; Rochet and Vives 2004).

Bearing in mind that illiquidity is off equilibrium and as such it is not a possible ex-post outcome, a bank liquidity assistance can have an effect on the formation of the depositors' expectations of a run, and therefore on the equilibrium. In fact, under this intervention subsidies to banks, even if never actually implemented, lower systemic financial fragility as they would allow banks to retain a larger amount of resources to distribute to the depositors in the final date, thus lowering their incentives to run. This means that a full liquidity assistance, that allows the banks to serve all depositors even when they all withdraw in the interim date, can rule out systemic self-fulfilling runs altogether. In other words, when feasibility is satisfied, no depositor has incentives to run, and the government, just by announcing an intervention, resolves systemic financial fragility at zero costs.

The more interesting case happens instead when a full intervention is not feasible, and the government can only implement a partial liquidity assistance. This consists of a set of wealth-specific subsidies that takes into account that the rich depositors' incentives to run are less sensitive to subsidization. Hence, a partial bank liquidity assistance results in the government ranking wealth groups in increasing order of wealth, and providing full liquidity assistance from the top of the ranking, until resources are exhausted. In that sense, depositors at the lower end of the wealth distribution are fully subsidized, and after a certain threshold the others receive zero. Interestingly, the resulting redistribution does not depend on a mere welfare motivation, but on the "trickle-up" effect of lowering systemic financial fragility for the whole economy.[4]

[4] Mitkov (2020) finds a similar result, but for very different reasons. In his framework, rich depositors receive lower transfers because the government wants to ensure that the ex-post payment schedule is the same across all the banks on which it intervenes.

The rest of the paper is organized as follows. In Sect. 2, we summarize our contribution to the literature. In Sect. 3, we lay down the environment of the model. In Sect. 4, we study the strategic complementarities in the depositors' decisions to run, and the banking equilibrium without government intervention. In Sect. 5, we characterize government intervention. In Sect. 6, we numerically run some comparative statics exercises to evaluate the effect of changing the wealth distribution on systemic financial fragility. Finally, Sect. 7 concludes.

2 Contribution to the literature

The present paper contributes to the literature in several respects. First, by developing a theory of banking system with heterogeneous depositors, it studies how systemic financial fragility is connected to deposit and wealth heterogeneity, which some new evidence suggests is a key driver of depositors' withdrawing behavior (Iyer et al. 2019). Importantly, this link is not conveyed through the raising of credit bubbles, which is a channel that applies well to the US (Kumhof et al. 2015) but is far from general.[5] In contrast, our argument is based on financial contagion (Allen and Gale 2000) in particular through expectation formation, which has been analyzed in the past in two-group environments (Dasgupta 2004; Goldstein 2005; Leonello 2018).[6]

The empirical literature on bank heterogeneity and systemic risk (Laeven et al. 2016) finds support for the argument that bank size has a positive impact on systemic risk. Nevertheless, the amount of theoretical work aimed at rationalizing this channel has been small, Davila and Walther (2020) being one of the few exceptions. Choi (2014) and Goldstein et al. (2020) are instead two examples that focus on the role of heterogeneity in affecting financial stability, but on the asset side of banks' balance sheets. Differently from these papers, we focus on the liability side, and in particular on depositors' heterogeneity and how it alters their incentives to run, while keeping banks homogeneous.

Second, our work contributes to the analysis of the economics of government intervention in the face of self-fulfilling run risk. In a recent paper, Allen et al. (2018) extend the bank-run framework of Goldstein and Pauzner (2005) by introducing a benevolent regulator who provides guarantees to banks' depositors. However, the authors study a homogeneous economy, which is not suitable to analyze heterogeneity and the redistributive implications of government intervention. Cooper and Kempf (2016), Mitkov (2020) and Davila and Goldstein (2021) develop banking models with deposit heterogeneity, and study government intervention after self-fulfilling runs modelled as sunspot-driven coordination failures.[7] Our contribution with respect to them is to

[5] Atkinson and Morelli (2010, 2015) and Bordo and Meissner (2012) find little evidence of a connection between inequality, household credit bubbles and financial crises, and Gu and Huang (2014) find that the relation holds only in Anglo-Saxon countries.

[6] In particular, we distinguish our contribution from Corsetti et al. (2004) and Bannier (2005) by abstracting from the presence of a larger depositor (a "Soros") who is so much richer than the others that can act strategically, possibly exploiting better information or a first-mover advantage.

[7] See Bertolai et al. (2019) for another model of government redistribution in the presence of run risk.

endogenize systemic financial fragility, and explicitly characterize its link to deposit heterogeneity.

Finally, the present paper contributes to the theoretical literature on bank runs as "global games" (Rochet and Vives 2004; Goldstein and Pauzner 2005) by studying the role of wealth heterogeneity and adapting the concept of Belief Constraint, that Sakovics and Steiner (2012) developed and applied to a canonical problem of investment subsidization. Guimaraes and Morris (2007) is an example of currency-crisis model with a global game, in which the authors study an extension with wealth heterogeneity. Differently from our work, they assume Cobb-Douglas utility function that allows linear aggregation of threshold strategies across population. Drozd and Serrano-Padial (2018) instead study a model of a debt-financed entrepreneur subject to enforcement externalities. Theoretically, their contribution lies in the characterization of an equilibrium in which the threshold strategies of the agents, differently from our work and from Sakovics and Steiner (2012), might cluster around more than one value.

3 A model of banking with heterogeneous depositors

3.1 Preferences and endowments

The economy lives for three dates, labeled $t = 0, 1, 2$, and is populated by a unitary continuum of bank depositors, divided into G groups indexed by j, each of equal mass.[8] The groups are heterogeneous with respect to the wealth that they deposit in the banking system: All depositors in group j have an initial endowment e^j at date 0, and nothing at dates 1 and 2. At date 1, a depositor i in wealth group j is hit by a private idiosyncratic shock θ^{ij}, that takes value 0 with probability $1 - \pi$ and 1 with probability π. The shock affects the point in time at which the depositor wants to consume, in accordance with the welfare function:

$$U(c_1^j, c_2^j, \theta^{ij}) = \theta^{ij} u(c_1^j) + (1 - \theta^{ij}) u(c_2^j). \tag{1}$$

The depositors gain utility from consumption either at date 1 or at date 2. If $\theta^{ij} = 1$, a depositor only wants to consume at date 1, while if $\theta^{ij} = 0$ they only want to consume at date 2. Thus, in line with the literature, we call type-0 and type-1 depositors late (or "patient") consumers and early (or "impatient") consumers, respectively. The law of large numbers holds, so π and $1 - \pi$ are the fractions of depositors in the whole economy who turn out to be early or late consumers. The utility functions $u(c)$ is twice continuously differentiable, increasing and concave. Moreover, $u(0) = 0$ and $\lim_{c \to 0} u'(c) = F$, with F arbitrarily large but finite. This is a modified Inada condition that has the same rationale of the standard condition that several models assume, including the original work by Diamond and Dybvig (1983): It ensures that the depositors really value the possibility of avoiding zero consumption.

[8] The assumption of groups of equal mass comes at no loss of generality for the main results concerning wealth inequality. This is because financial fragility will depend on the beliefs of each single atomistic depositor, which do not depend on the size of the groups to which they belong.

However, a standard Inada condition with $\lim_{c\to 0} u'(c) = +\infty$ would not be consistent with $u(0) = 0$. Then, the modified condition allows us to reconcile the two assumptions. A utility function satisfying these assumptions is $u(c) = ((c+\psi)^{1-\gamma} - \psi^{1-\gamma})/(1-\gamma)$. The constant ψ ensures that $u(0) = 0$ and $\lim_{c\to 0} u'(c) = \psi^{-\gamma}$, which can be arbitrarily large but finite as assumed.

3.2 Banks and technologies

The economy is also populated by a large number of competitive banks. The relationship that depositors have with their banks is exclusive, that is, they can can make a deposit only in one bank.[9] At date 0, the banks collect the initial endowments e^j of the depositors—which are the only liability on their balance sheets—and invest them so as to maximize their profits, subject to depositors' participation and to budget constraints. Perfect competition ensures that the banks solve the equivalent dual problem of maximizing the expected welfare of their depositors subject to budget constraints.

The banks invest the deposits in a productive asset yielding a stochastic return A at date 2 for each unit invested at date 0. This stochastic return takes values $R(1-\ell)$ with probability p, and 0 with probability $1-p$, where ℓ is the total fraction of depositors who withdraw at date 1 in the whole economy. The probability of success of the productive asset p represents the aggregate state of the economy, and is distributed uniformly over the interval [0, 1], with $(1-\pi)\mathbb{E}[p]R > 1$. Moreover, the productive asset can be liquidated at date 1, i.e. before its natural maturity, and yields 1 unit of consumption for each unit liquidated.[10] Intuitively, this productive asset represents an investment opportunity whose return in case of success depends on how much of the initial investment reaches maturity in the whole economy. Put differently, the common productive asset exhibits an investment externality across wealth groups.

The banks employ the productive asset to repay the depositors at date 1 and 2. As the banks observe the amounts deposited, they can perfectly discriminate across wealth groups. Put differently, they operate as "universal banks" that serve all wealth groups, but set up separate balance sheets for each one of them and offer group-specific deposit contracts. The deposit contracts state the uncontingent amount d^j that the depositors can withdraw at date 1 and the state-dependent amount $d_L^j(A)$ that they can withdraw at date 2, which is an equal share of the residual available resources.[11] As the realizations of the idiosyncratic shocks θ^{ij} are private information, the depositors must have the incentives to truthfully report them. This implies that the deposit contracts must satisfy the incentive compatibility constraint $d^j \leq d_L^j(R)$ in

9 Farhi et al. (2009) and Panetti (2017) show the inefficiency of competitive banking equilibria where this assumption is relaxed and deposit contracts are non-exclusive, and study the welfare implications of liquidity requirements.

10 Deidda and Panetti (2018) study the banking equilibrium when the liquidation value of the productive asset is smaller than 1, and the bank has to solve a liquidity management problem.

11 In order to rule out uninteresting run equilibria, we assume that the parameters are such that equilibrium early consumption d^j turns out to be smaller than $\min\{1/\pi, R\}$. Being well established in the literature, the assumption of standard deposit contract can be rationalized as a way to resolve conflicts between banks' managers and shareholders (Calomiris and Kahn 1991; Diamond and Rajan 2001) or as a consequence of asymmetric information (Flannery 1986; Dang et al. 2017).

every group j. The banks commit to the deposit contracts at date 0, and pay early withdrawals by liquidating the productive asset until their resources are exhausted. When this happens, and the banks are not able to fulfill their contractual obligations, they go into insolvency. In this case, they must liquidate all the productive assets at date 1, and equally share the proceeds inside each wealth group among all the depositors who withdraw early.[12]

We assume that the depositors cannot observe the true value of the realization of the aggregate state p, but receive at date 1 a noisy private signal $\sigma^{ij} = p + \eta^{ij}$. The term η^{ij} is an idiosyncratic noise, indistinguishable from the true value of p and drawn from a uniform distribution over the interval $[-\epsilon, +\epsilon]$, with ϵ positive but negligible.[13] Given the received signal, late consumers decide whether to withdraw from the bank at date 2, as the realization of the idiosyncratic shock would command, or "run on the bank" and withdraw at date 1. We assume that they take this decision following the threshold strategy:[14]

$$a^{ij}(\sigma) = \begin{cases} \text{wait} & \text{if } \sigma^{ij} \geq \sigma^{j*}, \\ \text{run} & \text{if } \sigma^{ij} < \sigma^{j*}. \end{cases} \tag{2}$$

3.3 Timing and definitions

The timing of actions is the following: At date 0, the banks collect the initial endowments, and choose the deposit contracts $\{d^j, d_L^j(A)\}$; At date 1, all depositors get to know their private types and signals, and the early consumers withdraw, while the late consumers, once observed their own signals, decide whether to run on their banks or not; Finally, at date 2, those late consumers who have not run at date 1 receive an equal share of the available resources.

We solve the model by backward induction, and characterize a perfect Bayesian equilibrium, in which a representative bank chooses wealth-specific deposit contracts. The definition of equilibrium is as follows:

Definition 1 Given the distributions of the idiosyncratic and aggregate shocks and of the private signals, a perfect Bayesian banking equilibrium is a set of deposit contracts $\{d^j, d_L^j(A)\}$ and depositors' threshold strategies, such that for every realization of signals and idiosyncratic shocks $\{\sigma^{ij}, \theta^{ij}\}$:

- The depositors' decisions to run maximize their expected welfare;
- The deposit contract maximizes the depositors' expected welfare, subject to budget constraints;
- The beliefs of the banks and depositors are updated according to the strategies employed and the Bayes rule.

[12] The assumption of equal shares at insolvency simplifies the analysis without altering its results.

[13] The two sources of heterogeneity in our framework, namely wealth and information, are not correlated one to the other. For an example of endogenous information acquisition in games with strategic complementarities, see Amir and Lazzati (2016).

[14] The focus on threshold strategies is common in the bank-run literature. In a similar environment with homogeneous wealth, Goldstein and Pauzner (2005) show that every equilibrium strategy is a threshold strategy.

3.4 Banking equilibrium with perfect information

As a benchmark for the results that follow, we start our analysis with the characterization of the banking equilibrium with perfect information, in which the bank can observe the realization of the private idiosyncratic shocks hitting the depositors. More formally, the bank solves:

$$\max_{d^j} \sum_j \left[\pi u(d^j) + (1-\pi) \int_0^1 pu \left(R(1-\pi) \frac{e^j - \pi d^j}{1-\pi} \right) dp. \right] \tag{3}$$

The bank knows that with probability π a depositor will turn out to be an early consumer and consume d^j, and with probability $1-\pi$ they will turn out to be a late consumer.[15] In this case, the total amount of available resources at date 2 depends on the realization of the aggregate state p, on the total number of late consumers in the whole economy, equal to $1 - (1/G)\sum_j \pi = 1 - \pi$, and on the amount of productive assets that are not liquidated to pay early consumption, $e^j - \pi d^j$. The first-order conditions with respect to early consumption d^j give the equilibrium conditions:

$$u'(d^j) = (1-\pi)\mathbb{E}[p]Ru'(R(e^j - \pi d^j)), \tag{4}$$

for every group j. Intuitively, this result shows that the bank provides an allocation such that the marginal rate of substitution between early and late consumption is equal to the expected return of the productive asset. Moreover, as the utility function $u(c)$ is concave, the equilibrium amounts d^j and $d_L^j(R) = R(e^j - \pi d^j)$ are both increasing in the initial endowment e^j, given that the ratio d^j/e^j is constant across wealth groups.[16]

Finally, the concavity of the utility function and the assumption that $(1-\pi)\mathbb{E}[p]R > 1$ imply that the incentive compatibility constraint is satisfied. In other words, a banking equilibrium without perfect information, i.e. in which a bank needs to ensure truth-telling, would be equivalent to the banking equilibrium with perfect information.

4 Systemic self-fulfilling runs

We now move to the analysis of the banking equilibrium in the presence of private signals regarding the aggregate state of the economy. To this end, we go by backward induction, and start by studying the Bayesian Nash equilibrium of the stage game in which the depositors choose their threshold strategies according to which they run. Then, we characterize the banking equilibrium.

[15] In equilibrium, by the modified Inada condition, both early and late consumption must be positive.

[16] To see that d^j is increasing in e^j, notice that the bank objective function is supermodular, as its cross derivative with respect to d^j and e^j is positive (see the definition of supermodular function in footnote 26). Also, with a simple change of variable, namely by letting $x^j = e^j - \pi d^j$, we can show that the objective function is supermodular in (x^j, e^j). This is equivalent to saying that $d_L^j(R) = R(e^j - \pi d^j)$ is increasing in e^j.

4.1 Endogenous threshold strategies

As in Ennis and Keister (2006), we assume that at date 1 the depositors arrive at the bank in random order, and know neither how many of them are in line nor their positions in the line. As a result, the depositors do not accept a deposit contract contingent on either their position in line or the number of early withdrawals.[17] In other words, when taking their withdrawal decisions, depositors do not know whether a run is under way or not.

Due to its commitment to pay an amount of early consumption d^j, the bank must liquidate the productive asset to pay early withdrawals until the resources are exhausted. As a consequence, if late consumers expect only early consumers to withdraw at date 1, they will withdraw at date 2 and receive the incentive-compatible consumption $d_L^j(R) > d^j$. However, if late consumers expect all other depositors to withdraw at date 1, they will rather withdraw at date 1 as well, because in that case they will be served pro-rata at date 1 instead of getting zero at date 2. This means that this economy, if there is common knowledge about the realization of the aggregate state, features a "no run" equilibrium and a "run" equilibrium as any Diamond-Dybvig environment.[18]

As we will show, the private signals allow us to resolve this multiplicity of equilibria by forcing the depositors to coordinate their actions: run under some range of signals, and not run under another. The effect of the signals is twofold: They provide private information about the aggregate state of the economy, and about the signals of the other depositors. Intuitively, obtaining a high signal increases the incentives for a late consumer to wait until date 2 and not withdraw (i.e. not "run on the bank") at date 1, because it induces the belief that the realization of the aggregate state is good, and the signals of the other depositors are also high (under the assumption that the volatility of the signal is negligible).

More formally, a late consumer i in group j receives a private signal σ^{ij} at date 1, and takes as given the deposit contract fixed at date 0. Based on these, they create posterior beliefs about how many depositors withdraw at date 1 in their own group as well as in the whole economy, and the probability of the realization of the aggregate state, and decides whether to withdraw at date 1 or not. Two regions of extremely high and extremely low signals arise. In these, the decision of a late consumer of any group is independent of their beliefs regarding the actions of the other depositors. In the "lower dominance region", the signal is so low that late consumers of all groups run irrespective of the behavior of the others. This happens below the threshold signal $\underline{\sigma} = \min_j \underline{\sigma}^j$, that is the lowest of the thresholds that make depositors indifferent between withdrawing or not, when all other depositors wait:

$$u(d^j) = \underline{\sigma}^j u\left(R(e^j - \pi d^j)\right). \tag{5}$$

[17] For the same reason, the depositors also do not accept a deposit contract that involves deposit pooling across wealth groups at a run, that would modify the bank insolvency threshold and create a further source of between-group externality.

[18] For this argument to hold, we need to assume that a government cannot credibly commit to suspend deposit convertibility in the case of a run. Ennis and Keister (2009) study the time inconsistency of suspension policies in a banking model with multiple equilibria. Cavalcanti and Monteiro (2016) study the role of suspension for the early acquisition of information about opportunistic bank behavior.

From here, it is easy to see that the threshold signal $\underline{\sigma}^j$ is decreasing in the initial endowment e^j and increasing in the early consumption d^j: The more a bank promises to an early consumer in group j, the larger is the set of signals below which the depositors in that group run irrespective of what the others do. In the "upper dominance region", instead, the signal is so high that late consumers always wait until date 2 to withdraw. Following Goldstein and Pauzner (2005), we assume that this happens above a threshold $\bar{\sigma}$, where the investment is safe, i.e. $p = 1$, and gives the same return $R(1-\pi)$ at date 1 and date 2. In this way, a late consumer is sure to get $R(e^j - \pi d^j)$ at date 2, irrespective of the behavior of all the other late consumers, and prefers to wait.

The existence of the lower and upper dominance regions, regardless of their size, ensures the existence of an equilibrium in the intermediate region $[\underline{\sigma}, \bar{\sigma}]$, where the late consumers decide whether to run or not based on their posterior beliefs. In this region, late consumers run if their signal is lower than a threshold signal σ^{j*}, which is the value of the signal that makes them indifferent between running or not given their beliefs. More formally, define the utility advantage of waiting versus running as:

$$v^j(n, n^j) = \begin{cases} \sigma^{ij} u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) - u(d^j) & \text{if } \pi \leq n^j < \frac{e^j}{d^j}, \\ -u\left(\frac{e^j}{n^j}\right) & \text{if } \frac{e^j}{d^j} \leq n^j \leq 1, \end{cases} \tag{6}$$

where n^j and n are the total fraction of depositors withdrawing at date 1 in group j and in the whole economy, respectively. By the law of large numbers, these fractions are given by:

$$n^j = \pi + (1-\pi)\text{prob}(\sigma^{ij} \leq \sigma^{j*}), \tag{7}$$

$$n = \sum_k n^k = \pi + (1-\pi)\sum_k \text{prob}\{\sigma^{ik} \leq \sigma^{k*}\}, \tag{8}$$

i.e. the fraction of depositors withdrawing at date 1 is the sum of the π early consumers who withdraw for sure plus those among the $1-\pi$ late consumers who get a signal below the threshold signal σ^{j*}.

The expression for $v^j(n, n^j)$ highlights that, when the fraction of depositors running is between π (i.e., when there is no run) and e^j/d^j (i.e. the maximum fraction of depositors that a bank can serve in a wealth group j according to the contract with the available resources), a late consumer receiving a signal σ^{ij} holds the belief that the productive asset yields a positive return with probability $\mathbb{E}[p] = \mathbb{E}[\sigma - \eta^{ij}] = \sigma^{ij}$. In that case, if they wait until date 2 depositors consume an amount of late consumption that—with a slight abuse of notation—we denote as $d_L^j(A, n, n^j)$, to keep track that it explicitly depends on the realization of the aggregate state, and on the total fraction of depositors withdrawing at date 1 in the whole economy and in group j. In other words, if they wait until date 2 depositors consume either $d_L^j(R, n, n^j) = R(1-n)\frac{e^j - n^j d^j}{1-n^j}$ or $d_L^j(0, n, n^j) = 0$, and if they withdraw they consume d^j. In contrast, when the fraction of depositors running is higher than e^j/d^j, the representative bank of wealth group j goes into insolvency: It is forced to liquidate all productive assets and equally share the proceeds among the depositors who withdraw. Hence, late consumers get zero if they wait, and e^j/n^j if they withdraw at date 1. As in Goldstein and Pauzner (2005), we can show that the function $v^j(n, n^j)$ exhibits both between- and within-group one-sided strategic complementarities. To see that, calculate:

$$\frac{\partial v^j}{\partial n^{\ell \neq j}} = \begin{cases} -R\sigma^{ij} u'\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right)\frac{e^j - n^j d^j}{1-n^j} & \text{if } \pi \leq n^j < \frac{e^j}{d^j}, \\ 0 & \text{if } \frac{e^j}{d^j} \leq n^j \leq 1, \end{cases} \tag{9}$$

and notice that the derivative in the first interval is always negative. As far as the within-group strategic complementarity, instead:

$$\frac{\partial v^j}{\partial n^j} = \begin{cases} R\sigma^{ij} u'\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right)\left[-\frac{e^j - n^j d^j}{1-n^j} + (1-n)\frac{e^j - d^j}{(1-n^j)^2}\right] & \text{if } \pi \leq n^j < \frac{e^j}{d^j}, \\ u'\left(\frac{e^j}{n^j}\right)\frac{e^j}{n^{j2}} > 0 & \text{if } \frac{e^j}{d^j} \leq n^j \leq 1. \end{cases} \tag{10}$$

Again, the derivative in the first interval is negative (i.e. we have one-sided strategic complementarity) as $n^j < 1$. Moreover, the derivative in the second interval is positive as an insolvent bank equally shares the proceeds from liquidation among all depositors running. Hence, the more depositors run, the lower the amount they will receive.

We can now show that there is a unique equilibrium in threshold strategies. Assume that all late depositors in group j run below the threshold σ^{j*}, and consider a late depositor who obtained signal σ^{ij}. To compute the expected utility differential between withdrawing at date 2 versus date 1, notice that since both state p and error terms η_{ij} are uniformly distributed, depositors' posterior distribution of p given σ^{ij} is uniformly distributed over $[\sigma^{ij} - \epsilon, \sigma^{ij} + \epsilon]$.[19] Also, the posterior belief of depositor i regarding the signal of agent $k \neq i$ is symmetric around σ^{ij}. Hence we can compute the expected utility differential, denoted $\mathbb{E}[v^j(n, n^j)|\sigma^{j*}]$. We derive the threshold signal σ^{j*} as the value of the signal such that $\mathbb{E}[v^j(n, n^j)|\sigma^{j*}] = 0$, i.e.:

$$\begin{aligned} &\int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} \sigma^{j*} u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) f(n^j) dn^j f(n) dn \\ &= \int_\pi^1 \left[\int_\pi^{\frac{e^j}{d^j}} u(d^j) f(n^j) dn^j + \int_{\frac{e^j}{d^j}}^1 u\left(\frac{e^j}{n^j}\right) f(n^j) dn^j\right] f(n) dn, \end{aligned} \tag{11}$$

The one-sided strategic complementarity, together with the fact that $v(n, n^j)$ is increasing in σ^{ij}, guarantees the uniqueness of a threshold signal σ^{j*} in between the dominance regions, below which a self-fulfilling run happens (Goldstein and Pauzner 2005). In this case, depositors run not because it is dominant strategy (as in the lower dominance region) but because the signal is sufficiently bad for them to expect all the other depositors to run. In a similar problem with a global game among heterogeneous

[19] This claim works even when the realized state is close to 1 as we assume positive but vanishing noise.

agents, Frankel et al. (2003) show that, as the noise ϵ of the signals vanishes, there exists a unique threshold signal σ^* around which the thresholds signals σ^{j*} tend to cluster, which is the solution to the system of equations of (11) for every group j. However, finding a solution to that system is cumbersome, as the expressions for σ^{j*} are highly non-linear. Instead, we solve the problem by applying the concept of "Belief Constraint" which states that the average critical beliefs are uniform, so we can characterize the critical type by averaging out the indifference conditions (Sakovics and Steiner 2012). The following proposition ensues:

Proposition 1 *The set of equilibrium threshold strategies characterizing the withdrawing decisions of the depositors is unique. As the volatility of the noise ϵ goes to zero, all threshold signals σ^{j*} converge to a common limit σ^*, which is characterized by the average indifference condition:*

$$\sum_j \mathbb{E}[v^j(n, n^j)|\sigma^*] = 0, \tag{12}$$

and gives:

$$\sigma^*(\mathbf{d}) = \frac{(1-\pi)\sum_j \left[\int_\pi^{\frac{e^j}{d^j}} u(d^j)dn^j + \int_{\frac{e^j}{d^j}}^1 u\left(\frac{e^j}{n^j}\right)dn^j\right]}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right)dn^j dn}, \tag{13}$$

where $\mathbf{d} = \{d^j\}_{j=1}^G$.

Proof In Appendix A. □

Intuitively, the proof of the proposition can be summarized as follows. In principle, every group j should have its own threshold signal σ^{j*} below which a signal triggers a self-fulfilling run. This threshold signals should be characterized by the wealth-specific indifference conditions for a late consumer between withdrawing early and waiting, given their beliefs. However, the presence of between-group strategic complementarities implies that the running behavior of a late consumer in a group j influences the running behavior of the late consumers in all the other groups, too. That would mean that we should solve for the groups-specific threshold signals σ^{j*} by solving a system of G indifference conditions in G unknowns. However, as the volatility of the noise ϵ goes to zero, all depositors tend to form the same posterior beliefs about the aggregate state. Moreover, the depositors have to form posterior beliefs about the behavior of all the other depositors, in their own group as well as in the others. The Laplacian Property (Morris and Shin 1998) ensures that the cumulative distribution functions of the random signals σ^{ij} in all groups j are uniformly distributed over the interval [0, 1]. Hence, the fraction of depositors withdrawing early in group j, which is given by (7), is a random variable uniformly distributed over the interval $[\pi, 1]$, and its probability distribution function is $f(n^j) = 1/(1-\pi)$.

To characterize the distribution of the total fraction of depositors running in the whole economy, we instead adapt to our environment the concept of "Belief Constraint" of Sakovics and Steiner (2012). The Belief Constraint states that the Laplacian property holds on average across the threshold types of the groups (from Sakovics and Steiner 2012). Hence, the total fraction of depositors withdrawing early in the whole economy, as given by (8), is also a random variable uniformly distributed over the interval $[\pi, 1]$, as the average cumulative distribution function of the signals is uniformly distributed over the interval [0, 1]. In other words, given their signals all depositors tend to assign the same probability to the future realization of the aggregate state and are all agnostic about how many depositors run in their own wealth group as well as in the others. Thus, all their threshold signals σ^{j*} must cluster around a common threshold signal $\sigma^*(\mathbf{d})$, which uniquely determines the probability of a systemic self-fulfilling run occurring in the economy. The characterization of this value should come from the solution of a system of G indifference conditions in one unknown. By averaging out the indifference conditions we can apply the Belief Constraint property (that only applies to the average strategic beliefs). Doing so allows us to retrieve the unique threshold signal $\sigma^*(\mathbf{d})$.

Importantly, the common threshold signal $\sigma^*(\mathbf{d})$ depends on the deposit contracts chosen by the representative bank for each wealth groups. The following corollary sheds light on this relationship:

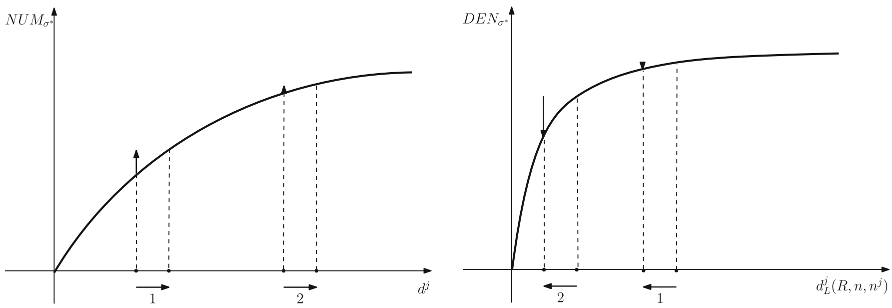
Corollary 1 *The threshold signal $\sigma^*(\mathbf{d})$ is an increasing and convex function of each d^j.*

Proof In "Appendix A". □

This result highlights the channels of financial contagion from one wealth group to the rest of the economy via expectation formation. As the bank promises a higher amount of early consumption, it increases a late consumer's expected utility of running before bankruptcy (the numerator of σ^*), and decreases the expected utility of waiting (the denominator of σ^*). Hence, the incentives to run are higher (σ^* increases) when d^j increases. Furthermore, σ^* is also convex in d^j. To understand the intuition we use Fig. 1. Figure 1a illustrates the effect of increasing d^j on the expected utility of running. For high d^j, a further increase in d^j (2) has a lower positive effect on the utility of running than for a lower d^j (1). Figure 1b illustrates instead the effect of increasing d^j on the expected utility of waiting. An increase in d_j lowers late consumption d_L^j. The effect on the expected utility is higher for low d_j (2) as in that case, an increase in d_j brings late consumption closer to zero, where marginal utility becomes very large by the modified Inada conditions. This effect is necessarily larger than the effect on the expected utility of running. Since σ^* balances the expected utility of running with the expected utility of waiting, then it must increase more when d_j is high than when it is low.

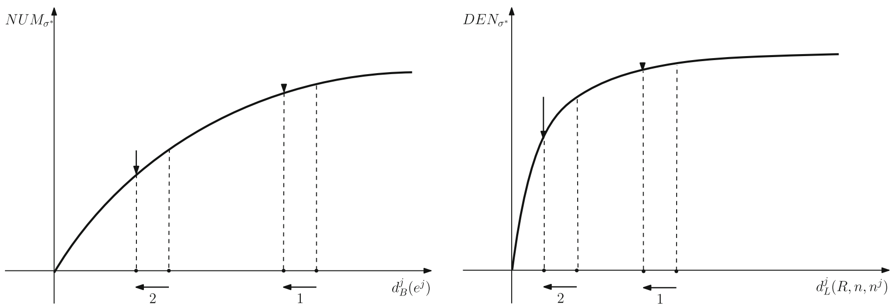
Finally, the expression for the endogenous threshold signal $\sigma^*(\mathbf{d})$ in (13) allows us to study how the endowments affect the probability of a systemic self-fulfilling run.

Corollary 2 *For any given set of deposit contracts $\mathbf{d}$, the threshold signal $\sigma^*(\mathbf{d})$ is a decreasing and convex function of the initial endowments e^j.*



(a) Expected utility of running **(b)** Expected utility of waiting

Fig. 1 Convexity of the threshold signal σ^* with respect to d^j



(a) Expected utility of running **(b)** Expected utility of waiting

Fig. 2 Convexity of the threshold signal σ^* with respect to e^j

Proof In "Appendix A". □

In the proof of the corollary, we show that increasing e^j has two effects on the threshold signal $\sigma^*(\mathbf{d})$. On the one hand, a higher e^j means that at insolvency a depositor of wealth group j receives a higher liquidation value (call it $d_B^j(e^j)$), and this increases $\sigma^*(\mathbf{d})$. On the other hand, a higher e^j increases the consumption of a late consumer who does not run at illiquidity, i.e. when the fraction of depositors running in their wealth group lies in the interval $[\pi, e^j/d^j]$, and this lowers $\sigma^*(\mathbf{d})$. This second channel dominates and its dominance increases with e^j: With a higher e^j those late consumers not running just before insolvency (i.e. when n^j approaches e^j/d^j) consume a positive amount instead of zero, and this has a large effect on their marginal utility due to concavity. Hence, the threshold signal $\sigma^*(\mathbf{d})$ is decreasing in e^j.

The intuition for why σ^* is convex in e^j is similar to that of the convexity in d^j, and we analyze it in Fig. 2. Figure 2a illustrates the effect of a drop in e^j on the expected utility of running (the numerator of σ^*) via a reduction in the bank liquidation value $d_B^j(e^j) = e^j/n^j$. For high e^j, a further drop in e^j (1) has a lower negative effect on the utility of running than for a lower e^j (2), by the concavity of the utility function. Figure 2b illustrates instead the effect of a drop in e^j on the expected utility of waiting (the denominator of σ^*) via lower late consumption $d_L^j(e^j)$. The effect on the expected utility is higher for low e_j (2) than for high e_j (1) as in the former case, a drop in e_j brings late consumption closer to zero, where marginal utility becomes very large by the modified Inada conditions. This effect is necessarily larger than the effect on the expected utility of running. Since σ^* balances the expected utility of running with the expected utility of waiting, then it must decrease more when e_j is low than when it is high.

4.2 Banking equilibrium

Having characterized the endogenous threshold strategy played by the late consumers at date 1, we proceed by backward induction and determine the deposit contract offered by the bank to each wealth group at date 0. To this end, the bank solves the following problem:

$$\max_{\{d^j\}_{j=1,\dots,G}} \sum_j \left[\int_0^{\sigma^*(\mathbf{d})} u(e^j)dp + \int_{\sigma^*(\mathbf{d})}^1 \left[\pi u(d^j) + (1-\pi)pu\left(R(e^j - \pi d^j)\right)\right] dp \right]. \tag{14}$$

Whenever the signal is between 0 and $\sigma^*(\mathbf{d})$ a systemic run happens, and all depositors receive the pro-rata wealth-specific return e^j from the liquidation of the productive assets available in portfolio. When instead the signal is between $\sigma^*(\mathbf{d})$ and 1, no systemic run happens, and the depositors turn out to be early consumers with probability π and late consumers with probability $1-\pi$, as in the banking equilibrium with perfect information.

To complete the characterization of the banking equilibrium, define the welfare gain from avoiding a run in a wealth group j when a depositor i receives a signal $\sigma^{ij} = \sigma^*(\mathbf{d})$ as:

$$\Delta U^j = \pi u(d^j) + (1-\pi)\sigma^*(\mathbf{d})u(R(e^j - \pi d^j)) - u(e^j), \tag{15}$$

which is decreasing in the initial endowment e^j as the effect of a higher e^j on the threshold signal $\sigma^*(\mathbf{d})$ is large and negative, as shown in Corollary 2. Then, the first-order condition with respect to d^j implicitly determines the equilibrium early consumption d^j:

$$\pi \int_{\sigma^*}^1 \left[u'(d^j) - (1-\pi)pRu'\left(R(e^j - \pi d^j)\right)\right] dp = \frac{\partial \sigma^*(\mathbf{d})}{\partial d^j} \sum_k \Delta U^k. \tag{16}$$

This Euler equation highlights that the endogeneity of the threshold signal $\sigma^*(\mathbf{d})$ forces the bank to impose a wedge between the marginal rate of substitution between early and late consumption and the expected return on the productive asset. To see that more clearly, rewrite (16) in terms of the marginal rate of substitution:

$$MRS^j \equiv \frac{u'(d^j)}{u'\left(R(e^j - \pi d^j)\right)} = \frac{1}{\pi(1-\sigma^*(\mathbf{d}))}\frac{1}{u'(R(e^j-\pi d^j))}\frac{\partial \sigma^*(\mathbf{d})}{\partial d^j}\sum_k \Delta U^k + (1-\pi)\mathbb{E}[p]R(1+\sigma^*(\mathbf{d})). \quad (17)$$

The right-hand side of (17) is higher than the expected return on the productive asset, namely $(1-\pi)\mathbb{E}[p]R$, which is equal to the marginal rate of substitution between early and late consumption in the banking equilibrium with perfect information. In other words, the endogeneity of the threshold signal $\sigma^*(\mathbf{d})$ forces the banks to increase the marginal rate of substitution. This means that the banking equilibrium exhibits lower amount of early consumption with respect to the banking equilibrium with perfect information, and as a consequence lower liquidity insurance.

The following lemma employs the Euler equation in (16) to derive the implications of the endowment heterogeneity for the heterogeneity of the deposit contract. This will be instrumental for the analysis of the effect of heterogeneity on the probability of a systemic self-fulfilling run.

Lemma 1 *In the banking equilibrium, early consumption d^j is an increasing and convex function of the initial endowment e^j.*

Proof In "Appendix A". □

The proof of the lemma shows that the derivative of the bank objective function in (14) with respect to early consumption d^j is increasing in the initial endowment e^j for the following reason. On the one hand, a higher e^j lowers the threshold signal σ^*, as shown in Corollary 2, and decreases the marginal utility of late consumption. On the other hand, the derivative of the bank objective function is decreasing in early consumption d^j, because of the concavity of the utility function and the fact that the threshold signal σ^* is a convex function of d^j, as proved in Corollary 1. Hence, optimality requires that increasing e^j (which increases the derivative of the bank objective function) must be counterbalanced by an increase in d^j (which lowers the derivative of the bank objective function). As far as convexity is concerned, in the proof we show that the marginal change in expected welfare induced by an increase in early consumption is a quasi-convex function of the initial endowment. This implies that the bank 's objective function exhibits quasi-convex differences in (d^j, e^j) which in turn implies that the policy function is convex (Jensen 2018). Put differently, an increase in e^j creates such a large increase in the expected welfare change that it must be optimally counterbalanced by a more than proportional increase in d^j.

With this result in hand, we are ready to state how the inequality in the distribution of the initial endowments impacts the probability of a systemic self-fulfilling run.

Proposition 2 *Higher inequality in the distribution of the initial endowments ceteris paribus brings about a higher probability of a systemic self-fulfilling run.*

Proof In "Appendix A". □

To prove this proposition, assume an increase in inequality. Marginally increase the endowment e^k for a wealth group k and lower for the same amount the endowment e^ℓ

for another wealth group for which $e^\ell < e^k$ so that the aggregate initial endowment $\sum_j e^j$ remains constant. Changing the distribution of the endowments has a direct and an indirect effect on the threshold signal σ^*. The direct effect obtains from the threshold signal being a decreasing convex function of the initial endowment e^j. Therefore, the increasing effect on σ^* induced by the drop in e^ℓ is larger than the decreasing effect on σ^* induced by the growth in e^k. At the same time, by the convexity of d^j in e^j, increasing the endowments of the rich increases their early consumption more than the corresponding decrease for the poor. Finally, by the convexity of σ^* in d^j, increasing the endowment of a rich wealth group k induces an increase in the threshold signal σ^* (through an increase in d^k) that is not counterbalanced by the effect of a lower d^ℓ induced by the lower endowment of a poor wealth group ℓ. Put differently, increasing the inequality in the distribution of the initial endowments has a stronger direct effect on the incentives to run of the poor, and an indirect stronger effect on the incentives to run of the rich. Both bring about a higher probability of a systemic self-fulfilling run.

5 Government intervention

Having characterized the banking equilibrium of the heterogeneous economy, in this section we study a government intervention against systemic self-fulfilling runs, and how it affects the formation of the depositors' expectations and the redistribution of resources across the economy. To this end, we assume the existence of an economy-wide benevolent government, who maximizes the total expected welfare of the depositors in the economy. This government is different from a social planner, in the sense that it operates in conjunction with markets (in this particular case, the banking system), and it can only influence its behavior through policy. We model this restriction by assuming that the bank collects the deposits and chooses the deposit contracts before government intervention, and the government perfectly observes the bank's behavior. The intervention consists of possibly wealth-specific lump-sum non-negative subsidies s^j financed by some government resources $\bar{e}$. An intervention is feasible if:

$$\sum_j s^j \leq \bar{e}. \tag{18}$$

5.1 Bank liquidity assistance

In this section, we start by analyzing the effect of an intervention that is unanticipated by the bank, and only affects systemic financial fragility via the change in the depositors' running behavior. We assume that the government implements an intervention in the spirit of a lender of last resort (LOLR).[20] More specifically, the government provides wealth-specific subsidies directly to the banks as long as they are illiquid but solvent, i.e. as long as the fraction of depositors running in each wealth group j

[20] In doing so, we abstract from the possibility that this intervention might bring about moral hazard, like in Martin (2006).

is lower than the fraction e^j/d^j. Notice that the government perfectly observes both the endowments e^j and the early consumption d^j promised by the banks. The budget constraint of the representative bank at date 1 reads:

$$X^j + s^j = n^j d^j, \tag{19}$$

where X^j is the amount of productive assets that needs to be liquidated to provide early consumption. Thus, the amount of productive assets that gets to maturity is equal to $e^j - X^j$, and affects the amount of consumption that a late consumer gets if they do not withdraw at date 1. Moreover, the subsidy affects the maximum fraction of depositors that can be served before the bank goes into insolvency, i.e. $(e^j + s^j)/d^j$. Thus, the advantage of waiting versus running in the presence of a subsidy reads:

$$v^j(n, n^j, s^j) = \begin{cases} \sigma u\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right) - u(d^j) & \text{if } \pi \le n^j < \frac{e^j+s^j}{d^j}, \\ -u\left(\frac{e^j}{n^j}\right) & \text{if } \frac{e^j+s^j}{d^j} \le n^j \le 1. \end{cases} \tag{20}$$

By the Belief Constraint, the endogenous threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ comes from the average indifference condition between running or not, and reads:

$$\sigma^*(\mathbf{d}, \mathbf{s}) = \frac{(1-\pi)\sum_j \left[\int_\pi^{\frac{e^j+s^j}{d^j}} u(d^j)dn^j + \int_{\frac{e^j+s^j}{d^j}}^1 u\left(\frac{e^j}{n^j}\right)dn^j\right]}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right)dn^j dn}. \tag{21}$$

From here, we can calculate the effect of a marginal increase of a subsidy s^j on the common threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$:

$$\frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j} = \frac{(1-\pi)\frac{u(d^j)-u\left(\frac{e^j}{e^j+s^j}d^j\right)}{d^j} - \sigma^*(\mathbf{d}, \mathbf{s})\int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u'\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right)\frac{R(1-n)}{1-n^j}dn^j dn}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right)dn^j dn}. \tag{22}$$

Intuitively, a subsidy has two effects. On the one hand, it increases the maximum fraction of depositors that the bank can serve before insolvency. This creates a positive difference between withdrawing early just before and after insolvency, that increases the depositors' incentives to run (the first part of the numerator of (22)). On the other hand, positive subsidies allow the bank to liquidate a lower amount of productive assets, that stay until maturity and pay higher late consumption at date 2, thus lowering the depositors' incentives to run. By the modified Inada condition the second effect dominates: Providing subsidies to late consumers who are not running just before insolvency would allow them to consume a positive amount instead of zero, and that would have a dominant positive effect on their marginal utility of waiting. Hence, the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ is decreasing in the subsidies s^j.

The previous result clarifies that it is possible for a government to choose a subsidy scheme that clears systemic self-fulfilling runs. Such an intervention would allow the bank to serve all depositors in all wealth groups at date 1, even in the case of a run. This is possible when the maximum fraction of depositors that can be served before insolvency $(e^j + s^j)/d^j$ is equal to 1, or $s^j = d^j - e^j$ for all groups j. If this full liquidity assistance is feasible, every depositor internalizes that there are sufficient resources to pay early withdrawals in the case of a systemic run, so no one runs and the bank can implement the equilibrium with perfect information at zero costs, i.e. $s^j = 0$ for all groups j.

We now focus on the more interesting case of a partial bank liquidity assistance. The government intervention can still lower systemic financial fragility, but not completely erase it. Then, the government choose an allocation of subsidies to maximize the expected welfare of the whole economy:

$$\max_{\{s^j\}_{j=1,\ldots,G}} \sum_j \Bigg[\int_0^{\sigma^*(\mathbf{d},\mathbf{s})} u(e^j)dp + \int_{\sigma^*(\mathbf{d},\mathbf{s})}^1 \Big[\pi u(d^j) + (1-\pi)pu(R(e^j - \pi d^j))\Big]dp \Bigg], \tag{23}$$

subject to the definition of $\sigma^*(\mathbf{d}, \mathbf{s})$ in (21), to its budget constraint in (18), and to $s^j \in [0, d^j - e^j]$ for all groups j. By solving this problem, we derive the following result:

Proposition 3 *The optimal partial bank liquidity assistance subsidizes wealth groups according to the statistics:*

$$\Psi^j = -\frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j} \sum_k \Delta U^k. \tag{24}$$

There exists a unique threshold group $\hat{j}$ such that all wealth groups with $\Psi^{(j)} > \Psi^{(\hat{j})}$ are fully subsidized (i.e. $s^j = d^j - e^j$), all wealth groups with $\Psi^{(j)} < \Psi^{(\hat{j})}$ receive zero (i.e. $s^j = 0$) and all wealth groups with $\Psi^{(j)} = \Psi^{(\hat{j})}$ receive $s^j \in (0, 1)$.

Proof In "Appendix A". □

Intuitively, as the government intervenes only when the bank is illiquid but solvent, the allocation of subsidies only maximizes their impact on the depositors' expectations, and therefore on the probability of a systemic self-fulfilling run. The government achieves this by calculating the statistic Ψ^j for each group j. This depends on the initial endowment in the following way:

Corollary 3 *The statistic Ψ^j is positive and decreasing in the initial endowments e^j.*

Proof In "Appendix A". □

The proof of this corollary is based on showing that the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ is decreasing in the subsidy s^j, and the welfare gains from avoiding a run ΔU^k in any group k is decreasing in the initial endowment e^j of a group j. On top of that, the marginal effect of a subsidy to group j on the probability of a systemic self-fulfilling run σ^* is not only negative, but also increasing in the initial endowment.

Corollary 3 has a crucial consequence for the allocation of subsidies in this partial intervention against bank illiquidity. The government finds it optimal to implement a redistributive subsidy scheme to minimize the occurrence of systemic self-fulfilling runs. It ranks poor wealth groups relatively higher, because they are "more systemic": The same subsidy has a larger dampening effect on the probability of a systemic self-fulfilling run when paid to them than when paid to rich wealth groups.

Finally, the third part of Proposition 3 suggests a practical rule to allocate subsidies: Rank wealth groups from the most to the least systemic according to (24), and start fully subsidizing them from top to bottom, until the government budget constraint clears. This means that the wealth groups at the bottom of the ranking, which are also the richest ones, might receive no subsidy. However, notice that the economy exhibits only two possible ex-post outcomes: run and no-run. This means that a government intervention against bank illiquidity is off equilibrium: It is announced, but never implemented. Rich wealth groups would benefit from this intervention anyway because the redistribution towards the poor is off equilibrium, but the resulting drop in systemic financial fragility occurs on the equilibrium path. Therefore, this redistribution (indeed, the mere announcement of it) has the "trickle-up" effect of lowering the probability of a systemic self-fulfilling run in the whole economy by subsidizing the poor.

5.2 Liquidity insurance with government intervention

In this section, we study how the banks' anticipation of government intervention affects their provision of liquidity insurance, i.e. their choice of d^j at date 0. Suppose first that there exists a subsidy scheme that, under the optimal deposit contracts, guarantees full bank liquidity assistance. As already argued, this is sufficient to rule out systemic self-fulfilling runs. Therefore, the representative bank is left with dealing with "fundamental" runs below the lower dominance regions only. More formally, the banking problem with bank full liquidity assistance reads:

$$\max_{\{d^j\}_{j=1,\ldots,G}} \sum_j \left[\int_0^{\underline{\sigma}} u(e^j)dp + \int_{\underline{\sigma}}^1 \left[\pi u(d^j) + (1-\pi)pu\left(R(e^j - \pi d^j)\right)\right]dp \right], \tag{25}$$

subject to the definition of the threshold for the lower dominance region:

$$\underline{\sigma} = \min_j \left\{ \frac{u(d^j)}{u(R(e^j - \pi d^j))} \right\}. \tag{26}$$

The first-order condition with respect to d^k implicitly determines the equilibrium deposit contract:

$$\pi \int_{\underline{\sigma}}^{1} \left[u'(d^k) - (1-\pi)pRu'\left(R(e^k - \pi d^k)\right)\right] dp = \frac{\partial \underline{\sigma}}{\partial d^k} \sum_{\ell} \Delta U^{\ell}, \tag{27}$$

for the group k from which $\underline{\sigma}$ is calculated. This expression is similar to the Euler equation of the banking equilibrium without government intervention, with the exception that the wedge between the marginal rate of substitution between early and late consumption and the expected return on the productive asset now depends on the welfare gains from avoiding a fundamental run, as represented by the marginal effect of d^k on the threshold signal $\underline{\sigma}$. For all the other groups $j \neq k$ instead, the deposit contract is characterized by:

$$u'(d^j) = (1-\pi)pRu'\left(R(e^j - \pi d^j)\right), \tag{28}$$

which is equal to the equilibrium of the economy with perfect information.

Now suppose there is no subsidy scheme that ensures full bank liquidity assistance once deposit contracts are optimally chosen. Then, government intervention cannot rule out systemic self-fulfilling runs, but only lower their occurrence. Yet, as the intervention only happens off equilibrium, a bank at date 0 chooses the deposit contract d^j by solving the very same problem as in (14). The only difference with the unregulated banking equilibrium lies in the fact that the optimization problem is subject to the expression for $\sigma^*(\mathbf{d}, \mathbf{s})$ in (21) that takes into account the effect of the subsidies on the threshold signal. This final proposition characterizes how the provision of liquidity insurance depends on full or partial government intervention:

Proposition 4 *In the banking equilibrium with bank liquidity assistance, the banks provide more liquidity insurance than in the banking equilibrium without government intervention, i.e.* $d^j_{FULL} > d^j_{PARTIAL} > d^j$ *for all groups* j.

Proof In "Appendix A". □

The proof of the lemma is based in part on showing that the amount of early consumption d^j offered by the banks and the subsidy that they receive and/or other groups receive are strategic complements in the banking problem. Then, when the government implements a partial bank liquidity assistance, higher subsidies lower the probability of a systemic self-fulfilling run, and the bank anticipates this by increasing liquidity insurance.

When a full liquidity assistance is instead feasible, the effect of the government intervention on liquidity insurance goes in a similar direction as when the intervention is partial, but for different reasons. A full liquidity assistance rules out systemic self-fulfilling runs, but leaves fundamental runs under the lower dominance region, whose occurrence is not affected by the subsidies. This means that we cannot infer what is the direct effect of this intervention on the provision of liquidity insurance. However, the wedge between the marginal rate of substitution between early and late consumption and the expected return on the productive asset with full bank liquidity assistance is zero for all groups $j \neq k$ and lower than in the banking equilibrium without government intervention for the group k from which $\underline{\sigma}$ is calculated. This is because in (27) the sum of the welfare gains $\sum_\ell \Delta U^\ell$ from avoiding a fundamental run is always lower than the sum of the welfare gains from avoiding a systemic self-fulfilling run with partial government intervention. Moreover, the marginal effect of increasing early consumption d^k is positive on both thresholds $\underline{\sigma}$ and σ^*, but is larger on the second one by the modified Inada condition. Hence, with full bank liquidity insurance the bank is free to offer higher liquidity insurance to all wealth groups.

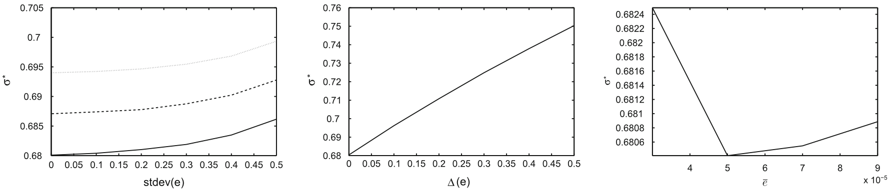
To sum up, the total effect of liquidity assistance on systemic financial fragility does not depend only on the feasibility of the intervention itself. On the one hand, it is true that full liquidity assistance, when feasible, completely resolves it. On the other hand, for given deposit contract, a partial liquidity assistance directly reduces systemic financial fragility. However, the anticipation of the intervention changes the provision of liquidity insurance, and this pushes in the direction of indirectly increasing systemic financial fragility, as Corollary 1 shows. Therefore, the total effect of a partial liquidity assistance is ambiguous. In the next section, we shed light on the total effect (direct and indirect) of a partial liquidity assistance in a numerical example.

6 A numerical example

In this final section, we present some properties of the model in a numerical example. The main goal is to study the comparative statics of the equilibrium relationship between the wealth distribution and systemic financial fragility. Moreover, we also analyze the total effect (direct and indirect) of a partial liquidity assistance on systemic financial fragility. To this end, we assume a utility function of the form $u(c) = ((c + \psi)^{1-\gamma} - \psi^{1-\gamma})/(1-\gamma)$, and set the parameters as follows: $\gamma = 3$ and $\psi = 4$, $R = 5$ and $\pi = 0.10$.

In the first exercise, we study the effect of increasing depositors' relative risk aversion on systemic financial fragility. In practice, we numerically solve the problem in (14) starting from a distribution of initial endowments [0.9, 1, 1.1], and gradually increase its standard deviation from 0.1 to 0.5 while keeping the aggregate endowment constant. Figure 3a plots the threshold signal $\sigma^*(\mathbf{d})$ as characterized by (13), and confirms the result of Proposition 2, namely a positive relationship between wealth inequality and systemic financial fragility. By raising γ to 3.1 (dashed line) and 3.2 (dotted line), we find also a positive relationship between relative risk aversion and systemic financial fragility. That is because the more risk averse depositors are, the more insurance against idiosyncratic uncertainty (i.e. higher d^j) they are provided, and therefore the higher the threshold signal $\sigma^*(\mathbf{d})$ by Corollary 1.

In the second exercise, we keep the volatility of the wealth distribution constant and increase its aggregate size by shifting all endowments by a constant amount. We solve again the problem in (14) starting from the distribution of the initial endowments [0.9, 1, 1.1], and gradually increase them all by 0.1 to 0.5. In principle, two effects should be at play here. On the one hand, according to Corollary 2 increasing the aggregate endowment should have the direct effect of lowering the threshold signal $\sigma^*(\mathbf{d})$. On the other hand, Lemma 1 shows that higher endowments bring about higher liquidity insurance and early consumption d^j, that should increase the threshold signal $\sigma^*(\mathbf{d})$ by Corollary 1. Figure 3b shows that the second effect dominates. For given



(a) Wealth inequality **(b)** Aggregate wealth **(c)** Government intervention

Fig. 3 The evolution of systemic financial fragility: comparative statics

volatility of the wealth distribution, a raise in aggregate wealth induces higher systemic financial fragility in the economy.

Finally, our third exercise studies the total effect (direct and indirect) of a partial liquidity assistance. To this end, we numerically solve a system of equations made of: (i) the banking problem (14) with endowments [0.9, 1, 1.1], modified with the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ in (21); (ii) the equilibrium conditions for the optimal subsidies in (24); (iii) the government budget constraint in (18). As a comparative-statics exercise, we let the size of government resources $\bar{e}$ vary in the interval [0.003, 0.009] percent.[21] Two channels should be at play here, too. First, as already argued in (22), higher subsidies should allow the banks to serve more depositors before insolvency, thus directly decreasing the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ and delaying a run. Second, by Proposition 4 higher subsidies should increase the provision of liquidity insurance and early consumption d^j, and as a consequence indirectly increase the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$. Figure 3c shows that the direct effect always dominates the indirect effect, hence partial liquidity assistance lowers systemic financial fragility. Interestingly, as the size of government resources increases, the economy gets to a point at which the direct effect dominates less the indirect effect, and systemic financial fragility increases with the size of government resources (while still being lower than its level without intervention). This seems to suggest that at least in the considered range of parameters, there exists an intermediate level of partial liquidity assistance that minimizes systemic financial fragility. Put differently, a government with a large budget (but not large enough for a full liquidity assistance) might prefer a smaller intervention to curb systemic financial fragility.

7 Concluding remarks

The present paper proposes a novel mechanism through which wealth inequality exacerbates systemic financial fragility via the self-fulfilling expectations of a systemic bank run. In our model, systemic financial fragility is fundamentally a consequence of the presence of the investment externality. In fact, it creates a channel of contagion

[21] We choose the lower bound of $\bar{e}$ only for numerical convenience and for the sake of exposition. By continuity, our results extends to the interval [0, 0.003] percent, too. Given the chosen parameters, in the numerical solutions the equilibrium ratios d^j/e^j are always larger than 1 (as expected) but extremely low for all groups j. Hence, in order to have $e^j + s^j < d^j$ for all groups j, so that subsidies are not sufficient for a full liquidity assistance, we need to assume low government resources. At the upper bound for $\bar{e}$, the fraction of depositors that a subsidized bank can serve in equilibrium is close to 99.8% for all wealth groups.

across wealth groups that otherwise would be separate one from the others. For the same reason, it is because of the investment externality that there exists a coordination failure among depositors not only within wealth groups, but also between them. Thus, the depositors who are unable to perfectly observe the fundamental of the economy, rely on noisy signals and on the formation of expectations regarding other depositors' signals to decide whether or not to run. We show that this coordination problem becomes more acute when wealth heterogeneity is high: Increasing wealth inequality has a direct and indirect effect on the depositors' incentives to run, and both bring about a higher probability of a systemic self-fulfilling run.

Notice that we build our argument in a framework without deposit insurance. This finds its justification in the increasing role of uninsured deposits in banks' balance sheets. In fact, total uninsured checkable, time and savings deposits held by U.S. chartered commercial banks today represent almost 40% of total bank liabilities, after reaching their lowest peak at around 10% in mid-2009.[22] Moreover, Allen et al. (2018) show that a deposit insurance scheme working in the same way as in the real world (i.e. guaranteeing a fixed repayment in any possible aggregate state and for any wealth level) would not completely isolate the economy from depositors' panic. In other words, our argument holds even in the presence of deposit insurance.

Do our results justify a direct government intervention to resolve the investment externality and against wealth inequality, for example through taxation? As far as the first is concerned, the answer is no. The representative bank in our model serves all depositors irrespective of their wealth group like a "universal" bank, and because of perfect competition it takes into account how the contractual choice that it makes for a group influences all the others. In that sense, a government intervention to resolve the externality would be needed only in a segmented banking system, for example one in which banks serve only specific wealth groups. In such a framework, a bank in group j would not internalize the effect of its own choice on the marginal costs of banks in other groups, and choose an inefficiently high early consumption d^j. Thus, a government would arguably need to impose restrictions on banks' deposit rates (e.g. an upper bound or a tax on early consumption d^j) much like as in Hellman et al. (2000). In any case, the mechanism through which the wealth distribution impacts systemic financial fragility, as characterized by Proposition 2, would hold exactly in the same way.

As already argued, the assumption of a universal bank is natural in this framework. Nevertheless, a universal bank can be an unstable business model. To see this, assume that there exists a bank-formation stage in which the wealth groups decide whether or not to create a coalition and form a universal bank at a cost. Further assume that there is no enforcement mechanism that forces the wealth groups to stay in the coalition. Under these assumptions, a wealth group that forms its own bank would benefit from lower systemic financial fragility but not incur in any coalition cost. In other words, a universal bank might not be coalition proof.[23] We leave for future research some further analyses of these issues.

22 Source: Financial Accounts of the United States.

23 This result is reminiscent of the free riding problem in climate agreements (Carraro 1997).

Regarding a direct government intervention against wealth inequality, we showed that lowering wealth inequality reduces systemic financial fragility. Therefore, a wealth redistribution might be Pareto improving. In fact, such an intervention would undoubtedly benefit poor groups, and might also make rich groups better off despite the wealth loss, because of the lower systemic financial fragility that they would enjoy. In the model, that would be equivalent to assuming a government intervention similar to the one in Sect. 5, but with three changes: (i) $\bar{e} = 0$, (ii) subsidies s^j can be negative, and (iii) must be in zero net supply, i.e. $\sum_j s^j = 0$. As also showed by Sakovics and Steiner (2012), this intervention would qualitatively yield the same results as the ones we derive: poor groups would be subsidized, and rich groups would be taxed.

Despite these considerations, it should be noted that financial fragility arises in wealth homogeneous economies too, where coordination failures are still possible. Moreover, a full liquidity assistance to the banks, if feasible, is extremely effective at ruling out self-fulfilling runs, independently of the level and of the heterogeneity of wealth in the economy. In other words, a wealth redistribution is neither necessary nor sufficient to eliminate the coordination failure leading to systemic financial fragility.

Appendices

A Proofs

Proof of Proposition 1 We start by proving the first part of the proposition. The utility advantage of waiting versus running is:

$$v^j(p, n, n^j) = \begin{cases} \sigma u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) - u(d^j) & \text{if } \pi \leq n^j < \frac{e^j}{d^j} \\ -u\left(\frac{e^j}{n^j}\right) & \text{if } \frac{e^j}{d^j} \leq n^j < 1, \end{cases} \tag{29}$$

where n^j and n are as in (7) and (8) and are the aggregate actions, i.e. the total fraction of depositors who are withdrawing at date 1 in group j and in the whole economy, respectively. Define $\Delta^j = (\sigma^{j*} - \sigma^*)$ as the difference between the threshold signal σ^{j*} of group j and the threshold signal σ^* of a generic group (which will turn out to be the unique equilibrium threshold). Given this definition, we can rescale the aggregate actions as:

$$\tilde{n}^j = \pi + (1-\pi)(1 - F(\sigma^{j*} - p)) = \pi + (1-\pi)(1 - F(\Delta^j - \zeta)) \equiv \tilde{n}^j(\zeta, \Delta^j), \tag{30}$$

$$\tilde{n} = \sum_k n^k = \pi + (1-\pi)\sum_k (1 - F(\sigma^{k*} - p)) \equiv \tilde{n}(\zeta, \Delta), \tag{31}$$

where Δ is the vector of Δ^j-s. Moreover, define $\vartheta(\tilde{n}, \Delta)$ as the inverse of $\tilde{n}(\zeta, \Delta)$ with respect to ζ. Finally, define:

$$H^j(\sigma^*, \Delta) = \mathbb{E}[v^j(\sigma^* + \vartheta(\tilde{n}, \Delta), \tilde{n}(\zeta, \Delta), \tilde{n}^j(\zeta, \Delta^j))]. \tag{32}$$

We follow Frankel et al. (2003) and prove by contradiction that the solution to the system of indifference conditions:

$$H^j(\sigma^*, \Delta) = 0, \tag{33}$$

for all $j = 1, \ldots, G$ is unique. Assume there exist two distinct solutions, namely (σ^*, Δ^*) and $(\sigma^{*\prime}, \Delta^{*\prime})$. We distinguish two cases: $\Delta^* = \Delta^{*\prime}$ and $\Delta^* \neq \Delta^{*\prime}$. Suppose first that $\Delta^* = \Delta^{*\prime}$, then it must be that $\sigma^* \neq \sigma^{*\prime}$ and without loss of generality, $\sigma^* < \sigma^{*\prime}$. Since $H^j(\sigma^*, \Delta)$ is increasing in σ^*, this implies that $H(\sigma^*, \Delta^*) < H(\sigma^{*\prime}, \Delta^{*\prime})$. However, given that both (σ^*, Δ^*) and $(\sigma^{*\prime}, \Delta^{*\prime})$ are solutions to the system, we should have that $H(\sigma^*, \Delta^*) = H(\sigma^{*\prime}, \Delta^{*\prime}) = 0$, and that is a contradiction.

Now suppose that $\Delta^* \neq \Delta^{*\prime}$ and $\sigma^* \leq \sigma^{*\prime}$. Choose $h \in \arg\max_j(\Delta^{j*\prime} - \Delta^{j*})$ and let $D = \max_j(\Delta^{j*\prime} - \Delta^{j*}) \geq 0$. Observe that $\Delta^{h*\prime} - \Delta^{j*\prime} \geq \Delta^{h*} - \Delta^{j*}$, for all $j = 1, \ldots, G$, with strict inequality for at least one j. Define $\tilde{\sigma} = \sigma^{*\prime} + D > \sigma^{*\prime} \geq \sigma^*$, hence:

$$H^h(\tilde{\sigma}, \Delta^*) \geq H^h(\sigma^*, \Delta^*) = 0. \tag{34}$$

In order to prove the contradiction, we have to show that:

$$H^h(\tilde{\sigma}, \Delta) \geq H^h(\sigma^{*\prime}, \Delta^{*\prime}) = 0. \tag{35}$$

To this end, rewrite:

$$\begin{aligned} H^h(\tilde{\sigma}, \Delta^*) &= \int_0^1 \int_0^1 v^h(p, n, n^h) f(n^h) dn^h f(n) dn \\ &= \int_{-\epsilon}^{\epsilon} v^h(\tilde{\sigma}^h - \eta^h, \tilde{n}(\Delta^{h*} - \eta^h, \Delta^*), n^h(\Delta^{h*} - \eta^h, \Delta^{h*})) f(\eta^h) d\eta^h, \end{aligned} \tag{36}$$

where $\tilde{\sigma}^h = \tilde{\sigma} + \Delta^{h*}$, and:

$$\begin{aligned} &H^h(\sigma^{*\prime}, \Delta^{*\prime}) \\ &\quad = \int_{-\epsilon}^{\epsilon} v^h(\sigma^{h*\prime} - \eta^h, \tilde{n}(\Delta^{h*\prime} - \eta^h, \Delta^{*\prime}), \end{aligned}$$

$$n^h(\Delta^{h*\prime} - \eta^h, \Delta^{h*\prime}))f(\eta^h)d\eta^h, \tag{37}$$

where $\sigma^{h*\prime} = \sigma^{*\prime} + \Delta^{h*\prime}$. It is easy to see that $\sigma^{h*\prime} = \tilde{\sigma}^h$, as $\tilde{\sigma}^h = \tilde{\sigma} + \Delta^{h*} = \sigma^{*\prime} + D + \Delta^{h*} = \sigma^{*\prime} + \Delta^{h*\prime} - \Delta^{h*} + \Delta^{h*} = \sigma^{*\prime} + \Delta^{h*\prime} = \sigma^{h*\prime}$. Moreover:

$$\tilde{n}(\Delta^{h*\prime} - \eta^h, \Delta^{*\prime}) \geq \tilde{n}(\Delta^{h*} - \eta^h, \Delta^*), \tag{38}$$

for all η^h, as:

$$\sum_j (1 - F(\Delta^{j*\prime} - \Delta^{h*\prime} + \eta^h)) \geq \sum_j (1 - F(\Delta^{j*} - \Delta^{h*} + \eta^h)) \tag{39}$$

holds due to the observation above. Similarly:

$$F(\Delta^{j*\prime} - \Delta^{h*\prime} + \eta^h) \leq F(\Delta^{j*} - \Delta^{h*} + \eta^h) \tag{40}$$

for all η^h. Hence, $H^j(\tilde{\sigma}, \Delta) \geq H^h(\sigma^{*\prime}, \Delta^{*\prime})$ because $H^j(\sigma, \Delta)$ is decreasing in $\tilde{n}(\zeta, \Delta)$ and $\tilde{n}^j(\zeta, \Delta)$. This gives a contradiction, and concludes the proof of the first part of the proposition.

For the second part of the proposition, we start by showing that, when ϵ is small, the system of indifference conditions $H^j(\sigma^*, \Delta)(\epsilon) = 0$ is well approximated by $H^j(\sigma^*, \Delta)(0) = 0$. Notice that, as $\epsilon \to 0$, we have that $\zeta = 0$ and $\vartheta(\tilde{n}, \Delta) = 0$. Hence:

$$\begin{aligned} &H^j(\sigma^*, \Delta)(\epsilon) \\ &\quad = \int_0^1 \int_{\pi}^{\frac{e^j}{d^j}} \Big[(\sigma^* + \vartheta(\tilde{n}, \Delta))u \\ &\quad \times \left(\frac{R(1 - \tilde{n}(\zeta, \Delta))(e^j - \tilde{n}^j(\zeta, \Delta)d^j)}{1 - \tilde{n}^j(\zeta, \Delta)}\right) - u(d^j)\Big] \\ &\quad \times f(\tilde{n}^j)d\tilde{n}^j(\zeta, \Delta)f(\tilde{n})d\tilde{n}(\zeta, \Delta) \\ &\quad - \int_0^1 \int_{\frac{e^j}{d^j}}^1 u\left(\frac{e^j}{d^j}\right) f(\tilde{n}^j)d\tilde{n}^j(\zeta, \Delta)f(\tilde{n})d\tilde{n}(\zeta, \Delta), \end{aligned} \tag{41}$$

$$\begin{aligned} H^j(\sigma^*, \Delta)(0) &= \int_0^1 \int_{\pi}^{\frac{e^j}{d^j}} \left[\sigma^* u \left(\frac{R(1 - \tilde{n}(0, \Delta))(e^j - \tilde{n}^j(0, \Delta)d^j)}{1 - \tilde{n}^j(0, \Delta)}\right) u(d^j)\right] \\ &\quad \times f(\tilde{n}^j)d\tilde{n}^j(0, \Delta)f(\tilde{n})d\tilde{n}(0, \Delta) \\ &\quad - \int_0^1 \int_{\frac{e^j}{d^j}}^1 u\left(\frac{e^j}{d^j}\right) f(\tilde{n}^j)d\tilde{n}^j(0, \Delta)f(\tilde{n})d\tilde{n}(0, \Delta). \end{aligned} \tag{42}$$

The intervals of integration of the two functions are the same. Moreover, the integrands are both Lipschitz continuous in σ^*. Hence, there exists a constant C_1 such that:

$$|H^j(\sigma^*, \Delta)(\epsilon) - H^j(\sigma^*, \Delta)(0)| \leq C_1\epsilon. \tag{43}$$

In other words, as ϵ goes to zero, the two systems of equations coincide. To see that also the solutions of the two systems of equations coincide, let σ^* and Δ^* be the solution of the system of indifference conditions $H^j(\sigma^*, \Delta)(0) = 0$. Given any neighbourhood N of (σ^*, Δ^*), the function $H^j(\sigma^*, \Delta)(0)$ is uniformly bounded from 0 by some ι on $S \setminus N$. Choosing $\bar{\epsilon}$ such that $|H^j(\sigma^*, \Delta)(\epsilon) - H^j(\sigma^*, \Delta)(0)| \leq \iota$ for all $\epsilon < \bar{\epsilon}$, the system of equations $H^j(\sigma^*, \Delta)(\epsilon) = 0$ has no solution outside of N.

Finally, by taking the average of the indifference condition we can use the Laplacian property. This allows us to characterize the unique threshold signal $\sigma^*(\mathbf{d})$ in (13),

$$\frac{1}{G}\sum_j H^j(\sigma^*, \Delta)(0) =$$
$$\frac{1}{G}\sum_j \Bigg[\int_0^1 \int_\pi^{\frac{e^j}{d^j}} \left[\sigma^* u\left(\frac{R(1-\tilde{n}(0,\Delta))(e^j - \tilde{n}^j(0,\Delta)d^j)}{1-\tilde{n}^j(0,\Delta)}\right) - u(d^j)\right]$$
$$\times f(\tilde{n}^j(0,\Delta))d\tilde{n}^j(0,\Delta) f(\tilde{n}(0,\Delta))d\tilde{n}(0,\Delta) +$$
$$- \int_0^1 \int_{\frac{e^j}{d^j}}^1 u\left(\frac{e^j}{d^j}\right) f(\tilde{n}^j(0,\Delta))d\tilde{n}^j(0,\Delta) f(\tilde{n}(0,\Delta))d\tilde{n}(0,\Delta)\Bigg]. \tag{44}$$

By the Laplacian Property, $\tilde{n}^j(0, \Delta) \sim U[\pi, 1]$, hence the probability distribution $f(\tilde{n}^j(0, \Delta)) = 1/(1-\pi)$ is independent of Δ. In a similar way, by the Belief Constraint (Sakovics and Steiner 2012), the Laplacian Property holds on average, meaning that also $\tilde{n}(0, \Delta) \sim U[\pi, 1]$, therefore the probability distribution $f(\tilde{n}(0, \Delta)) = 1/(1-\pi)$ is independent of Δ. Thus, the average indifference condition takes the form:

$$\sum_j \int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} \sigma^* u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) dn^j dn =$$
$$\sum_j \int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u(d^j)dn^j + \int_{\frac{e^j}{d^j}}^1 u\left(\frac{e^j}{n^j}\right) dn^j dn. \tag{45}$$

Rearranging this expression, we get threshold signal $\sigma^*(\mathbf{d})$ in (13). This ends the proof. □

Proof of Corollary 1 We study the sign of:

$$\frac{\partial \sigma^*(\mathbf{d})}{\partial d^j} = \frac{1}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) dn^j dn}$$
$$\times \Bigg[(1-\pi)u'(d^j)\left(\frac{e^j}{d^j} - \pi\right)$$
$$+ \sigma^*(\mathbf{d})\int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u'(d_L^j(R, n, n^j))\frac{R(1-n)n^j}{1-n^j} dn^j dn\Bigg] \tag{46}$$

This is clearly positive, as the utility function $u(c)$ is increasing and e^j/d^j is larger than π.

To show that σ^* is convex in d^j, calculate:

$$\frac{\partial^2\sigma^*}{\partial d^{j2}} = \frac{1}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) dn^j dn} \Bigg[(1-\pi)u''(d^j)\left(\frac{e^j}{d^j} - \pi\right)$$
$$- (1-\pi)u'(d^j)\frac{e^j}{d^{j2}} + 2\frac{\partial\sigma^*}{\partial d^j}\int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u'(d_L^j(R,n,n^j))\frac{R(1-n)n^j}{1-n^j} dn^j dn +$$
$$- \sigma^*\frac{\partial\sigma^*}{\partial d^j}\int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u''(d_L^j(R,n,n^j))\left(\frac{R(1-n)n^j}{1-n^j}\right)^2 dn^j dn\Bigg]. \tag{47}$$

The first two terms in the square brackets are negative, and the last two are positive. By the modified Inada condition:

$$\lim_{n^j \to \frac{e^j}{d^j}} u'\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) = - \lim_{n^j \to \frac{e^j}{d^j}} u''\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right)$$
$$= \lim_{c\to 0} u'(c) = F. \tag{48}$$

Hence, the derivative is positive. This ends the proof. □

Proof of Corollary 2 To prove that the threshold signal $\sigma^*(\mathbf{d})$ is decreasing in e^j, calculate:

$$\frac{\partial\sigma^*(\mathbf{d})}{\partial e^j} = \frac{1-\pi}{DEN_{\sigma^*}}\Bigg[\int_{\frac{e^j}{d^j}}^1 u'\left(\frac{e^j}{n^j}\right)\frac{1}{n^j} dn^j +$$
$$- \sigma^*(\mathbf{d})\int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u'\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right)\frac{R(1-n)}{1-n^j} dn^j dn\Bigg], \tag{49}$$

where DEN_{σ^*} is the denominator of $\sigma^*(\mathbf{d})$. By the modified Inada condition, the derivative is negative.[24] For the second part of the proof regarding the convexity, calculate instead:

$$\frac{\partial^2\sigma^*(\mathbf{d})}{\partial e^{j2}} = \frac{1-\pi}{DEN_{\sigma^*}}\Bigg[-\frac{u'(d^j)}{e^j} + \int_{\frac{e^j}{d^j}}^1 u''\left(\frac{e^j}{n^j}\right)\frac{1}{n^{j2}} dn^j +$$
$$- \frac{\partial\sigma^*(\mathbf{d})}{\partial e^j}\left[\int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u'\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right)\frac{R(1-n)}{1-n^j} dn^j dn\right] +$$

[24] Notice that $u(c)$ has a kink at $c = 0$, so it is not differentiable at that point. The derivative tends to infinity, without ever getting there.

$$
\begin{aligned}
& - \sigma^*(\mathbf{d}) \left[\int_{\pi}^{1} \int_{\pi}^{\frac{e^j}{d^j}} u'' \left(R(1-n) \frac{e^j - n^j d^j}{1-n^j} \right) \left[\frac{R(1-n)}{1-n^j} \right]^2 dn^j dn \right] + \\
& - \frac{\partial \sigma^*(\mathbf{d})}{\partial e^j} \left[\int_{\pi}^{1} \int_{\pi}^{\frac{e^j}{d^j}} u' \left(R(1-n) \frac{e^j - n^j d^j}{1-n^j} \right) \frac{R(1-n)}{1-n^j} dn^j dn \right] \Bigg].
\end{aligned} \tag{50}
$$

As $\sigma^*(\mathbf{d})$ is decreasing in e^j and the utility function is concave, this expression must be positive by the modified Inada condition. Hence $\sigma^*(\mathbf{d})$ is a convex function of e^j. This ends the proof. □

Proof of Lemma 1 We apply the implicit function theorem to the Euler equation (16). In particular:

$$
\begin{aligned}
\frac{\partial FOC}{\partial e^j} = & -\pi \int_{\sigma^*(\mathbf{d})}^{1} (1-\pi) p R^2 u''(R(e^j - \pi d^j)) dp + \\
& - \pi \frac{\partial \sigma^*(\mathbf{d})}{\partial e^j} \Big[u'(d^j) - (1-\pi)\sigma^*(\mathbf{d}) R u'(R(e^j - \pi d^j)) \Big] + \\
& - \frac{\partial^2 \sigma^*(\mathbf{d})}{\partial d^j \partial e^j} \sum_k \Delta U^k - \frac{\partial \sigma^*(\mathbf{d})}{\partial d^j} \Big[(1-\pi)\sigma^* R u'(R(e^j - \pi d^j)) - u'(e^j) \Big] + \\
& - \frac{\partial \sigma^*(\mathbf{d})}{\partial d^j} (1-\pi) \frac{\partial \sigma^*(\mathbf{d})}{\partial e^j} \sum_k u(R(e^k - \pi d^k)).
\end{aligned} \tag{51}
$$

This expression is positive, as all terms are negative and:

$$
\begin{aligned}
\frac{\partial^2 \sigma^*(\mathbf{d})}{\partial d^j \partial e^j} = & \frac{1}{DEN_{\sigma^*}} \Bigg[(1-\pi) \frac{u'(d^j)}{d^j} + \frac{\partial \sigma^*(\mathbf{d})}{\partial e^j} \int_{\pi}^{1} \int_{\pi}^{\frac{e^j}{d^j}} u'(d_L^j(R, n, n^j)) \\
& \times \frac{R(1-n)n^j}{1-n^j} dn^j dn \\
& + \sigma^*(\mathbf{d}) \int_{\pi}^{1} \int_{\pi}^{\frac{e^j}{d^j}} u''(d_L^j(R, n, n^j)) \left(\frac{R(1-n)}{1-n^j} \right)^2 n^j dn^j dn + \\
& - \frac{\partial \sigma^*(\mathbf{d})}{\partial d^j} \int_{\pi}^{1} \int_{\pi}^{\frac{e^j}{d^j}} u'(d_L^j(R, n, n^j)) \frac{R(1-n)n^j}{1-n^j} dn^j dn \Bigg]
\end{aligned} \tag{52}
$$

is also negative by the modified Inada condition. Furthermore:

$$
\begin{aligned}
\frac{\partial FOC}{\partial d^j} = & \pi \int_{\sigma^*(\mathbf{d})}^{1} \Big[u''(d^j) + (1-\pi)\pi p R^2 u''(R(e^j - \pi d^j)) \Big] dp + \\
& - \frac{\partial \sigma^*(\mathbf{d})}{\partial d^j} \pi \Big[u'(d^j) - (1-\pi)\sigma^*(\mathbf{d}) R u'(R(e^j - \pi d^j)) \Big] +
\end{aligned}
$$

$$
- \frac{\partial^2 \sigma^*(\mathbf{d})}{\partial d^{j2}} \sum_k \Delta U^k - \frac{\partial \sigma^*(\mathbf{d})}{\partial d^j} \Big[\pi u'(d^j) - (1-\pi)\sigma^* \pi R u'(R(e^j - \pi d^j)) \Big] + \\
- \left(\frac{\partial \sigma^*(\mathbf{d})}{\partial d^j} \right)^2 (1-\pi) \sum_k u(R(e^k - \pi d^k)). \tag{53}
$$

This expression is also negative as $\sigma^*(\mathbf{d})$ is convex in d^j. Using these two results:

$$
\frac{\partial d^j}{\partial e^j} = -\frac{\partial FOC/\partial e^j}{\partial FOC/\partial d^j} \tag{54}
$$

is positive.

To prove convexity, we follow Jensen (2018). If the objective function $u(x, y)$ exhibits quasi-convex differences, then $x^* = \text{argmax}_x u(x, y)$ is convex in y. The objective function $u(x, y)$ exhibits quasi-convex differences if and only if its partial derivative with respect to x is quasi-convex in (x, y). A differentiable function $f(x, y)$ is quasi-convex in (x, y) if and only if for any x_1, x_2 such that $f(x_1) \leq f(x_2)$ we have that the following holds (Boyd and Vandenberghe 2004):

$$
\nabla f(x_2)^T (x_1 - x_2) \leq 0. \tag{55}
$$

In our case, (55) reads:

$$
\left.\frac{\partial FOC}{\partial d^j}\right|_{(d_2^j, e_2^j)} (d_1^j - d_2^j) + \left.\frac{\partial FOC}{\partial e^j}\right|_{(d_2^j, e_2^j)} (e_1^j - e_2^j) \leq 0. \tag{56}
$$

Given that the first-order condition is increasing in e^j and decreasing in d^j, then the pairs (d_1^j, e_1^j) and d_2^j, e_2^j that satisfy $FOC(d_1^j, e_1^j) \leq FOC(d_2^j, e_2^j)$ are such that $d_1^j \geq d_2^j$ and $e_2^j \geq e_1^j$. Hence (55) holds. This ends the proof. □

Proof of Proposition 2 Assume an increase in inequality: Marginally increase the endowment e^k for a wealth group k and lower for the same amount the endowment e^ℓ for another wealth group for which $e^\ell < e^k$ so that the aggregate initial endowment $\sum_j e^j$ remains constant. The effect of this change on $\sigma^*(\mathbf{d})$ is represented by the total differential:

$$
\begin{aligned}
d\sigma^*(\mathbf{d}) &= \left[\frac{\partial \sigma^*(\mathbf{d})}{\partial e^k} + \frac{\partial d^k}{\partial e^k} \frac{\partial \sigma^*(\mathbf{d})}{\partial d^k} \right] de^k + \left[\frac{\partial \sigma^*(\mathbf{d})}{\partial e^\ell} + \frac{\partial d^\ell}{\partial e^\ell} \frac{\partial \sigma^*(\mathbf{d})}{\partial d^\ell} \right] de^\ell \\
&= \left[\frac{\partial \sigma^*(\mathbf{d})}{\partial e^k} - \frac{\partial \sigma^*(\mathbf{d})}{\partial e^\ell} \right] de^k + \left[\frac{\partial d^k}{\partial e^k} \frac{\partial \sigma^*(\mathbf{d})}{\partial d^k} - \frac{\partial d^\ell}{\partial e^\ell} \frac{\partial \sigma^*(\mathbf{d})}{\partial d^\ell} \right] de^k.
\end{aligned} \tag{57}
$$

This expression is positive as $\sigma^*(\mathbf{d})$ is increasing and convex in d^j and decreasing and convex in e^j, and d^j is increasing and convex in e^j. This ends the proof. □

Proof of Proposition 3 Attach the Lagrange multipliers λ^j and χ^j to the upper and lower bounds of s^j. The first-order condition with respect to s^j then reads:

$$-\frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j} \sum_k \Delta U_B^k - \lambda^j + \chi^j = \xi, \tag{58}$$

for all $j = 1, \ldots, G$, where ΔU_B^k is defined as in (15). Then, the government intervention scheme satisfies the equilibrium condition:

$$-\frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j} \sum_k \Delta U^k - \lambda^j + \chi^j = -\frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^\ell} \sum_k \Delta U^k - \lambda^\ell + \chi^\ell, \tag{59}$$

for any two groups j and ℓ. Calculate Ψ^j according to (24), which is obviously positive for every group j, because we proved that $\sigma^*(\mathbf{d}, \mathbf{s})$ is a decreasing function of the subsidy s^j and $u(c)$ is increasing. Then, rank the groups by decreasing Ψ^j. For the condition (59) to hold, it must be the case that:

$$-\lambda^{(1)} + \chi^{(1)} < -\lambda^{(2)} + \chi^{(2)} < \cdots < -\lambda^{(G)} + \chi^{(G)}, \tag{60}$$

where (j) indicates the j-th group in the ranking. Assume that $-\lambda^{(1)} + \chi^{(1)} > 0$. For this to be true, it must be that $\lambda^{(1)} = 0$ and $\chi^{(1)} > 0$, meaning that the group with the highest Ψ^j gets the lowest possible subsidy, or $s^{(1)} = 0$. But if $-\lambda^{(1)} + \chi^{(1)} > 0$, also $-\lambda^j + \chi^j > 0$ for all groups j. This means that all groups get the lowest possible subsidy, or $s^j = 0$ for all groups j, which cannot be an equilibrium for the same argument that we make above. Hence, we must have that $-\lambda^{(1)} + \chi^{(1)} \leq 0$. On the contrary, assume that $-\lambda^{(G)} + \chi^{(G)} < 0$. Then $\chi^{(G)} = 0$ and $\lambda^{(G)} > 0$, implying that $s^{(G)} = d^{(G)} - e^{(G)} > 0$. However, if $-\lambda^{(G)} + \chi^{(G)} < 0$, also $-\lambda^j + \chi^j < 0$ for all groups j, and $s^j = d^{(j)} - e^{(j)}$ for all groups j. This is not possible, as we ruled out the possibility of complete subsidization. Thus, the only possible equilibrium features $-\lambda^{(G)} + \chi^{(G)} \geq 0$: Some groups are fully subsidized and some others get zero. This implies that there exists a unique threshold group $\hat{j}$ for which there is indifference. This ends the proof. □

Proof of Corollary 3 To prove the corollary, we calculate:

$$\frac{\partial \Psi^j}{\partial e^j} = -\frac{\partial^2 \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j \partial e^j} \sum_k \Delta U^k - \frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j} \sum_k \frac{\partial \Delta U^k}{\partial e^j}. \tag{61}$$

This expression is negative. To see that, notice that the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ is decreasing in the subsidy s^j, and the welfare gains from avoiding a run ΔU^k in any group k is decreasing in the initial endowment e^j of a group j. Moreover, from (22):[25]

[25] Notice that the utility function $u(c)$ has a kink at $c = 0$, hence $u'(0)$ is undefined.

$$\frac{\partial^2\sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j \partial e^j} = \frac{(1-\pi)}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^jd^j}{1-n^j}\right) dn^j dn} \times$$
$$\times \left[-u'\left(\frac{e^j}{e^j+s^j}d^j\right)\frac{s^j}{(e^j+s^j)^2} + \right.$$
$$-\frac{\partial\sigma^*(\mathbf{d},\mathbf{s})}{\partial e^j}\left[\int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u'\left(R(1-n)\frac{e^j+s^j-n^jd^j}{1-n^j}\right)\frac{R(1-n)}{1-n^j} dn^j dn\right] +$$
$$-\sigma^*(\mathbf{d},\mathbf{s})\int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u''\left(R(1-n)\frac{e^j+s^j-n^jd^j}{1-n^j}\right)\left(\frac{R(1-n)}{1-n^j}\right)^2 dn^j dn +$$
$$\left. -\frac{\partial\sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j}\left[\int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u'\left(R(1-n)\frac{e^j+s^j-n^jd^j}{1-n^j}\right)\frac{R(1-n)}{1-n^j} dn^j dn\right]\right]. \tag{62}$$

Again, the threshold signal $\sigma^*(\mathbf{d},\mathbf{s})$ is decreasing both in the initial endowment e^j and in the subsidy s^j. Then, as the utility function is concave, (62) must be positive: The marginal utility of consumption of a late consumer who waits until date 2 and consumes just before insolvency (i.e. as n^j approaches $(e^j+s^j)/d^j$) tends to be large by the modified Inada condition. □

Proof of Proposition 4 We split the proof in two parts.

Partial liquidity assistance When the government intervenes with a partial liquidity assistance, the banking problem reads:

$$\max_{d^j} \sum_j \left[\int_0^{\sigma^*(\mathbf{d},\mathbf{s})} u(e^j)dp + \int_{\sigma^*(\mathbf{d},\mathbf{s})}^1 \left[\pi u(d^j) + (1-\pi)pu(R(e^j-\pi d^j))\right]dp\right], \tag{63}$$

subject to the expression for $\sigma^*(\mathbf{d},\mathbf{s})$ in (21). To prove that d^j is non-decreasing in s^k for any k, we need to prove that the bank objective function is supermodular in d^j and s^k, which is true if the cross-derivative of the bank objective function with respect to d^j and s^k is positive.[26] The only place in the bank objective function where d^j and s^k interact is in the threshold signal $\sigma^*(\mathbf{d},\mathbf{s})$, which has a negative effect on the objective function. Hence, in order to prove supermodularity we just need to prove that the cross derivative of $\sigma^*(\mathbf{d},\mathbf{s})$ with respect to d^j and s^k is negative. Differentiating (22) with

[26] Let X be an open sublattice of $\mathbb{R}^m$. A twice-continuously differential function $F: X \to \mathbb{R}$ is supermodular (submodular) on X if and only if for all $\mathbf{x} \in X$ we have that $\partial^2 F/\partial x_i \partial x_j \geq (\leq)\ 0$ for any $i, j = 1, \ldots, m$ and $i \neq j$ (Topkis 1998).

respect to d^j, we obtain:

$$\frac{\partial^2\sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j\partial d^j} = \frac{(1-\pi)}{\sum_j\int_\pi^1\int_\pi^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^jd^j}{1-n^j}\right)dn^j dn} \times\left[-\frac{u(d^j)-u\left(\frac{e^j}{e^j+s^j}d^j\right)}{d^{j2}}+\frac{u'(d^j)-u'\left(\frac{e^j}{e^j+s^j}d^j\right)\frac{e^j}{e^j+s^j}}{d^j}+ -\frac{\partial\sigma^*(\mathbf{d},\mathbf{s})}{\partial d^j}\left[\int_\pi^1\int_\pi^{\frac{e^j+s^j}{d^j}} u'(d_L(R,n,n^j))\frac{R(1-n)}{1-n^j}dn^j dn\right] + \sigma^*\int_\pi^1\int_\pi^{\frac{e^j+s^j}{d^j}} u''(d_L(R,n,n^j))\frac{R^2(1-n)^2n^j}{(1-n^j)^2}dn^j dn + \frac{\partial\sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j}\left[\int_\pi^1\int_\pi^{\frac{e^j+s^j}{d^j}} u'(d_L(R,n,n^j))\frac{R(1-n)n^j}{1-n^j}dn^j dn\right]\right]. \tag{64}$$

This expression is negative: The first two terms in the square brackets are finite, but the last three terms are negative and large by the modified Inada condition. To prove that d^j is non-decreasing in s^k for any $k \neq j$, calculate instead:

$$\frac{\partial^2\sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j\partial d^k} = \frac{(1-\pi)}{\sum_j\int_\pi^1\int_\pi^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^jd^j}{1-n^j}\right)dn^j dn} \times\left[-\frac{\partial\sigma^*}{\partial d^k}\left[\int_\pi^1\int_\pi^{\frac{e^j+s^j}{d^j}} u'(d_L(R,n,n^j))\frac{R(1-n)}{1-n^j}dn^j dn\right] + \frac{\partial\sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j}\left[\int_\pi^1\int_\pi^{\frac{e^k+s^k}{d^k}} u'(d_L(R,n,n^k))\frac{R(1-n)n^k}{1-n^k}dn^k dn\right]\right]. \tag{65}$$

For the same reasons as above, this expression is negative.

Full liquidity assistance We already proved that d^j is higher in the equilibrium with perfect information than in the banking equilibrium with systemic self-fulfilling runs. Then, we just need to prove that $d^k_{FULL} > d^k$. To this end, evaluate the first-order condition of the banking problem with government intervention in (27) at d^k, and compare it to (16). Clearly, as $\underline{\sigma} < \sigma^*$ by construction, it is sufficient to prove that

$\partial\sigma^*(\mathbf{d},\mathbf{s})/\partial d^k > \partial\underline{\sigma}/\partial d^k$, where:

$$\frac{\partial\underline{\sigma}}{\partial d^k} = \frac{u'(d^k) + \underline{\sigma}\pi R u'(R(e^k - \pi d^k))}{u(R(e^k - \pi d^k))}, \tag{66}$$

and $\partial\sigma^*(\mathbf{d},\mathbf{s})/\partial d^k$ is equal to (46) with the addition of subsidies. Rearranging the inequality, we obtain:

$$\begin{aligned} u(R(e^k - \pi d^k)) \int_\pi^1 \int_\pi^{\frac{e^k+s^k}{d^k}} \left[u'(d^k) + \sigma^* u'(d_L^k(R,n,n^k)) \frac{R(1-n)n^k}{1-n^k} dn^k dn \right] \\ > \left[u'(d^k) + \underline{\sigma}\pi R u'(R(e^k - \pi d^k)) \right] \left[\int_\pi^1 \int_\pi^{\frac{e^k+s^k}{d^k}} u(d_L^k(R,n,n^k)) dn^k dn \right], \end{aligned} \tag{67}$$

and:

$$\begin{aligned} & u'(d^k) \int_\pi^1 \int_\pi^{\frac{e^k+s^k}{d^k}} u(R(e^k - \pi d^k)) dn^k dn \\ & + \int_\pi^1 \int_\pi^{\frac{e^k+s^k}{d^k}} \sigma^* u(R(e^k - \pi d^k)) u'(d_L^k(R,n,n^k)) \\ & \times \frac{R(1-n)n^k}{1-n^k} dn^k dn > u'(d^k) \int_\pi^1 \int_\pi^{\frac{e^k+s^k}{d^k}} u(d_L^k(R,n,n^k)) dn^k dn \\ & + \int_\pi^1 \int_\pi^{\frac{e^k+s^k}{d^k}} \underline{\sigma} u(d_L^k(R,n,n^k)) u'(R(e^k - \pi d^k)) \pi R dn^k dn. \end{aligned} \tag{68}$$

The first element on the left-hand side of this expression is larger than the first element on the right-hand side, because $R(e^k - \pi d^k) > d_L^k(R,n,n^k)$ by definition. The second term on the right-hand side is finite, while the second term on the left-hand side is large by the modified Inada condition. This ends the proof. □

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