



Competitive pricing despite search costs when lower price signals quality

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Abstract

The Diamond paradox demonstrates that when learning prices is costly for consumers, each firm has market power. However, making firms privately informed about their quality and cost restores competitive pricing if quality and cost are negatively correlated. Such correlation arises from, e.g. regulation, differing equipment or skill, or economies of scale. If good quality firms have lower costs, then they can signal quality by cutting prices, in which case bad quality firms must cut prices to retain customers. This price-cutting race to the bottom ends in an equilibrium in which all firms price nearly competitively and cheap talk reveals quality.

Keywords Price signalling · Diamond paradox · Incomplete information · Price war

JEL classification D82 · C72 · D41

The Diamond paradox illustrates starkly the possibility that when consumers find it costly to learn prices (e.g. the few seconds to click on a website constitute a small cost), each firm has market power. In the unique equilibrium of the Diamond paradox, all

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firms set the same price. Due to equal prices, no consumer engages in costly learning. Since no consumers learn, the price that the firms choose is the monopoly price.

Consumer search is an important part of many markets. Yet, if consumers do not expect any variability in price or quality, then there is no incentive to search. Even if price and quality differ, consumers may be indifferent between good quality at a high price and bad quality at a cheap price. In this situation, which arises when a high quality firm has a higher cost and consumers are homogeneous, there is still no search.

Moreover, heterogeneity of consumers (who prefer one price-quality combination enough to search for it) need not put downward pressure on prices. A high quality, high cost firm signals its privately known quality by increasing its price, which drives some consumers to switch to a low quality seller. The latter can then profitably raise its price above the monopoly level to hold up the switchers whose search cost is sunk.

This paper identifies a setting in which consumer search forces firms to radically reduce prices. If quality and cost are negatively related and private, then higher quality is signalled through a lower price. While this might seem counterintuitive, there are markets where it has been empirically verified that higher quality is indeed correlated with lower prices. Examples are mutual funds, private label foods and some categories of electronics.¹ The negative association of cost and quality can be caused for example by regulation that raises the cost of bad quality, by economies of scale or differing equipment or skill that both reduces cost and improves quality. Combining higher quality with lower costs, not only do high quality firms price below their monopoly level, but all firms set a price that is close to perfectly competitive. Private information thus neutralises the market power coming from costly consumer learning.

In the markets this paper studies, there are at least two firms, each of which draws an independent type, either *good* or *bad*. The good type has lower marginal cost and higher quality than the bad. Each firm knows its own type, but other players only have a common prior over a firm's type. First the firms simultaneously set prices. Second, each consumer observes the price of one firm and chooses either to buy from this firm, leave the market or pay a small cost to learn the price of another firm. Finally, each consumer who learned chooses either to buy from one of the firms whose price he knows or leave the market. The consumers have a distribution of valuations. A higher-valuation consumer values high quality relatively more. Consumers update their beliefs about the type of a firm whose price they see, using Bayes' rule whenever possible. The equilibrium concept is perfect Bayesian equilibrium (PBE). A unique equilibrium remains after refining with the Intuitive Criterion.

In equilibrium, prices are close to competitive due to a *race to the bottom* consisting of two forces. One is downward price signalling: a good quality firm reduces price to distinguish itself from bad quality firms and attract greater demand. The second force is that a bad quality firm cuts price to retain its customers and attract those at other

¹ Nelson et al. (2009) find that among medical innovations, cost reduction is positively correlated with quality improvement. Bloom et al. (2013) show that better management practices cause higher quality, profit and TFP, thus lower unit cost. Among mutual funds, Gil-Bazo and Ruiz-Verdú (2009) show not just a correlation, but that higher fees even predict lower future before-fee returns. Olbrich and Jansen (2014) find negative correlation of price and quality among private label foods, Caves and Greene (1996) for 55 out of 196 product categories, Bartelink (2016) for public tender offers, Reuter and Caulkins (2004) for street heroin. Sheen (2014) shows that larger firms have both higher quality and lower price.

firms if those also have bad quality. The bad quality firms are in Bertrand competition over the consumers who learn more than one price (consumers who initially see a price indicating a bad quality firm).

After the bad quality firms undercut each other's price, the good types must cut price further to separate themselves. Then, the bad quality firms again undercut each other, etc. The race to the bottom ends when both types price at the marginal cost of the bad quality firm, same as under complete information Bertrand competition between two bad types. Bertrand competition between two known good quality firms leads to a lower price than under incomplete information, but between a known good quality and bad quality firm to a higher price.

Averaging across the Bertrand prices for different type combinations according to the prior used for the incomplete information environment, the expected price may be higher or lower than under incomplete information, whether with costly search or freely observed prices. The *ex ante* expected price is higher under complete information iff the quality difference between the types is large enough relative to the cost difference. If the cost and quality differences between the types go to zero, then the prices in the complete and incomplete information Bertrand and the incomplete information costly search environments converge to the same level. That level is competitive—drastically different from costly search under complete information, where the unique equilibrium of Diamond (1971) features the monopoly price and no consumer learning.

In the current paper, consumers learn if they initially find themselves at a bad quality firm. Consumer learning makes the bad types undercut each other's price down to their competitive level. As long as prices are above the marginal cost of the bad quality type, good quality firms signal their type by a price strictly below that of bad quality firms. At price equal to the bad type's cost, cheap talk can distinguish the qualities, because the bad type has no incentive to raise demand by claiming to be good.²

The equilibrium prices of this paper contrast with a privately informed monopolist, and with competition when quality is learned together with the price. The good type of a monopolist still signals its quality by reducing its price to a level that the bad type prefers not to mimic. However, the bad type has no incentive to cut price below its monopoly level.

In competition, when paying a learning cost leads to observing both price and quality, bad quality firms are still in a Bertrand-like situation and compete to a low price. However, a good quality firm has no incentive to cut price to signal, because customers see its quality and stay with it even at a high price.

The Bertrand pricing found in the present work implies that profit is lower for firms with private information, and total and consumer surplus are higher. The competitive outcome does not depend on the number of firms, nor on whether the firms observe each other's cost or quality, but relies on consumers not observing these. Thus firms are better off and consumers worse off when consumers have more information.

The next section sets up the model. The benchmark of monopoly signalling to heterogeneous buyers is studied in Sect. 2.1 and competition under zero search cost in Sect. 2.2. Section 3.1 constructs an equilibrium with near-competitive pricing and

² If prices are chosen from a discrete grid (as in reality: one cent increments), then a good quality firm sets price strictly below the bad in equilibrium, and cheap talk can be dispensed with.

shows that this equilibrium is the unique one that survives the Intuitive Criterion of Cho and Kreps (1987). The robustness of the results to relaxing various assumptions is discussed in Sect. 3.2. Section 3.3 compares the present work to the previous theoretical literature. The final section concludes with the policy implications.

1 Price competition under costly learning of prices

There are two firms, indexed by $i \in \{X, Y\}$, each with a type $\theta \in \{G, B\}$, interpreted, respectively, as good and bad. Types are i.i.d. with $\Pr(G) = \mu_0 \in (0, 1)$. There is a continuum of consumers of mass 1 with types $v \in [0, \bar{v}]$ distributed according to the strictly positive continuous pdf f_v , with cdf F_v , independently of firm types. Firms and consumers know their own type, but not the types of other players. There is a common prior belief over the types of all players. The timeline of the game is as follows.

1. Nature draws independent types for firms and consumers, and assigns half the consumers to one firm, half to the other, independently of types. Each player observes his own type, but not the types of the others.
2. Firms simultaneously choose prices and cheap talk messages.³
3. Each consumer observes the price and message of his assigned firm and chooses either to buy from this firm, learn the price and message of the other firm, or leave the market.
4. Each consumer who chose to learn observes both firms' prices and cheap talk and chooses either to buy from his assigned firm, buy from the other firm, or leave the market.

A type G firm has marginal cost c_G , normalised to 0, and type B has $c_B > 0$. The quality of a type G firm is better, in the sense that a type v consumer values firm type B 's product at v and G 's product at $h(v) \geq v$, with $h' \geq 1$ and $h(\bar{v}) < \infty$. A discussion of the assumptions is in Sect. 3.2. To ensure that demand for B 's good is positive, but not all consumers buy at price equal to the bad type's cost, assume $\bar{v} > c_B \geq h(0)$. Consumers and firms are risk-neutral. Each consumer has unit demand.

After the firms' cost and quality are determined, the firms simultaneously set prices $P_X, P_Y \in \mathbb{R}_+$ and choose cheap talk messages $t_X, t_Y \in \{G, B\}$. A behavioural strategy of firm i maps its type to $\Delta(\mathbb{R}_+ \times \{G, B\})$. The probability that type θ of firm i puts on message t and prices below P is denoted $\sigma_i^\theta(P, t)$, so $\sigma_i^\theta(\cdot, t)$ is the cdf of price.

The corresponding pdf is $\frac{d\sigma_i^\theta(P, t)}{dP}$ if it exists.

A consumer sees the price and cheap talk of his assigned firm and can learn those of the other firm at cost $s > 0$. Assume that $s \leq \mu_0(h(c_B) - c_B)$, i.e. the learning cost is small relative to the prior probability of the good type and the quality difference for a consumer with $v = c_B$. This implies that a consumer willing to buy from B at price c_B is also willing to search for G when expecting the same price. The cost difference $c_B - 0$ between the types, as well as the quality difference $h(v) - v$ may be small, provided the learning cost is even smaller.

³ The cheap talk is needed for types to separate when they both price at the marginal cost of the bad type. Otherwise equilibrium does not exist, unless prices are restricted to a discrete grid. The proof is in an earlier version of this paper, available on <https://sanderheinsalu.com/>.

After seeing the price of his assigned firm, a consumer decides whether to buy from this firm (denoted b), learn the other firm's price (ℓ) or not buy at all (n). Upon learning the price of the other firm, the consumer decides whether to buy from firm X (denoted b_X), firm Y (b_Y) or not at all (n_ℓ). A consumer's behavioural strategy maps his valuation, the price(s) and cheap talk message(s) to a decision via the functions $\sigma_1 : [0, \bar{v}] \times \mathbb{R}_+ \times \{G, B\} \rightarrow \Delta \{b, n, \ell\}$ and $\sigma_2 : [0, \bar{v}] \times \mathbb{R}_+^2 \times \{G, B\}^2 \rightarrow \Delta \{b_X, b_Y, n_\ell\}$. For example, $\sigma_2(v, P_i, P_j, t_i, t_j)(b_j)$ is the probability that a consumer type v initially at firm i buys from $j \neq i$ after learning P_j, t_j . A consumer's posterior belief about firm i after observing its price P and message t is denoted $\mu_i(P, t)$.

A type θ firm's *ex post* payoff if mass D of consumers buy from it at price P is $(P - c_\theta)D$. Assume that the full-information monopoly profit $P[1 - F_v(h^{-1}(P))]$ of the good type firm strictly increases in P on $[0, c_B]$, so that the full-information monopoly price P_G^m of G is above c_B .

The solution concept used is perfect Bayesian equilibrium (PBE), hereafter simply called equilibrium. The formal definition is notationally cumbersome and relegated to "Appendix A", but the idea is standard: each firm maximises profit given the strategies of the consumers and the other firm, the consumers maximise their profits given their beliefs, and the beliefs are derived from Bayes' rule when possible. Later, a unique equilibrium is selected using the Intuitive Criterion.

The following section outlines the two benchmarks for the main model: price signalling by a monopoly, and competition with zero search cost.

2 Benchmarks: monopoly and costless search

2.1 Signalling by a monopoly

The baseline model is simplified by omitting one of the firms (and the firm subscripts). The timeline reduces to: (1) Nature draws independent types for the firm and the consumers. Each player observes his own type, but not the types of the others, (2) the firm chooses a price and a cheap talk message, (3) each consumer observes the price and message and chooses to buy or leave the market. The definition of equilibrium simplifies correspondingly. The equilibrium is characterised in Proposition 1 below. The full-information monopoly price of B is denoted $P_B^m := \arg \max_P (P - c_B)[1 - F_v(P)] > c_B$. To simplify the exposition, assume P_B^m is unique and that the full-information monopoly profit of G strictly increases in price on $[0, \min \{P_B^m, P_G^m\}]$.

Proposition 1 *If $\mu_0 h(0) < c_B$, then in the unique equilibrium satisfying the Intuitive Criterion, type B chooses P_B^m and type G chooses $P_G \in (c_B, P_B^m)$ s.t. $(P_G - c_B)[1 - F_v(h^{-1}(P_G))] = (P_B^m - c_B)[1 - F_v(P_B^m)]$.*

The proofs of this and subsequent results are in "Appendix B".

The assumption $\mu_0 h(0) < c_B$ ensures that not all consumers buy at the prior belief when the price equals the cost of the bad type. If all customers buy at the prior, then pooling on any $P \in [c_B, \mu_0 h(0)]$ survives the Intuitive Criterion, because the good

type has no incentive to price signal downwards to improve belief when this does not increase demand.

As usual in signalling games, there are many pooling, separating and semi-pooling equilibria, but applying the Intuitive Criterion leaves only the least-cost separating equilibrium. The prices are bounded away from competitive, unlike in the main model of this paper. In particular, the bad type sets its monopoly price. Downward price signalling generally causes G to set a price below its full-information monopoly price, so unobservable types have some of the same pro-competitive effect with one firm as with two. However, only the combination of competition and unobservable types reduces prices to the competitive level.

Next, the second benchmark (competition without search frictions) will be considered. After that, the main model is solved in Sect. 3.1.

2.2 Competition with costless search

The only difference from Sect. 1 is the zero search cost $s = 0$. The only departure from Bertrand competition with heterogeneous buyers is that consumers choose whether to learn both firms' prices and messages, or just their initial firm's.⁴ Learning about both firms is weakly dominant, but the equilibrium of the main model constructed in Sect. 3.1 survives when consumers initially at a good type firm choose not to learn. Given the unchanged consumer strategy, firms' best responses remain the same as under $s > 0$: both set price c_B and reveal their type by cheap talk. Then, consumers at a type G firm get no benefit from learning.

For the remainder of this section, the focus is on weakly undominated strategies. The next result characterises the essentially unique equilibrium in which all consumers learn that passes the Intuitive Criterion. The only non-uniqueness in the equilibrium is due to switching the cheap talk messages or including the endpoint c_B in the support of the atomless price distribution σ_i^{G*} , with $t_G \neq t_B$ when $P = c_B$. The distribution of payoffs remains unchanged.

Proposition 2 *In any equilibrium in weakly undominated strategies that satisfies the Intuitive Criterion, type B chooses $P_B = c_B$ and type G mixes atomlessly on $[\underline{P}_G, c_B)$, where*

$$\underline{P}_G \left[1 - F_v(h^{-1}(\underline{P}_G)) \right] = (1 - \mu_0)c_B \left[1 - F_v(h^{-1}(c_B)) \right].$$

The mixing cdf σ_j^{G*} satisfies $\sum_{t_j \in \{G, B\}} \sigma_j^{G*}(P, t_j) = \frac{1}{\mu_0} - \frac{(1-\mu_0)c_B[1-F_v(h^{-1}(c_B))]}{\mu_0 P [1-F_v(h^{-1}(P))]}$ on its support.

In the equilibrium in Proposition 2, as in the monopoly case in Sect. 2.1, cheap talk messages are redundant, because prices perfectly reveal the types of the firms. Consumers ignore the messages, in contrast to the case of costly search studied in Sect. 3.1 where both firms set the same price c_B and separate using cheap talk. Thus, price search frictions create a meaningful role for advertising: cheap talk can credibly

⁴ This is a special case of the *optional monitoring* defined in Miyahara and Sekiguchi (2013).

communicate product quality when prices do not reveal it. Prices alone are uninformative under costly search and competition because consumers facing a low price decide not to learn, while those facing a high price do learn, which compresses the price distribution to a single point. Costless search makes it a best response for all consumers to learn, creating downward pressure on the lowest prices as well as the highest. The price distribution remains non-degenerate, thus informative.

The next section constructively proves equilibrium existence in the main model with costly search by guessing and verifying.

3 Main results

3.1 Equilibrium

This section constructs an equilibrium in which consumers put probability one on a firm being the good type if the price is strictly below the bad type’s cost, probability one on the bad type if the price is strictly above the bad type’s cost, and ignore cheap talk in these cases. At price equal to the bad type’s cost, consumers interpret the cheap talk as the truth (are certain that the firm’s type equals its message). Figure 1 depicts the belief and strategy of the consumers. Both firms set price equal to the bad type’s cost and claim their type in cheap talk. A consumer who believes that his initial firm is the good type either buys (when his valuation for the good type is above the price) or leaves the market. A consumer believing himself to face the bad type learns when his expected valuation for the other firm is above s , otherwise leaves the market. After learning, all consumers buy from the lower-priced firm or leave the market (Fig. 1 bottom), breaking ties in favour of the firm claiming to be the good type and in favour of buying, with the remaining ties broken uniformly randomly. The gain from trade that consumer type v expects from buying from firm i at price P is denoted

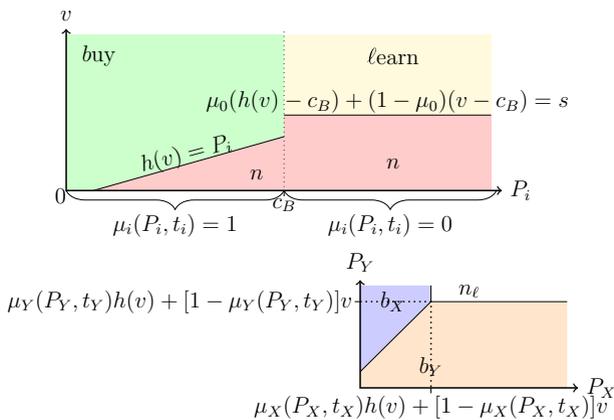


Fig. 1 Belief and strategy of consumers initially at firm i

$w(v, i, P, t) := \mu_i(P, t)h(v) + (1 - \mu_i(P, t))v - P$. The formal definition of the **guessed equilibrium** is the following:

1. Beliefs: if $P < c_B$, then $\mu_i(P, t) = 1$; if $P > c_B$, then $\mu_i(P, t) = 0$; and if $P = c_B$, then $\mu_i(P, t) = \mathbf{1}\{t = G\}$ ⁵ for $i \in \{X, Y\}$.
2. Each firm i and type θ sets price c_B and sends message $t_i = \theta$.
3. If $\mu_i(P, t) = 1$, then $\sigma_1^*(v, P, t)(b) = \mathbf{1}\{h(v) \geq P\} = 1 - \sigma_1^*(v, P, t)(n)$.
4. If $\mu_i(P, t) = 0$, then $\sigma_1^*(v, P, t)(\ell) = \mathbf{1}\{\mu_0(h(v) - c_B) + (1 - \mu_0)(v - c_B) \geq s\} = 1 - \sigma_1^*(v, P, t)(n)$.
5. If $w(v, i, P_i, t_i) \geq \max\{0, w(v, j, P_j, t_j)\}$, then $\sigma_2^*(v, P_i, P_j, t_i, t_j)(b_i) = 1$, and if in addition $w(v, i, P_i, t_i) > w(v, j, P_j, t_j)$, then $\sigma_2^*(v, P_j, P_i, t_j, t_i)(b_i) = 1$. However, if $\max\{w(v, i, P_i, t_i), w(v, j, P_j, t_j)\} < 0$, then $\sigma_2^*(v, P_i, P_j, t_i, t_j)(n_\ell) = 1$.

“Appendix A” proves that no player can profitably deviate from the guessed equilibrium. The idea of the proof is as follows. Consumers are clearly best responding to their belief, which is consistent with firm strategies. The learning cost is less than the expected quality difference, so that consumers who believe they are at a bad type prefer to learn. The bad type does not price below c_B , because it guarantees nonpositive profit. If all consumers at a bad type learn and find the other firm to be a good type, then all consumers leave the bad type. Conditional on the other firm being a bad type, the two bad types are in Bertrand competition over the consumers who learn. So the bad types undercut each other’s price until $P_i = c_B$. At $P_i = c_B$, a bad type gains nothing from claiming to be the good type and thereby increasing demand. Neither type increases price above c_B , because the resulting fall in belief makes consumers learn and leave, so reduces demand and expected profit to zero, regardless of the type of the other firm. At prices less than c_B , the Diamond paradox reasoning applies to the good types: each can raise its price above that of the rival by less than s without losing demand. A price slightly greater than that expected from the other firm does not motivate consumers to learn, unless their belief also decreases after the price increases.

The guessed equilibrium already partly resolves the Diamond paradox, because the price is below the monopoly level and search occurs. Prices in the guessed equilibrium are close to competitive. Both types price the same as under Bertrand competition between the B types with zero search cost and complete information. The price in the guessed equilibrium is higher than when two known G types Bertrand compete and $s = 0$, but lower than when a known G type competes with a known B type. When the quality and cost difference between the types is small, all three Bertrand prices are close to that in the guessed equilibrium.

For a stronger resolution of the Diamond paradox, subsequent results will show that the guessed equilibrium introduced above is the unique one that survives the Intuitive Criterion. As a first step towards proving uniqueness, the following lemma shows that the good type’s price is lower and demand higher than the bad type’s in any equilibrium. Given the ranking of the costs and qualities of the types, the results are intuitive—the lower-cost type G sets a lower price and the higher quality type G receives higher demand. Based on Lemma 3, there cannot be two prices on which both types put positive probability and at one of which, demand is positive.

⁵ The indicator function $\mathbf{1}\{X\}$ equals 1 if condition X holds, and 0 otherwise.

Lemma 3 *In any equilibrium, for any P_θ, t_θ in the support of $\sigma_i^{\theta*}$, $D_i(P_G, t_G) \geq D_i(P_B, t_B)$, and if in addition $0 < D_i(P_B, t_B) \leq D_i(P_G, t_G)$, then $P_G \leq P_B$.*

The next lemma shows that pooling fails the Intuitive Criterion. The lemma also proves the natural result that the good type makes positive profit.

Lemma 4 *Any equilibrium satisfying the Intuitive Criterion has disjoint supports of σ_i^{G*} and σ_i^{B*} , and has $\pi_{iG}^* > 0$ for $i \in \{X, Y\}$.*

The intuition for the proof of Lemma 4 is that for any candidate pooling equilibrium price, there is a cutoff price below which a bad type firm makes less profit than in the candidate equilibrium even under the most favourable consumer belief (probability 1 of the good type). At prices close to this cutoff, under the most favourable belief, the good type makes strictly more profit than at the candidate pooling price, because the good type has strictly lower cost than the bad who is indifferent at the cutoff. If the good type, but not the bad, deviates to a price, then the Intuitive Criterion sets consumers' belief to certainty of the good type after such a deviation, which in turn motivates a good firm to set that price.

Lemma 4 provides the first component of the race to the bottom, namely the good types separating from the bad by setting a lower price. The next lemma establishes a lower bound on the equilibrium price by showing that the good types price weakly above the cost of the bad type.

Lemma 5 *For any $i \in \{X, Y\}$, $P_i < c_B$ and $t \in \{G, B\}$, in any equilibrium satisfying the Intuitive Criterion, $\sigma_i^{G*}(P_i, t) = 0$.*

The intuition for Lemma 5 is that the firms' good types are in a *race to the top* at prices in $[0, c_B)$. Neither firm's good type loses customers to the other firm when raising price slightly, because the small price difference does not motivate customers to pay the learning cost. The reason that a good type does not increase price strictly above c_B is that belief and demand drop discretely.⁶

In the unique⁷ equilibrium surviving the Intuitive Criterion, each type sets price c_B and the types separate using cheap talk, as shown in the following Theorem. The proof provides the second component of the race to the bottom: a bad type reduces price to deter its customers from learning, and to undercut the other firm's bad type. The motive for a customer to learn comes from the good types separating (the first component of the race to the bottom, Lemma 4), which makes the other firm's price or message informative, enabling the customer to choose the better quality firm.

Theorem 6 *In the unique equilibrium satisfying the Intuitive Criterion, both types of both firms set price c_B and send different messages with probability 1.*

Theorem 6 shows that the unique equilibrium that satisfies the Intuitive Criterion is the guessed equilibrium from above. Prices are close to competitive. The equilibrium is robust to changing the prior, the learning cost, the distribution of consumer valuations

⁶ The discrete drop of belief is due to discrete types. The results are robust to a continuum of types.

⁷ Uniqueness is up to permutation of the cheap talk messages. Formally, there are two equilibria: in one, each type θ sends message $t_\theta = \theta$; in the other, each θ sends $t_\theta \neq \theta$.

and the good type's cost in a range of parameters.⁸ Even outside this range, in general the equilibrium remains the same or is continuous in the parameters.

The equilibrium in Theorem 6 is distinct from signalling by a monopoly, because a bad type monopolist does not have an incentive to cut price when the good type's price is low enough. This is because there is no competing firm for the customers to learn about and leave to. Thus, the bad type sets its monopoly price.

Section 3.2 below discusses the difference between Theorem 6 and competition when the type is learned together with the price. The comparisons of the guessed equilibrium to the benchmarks in Sect. 2 and to observable type show that the combination of signalling and multiple firms is necessary as well as sufficient to overcome the effect of the positive learning cost.

3.2 Discussion

The results remain qualitatively similar⁹ if the monopoly price of the good type is below the bad type's cost, consumers have a distribution of learning costs (which may extend to large negative or positive levels), or consumers are homogeneous. Customers may also hold somewhat incorrect beliefs without affecting the results. Firms may know each other's cost or quality. More than two firms or types lead to similar results as in the baseline model. Multidimensional types reduce to the two-type case.

Other ways to signal quality (advertising, warranties) may modify the results, depending on the noisiness and cost of the signal. If ads reveal prices, then competition increases and the good types mix over prices below the bad types' cost. The bad types still price at cost. If ads do not reveal prices and are a noisy signal of advertising expenditure, then ads seen before the prices only change the prior. Ads seen after the prices have no effect, because the prices already reveal the types. Suppose that ads are perfect signals of the money spent on them. Then, the relative cost to the types per unit of ads vs per unit of price decrease determines which signalling channel the good type uses.

If consumers see their initial firm's type together with the price (instead of having to infer the type from the price), but still have to pay a cost to learn the price, message and type of the rival firm, then the good type sets its monopoly price or both types price above the competitive level. The good type can price above the bad type's cost by at least the quality difference plus the learning cost, because consumers observe the quality and stay at the good type at such a price. If this price is above the good type's monopoly level, then the good type sets its monopoly price. The bad type prices competitively if the lowest-value consumer who has positive gains from trade with the bad type prefers the good type at its monopoly price to the bad type at price equal to its cost. Otherwise the bad type has some captive customers, thus makes positive profit and only sets prices bounded away from its cost. Above-cost pricing by the bad type loosens the good type's constraint on price increases. The good types may be able

⁸ The range is the nonempty open set of parameters defined by $h(v) \geq v$, $h' \geq 1$, $h(\bar{v}) < \infty$, $s \leq \mu_0(h(c_B) - c_B)$, $c_B \geq h(0) > 0$ and $\frac{d}{dP} P[1 - F_v(h^{-1}(P))] > 0$ for $P \in [0, c_B + \epsilon]$.

⁹ The robustness checks and extensions mentioned here are covered in greater detail in an earlier version of this paper, available on <https://sanderheinsalu.com/>.

to raise prices to their monopoly level at the same time as the bad type makes positive profit.

3.3 Literature

The foremost article on costly learning of prices is Diamond (1971), in which competing firms set the monopoly price. A number of solutions to the Diamond paradox have been proposed. When a positive fraction of consumers can learn at zero cost, as in Butters (1977), Stahl (1996), Klemperer (1987) and Benabou (1993), firms put a positive probability on the competitive price. However, with positive learning costs for some consumers, firms mix over prices above the competitive level. Both mixing and above-competitive pricing differ from the current work. A similar idea to zero learning cost is that consumers observe multiple prices with positive probability, for example because firms send them price advertisements (Salop and Stiglitz 1977; Burdett and Judd 1983; Robert and Stahl 1993). If consumers have private taste shocks, then that generates search and mixed below-monopoly pricing (Wolinsky 1986; Anderson and Renault 1999; Zhou 2014).

Learning, or the motive to learn, is exogenous for at least some consumers in the above papers. In the current work, the learning motive is always endogenous for all consumers. They pay to observe a firm's price and cheap talk in order learn the firm's quality, because learning gives the option to buy better quality. Price and cheap talk are informative because the firms play a separating equilibrium. The firms in turn separate because the consumers learn. If all firms pooled, then no consumer would have any incentive to pay the learning cost.

With consumer taste shocks (horizontal differentiation of firms), some consumers initially at each firm learn another firm's price and leave. This differs from the current work, which models vertical differentiation and shows that consumers initially at a good firm do not learn or switch.

Prices below the monopoly level also occur with repeat purchases, as in Salop and Stiglitz (1982), Bagwell and Ramey (1992) and Poeschel (2018), where in some equilibria, raising the price is punished in subsequent stage games. However, the markets described by repeated games with high discount factors differ from the markets studied here, which involve infrequent buying (repair services, insurance, durable goods such as cars) and are thus closer to one-shot interactions. The present article does not rely on repeat purchases, a zero learning cost, multiple free price observations or taste shocks. Firms do not mix in the current paper.

To the author's knowledge, this work is the first to combine consumer learning costs and signalling in the sense of Spence (1973).¹⁰ Signalling relies on private information about vertical differentiation, and to the author's knowledge, the present paper is the first to combine privately known quality differences with sequential search, either costly or costless. Public quality differences are combined with non-sequential consumer search in Wildenbeest (2011).

¹⁰ Among the thousands of Google Scholar results citing Diamond (1971) or Spence (1973), only 79 cite both. These are either review articles or only tangentially related.

The benchmark of Bertrand competition considered in this paper is similar to costless simultaneous (non-sequential) search, which unsurprisingly yields competitive prices, as in Janssen and Roy (2010, 2015), Sengupta (2015) and Heinsalu (2019). They find that incomplete information may increase or decrease the price set by each type, as well as the *ex ante* expected price, under both positive and negative correlation of cost and quality. A formal comparison of Diamond and Bertrand environments under positive correlation is in “Appendix C”.

Only with costly search and negatively correlated quality and production cost does incomplete information reduce price and profit unambiguously. The outcome is similar to freely observed prices under negatively associated private cost and quality, for which Janssen and Roy (2015) find competitive pricing, as may be expected. By contrast, costly search with positively related quality and cost results in a price weakly above the monopoly level, whether information is complete or incomplete (as in “Appendix C” and the Online Appendix). Perfectly observable prices with positively related private cost and quality result in full surplus extraction from homogeneous consumers for some parameter values (Janssen and Roy 2015), but not from heterogeneous consumers (Heinsalu 2019).

Downward price signalling by a single firm has been studied in Shieh (1993). A similar idea is found in Simester (1995), where multi-product firms (whose prices for all products are positively correlated) signal by a low price on one product. Kihlstrom and Riordan (1984) also allow quality and cost to be negatively correlated in a signalling context. Pricing is not competitive in these articles, because the signaller is a monopolist.

Both horizontal and privately known vertical differentiation (Hotelling with quality uncertainty) when prices are seen costlessly is studied in Daughety and Reinganum (2007, 2008). Quality and cost are positively correlated and incomplete information raises price and profit, unlike in the present work.

If firm types only differ in their private marginal cost, but not quality, and consumers observe both firms’ prices for free, then the high cost type prices at its marginal cost. The low cost type mixes over a range of prices strictly above its marginal cost and weakly below the price of the high-cost type (Spulber 1995). If instead the consumers have a positive learning cost, then the low-cost firm prices at its monopoly level and the high-cost firm s above that (just enough to deter consumers from learning) or at its monopoly price, whichever is lower. This outcome resembles the Diamond paradox. There is no quality difference to signal, so no reason for the low-cost firm to cut price. Thus, the race to the bottom does not start.

4 Conclusion

The famous paradox of Diamond (1971) is that a market with multiple firms is not competitive if consumers have to pay a cost to learn prices. However, as shown in the current article, negatively correlated private production cost and quality restore competitive pricing. This result is robust to a wide range of quality and cost differences, prior distributions and learning costs.

Several mechanisms make cost and quality negatively correlated across firms, e.g. economies of scale, regulation, or differing managerial talent. These mechanisms operate in many markets, including oligopolistic ones in which price is close to the marginal cost of at least some firms, for example among car manufacturers and airlines. Private information about cost and quality, as well as prices close to the competitive level are empirically reasonable in skilled services, construction and insurance, among others.

The previous literature resolves the Diamond paradox assuming either (a) zero learning cost for a positive fraction of consumers, (b) that consumers observe multiple prices at once, (c) large private taste shocks, or (d) repeat purchases. The current work models markets in which a given consumer purchases rarely, e.g. cars, insurance, repair services, and in which the vertical quality difference is more important than the horizontal taste shock. The predictions of the current article differ from zero search costs and observing multiple prices, because the firms set deterministic prices instead of mixing, and the mark-up and profit are larger for a lower-price firm. The present article assumes no repeat buying of the same good (insurance policies and car models change by the time the consumer purchases a replacement), which distinguishes the model from the literature on repeat purchases. With taste shocks, prices decrease in the number of firms and the degree of product differentiation. In this article, prices stay constant when the number of firms rises above two or when the quality difference changes within some bounds.

If lower cost implies higher quality, then a low price is a credible signal of quality, because it is differentially costly to the firm types. In some markets, other costly signals are available, e.g. warranties or advertising. In other applications like insurance, warranties are uncommon, so price signalling is more likely. Even if feasible, signalling by ads or warranties may not be optimal, for example when price signals are cheaper or more precise.

Total and consumer surplus are strictly greater when the price is competitive and consumers pay the learning cost than when there is monopoly pricing and no learning. A regulator maximising total or consumer surplus should encourage the race to the bottom in prices, for example by punishing low quality or checking the quality of a firm with a larger market share more frequently. The regulator should not facilitate verifiable disclosure of quality, because this would allow both types of firms to increase prices, possibly to monopoly levels. This ability to raise prices after disclosing quality explains the large sums firms spend on certification and ratings.

Similarly, industry policy should focus on improving the quality of the low-cost firms, rather than reducing the cost of the high-cost enterprises. The optimal policy is more nuanced than supporting national champions (the lowest-cost, largest producers), because the assistance in improving quality should be targeted only to firms whose quality and cost are uncertain, e.g. start-ups, firms launching a novel product.

An implication of this article for competition policy is that a merger to duopoly need not increase price above the competitive level if there is uncertainty about the (negatively correlated) costs and qualities of the duopolists.

A Equilibrium definition and existence

A consumer's posterior belief about firm i after observing its price P and message t and expecting the firm to choose strategy σ_i^* is

$$\mu_i(P, t) := \frac{\mu_0 \frac{d}{dP} \sigma_i^{G*}(P, t)}{\mu_0 \frac{d}{dP} \sigma_i^{G*}(P, t) + (1 - \mu_0) \frac{d}{dP} \sigma_i^{B*}(P, t)} \quad (1)$$

whenever the denominator is positive. A discontinuity of height h_θ in the cdf $\sigma_i^{\theta*}$ is interpreted in the pdf as a Dirac δ function times h_θ . Therefore, an atom in $\sigma_i^{G*}(\cdot, t)$, but not $\sigma_i^{B*}(\cdot, t)$ at P results in $\mu_i(P, t) = 1$, and an atom in $\sigma_i^{B*}(\cdot, t)$, but not $\sigma_i^{G*}(\cdot, t)$ yields $\mu_i(P, t) = 0$. If $\sigma_i^{\theta*}$ has an atom of size h_θ at P for $\theta \in \{G, B\}$, then $\mu_i(P, t) = \frac{\mu_0 h_G}{\mu_0 h_G + (1 - \mu_0) h_B}$. Finally, if the denominator of (1) is zero, then belief is arbitrary.

The demand that firm i expects at price P and message t given the expected strategies of firm j and the consumers is

$$\begin{aligned} D_i(P, t) := & \frac{1}{2} \int_0^{\bar{v}} \sigma_1^*(v, P, t)(b) \\ & + \int_0^\infty \sum_{t_j \in \{G, B\}} \{ \sigma_1^*(v, P, t)(\ell) \sigma_2^*(v, P, P_j, t, t_j)(b_i) \\ & + \sigma_1^*(v, P_j, t)(\ell) \sigma_2^*(v, P_j, P, t_j, t)(b_i) \} [\mu_0 d\sigma_j^{G*}(P_j, t_j) \\ & + (1 - \mu_0) d\sigma_j^{B*}(P_j, t_j)] dF_v(v). \end{aligned} \quad (2)$$

The first term in (2) reflects the consumers initially at i who buy immediately. The second term describes consumers who buy from i after learning both prices and messages, which consists of (the first term in the curly braces) consumers at i who learn and then buy from i and (the second term in the braces) the consumers initially at j who learn and then buy from i .

Denote $\mu_0 d\sigma_j^{G*}(P_j, t_j) + (1 - \mu_0) d\sigma_j^{B*}(P_j, t_j)$ by $d\sigma_j^{\mu_0*}(P_j, t_j)$, and recall $w(v, i, P, t) := \mu_i(P, t)h(v) + (1 - \mu_i(P, t))v - P$.

Definition 1 An equilibrium consists of $\sigma_X^*, \sigma_Y^*, \sigma_1^*, \sigma_2^*$ and μ_X, μ_Y satisfying the following for $\theta \in \{G, B\}$, $v \in [0, \bar{v}]$, $i, j \in \{X, Y\}$, $i \neq j$:

- if $w(v, i, P_i, t_i) \geq \max\{0, w(v, j, P_j, t_j)\}$, then $\sigma_2^*(v, P_i, P_j, t_i, t_j)(b_i) = 1$, and if in addition $w(v, i, P_i, t_i) > w(v, j, P_j, t_j)$, then $\sigma_2^*(v, P_j, P_i, t_j, t_i)(b_i) = 1$,
- if $\max\{w(v, i, P_i, t_i), w(v, j, P_j, t_j)\} < 0$, then $\sigma_2^*(v, P_i, P_j, t_i, t_j)(n_\ell) = 1$,
- if $w(v, i, P_i, t_i) > \max\{0, \int_0^\infty \sum_{t_j \in \{G, B\}} \max\{w(v, i, P_i, t_i), w(v, j, P_j, t_j)\} d\sigma_j^{\mu_0*}(P_j, t_j) - s\}$, then $\sigma_1^*(v, P_i, t_i)(b) = 1$,
- if $w(v, i, P_i, t_i) \leq \int_0^\infty \sum_{t_j \in \{G, B\}} \max\{0, w(v, i, P_i, t_i), w(v, j, P_j, t_j)\} d\sigma_j^{\mu_0*}(P_j, t_j) - s \geq 0$, then $\sigma_1^*(v, P_i, t_i)(\ell) = 1$,

- (e) if $\max\{w(v, i, P_i, t_i), \int_0^\infty \sum_{t_j \in \{G, B\}} \max\{0, w(v, j, P_j, t_j)\} d\sigma_j^{\mu_0^*}(P_j, t_j) - s\} < 0$, then $\sigma_1^*(v, P_i, t_i)(n) = 1$,
- (f) if (P_i, t_i) is in the support of $\sigma_i^{\theta^*}$, then $(P_i, t_i) \in \arg \max_{P, t} (P - c_\theta) D_i(P, t)$, where $D_i(P, t)$ is given by (2),
- (g) if (P, t) is in the support of $\sigma_i^{G^*}$ or $\sigma_i^{B^*}$, then $\mu_i(P, t)$ is derived from (1).

Consumers are clearly best responding to their beliefs in parts 3–5 of the guessed equilibrium. Beliefs in part 1 are consistent with part 2. It remains to check whether firms are best responding in part 2. First, deviations of type G to lower prices are ruled out. The preparative Lemma 7 derives the profit function of G from setting $P < c_B$.

Lemma 7 *In the guessed equilibrium, the profit of type G from $P < c_B$ and t is*

$$\frac{1}{2} P \left[1 - F_v(h^{-1}(P)) + (1 - \mu_0) \int_{h^{-1}(P)}^{\bar{v}} \sigma_1^*(v, c_B, B)(\ell) dF_v(v) \right]. \tag{3}$$

Proof The profit (3) is derived from (2) by substituting in the consumers’ strategies in the guessed equilibrium: $\sigma_1^*(v, P_i, t)(b) = 1$ and $\sigma_1^*(v, P_i, t)(\ell) = 0$ for consumers initially at i , because $P_i < c_B$ and $\mu_i(P_i, t) = 1$. Consumers with $v \geq h^{-1}(P_i)$ buy from i , and they form a fraction $1 - F_v(h^{-1}(P_i))$ of the mass of consumers initially at i .

If firm j is type G , then $P_j = c_B, t_j = G$ and $\sigma_1^*(v, P_j, t_j)(\ell) = 0$. With probability $1 - \mu_0$, firm j is type B , in which case consumer v at firm j learns P_i with probability $\sigma_1^*(v, c_B, B)(\ell)$ and then buys if $v \geq h^{-1}(P_i)$. □

Next, the technical Lemma 8 simplifies (3) by showing that if $\sigma_i^{B^*}$ puts probability 1 on $(P, t) = (c_B, B)$ and $\sigma_i^{G^*}$ puts probability 1 on (c_B, G) for $i \in \{X, Y\}$, then $\sigma_1^*(v, c_B, B)(\ell)$ is a step function increasing in v .

Lemma 8 *For customers initially at a type B firm, there exists $v_{01} \in [h^{-1}(c_B), \bar{v}]$ s.t. $\sigma_1^*(v, c_B, B)(\ell) = 0$ for $v < v_{01}$ and 1 for $v > v_{01}$.*

Proof Suppose firm i has type B . Due to $P_j \leq c_B$, in Definition 1(d), $v - c_B$ may be dropped under the max w.l.o.g. If $\int_0^\infty \sum_{t_j \in \{G, B\}} \max\{0, w(v, j, P_j, t_j)\} [\mu_0 d\sigma_j^{G^*}(P_j, t_j) + (1 - \mu_0) d\sigma_j^{B^*}(P_j, t_j)] - s \geq 0$, for consumer v , then for all $\hat{v} > v$, the inequality is strict.

If $w(v, j, P_j, t_j) \leq 0$, then $v - c_B < 0$, so the first inequality in Definition 1(d) holds. If $w(v, j, P_j, t_j) > 0$, then 0 may be dropped under the max w.l.o.g. Then from $h' \geq 1$ and $\int_0^\infty \sum_{t_j \in \{G, B\}} [\mu_0 d\sigma_j^{G^*}(P_j, t_j) + (1 - \mu_0) d\sigma_j^{B^*}(P_j, t_j)] = 1$, the first inequality in Definition 1(d) holds for all $\hat{v} \geq v_1$. So if $\sigma_1^*(v, c_B, B)(\ell) > 0$, then for all $\hat{v} \geq v$, $\sigma_1^*(\hat{v}, c_B, B)(\ell) = 1$. Taking $v_{01} := \inf \{v : \sigma_1^*(v, c_B, B)(\ell) > 0\}$ ensures that $\sigma_1^*(\hat{v}, c_B, B)(\ell) = 0$ for $\hat{v} < v_{01}$ and 1 for $\hat{v} > v_{01}$.

To prove $v_{01} \geq h^{-1}(c_B)$, note that $h^{-1}(x) < x \ \forall x$, so $h^{-1}(c_B) - c_B < 0$. If $P_j \geq c_B$, then $w(h^{-1}(c_B), j, P_j, t_j) \leq 0$ for any t_j . The $-s$ term in Definition 1(d) then ensures $\sigma_1^*(h^{-1}(c_B), c_B, B)(\ell) = 0$. □

Downward price deviations by a type G firm are ruled out in the following Lemma. After that, the incentives of firm type B are discussed, and then, the deviations of G to $P_G > c_B$ are ruled out.

Lemma 9 *A type G firm’s best response to the strategies of other players in the guessed equilibrium satisfies $P \geq c_B$.*

Proof Based on Lemma 8, $\sigma_1^*(v, c_B, B)(\ell) = 0$ for all $v \leq h^{-1}(c_B) \geq h^{-1}(P)$, where $P \leq c_B$. Therefore (3) reduces to $\frac{1}{2}P[1 - F_v(h^{-1}(P))] + (1 - \mu_0)[1 - F_v(v_{01})]$, with v_{01} independent of P .

The assumption $P_G^m := \arg \max_P P[1 - F_v(h^{-1}(P))] \geq c_B$ then implies $\arg \max_P \frac{1}{2}P[1 - F_v(h^{-1}(P))] + (1 - \mu_0)[1 - F_v(v_{01})] \geq c_B$, because if $P_G^m D(P_G^m) \geq PD(P)$ for all $P \leq P_G^m$, then for any $\bar{D} > 0$ and $P \leq P_G^m$, we have $P_G^m D(P_G^m) + P_G^m \bar{D} \geq PD(P) + P\bar{D}$. So type G optimally sets a price $P \geq c_B$. \square

Lemma 10 *In the guessed equilibrium, a type B firm’s best response to the strategies of other players is $P = c_B$ and $t = B$.*

Proof A type B firm clearly does not deviate to $P < c_B$ with any message. Consider B’s deviations to $P > c_B$ and some $t \in \{G, B\}$. Parts 1 and 4 of the guessed equilibrium ensure that each customer initially at firm i charging $P > c_B$ either leaves the market or learns the price and message of j . By part 5 of the guessed equilibrium, a customer who learns at firm i will choose firm j , which has both a lower price $P_j = c_B < P$ and a higher belief $\mu_j(c_B, t_j) \geq 0 = \mu_i(P, t)$ for any t_j, t .

At $P = c_B$, type B is indifferent between demand levels and thus between messages $t \in \{G, B\}$. Therefore, $t = B$ is part of a best response. \square

Having ruled out deviations by B, the final step (Lemma 11) is to eliminate deviations by a type G firm.

Lemma 11 *A type G firm’s best response to the strategies of other players in the guessed equilibrium is $P = c_B$ and $t = G$.*

Proof Lemma 9 established $P \geq c_B$. If firm i ’s type G sets $P > c_B$, with any $t \in \{G, B\}$, then it gets zero demand in the guessed equilibrium, because $\mu_i(P, t) = 0$ and the other firm j is expected to set price $P_j \leq c_B < P$. At $P = c_B$, message $t = B$ leads to belief $\mu_i(c_B, B) = 0$, but message $t = G$ to $\mu_i(c_B, G) = 1$, thus greater demand. Therefore, (c_B, G) is the unique best response for type G. \square

B Proofs omitted from the main text

Proof of Proposition 1 Both types obtain positive profit in any equilibrium, because by the assumption $\bar{v} > c_B$, setting $P = c_B + \epsilon$ for $\epsilon > 0$ small enough results in positive demand at any message t , even at the worst belief $\mu(P, t) = 0$. Positive profit implies for any t that $\sigma^{G*}(0, t) = 0$ and for all $P \leq c_B, \sigma^{B*}(P, t) = 0$, i.e. weakly dominated strategies are never played in any equilibrium. Because demand is positive for both types, by Lemma 3 (the proof of which does not depend on any other results or on two firms), $P_G \leq P_B$ for any P_θ, t_θ in the support of $\sigma^{\theta*}$.

Denote the profit of type θ in a candidate equilibrium by π_θ^* . Define

$$P_1 := \sup \left\{ P : (P - c_B)[1 - F_v(h^{-1}(P))] < \pi_B^* \right\},$$

so type B does not set $P < P_1$ for any message t even at $\mu(P_1, t) = 1$. Clearly $P_1 > c_B$.

Positive f_v implies continuous F_v . Differentiable strictly increasing h implies continuous $(P - c_B)[1 - F_v(h^{-1}(P))]$, thus the sup defining P_1 is a max and $(P_1 - c_B)[1 - F_v(h^{-1}(P_1))] = \pi_B^*$.

Suppose that there exist (semi)pooling P_0, t_0 s.t. $\sigma^{\theta^*}(P_0, t_0) > 0$ for both types. Then $(P_0 - c_B)D(P_0, \mu(P_0, t_0)) = \pi_B^*$ and $P_0D(P_0, \mu(P_0, t_0)) = \pi_G^* = \pi_B^* + c_B D(P_0, \mu(P_0, t_0)) < \pi_B^* + c_B[1 - F_v(h^{-1}(P_1))] = \pi_B^* + c_B D(P_1, 1) = P_1 D(P_1, 1)$, so for $\epsilon > 0$ small enough, G strictly prefers $P_1 - \epsilon$ at belief 1 to P_0 at belief $\mu(P_0, t_0)$. The strict preference of B not to deviate to $P < P_1$ and G to deviate justifies belief 1 at any $P < P_1$ by the Intuitive Criterion, and contradicts (semi)pooling on P_0 .

Because the types separate, belief is 0 at any P, t in the support of σ^{B^*} . Thus, belief threats cannot deter B from deviating to P_B^m and any message t . The assumption that P_B^m is unique ensures that B chooses a pure price. The cheap talk message t is arbitrary.

The assumption that the full-information monopoly profit of G strictly increases in price on $[0, \min \{P_B^m, P_G^m\}]$ ensures that G chooses a pure $P_G = \min \{P_1, P_G^m\}$. If $P_1 \leq P_G^m$ and $\mu(P_1, t_G) < 1$ for the message t_G that type G sends, then the best response of G does not exist, because of the open set problem. Thus in any equilibrium if $P_1 \leq P_G^m$, then $\mu(P_1, t_G) = 1$ and $P_G = P_1$. Therefore, the equilibrium is unique up to changing the cheap talk messages. Consumer beliefs are constant in the cheap talk, so omitting it does not change the equilibrium prices. \square

Proof of Proposition 2 Type G setting $P \in (0, c_B)$ obtains positive demand and profit with probability at least $1 - \mu_0$ (when the rival firm is a bad type). Thus in any equilibrium, the price, demand and profit of G are positive. Then $P_G \leq P_B$ for any P_θ in the support of $\sigma_i^{\theta^*}$ by Lemma 3, the proof of which is independent of other results.

The Intuitive Criterion rules out (semi)pooling and sets belief to 1 for any $P < c_B$ by the same argument as in Proposition 1 and Lemma 4. All consumers learn, so leave a type B firm if the other firm is type G . Bertrand competition between the bad types then implies $P_B = c_B$ in any equilibrium.

Suppose $\inf \left\{ P : \exists t, \sigma_j^{G^*}(P, t) > 0 \right\} =: \underline{P}_{jG} < \underline{P}_{iG}$. Then, the profit of jG (firm j 's type G) from $P \in (\underline{P}_{jG}, \underline{P}_{iG})$ is $P[1 - F_v(h^{-1}(P))]$, which strictly increases in P by assumption. This contradicts the optimality of \underline{P}_{jG} . Therefore, $\underline{P}_{jG} = \underline{P}_{iG}$, denoted \underline{P}_G from here on. Gaps in the support of $\sigma_j^{G^*}$ are ruled out by the same reasoning.

Suppose that $\sigma_i^{G^*}$ has an atom at some P_{iG} . Then by the standard Bertrand undercutting argument, there exist $\epsilon_1, \epsilon_2 > 0$ s.t. jG profitably deviates from any $P_j \in [P_{iG}, P_{iG} + \epsilon_1)$ to $P = P_{iG} - \epsilon_2$. This rules out atoms in $\sigma_i^{G^*}$.

Suppose $\sup \left\{ P : \exists t, \sigma_j^{G^*}(P, t) > 0 \right\} =: \bar{P}_{jG} < \bar{P}_{iG} \leq c_B$. Because $\mu_i(P, t) = 1$ for any $P < c_B$ and any t , the profit of iG from $P \in (\bar{P}_{jG}, c_B)$ is $(1 - \mu_0)P[1 -$

$F_v(h^{-1}(P))$], which strictly increases in P by assumption. This contradicts the optimality of $P \in (\bar{P}_{jG}, c_B)$, and a gap in the support of σ_i^{G*} was ruled out above. Therefore, $\bar{P}_{jG} = \bar{P}_{iG} = c_B$.

Profit must be constant on the support of σ_i^{G*} , i.e. $\mu_0 P[1 - F_v(h^{-1}(P))][1 - \sum_{t_j \in \{G, B\}} \sigma_j^{G*}(P, t_j)] + (1 - \mu_0)P[1 - F_v(h^{-1}(P))] = (1 - \mu_0)c_B[1 - F_v(h^{-1}(c_B))]$. Rearranging yields $\sum_{t_j \in \{G, B\}} \sigma_j^{G*}(P, t_j) = \frac{1}{\mu_0} - \frac{(1 - \mu_0)c_B[1 - F_v(h^{-1}(c_B))]}{\mu_0 P[1 - F_v(h^{-1}(P))]}$. Taking $P \in \{\underline{P}_G, \bar{P}_G\}$ yields $\underline{P}_G [1 - F_v(h^{-1}(\underline{P}_G))] = (1 - \mu_0)c_B [1 - F_v(h^{-1}(c_B))]$. \square

Proof of Lemma 3 In any equilibrium, the incentive constraints (ICs) $P_G D_i(P_G, t_G) \geq P D_i(P, t)$ and $(P_B - c_B)D_i(P_B, t_B) \geq (P - c_B)D_i(P, t)$ hold for any P, P_θ, t, t_θ with (P_θ, t_θ) in the support of $\sigma_i^{\theta*}$. Demand and price are nonnegative and finite by definition. From $(P_B - c_B)D_i(P_B, t_B) \geq (P_G - c_B)D_i(P_G, t_G)$ and $P_G D_i(P_G, t_G) \geq P_B D_i(P_B, t_B)$, we get $(P_B - c_B)D_i(P_B, t_B) \geq P_G D_i(P_G, t_G) - c_B D_i(P_G, t_G) \geq P_B D_i(P_B, t_B) - c_B D_i(P_G, t_G)$, so $D_i(P_B, t_B) \leq D_i(P_G, t_G)$.

If $0 < D_i(P_B, t_B) \leq D_i(P_G, t_G)$ and $(P_B - c_B)D_i(P_B, t_B) \geq (P_G - c_B)D_i(P_G, t_G)$, then $P_B - c_B \geq P_G - c_B$, so $P_B \geq P_G$. \square

Proof of Lemma 4 Suppose $\pi_{iG}^* = 0$ and use the Intuitive Criterion to derive a contradiction. Fix some $P_i \in (0, \min\{s, c_B\})$ and $t \in \{G, B\}$. Set belief to $\mu_i(P_i, t) = 1$. No consumer learns at P_i, t and belief $\mu_i(P_i, t) = 1$, because firm j is expected to have weakly lower quality and a price $P_j \geq 0$ lower by at most s . The greatest possible price decrease $|P_j - P_i| < s$ from switching to j does not justify paying the learning cost s . By assumption, $h(P_i) > P_i > 0$, so consumers with valuations $v \leq P_i$ buy at $P_i, t, \mu_i(P_i, t)$ and yield positive demand and profit to type G . Type B can ensure nonnegative profit by setting $P \geq c_B$, thus must get nonnegative equilibrium profit $\pi_{iB}^* \geq 0$. Choosing P_i, t gives B positive demand, so strictly negative profit. Thus, belief $\mu_i(P_i, t) = 1$ is justified and any supposed equilibrium with $\pi_{iG}^* = 0$ is eliminated.

Next, the Intuitive Criterion is used to eliminate pooling and semi-pooling on any $P_{i0} > c_B, t_{i0}$. By $\pi_{iG}^* > 0$, demand is positive at P_{i0}, t_{i0} , so $\pi_{iB}^* > 0$. Lemma 3 implies that any price in the support of σ_i^{B*} is above P_{i0} and any price in the support of σ_i^{G*} is below P_{i0} .

Denote demand at the fixed belief μ by $D_i^\mu(P)$; it does not depend on t due to the fixed μ . Demand $D_i^\mu(P)$ increases in μ and decreases in P , so the profit $(P - c_\theta)D_i^\mu(P)$ as a function of P does not have upward jumps. At $P = c_B$, the profit of B is $(c_B - c_B)D_i^\mu(c_B) = 0$ for any μ , but at $P_{i0} > c_B$, the equilibrium profit is $\pi_{iB}^* > 0$. Pooling implies $\mu_i(P_{i0}, t_{i0}) < 1$, so $D_i(P_{i0}, t_{i0}) < D_i^1(P_{i0})$ and therefore $(P_{i0} - c_B)D_i^1(P_{i0}) > \pi_{iB}^* > 0$. Thus, there exists $P_{d*} \in (c_B, P_{i0})$ s.t. for any $P < P_{d*}$, $(P - c_B)D_i^1(P) < \pi_{iB}^*$. Focus on the maximal such P_{d*} . The lack of upward jumps in $(P - c_B)D_i^1(P)$ implies that for any $\epsilon > 0$ there exists $\delta > 0$ s.t. $(P_{d*} - \delta - c_B)D_i^1(P_{d*} - \delta) \geq \pi_{iB}^* - \epsilon$. If δ is small enough s.t. $\epsilon < D_i^1(P_{d*} - \delta) - D_i(P_{i0}, t_{i0})$, then type G strictly prefers to deviate to $P_{d*} - \delta$ and t at $\mu_i(P_{d*} - \delta, t) = 1$, because $(P_{d*} - \delta - 0)D_i^1(P_{d*} - \delta) \geq \pi_{iB}^* + (c_B - 0)D_i^1(P_{d*} - \delta) - \epsilon > \pi_{iB}^* + (c_B - 0)D_i(P_{i0}, t_{i0}) = \pi_{iG}^*$. By the definition of P_{d*} , type B strictly prefers the equilibrium

to $P_{d^*} - \delta$, which justifies $\mu_i(P_{d^*} - \delta, t) = 1$ and eliminates pooling on any $P_{i0} > c_B$ and t_{i0} .

Pooling and semi-pooling on $P_{i0} = c_B$ and some t_{i0} is eliminated by the Intuitive Criterion as follows. For $\epsilon > 0$ small and some t_i , set $\mu_i(c_B - \epsilon, t_i) = 1$. Due to pooling, $\mu_i(c_B, t_{i0}) < 1$, so $D_i^1(c_B) > D_i^{\mu_i(c_B, t_{i0})}(c_B) = D_i(c_B, t_{i0})$. At $c_B - \epsilon$ and t_i , demand is $D_i^1(c_B - \epsilon) > D_i^{\mu_i(c_B, t_{i0})}(c_B - \epsilon) = D_i(c_B, t_{i0})$. Thus for ϵ small enough, G strictly prefers $c_B - \epsilon, t_i$ to c_B, t_{i0} . By $\pi_{iG}^* > 0$, demand is positive at c_B, t_{i0} , so B strictly prefers c_B, t_{i0} to $c_B - \epsilon, t_i$. These strict preferences justify $\mu_i(c_B - \epsilon, t_i) = 1$ and eliminate pooling on c_B, t_{i0} .

Pooling and semi-pooling on $P_{i0} < c_B$ and some t_{i0} cannot occur, because $\pi_{iG}^* > 0$ implies $D_i(P_{i0}, t_{i0}) > 0$, which would yield $\pi_{iB}^* < 0$. \square

Proof of Lemma 5 By Lemma 4, $\pi_{iG}^* > 0$ for both $i \in \{X, Y\}$, and type B strictly prefers its equilibrium price to any $P_i < c_B$ for any t_i . Thus if G strictly prefers P_i, t_i to its equilibrium action when $\mu_i(P_i, t_i) = 1$, then the equilibrium fails the Intuitive Criterion.

Denote by $\underline{P}_{i\theta} := \inf \{P : \sigma_i^{\theta*}(P, t) = 0 \forall t\}$ the lower bound of the prices that firm i 's type θ sets. Assume w.l.o.g. that $\underline{P}_{iG} \leq \underline{P}_{jG}$. If firm i raises price to $\underline{P}_{iG} + \epsilon$ for $\epsilon \in (0, \min\{s, c_B - \underline{P}_{iG}\})$ and belief is $\mu_i(\underline{P}_{iG} + \epsilon, t) = 1$ for some t , then consumers initially at firm i still choose $\sigma_1(v, \underline{P}_{iG} + \epsilon, t)(\ell) = 0$, because a price difference less than s does not justify the learning cost. The customers at j who chose ℓ anticipating σ_i^{G*} do not know about G 's deviation, so still choose ℓ . Upon learning $\underline{P}_{iG} + \epsilon, t$, a customer initially at j 's type B has a choice between B at $P_B \geq c_B$ and G at $\underline{P}_{iG} + \epsilon < c_B$, so still buys from i 's type G . If a customer initially at j 's type G learns both firms' prices and messages and believes $\mu_i(\underline{P}_{iG} + \epsilon, t) = 1$, then he still buys from i if $P_{jG} \geq \underline{P}_{iG} + \epsilon$. If $P_{jG} \leq \underline{P}_{iG} + \epsilon$, then no customer facing P_{jG} learns, because firm i has weakly lower quality and a price lower by at most $\epsilon < s$. At prices $P \leq c_B$, the profit of G is then given by (3). By Lemmas 8–9, G strictly prefers to increase price. \square

Proof of Theorem 6 The assumptions $\bar{v} > c_B$ and $f_v > 0$ ensure that there exists $\epsilon > 0$ s.t. total demand is positive at $P_i = c_B + \epsilon$ for any P_j, t_j, t_i . Suppose by way of contradiction that σ_j^{B*} puts positive probability on P_j, t_j at which $D_j(P_j, t_j) = 0$. Then the expected demand for firm i is positive at $P_i = c_B + \epsilon$, implying that both types of firm i make positive profit, thus $\underline{P}_{iB} > c_B$. By Lemma 4, the supports of σ_i^{B*} and σ_i^{G*} are disjoint, so $\mu_i(P, t) = 0$ for any (P, t) in the support of σ_i^{B*} . Any P_{jd}, t_{jd} with $P_{jd} \in (c_B, \underline{P}_{iB})$ then attracts positive demand in expectation, because $\mu_j(P_{jd}, t_{jd}) \geq \mu_i(P, t) = 0$. Firm j 's type B deviates from any zero-demand price to P_{jd}, t_{jd} and makes positive profit. This contradicts $D_j(P_j, t_j) = 0$ for any P_j, t_j in the support of σ_j^{B*} . Therefore, demand is positive in expectation for both types of both firms in any equilibrium satisfying the Intuitive Criterion.

By Lemma 3, positive demand implies $P_G \leq P_B \geq c_B$ for any P_θ in the support of $\sigma_i^{\theta*}$. As shown next, all consumers initially at type B of at least one firm learn before buying. Assume $\bar{P}_{iG} \geq \bar{P}_{jG}$ w.l.o.g. Then $\underline{P}_{iB} \geq \bar{P}_{jG}$, so a consumer with valuation v facing $P_{iB} \geq \underline{P}_{iB}$ and any t_i gets payoff $v - P_{iB}$ from buying immediately. On the other hand, learning yields consumer v a payoff of at least $h(v) - \bar{P}_{jG}$ with probability

μ_0 , and $v - P_{iB}$ with probability $1 - \mu_0$. If $\mu_0[h(v) - v] \geq s$, then consumer v prefers learning to buying. Consumers $v < P_{iB}$ do not buy at P_{iB} and $\mu_i(P_{iB}, t) = 0$, thus either leave the market or learn. For $v \geq P_{iB} \geq c_B$, the assumption $\mu_0[h(c_B) - c_B] \geq s$ implies learning instead of immediate buying.

Having $\bar{P}_{iB} > \bar{P}_{jB}$ when all consumers at i learn or leave contradicts positive demand for i . The previous paragraph proves that all customers at j learn or leave when $\bar{P}_{iB} \leq \bar{P}_{jB}$. Given that all consumers who end up buying from type B have learned both firms' prices and messages, the B types are in Bertrand competition. The following undercutting argument then shows that B prices at c_B with certainty. Having $\bar{P}_{iB} \neq \bar{P}_{jB}$ contradicts positive demand for one firm. If σ_j^{B*} has an atom at $\bar{P}_{jB} = \bar{P}_{iB} > c_B$, then for small enough $\epsilon > 0$, firm i 's type B strictly prefers $\bar{P}_{iB} - \epsilon$ to \bar{P}_{iB} . Supposing σ_j^{B*} has no atom at $\bar{P}_{jB} > c_B$ implies probability 1 of $\bar{P}_{iB} > P_{jB}$, which contradicts $D_i(\bar{P}_{iB}, t) > 0$.

Lemmas 5 and 3 with $\bar{P}_{iB} = c_B$ imply that both types price at c_B with certainty. Lemma 4 proves disjoint supports of σ_i^{B*} and σ_i^{G*} , so $t_{iB} \neq t_{iG}$ with certainty. \square

C Positively correlated cost and quality

Consumers are assumed homogeneous and cheap talk absent, both for simplicity and for better comparability to the literature. Adding cheap talk does not change the prices or consumers' strategies. The Online Appendix studies the heterogeneous consumer case. In this section, all consumers have valuation $v_B > c_B$ for type B and $v_G := h(v_B) > v_B$ for G . The marginal cost of G is $c_G > c_B$.

There are multiple separating equilibria with the same outcome: $P_{i\theta} = v_\theta$ for $i \in \{X, Y\}, \theta \in \{B, G\}$, no consumers learn, all buy at $P \leq v_B$, fraction $\frac{v_B - c_B}{v_G - c_B}$ buy at $P > v_B$. Beliefs that support these strategies are $\mu_i(P) = \frac{P - v_B}{v_G - v_B}$ for $P \in [v_B, v_G]$, and arbitrary beliefs for $P > v_G$ and $P < v_B$. Other equilibria with the same outcome have $\mu_i(P) \leq \frac{P - v_B}{v_G - v_B}$ for $P \in [v_B, v_G)$, $\mu_i(v_G) = 1$ and fraction less than $\frac{v_B - c_B}{P - c_B}$ of consumers buying at $P \in (v_B, v_G)$. The fraction is 0 if $\mu_i(P) < \frac{P - v_B}{v_G - v_B}$. The beliefs in all these separating equilibria pass the Intuitive Criterion, because if G wants to deviate to price $P_d \in (v_B, v_G)$ with belief $\mu_i(P_d) = 1$, then B strictly prefers P_d , which yields the same demand as $P_{iB} = v_B$, but strictly greater margin. The equilibrium outcome is the natural analogue of Diamond (1971). The uniqueness of this outcome is shown next.

An equilibrium with $P_{iB} < v_B$ and $P_{iB} \leq P_{jB}$ cannot exist, because if consumers who see P_{iB} do not learn, then firm i 's type B can raise its price by $\epsilon \in (0, s)$. For consumers who see P_{iB} to learn, they must expect $\mu_0(v_G - P_{jG}) + (1 - \mu_0)(v_B - P_{jB}) - s \geq v_B - P_{iB}$, i.e. $\mu_0(v_G - P_{jG} - v_B + P_{jB}) \geq P_{jB} - P_{iB} + s > 0$. However, if $v_G - P_{jG} > v_B - P_{jB}$, then demand is weakly greater at P_{jG} than at P_{jB} , thus type B of firm j strictly prefers to deviate to P_G .

An equilibrium where firm i does not pool and $P_{iG} < v_G$ cannot exist, because all consumers would buy at P_{iG} . In this case, demand is weakly greater at P_{iG} than at

P_{iB} , so type B of firm i strictly prefers to deviate to P_{iG} . Thus the only non-pooling equilibria feature $P_{i\theta} = v_\theta$.

Pooling on any $P_{i0} \leq \mu_0 v_G + (1 - \mu_0)v_B$ fails the Intuitive Criterion: at the deviation price $P_d = v_G$ and the most favourable belief $\mu_i(v_G) = 1$, a (mixed) best response of the consumers exists for which G prefers to deviate from P_{i0} and B prefers not to.

The unique separating outcome (no learning, $P_{i\theta} = v_\theta$, some consumers do not buy at $P_{iG} = v_G$) stands in contrast to Janssen and Roy (2015), regardless of which equilibrium characterisation in their Proof of Proposition 2 is used. The first paragraph of Janssen and Roy (2015) Proof of Proposition 2 describes the unique symmetric D1 equilibrium as follows.

- (a) If $\frac{v_B - c_B}{v_G - c_B} > \frac{1}{2}$, then $P_{iG} = v_G$. If both firms are type G , then some consumers do not buy, otherwise all buy.
- (b) If $\frac{v_B - c_B}{v_G - c_B} \leq \frac{1}{2}$, then $P_{iG} = \max\{c_G, c_B + 2(v_G - v_B)\}$ and all consumers buy.

From the second paragraph on, Janssen and Roy (2015) Proof of Proposition 2 claims:

- (a) If $\frac{v_B - c_B}{v_G - c_B} \geq \frac{1}{2}$, then $P_{iG} = c_B + 2(v_G - v_B)$, type B mixes over $P_{iB} \in [c_B + \mu_0(v_G - v_B), c_B + v_G - v_B]$, all consumers buy at the lowest price, breaking ties uniformly randomly.
- (b) If $\frac{v_B - c_B}{v_G - c_B} < \frac{1}{2}$, then $P_{iG} = v_G$, type B mixes over $P_{iB} \in [c_B + \mu_0(v_B - c_B), v_B]$. If both firms charge v_G , then a consumer buys from each with probability $\frac{v_B - c_B}{v_G - c_B}$ and leaves the market with probability $\frac{v_G - c_B - 2v_B + 2c_B}{v_G - c_B}$. If at least one firm charges $P \leq v_B$, then the consumer buys at the lowest price with certainty.

Unlike in the incomplete information Bertrand model of Janssen and Roy (2015), the equilibrium under costly learning in this section features $P_{iB} = v_B$ (instead of B mixing on lower prices), zero consumer surplus, consumers never switching (as opposed to always switching when the types of the firms differ), and not all consumers buying when the types of the firms differ. Depending on the parameters in Janssen and Roy (2015), the equilibria also differ in P_{iG} and the probability of consumers purchasing when both firms are type G .

Several of the differences are the expected ones between Bertrand and Diamond environments—no search, monopoly pricing and the corresponding surplus extraction. This paper's low price resembles the Bertrand competition with negative correlation in the appendix of Janssen and Roy (2015), but in their Bertrand environment, a competitive outcome is to be expected, unlike here.

With heterogeneous consumers and positively related cost and quality, the Online Appendix shows that the results are analogous to the current section and Diamond (1971), thus contrasting Sect. 3. In particular, type B still prices above its complete-information monopoly level, G prices above B by at least the quality difference between the types, and consumers do not learn at B . The heterogeneous consumer case differs from homogeneous in that G may set a price different from its complete-information monopoly level, some consumers learn at G and switch to B given the chance, and at both types of firms, low-valuation consumers leave the market.

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