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RESEARCH ARTICLE

Enlarging the collective model of household behavior: A revealed preference analysis

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Abstract We use a comprehensive model of strategic household behavior in which the spouses' expenditure on each public good is decomposed into autonomous spending and coordinated spending *à la* Lindahl. We obtain a continuum of semi-cooperative regimes parameterized by the relative weights put on autonomous spending, by each spouse and for each public good, nesting full cooperative and noncooperative regimes as limit cases. Testing is approached through revealed preference analysis, by looking for rationalizability of observed data sets, with the price of each public good lying between the maximum and the sum of the hypothesized marginal willingness to pay of the two spouses. Once rationalized, an observed data set always allows to identify the sharing rule, except when both spouses contribute in full autonomy to some public good (a situation of local income pooling).

Keywords Semi-cooperative household behavior · Revealed preference analysis · Rationalizability · Sharing rule identification

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1 Introduction

In a previous issue of this Journal[1], we have argued that the typical household spending behavior could not be reduced to the two extreme kinds that are assumed by all economic models of household behavior, both theoretical and empirical. This is either to assume fully efficient cooperation or a complete absence of collaboration.

Assuming full cooperation, the most popular model of household behavior, known as the 'collective model' and initiated by Chiappori (1988, 1992), has the merit to integrate the fact that within-household decision-making is generally a multi-person process. This model has generated testable restrictions in spite of being parsimonious in describing the decision process itself. An earlier approach to full cooperation was to assume Nash bargaining (with Nash cooperative solution) as applied to household behavior. In this line, some theoretical models[2] have formalized the possibility for a household agreement to be a noncooperative equilibrium in a game of voluntary contributions to public goods, but as the threat (or disagreement) point of a bargaining model.[3] Assuming full non-cooperation, recent work[4] has shown that generically only two types of noncooperative equilibrium can emerge in such a noncooperative game: "separate spheres" proper and "separate spheres up to one public good" (*i.e.,* spouses do not contribute jointly to more than one public good within the household). Income pooling holds[5] at this second type of equilibrium, meaning that only the household total income matters, not its distribution, to determine household demands. However most empirical studies reject the income pooling hypothesis[6] and the separate spheres equilibrium remains a very particular case (say, justified by a traditional gender partition).

What we have done in our previous paper is to introduce a comprehensive approach opening the possibility of testing semi-cooperative spending agreements. We have defined a concept of *household θ-equilibrium* in a strategic model of household consumption, where the given vector parameter θ determines the degree of autonomy of each spouse for each public good within the household, the two extreme regimes remaining full cooperation (θ identically nil) and full non-cooperation (θ identically one). Hence, by varying this vector parameter, we get a continuum of household consumption models between the fully cooperative and the fully noncooperative model. Our purpose in the present paper is to further explore methodologically the testability of this comprehensive equilibrium concept.

1 d'Aspremont and Dos Santos Ferreira (2014).

2 See Ulph (2006), Lundberg and Pollak (1993) and Chen and Woolley (2001). For a survey, see Donni (2007).

3 In earlier models the disagreement point was taken to be divorce (Manser and Brown 1980; McElroy and Horney 1981).

4 See Lechene and Preston (2005, 2011) and Browning et al. (2010).

5 Income pooling is already present in unitary models (Samuelson 1956, and Becker 1974) as well as in divorce threat bargaining models (Manser and Brown 1980; McElroy and Horney 1981)

6 For references, see Browning and Chiappori (1998) and Vermeulen (2002).

In testing behavioral models, two general approaches have been used in the literature. The first, the one used in our previous work, is to take a flexible parameterization of the demand system and to derive testable local properties, such as properties of the (pseudo-) Slutsky matrix. In the collective case, Browning and Chiappori (1998) show that the pseudo-Slutsky matrix can be written as the sum of a symmetric negative semidefinite matrix and a rank 1 'deviation' matrix, and can be used to discriminate the collective model from the (less general) unitary model. In the fully noncooperative case, the rank of the deviation matrix generally increases (Lechene and Preston 2011), and increases even more in the semi-cooperative case that we have introduced. Unfortunately, each kind of effects increases the requirements for empirical testing. A more general argument against the parametric tests approach is that the functional structure used is not verifiable *per se* and the tested model may be rejected due to misspecification. For that reason, Cherchye et al. (2007) have adopted a second approach, a non-parametric one, to test the collective model. No functional specification is assumed and observed quantity and price data are *rationalized* by means of revealed preference axioms. The same non-parametric approach is used to test the fully noncooperative model by Cherchye et al. (2011) and to test noncooperative models with caring[7] by Cherchye et al. (2015).

Here we shall follow the same alternative approach (using the same revealed preference axiom) for our comprehensive model and derive necessary and sufficient conditions for the θ-rationalizability of an observed data set. These conditions coincide with the necessary and sufficient conditions derived by Cherchye et al. (2011) for full cooperative rationalizability (when θ is identically nil) and they coincide with the ones they derive for full noncooperative rationalizability (when θ is identically one). As a practically useful corollary, we can show as well that, taking as given the degrees of autonomy θ, these conditions are verifiable by simple Mixed Integer Programming methods, combining linear constraints with binary integer variables. These methods are also useful to identify the prevailing degrees of autonomy.

An important property that we get is that each spouse income can be empirically identified on the basis of the observed data set if this set is θ-rationalized, except when $\theta_k^A = \theta_k^B = 1$ for some public good k to which both spouses contribute. In that case household demand depends locally only on aggregate household income. In other words, the *sharing rule* is identifiable except in cases of *local income pooling*.[8]

Another property is non-nestedness. Cherchye et al. (2011) construct two data sets to show that one type of rationalizability (cooperative or noncooperative) does not imply the other. We shall exhibit another data set which is θ-rationalizable, but not for

[7] Each individual is assumed to have a (personal) Bergson–Samuelson Social Welfare Function (SWF) through which she/he "cares" about the utility of each member in her/his household. This is in contrast with Samuelson (1956) consensus model where all members care and have the same SWF and with Becker (1974) where only one member (say the husband) cares. By varying the degree of intrahousehold caring for each individual member, Cherchye et al. (2015) also obtain a continuum of household consumption models between the fully cooperative model and the fully noncooperative model without caring.

[8] Cherchye et al. (2015) have already shown that the sharing rule is identifiable for fully cooperative-rationalized data sets and not for noncooperative-rationalized data sets. Sharing rule identifiability also fails in models with caring, except when full cooperation is rationalized (Cherchye et al. 2015).

these two limit cases. This does not exclude, as we will show, that the same data set can be θ-rationalized for a large set of θ's, including the extremes.

Our paper is organized as follows. In the next section, we start by recalling the model and the concept of household θ-equilibrium introduced in d'Aspremont and Dos Santos Ferreira (2014). Then, using the generalized axiom of revealed preference (GARP), we derive the necessary and sufficient conditions for θ-rationalizability of an observed data set and interpret these conditions in terms of marginal willingness to pay for the public goods within the household. The property of sharing rule identifiability is finally shown to hold for all θ's except when identically one for some public good to which both spouses contribute. Section 3 is devoted to empirical verifiability from a methodological point of view. First, we show that a simple mixed integer program can be used. Then, using this method, we construct examples to study non-nestedness. In Sect. 4 we conclude.

2 Semi-cooperative household behavior

In this section we define a comprehensive concept of household equilibrium which will encompass, as two limit cases, the concepts of cooperative and noncooperative equilibrium already well-studied in the literature. This is achieved by introducing cooperation via a decentralized mechanism *à la* Lindahl. Our concept allows to cover a continuum of intermediate cases of semi-cooperative equilibria.

2.1 The model

Consider a two-adult household and denote by A the wife and by B the husband. Household consumption consists in two kinds of goods, private or public within the household. We assume that this classification is given and may result from a contractual arrangement, supposed to be initially made by the spouses. Let $\left(q^A, q^B\right) \in \mathbb{R}_+^{2n}$ be the vector of consumption by the two spouses of the n private goods and $Q \in \mathbb{R}_+^m$ the consumption vector of the m public goods. The preferences of each spouse J ($J = A, B$) are represented by the utility function $U^J\left(q^J, Q\right)$, defined on $\mathbb{R}_+^{n+m}$, continuous, increasing and concave. The vector of private good prices $p \in \mathbb{R}_{++}^n$ and the vector of public good prices $P \in \mathbb{R}_{++}^m$ are given. The first private good, assumed to be always desired, is taken as numéraire ($p_1 = 1$). Although only the household income Y may be observable, each spouse J is supposed to know her/his income $Y^J \geq 0$, with $Y^A + Y^B = Y > 0$.

As in d'Aspremont and Dos Santos Ferreira (2014), we assume that the two spouses have preliminarily agreed on some mechanism to share the financing of public consumption, but behave strategically in their consumption decisions. The model is based on a "mechanism design" reformulation of the Lindahl equilibrium for Nash-implementation (see Hurwicz 1979, and Walker 1981), which we extend to semi-cooperation. A household arrangement is formalized as a mechanism defined by personalized pricing rules for the (within household) public goods. For each public good, the pricing rule applying to a spouse will only depend on the strategies of the other spouse. As these strategies are taken as given by the Nash assumption, such a pricing rule is unilaterally non-manipulable.

Formally, for each public good k, each spouse, say the wife, makes two announcements: her desired household consumption Q_k^A and her voluntary contribution g_k^A, taking as given (by the Nash assumption) the desired consumption Q_k^B and the voluntary contribution g_k^B of her husband. Feasibility requires $g_k^J \leq Q_k^J \leq g_k^J + g_k^{-J}$ $(J = A, B)$ implying $Q_k^J = g_k^J + g_k^{-J}$ at equilibrium (since U^J is increasing in Q_k^J). This means that she is ready to pay $P_k g_k^A$ for good k, for instance by buying it directly in the market place, leaving to her husband (according to the mechanism) a non-manipulable fraction $\left(Q_k^A - g_k^A\right)/Q_k^A$ of her desired household expenditure for this good. Symmetrically, the husband will be ready to pay $P_k g_k^B$ for good k, leaving to his wife a non-manipulable fraction $\left(Q_k^B - g_k^B\right)/Q_k^B$ of his desired household expenditure for the same good.

Two types of budgetary arrangement correspond to the two traditional approaches, the *noncooperative game with voluntary contributions to public goods* and the *(collectively efficient) Lindahl equilibrium mechanism*. The first budgetary arrangement consists in requiring both spouses to contribute fully autonomously and voluntarily to each public good k, by spending $P_k g_k^A$ and $P_k g_k^B$, respectively. If this stands for all public goods, we should end up with a noncooperative equilibrium in the game with voluntary contributions to public goods. The second budgetary arrangement (leading to collective efficiency) consists in letting each spouse, say the husband, decide on his contribution to a common fund allocated to public good k, facing a non-manipulable (Lindahl) price $\left(\left(Q_k^A - g_k^A\right)/Q_k^A\right) P_k$, so that he will have to pay a tax $\left(\left(Q_k^A - g_k^A\right)/Q_k^A\right) P_k Q_k^B$ to finance his desired household consumption Q_k^B of public good k. If this stands for all public goods, we should end up with a *Lindahl equilibrium* since, at equilibrium for any public good k, the sum of the two prices will be equal to P_k and g_k^A and g_k^B will be either both positive or both nil.[9]

Intermediate, semi-cooperative, schemes are however possible, some degree of autonomy being preserved for each spouse in purchasing each public good. For that, assume that the initial marriage agreement fixes, for each spouse J and each public good k, the proportions θ_k^J and $1 - \theta_k^J$ (with $\theta_k^J \in [0, 1]$) applying to each one of the two financing schemes, respectively.[10] Given these agreed proportions, the announcements of the other spouse and the market prices, each spouse J is confronted with the following budget constraint:

$$pq^J + \sum_{k=1}^{m} \left(\theta_k^J P_k g_k^J + \left(1 - \theta_k^J\right) \frac{Q_k^{-J} - g_k^{-J}}{Q_k^{-J}} P_k \left(g^J + g^{-J}\right) \right) \leq Y^J. \qquad (1)$$

[9] Suppose $g_k^B = 0$ and $g_k^A > 0$. Since $Q_k^A = g_k^A + g_k^B$, $\left(\left(Q_k^A - g_k^A\right)/Q_k^A\right) P_k = 0$. This cannot be optimal as $U^B\left(q^B, Q^B\right)$ would increase if g_k^B (and Q_k^B) is increased (for free).

[10] The initial marriage agreement could be viewed as the result of a preliminary stage where each spouse J chooses strategically the degrees of autonomy θ_k^J for all k. This creates a moral hazard problem for the Lindahl mechanism since θ identically zero may not be the equilibrium strategy of the first-stage game, although some other arrangement, with positive θ, may well be implementable. If there is incomplete information, efficiency may be even more difficult to reach (see Gori and Villanacci 2011).

On the left-hand side of this inequality, the first term is the spouse's expenditure on private goods, and the second term is the sum of the spouse's expenditure on each public good k, decomposed into an autonomous spending $\theta_k^J P_k g_k^J$ and a coordinated spending *à la* Lindahl $\left(1-\theta_k^J\right)\left[\left(Q_k^{-J}-g_k^{-J}\right)/Q_k^{-J}\right]P_k\left(g^J+g^{-J}\right)$. This is the budget constraint that each spouse faces while maximizing her/his utility. This type of mixed budgetary arrangement might be organized, for instance, by having several bank accounts. For example, the housing rent may be paid via a joint account, but some charges paid on one spouse's account. The family car may be bought using the joint account and maintenance costs paid on an individual account.

2.2 The household θ-equilibrium

A game is thus defined where the payoffs are the spouses' utility functions. The strategies of each spouse J are the quantities $\left(q^J, g^J, Q^J\right) \in \mathbb{R}_+^{n+2m}$, denoting respectively the quantities of private goods, the voluntary contributions and the desired household consumptions for the various public goods. For each spouse J, these strategies have to satisfy the budget constraint plus feasibility constraints, whereby the desired quantities Q^J should be less than or equal to the aggregate voluntary contributions $g^J + g^{-J}$. This leads to the following definition.

Definition 1 A vector $\left(q^A, g^A, Q^A, q^B, g^B, Q^B\right) \in \mathbb{R}_+^{2(n+2m)}$ is a *household θ-equilibrium* with degrees of autonomy $\left(\theta^A, \theta^B\right) \in [0,1]^{2m}$ if, for $J = A, B$, the strategy $\left(q^J, g^J, Q^J\right)$ solves the program:

$$\begin{aligned} &\max_{\left(\underline{q}^J, \underline{g}^J, \underline{Q}^J\right)\in\mathbb{R}_+^{n+2m}} U^J\left(\underline{q}^J, \underline{Q}^J\right) \\ &s.t.\quad p\underline{q}^J + \sum_{k=1}^{m}\left(\theta_k^J P_k \underline{g}_k^J + \left(1-\theta_k^J\right)\frac{Q_k^{-J}-g_k^{-J}}{Q_k^{-J}}P_k\left(\underline{g}_k^J + g_k^{-J}\right)\right) \le Y^J, \\ &and\quad \underline{g}^J \le \underline{Q}^J \le \underline{g}^J + g^{-J}. \end{aligned} \tag{2}$$

Existence of a household θ-equilibrium for every $\left(\theta^A, \theta^B\right) \in [0,1]^{2m}$ has been proved in d'Aspremont and Dos Santos Ferreira (2014, Proposition 1), when the utility functions of the two spouses are strongly quasi-concave. The corresponding equilibrium outcome coincides, if $\left(\theta^A, \theta^B\right) \equiv (0,0)$, with the Lindahl equilibrium outcome and, if $\left(\theta^A, \theta^B\right) \equiv (1,1)$, with the outcome of a noncooperative equilibrium of the game with voluntary contributions to public goods.[11]

[11] As stated in d'Aspremont and Dos Santos Ferreira (2014, Proposition 2), the outcome of any θ-equilibrium with *separate spheres* (*i.e.,* with $g_k^A g_k^B = 0$ for any public good k) coincides, even for $\left(\theta^A, \theta^B\right) \neq (1,1)$, with the outcome of an equilibrium of the game with voluntary contributions to public goods.

3 Revealed preference analysis

Two approaches have been used to test for household behavior. One is to assume sufficient differentiability of the demand system (a parameterized system for empirical applications) and to derive testable local properties, such as properties of the Slutsky matrix. This is the approach of Browning and Chiappori (1998) to discriminate the collective model from the (less general) unitary model, an approach extended by d'Aspremont and Dos Santos Ferreira (2014) for the present comprehensive model. Another approach, that we shall here adopt, is the revealed preference approach which consists in rationalizing given data sets in terms of a particular model. Such rationalization is based on global conditions and is non-parametric (see Cherchye et al. 2007, for the collective model, and Cherchye et al. 2011, for the noncooperative model).

3.1 θ-rationalizability: a necessary and sufficient condition

For the present model, we use the following general definition of rationalizability. This is a generalization of both cooperative rationalizability (θ identically 0) and of noncooperative rationalizability (θ identically 1) as defined in Cherchye et al. (2011), and, more fundamentally, of the basic definition of rationalizability for the individual consumer model.

Definition 2 An observed data set $(p_t, P_t, q_t, Q_t)_{t\in T}$ is θ-*rationalizable* (for given degrees of autonomy $\left(\theta^A, \theta^B\right) \in [0, 1]^{2m}$) if there exist pairs of continuous, increasing and concave utility functions $\left(U^A, U^B\right)$ defined on $\mathbb{R}_+^{n+m}$, of individual incomes $\left(Y_t^A, Y_t^B\right)_{t\in T} \in \mathbb{R}_+^{2|T|}$, of individual private consumptions $\left(q_t^A, q_t^B\right)_{t\in T} \in \mathbb{R}_+^{2n|T|}$ and of voluntary contributions to public goods $\left(g_t^A, g_t^B\right)_{t\in T} \in \mathbb{R}_+^{2m|T|}$, such that, for any $t \in T$,

$$Y_t^A + Y_t^B = p_t q_t + P_t Q_t,\ q_t^A + q_t^B = q_t,\ g_t^A + g_t^B = Q_t$$

and such that $\left(q_t^A, g_t^A, Q_t, q_t^B, g_t^B, Q_t\right)$ is a household θ-equilibrium.

It should be noticed that this approach, as applied to the household, involves an identification problem as long as the observed data set contains only aggregate information on the household. In addition, any rationalization is determined only up to the permutation of the spouses' decisions $\left(q_t^A, g_t^A\right)$ and $\left(q_t^B, g_t^B\right)$.[12]

Varian (1982) has established the connection between rationalizability for the individual consumer model and a property called the generalized axiom of revealed preference (GARP). In the present model this can be defined for a data set where the vector of public good prices may be the observed prices themselves or prices derived from the observed prices. It is the same definition as the one introduced in Cherchye et al. (2011).

[12] One possible way of tackling this problem would be to use the exclusivity assumption suggested by Chiappori and Ekeland (2009), "whereby each member is the exclusive consumer of at least one good."

Definition 3 A data set $(p_t, \tau_t, q_t, Q_t)_{t \in T}$ satisfies the *generalized axiom of revealed preferences (GARP)* if there exists a *transitive* binary relation $\mathcal{R}$ such that, for any $s, t \in T$, $p_t q_t + \tau_t Q_t \geq p_t q_s + \tau_t Q_s$ implies $(q_t, Q_t) \mathcal{R} (q_s, Q_s)$, and $(q_t, Q_t) \mathcal{R} (q_s, Q_s)$ implies $p_s q_s + \tau_s Q_s \leq p_s q_t + \tau_s Q_t$.

We can now establish a necessary and sufficient condition for an observed data set to be θ-rationalizable.

Theorem 1 *Consider the observed data set* $\mathcal{D} = (p_t, P_t, q_t, Q_t)_{t \in T} \in \mathbb{R}_{++}^{2(n+m)|T|}$ *and take as given the vector pair* $\left(\theta_t^A, \theta_t^B\right) \in [0,1]^{2m}$ *of degrees of autonomy. The following conditions are equivalent:*

(i) The data set $\mathcal{D}$ *is* θ*-rationalizable.*
(ii) For any $t \in T$*, there exist vector pairs* $\left(q_t^A, q_t^B\right) \in \mathbb{R}_+^{2n}$*,* $\left(g_t^A, g_t^B\right) \in \mathbb{R}_+^{2m}$ *and* $\left(\tau_t^A, \tau_t^B\right) \in \mathbb{R}_+^{2m}$ *such that:*

$$q_t^A + q_t^B = q_t,\ g_t^A + g_t^B = Q_t; \tag{T1.1}$$

for $J = A, B$ *and* $k = 1, \ldots, m$*,*

$$\tau_{tk}^J \leq \theta_k^J P_{tk} + \left(1 - \theta_k^J\right)\left(g_{tk}^J / Q_{tk}\right) P_{tk},\ \text{with equality if } g_{tk}^J > 0; \tag{T1.2}$$

and, for $J = A, B$*,*

$$\text{the hypothesized data set } \mathcal{D}^J = \left(p_t, \tau_t^J, q_t^J, Q_t\right)_{t \in T} \text{ satisfies GARP.} \tag{T1.3}$$

Proof Sufficiency ((ii)⟹(i)). Take any $J \in \{A, B\}$. By Afriat's theorem (see Varian 1982, p. 969), $\mathcal{D}^J$ satisfies GARP (T1.3) if and only if there exist numbers $\left(U_t^J, \lambda_t^J\right)_{t \in T} \in (\mathbb{R} \times \mathbb{R}_{++})^{|T|}$, such that

$$U_s^J \leq U_t^J + \lambda_t^J \left[\left(p_t, \tau_t^J\right)\left(q_s^J - q_t^J, Q_s - Q_t\right)\right] \tag{3}$$

for any $s, t \in T$ (*i.e.,* Afriat's inequalities hold). Define J's utility function as the piecewise linear function

$$U^J\left(q^J, Q\right) \equiv \min_{t \in T}\left\{U_t^J + \lambda_t^J \left[\left(p_t, \tau_t^J\right)\left(q^J - q_t^J, Q - Q_t\right)\right]\right\}. \tag{4}$$

so that U^J is continuous, increasing, concave, and such that for any $t \in T$, $U_t^J = U^J\left(q_t^J, Q_t\right)$. We prove that, for any t, $\left(q_t^J, g_t^J\right)$ maximizes $U^J\left(q_t^J, g_t^J + g_t^{-J}\right)$ under the budget constraint (1) with income $Y_t^J \equiv p_t q_t^J + \sum_{k=1}^m P_{tk} g_{tk}^J$. More precisely, $U^J\left(q_t^J, g_t^J + g_t^{-J}\right) = U_t^J \geq U^J\left(q^J, g^J + g_t^{-J}\right)$ for any $\left(q^J, g^J\right)$ satisfying:

$$p_t q^J + \sum_{k=1}^m \left(\theta_k^J P_{tk} g_k^J + \left(1 - \theta_k^J\right) \frac{Q_{tk} - g_{tk}^{-J}}{Q_{tk}} P_{tk}\left(g_k^J + g_{tk}^{-J}\right)\right) \leq Y_t^J. \tag{5}$$

Adding $\sum_{\{k:\, g_{tk}^J>0\}} \theta_k^J P_{tk} g_{tk}^{-J}$ to both sides of this inequality, and using (T1.1) and (T1.2), we obtain:

$$\begin{aligned}
& p_t q^J + \sum_{\{k:\, g_{tk}^J>0\}} \left(\underbrace{\theta_k^J P_{tk} \overbrace{\left(g_k^J + g_{tk}^{-J}\right)}^{Q_k^J} + \left(1-\theta_k^J\right) \frac{Q_{tk} - g_{tk}^{-J}}{Q_{tk}} P_{tk} \overbrace{\left(g_k^J + g_{tk}^{-J}\right)}^{Q_k^J}}_{\tau_{tk}^J Q_k^J} \right) \\
& \quad + \sum_{\{k:\, g_{tk}^J=0\}} \underbrace{\theta_k^J P_{tk} g_k^J}_{\tau_{tk}^J g_k^J} \\
& \quad \le p_t q_t^J + \sum_{\{k:\, g_{tk}^J>0\}} \left(\underbrace{\theta_k^J P_{tk} g_{tk}^{-J} + \overbrace{\theta_k^J P_{tk} g_{tk}^J + \left(1-\theta_k^J\right) P_{tk} \frac{g_{tk}^J}{Q_{tk}} Q_{tk}}^{P_{tk} g_{tk}^J}}_{\tau_{tk}^J Q_{tk}} \right). \qquad (6)
\end{aligned}$$

By adding $\sum_{\{k:\, g_{tk}^J=0\}} \tau_{tk}^J g_{tk}^{-J}$ to both sides of this inequality and using (T1.1), we see that it implies:

$$\begin{aligned}
& p_t q^J + \sum_{\{k:\, g_{tk}^J>0\}} \tau_{tk}^J Q_k^J + \sum_{\{k:\, g_{tk}^J=0\}} \tau_{tk}^J \overbrace{\left(g_k^J + g_{tk}^{-J}\right)}^{Q_k^J} \\
& \quad \le \; p_t q_t^J + \sum_{\{k:\, g_{tk}^J>0\}} \tau_{tk}^J Q_{tk} + \sum_{\{k:\, g_{tk}^J=0\}} \tau_{tk}^J \overbrace{g_{tk}^{-J}}^{Q_{tk}}, \qquad (7)
\end{aligned}$$

an inequality which can be written as $\left(p_t, \tau_t^J\right)\left(q^J - q_t^J, Q^J - Q_t\right) \le 0$, so that

$$\begin{aligned}
U^J\left(q^J, Q^J\right) &\equiv \min_{s\in T} \left\{ U_s^J + \lambda_s^J \left[\left(p_s, \tau_s^J\right)\left(q^J - q_s^J, Q^J - Q_s\right) \right] \right\} \\
&\le U_t^J + \lambda_t^J \left[\left(p_t, \tau_t^J\right)\left(q^J - q_t^J, Q^J - Q_t\right) \right] \le U_t^J.
\end{aligned}$$

Deviating from $\left(q_t^J, g_t^J\right)$ can only decrease $U^J\left(q^J, Q^J\right)$. As this is true for $J = A, B$, we may conclude that $\left(q_t^A, g_t^A, Q_t, q_t^B, g_t^B, Q_t\right)$ is a household θ-equilibrium, and hence that the data set $\mathcal{D}$ is θ -rationalizable.

Necessity ((i)⟹(ii)). Suppose $\mathcal{D}$ is θ-rationalizable. Clearly, condition (T1.1) is then fulfilled for the corresponding quantities q_t^J and g_t^J, $J \in \{A, B\}$ and $t \in T$. Take any $J \in \{A, B\}$. By the FOC of spouse J's program at a household equilibrium (Lemma 1 of d'Aspremont and Dos Santos Ferreira 2014, extended to the case of non-differentiability), we have

$$\text{for } h = 1, \ldots, n, \; U^J_{q^J_{th}}\left(q^J_t, Q_t\right) \leq \lambda^J_t p_{th}, \text{ with equality if } q^J_{th} > 0,$$

$$\text{for } k = 1, \ldots, m, \; U^J_{Q_{tk}} \leq \lambda^J_t \left(\theta^J_k P_{tk} + \left(1 - \theta^J_k\right) \frac{g^J_{tk}}{Q_{tk}} P_{tk}\right), \text{ with equality if } g^J_{tk} > 0,$$

for some subgradient $\left(U^J_{q^J_t}, U^J_{Q_t}\right) \in \partial U^J\left(q^J_t, Q_t\right)$ and some positive Lagrange multiplier λ^J_t. Take $\tau^J_{tk} \equiv U^J_{Q_{tk}}/\lambda^J_t \leq \theta^J_k P_{tk} + \left(1 - \theta^J_k\right)\left(g^J_{tk}/Q_{tk}\right) P_{tk}$, with equality if $g^J_{tk} > 0$, so that condition (T1.2) is satisfied. By concavity of the utility function U^J, for any $s, t \in T$,

$$U^J\left(q^J_s, Q_s\right) - U^J\left(q^J_t, Q_t\right) \leq \widetilde{U}^J_{q^J_t} \cdot \left(q^J_s - q^J_t\right) + \widetilde{U}^J_{Q_t} \cdot \left(Q_s - Q_t\right),$$

where $\left(\widetilde{U}^J_{q^J_t}, \widetilde{U}^J_{Q_t}\right) \in \partial U^J\left(q^J_t, Q_t\right)$ is *any* subgradient of U^J at $\left(q^J_t, Q_t\right)$. We thus obtain

$$\underbrace{U^J\left(q^J_s, Q_s\right)}_{U_s} \leq \underbrace{U^J\left(q^J_t, Q_t\right)}_{U_t} + \lambda^J_t\left[\left(p_t, \tau^J_t\right)\left(q^J_s - q^J_t, Q_s - Q_t\right)\right],$$

for any $s, t \in T$. We get Afriat's inequalities. By Afriat's theorem, we conclude that GARP applies to the hypothesized data set $\mathcal{D}^J = \left(p_t, \tau^J_t, q^J_t, Q_t\right)_{t \in T}$, so that condition (T1.3) is also satisfied, completing the proof. □

3.2 Marginal willingness to pay for public goods

In the case of differentiability of the utility functions, the first-order conditions for utility maximization at a household θ-equilibrium for each spouse J and any public good k is given by:

$$\frac{\partial U^J\left(q^J, Q\right)/\partial Q_k}{\partial U^J\left(q^J, Q\right)/\partial q_1} \leq \theta^J_k P_k + \left(1 - \theta^J_k\right) \frac{g^J_k}{Q_k} P_k, \tag{8}$$

with equality if $g^J_k > 0$. Thus, the numbers τ^J_{tk} in condition (T1.2) of the theorem can be interpreted as instances of J's marginal willingness to pay (MWTP) for public good k. By adding the MWTPs for the two spouses, we obtain:

$$\tau^A_{tk} + \tau^B_{tk} = \left(1 + \theta^A_k \frac{g^B_{tk}}{Q_{tk}} + \theta^B_k \frac{g^A_{tk}}{Q_{tk}}\right) P_{tk} \geq P_{tk}, \tag{9}$$

with equality if and only if $\theta^A_k g^B_{tk} = \theta^B_k g^A_{tk} = 0$. In particular, the equality is always satisfied in the cooperative case, where $\theta^A_k = \theta^B_k = 0$ for every k . The equality

$\tau_{tk}^A + \tau_{tk}^B = P_{tk}$ is then an expression of the Bowen–Lindahl–Samuelson condition. Hence, θ-rationalizability coincides in this case with cooperative rationalizability, as defined by Cherchye et al. (2011), our Theorem implying their Theorem 2.

If we now consider the noncooperative case, where $\theta_k^A = \theta_k^B = 1$ for every k, we see that

$$\max\left\{\tau_{tk}^A, \tau_{tk}^B\right\} \le P_{tk}, \tag{10}$$

the equality $\tau_{tk}^J = P_{tk}$ being satisfied for any k such that $g_{tk}^J > 0$. Thus, θ-rationalizability coincides in this case with noncooperative rationalizability, as defined by Cherchye et al. (2011), our Theorem implying their Theorem 3.

Putting together inequalities (9) and (10), we obtain, for any public good k:

$$\max\left\{\tau_{tk}^A, \tau_{tk}^B\right\} \le P_{tk} \le \tau_{tk}^A + \tau_{tk}^B, \tag{11}$$

a generalization of conditions (C.2) and (NC.2), for the cooperative and noncooperative cases, respectively, in Cherchye et al. (2011).

3.3 Sharing rule identifiability

An important property, verified by the collective model (see Cherchye *et al.,* 2007), is that the *sharing rule* be identifiable, *i.e.,* that each spouse income, Y_t^A and Y_t^B, be empirically identified on the basis of the observed data set, once rationalized (hence on the basis of the hypothesized data sets $\mathcal{D}^A$ and $\mathcal{D}^B$). This rationalization is of course conditional on the supposed degrees of autonomy $\left(\theta^A, \theta^B\right)$, as it is usual when assuming full cooperation ($\theta^A = \theta^B \equiv 0$) or full non-cooperation ($\theta^A = \theta^B \equiv 1$).

In our comprehensive model, by the theorem, if the observed data set is θ-rationalizable, then there exist vector pairs $\left(q_t^A, q_t^B\right) \in \mathbb{R}_+^{2n}$, $\left(g_t^A, g_t^B\right) \in \mathbb{R}_+^{2m}$ and $\left(\tau_t^A, \tau_t^B\right) \in \mathbb{R}_+^{2m}$ that satisfy the set of conditions (ii) of the theorem and that, as we will see below, are computable. This implies that sharing rule identifiability extends to our comprehensive model except when $\theta_k^A = \theta_k^B = 1$ for some public good k to which both spouses contribute ($g_{tk}^A > 0$, $g_{tk}^B > 0$ and $\tau_{tk}^A = \tau_{tk}^B = P_{tk}$). Indeed, in that case, any positive pair $\left(g_{tk}^A, g_{tk}^B\right)$ such that $g_{tk}^A + g_{tk}^B = Q_{tk}$ trivially satisfies, through (T1.1), the whole set of conditions (ii), so that the voluntary contributions cannot be identified. Since spouse J's income ($J = A, B$) is simply $Y_t^J = p_t q_t^J + \sum_k P_{tk} g_{tk}^J$, the sharing rule cannot be identified either. We thus observe a situation of *local income pooling* (household demands for all goods are locally independent of individual incomes and only depend on aggregate household resources).

This is the only bad case in our model. If $\theta_k^A = \theta_k^B = 1$ for some public good k, but $g_{tk}^J = 0$ and $g_{tk}^{-J} = Q_{tk}$ (and $\tau_{tk}^A \le \tau_{tk}^B = P_{tk}$) for some J, then the voluntary contributions to public good k are clearly identified. Also if, for some public good k, $\theta_k^J < 1$ for some J, then by condition (T1.2) either $\tau_{tk}^J = \theta_k^J P_{tk} + \left(1 - \theta_k^J\right)\left(g_{tk}^J / Q_{tk}\right) P_{tk}$, so that

$$g_{tk}^J = \frac{\tau_{tk}^J / P_{tk} - \theta_k^J}{1 - \theta_k^J} Q_{tk}, \tag{12}$$

or $g_{tk}^J = 0$ (and $\tau_{tk}^J \leq \theta_k^J P_{tk}$). Again this allows to identify the voluntary contribution g_{tk}^J by spouse J to public good k (and $g_{tk}^{-J} = Q_{tk} - g_{tk}^J$ for the other spouse), without excluding the case of joint contribution.

In conclusion, in our approach, the failure of identifiability of the sharing rule is confined to the case where both spouses contribute *fully autonomously* to some public good. Such failure of the sharing rule identifiability, outside the case of separate spheres (when $g_{tk}^J = 0$ and $g_{tk}^{-J} = Q_{tk}$ for any k), was already emphasized by Cherchye et al. (2011) in the noncooperative case and still appears in noncooperative regimes with caring, as introduced by Cherchye et al. (2015). In these models, sharing rule identifiability only holds in the fully cooperative case.

4 Empirical verifiability

We shall not analyze here a particular sample of two-person households as done, for instance, in Cherchye et al. (2011) for the noncooperative regime and in Cherchye et al. (2015) for the noncooperative model with caring, both using the Russian Longitudinal Monitoring Survey (RLMS). Still, from a methodological point of view, it is important to investigate the empirical applicability of the theorem. This we will do first in a general perspective, second by constructing illustrative examples.

4.1 A mixed integer programming formulation

Two important problems have to be solved. The first is the computational verifiability of the conditions of the theorem, taking as given the degrees of autonomy θ. The second is the identification of these degrees of autonomy. To solve the first problem, we can refer to Cherchye et al. (2011), where the noncooperative rationalizability of an observed data set can be verified by solving a Mixed Integer Programming (MIP) problem. Following this approach, we can handle the GARP conditions by defining binary variables $x_{ts}^J \in \{0, 1\}$ for any pair $(t, s) \in T^2$, where $x_{ts}^J = 1$ means that $\left(q_t^J, Q_t\right) \mathcal{R}^J \left(q_s^J, Q_s\right)$ for the hypothesized set $\mathcal{D}^J = \left(p_t, \tau_t^J, q_t^J, Q_t\right)_{t \in T}$ and the revealed preference relation $\mathcal{R}^J$, for $J = A, B$. By applying Definition 3, we have:

$$p_t q_t^J + \tau_t^J Q_t - p_t q_s^J - \tau_t^J Q_s \leq C x_{ts}^J - \varepsilon \tag{P1}$$

(where C and ε are positive constants, arbitrarily large and small, respectively), ensuring that $x_{ts}^J = 1$ whenever the LHS is nonnegative. Also,

$$p_s q_s^J + \tau_s^J Q_s - p_s q_t^J - \tau_s^J Q_t \leq C \left(1 - x_{ts}^J\right), \tag{P2}$$

ensuring that the LHS is non-positive whenever $x_{ts}^J = 1$. Thirdly, in order to ensure transitivity of $\mathcal{R}^J$, we have in addition:

$$x_{tv}^J + x_{vs}^J \leq 1 + x_{ts}^J. \tag{P3}$$

It should be noticed that constraints (P1)-(P3) are equivalent to condition (T1.3) of the theorem, covering all possible regimes.

To follow, we have to introduce as variables not only the nonnegative quantities of private goods q_t^A and q_t^B (as in Cherchye et al. 2011), but also the spouses' nonnegative contributions to public goods g_t^A and g_t^B, both constrained by the equalities (T1.1). As to the variables τ_{tk}^J representing the marginal willingnesses to pay for the public goods, each one is constrained as follows:

$$0 \leq \tau_{tk}^J \leq P_{tk}. \tag{P4}$$

These variables are in addition linked by condition (T1.2) to the contributions g_{tk}^J. In order to integrate this condition in the MIP problem, we define binary variables $z_{tk}^J \in \{0, 1\}$, such that $z_{tk}^J = 1$ if and only if $g_{tk}^J = 0$, and write:

$$- z_{tk}^J + \varepsilon \leq g_{tk}^J \leq C\left(1 - z_{tk}^J\right) \tag{P5a}$$

$$- C z_{tk}^J \leq \tau_{tk}^J - \left(\theta_k^J + \left(1 - \theta_k^J\right)\frac{g_{tk}^J}{Q_{tk}}\right) P_{tk} \leq 0. \tag{P5b}$$

We recall that C and ε are positive constants, arbitrarily large and small, respectively.

To summarize,

Proposition 2 *For given degrees of autonomy $\left(\theta_t^A, \theta_t^B\right) \in [0, 1]^{2m}$, the observed data set $\mathcal{D} = (p_t, P_t, q_t, Q_t)_{t \in T} \in \mathbb{R}_{++}^{2(n+m)|T|}$ is θ-rationalizable if and only if the MIP problem with unknowns q_t^J, g_t^J and τ_t^J, binary variables x_{ts}^J and z_t^J, inequalities (P1) to (P5), equalities (T1.1), and the non-negativity constraints on the quantity variables, is feasible.*

The second problem concerning the empirical application of the theorem is the identification of the degrees of autonomy θ. The standard approach to this problem considers only the two extreme cases where the θ_k^J's are either all equal to 0 (the fully cooperative regime) or all equal to 1 (the noncooperative regime), and then checks if rationalizability is ensured or not using the MIP problem. Presently, the most convenient way to take other values of the degrees of autonomy into account is probably to follow the suggestion of Cherchye et al. (2015), namely "to conduct a grid search that checks the above problem (through MIP methods) for a whole range of possible values" (p. 21).

5 Examples

Cherchye et al. (2011) emphasized the importance of the non-nestedness property exhibited by the cooperative and noncooperative regimes in the revealed preference approach in contrast with the parametric approach: "a data set that satisfies the cooperative condition does not necessarily satisfy the noncooperative condition, and *vice versa*" (p. 1084). The non-nestedness property "is not a theoretical curiosity but also has empirical relevance" (*ibid.*). Indeed, given a set of observations to be rationalized, we may then hope to be able to falsify the rationalizability in terms of all regimes outside the relevant one (or, in our context, outside some neighborhood of the relevant one).

The non-nestedness property is shown in Cherchye et al. (2011) by using two examples, the first exhibiting a data set that is cooperatively but not noncooperatively rationalizable, the second the other way round. By combining these two examples, we can easily construct (Example 1 below) a data set which is semi-cooperatively rationalizable, but neither cooperatively nor noncooperatively rationalizable, thus extending the non-nestedness property to semi-cooperation.[13]

A general problem with the empirical application of the revealed preference approach to household behavior remains however. This is the multiplicity of potential ways of rationalizing the same set of observed data. As already mentioned, since the data are restrained to household aggregates, this multiplicity results from the difficulty in identifying who is who in the couple, also from the occurrence of income pooling and more generally from other sources of non-uniqueness for given degrees of autonomy. In addition, varying the degrees of autonomy can only increase this indeterminacy. However, referring to the falsifying method which results from the proposition above, we can at least establish bounds on the values of the θ_k^J's (or on the values of other parameters such as the income shares Y_t^J/Y_t) that are incompatible with rationalization of the observed data set.[14]

The objective of our second example is to illustrate this indeterminacy. Some observed data set can be θ-rationalized for a large set of degrees of autonomy, that may even include the two limit cases.

Example 1 We assume 7 observations, 7 public goods and no private goods. Observed prices and quantities are as follows (different lines corresponding to different observations, and different columns to different goods):

[13] Of course, rationalizability of any kind may also fail, as it can be easily shown using Example 1 in Cherchye et al. (2007).

[14] For an application to the sharing rule recovery in the collective model along these lines, see Cherchye et al. (2011).

$$P = \begin{bmatrix} 7 & 4 & 4 & 1 & 1 & 1 & 1 \\ 4 & 7 & 4 & 1 & 1 & 1 & 1 \\ 4 & 4 & 7 & 1 & 1 & 1 & 1 \\ 7 & 7 & 7 & 1 & \varepsilon & \varepsilon & \varepsilon \\ 7 & 7 & 7 & \varepsilon & \varepsilon & \varepsilon & 1 \\ 7 & 7 & 7 & \varepsilon & 1 & 1 & \varepsilon \\ 7 & 7 & 7 & 1 & \varepsilon & 1 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & 100 & 11 & \varepsilon & 20 \\ \varepsilon & \varepsilon & \varepsilon & 20 & \varepsilon & 11 & 100 \\ \varepsilon & \varepsilon & \varepsilon & 5 & 10 & 10 & 5 \\ \varepsilon & \varepsilon & \varepsilon & 10 & 4 & 4 & 10 \end{bmatrix},$$

with ε positive but arbitrarily small. In each one of these two matrices the north-west 3×3 block corresponds to Example 1 and the south-east 4×4 block to Example 2 in Cherchye et al. (2011). In the complementary blocks, we have introduced the maximal prices and the minimal quantities in the given data of those two examples. The three first observations suffice to recover the authors' conclusion (concerning example 1) that noncooperative rationalization is impossible. Similarly, the last four observations suffice to recover their conclusion (concerning example 2) that cooperative rationalization is impossible. In this example, it seems natural to conjecture that the present data set is θ-rationalizable for the degrees of autonomy

$$\theta^A = \theta^B = \begin{bmatrix} 0\,0\,0\,1\,1\,1\,1 \end{bmatrix},$$

that is for the two spouses behaving cooperatively with respect to the first three public goods, and noncooperatively with respect to the last four. Using a MIP algorithm,[15] it is easy to verify this conjecture.

Example 2 Take the case of 3 observations, 3 public goods and no private goods. We know that, for any observation $t \in \{s, v, w\}$ and for any public good $k \in \{1, 2, 3\}$, $\tau_{tk}^J Q_{tk} = \theta_k^J P_{tk} Q_{tk} + \left(1 - \theta_k^J\right) P_{tk} g_{tk}^J$. For simplicity, we assume that $\theta_k^J = \theta_k^{-J} = \theta_k$, for all k. Then, since $g_{tk}^J + g_{tk}^{-J} = Q_{tk}$, we get: $\tau_{tk}^J Q_{tk} + \tau_{tk}^{-J} Q_{tk} = (1 + \theta_k)\, P_{tk} Q_{tk}$, hence

$$\tau_{tk}^A + \tau_{tk}^B = (1 + \theta_k)\, P_{tk} \geq P_{tk}.$$

Let $\tau_{tk}^A = \gamma_{tk} P_{tk}$ and $\tau_{tk}^B = (1 + \theta_k - \gamma_{tk})\, P_{tk}$, with $\theta_k \leq \gamma_{tk} \leq 1$. As $\gamma_{tk} \leq 1$ and $\theta_k \leq 1 + \theta_k - \gamma_{tk} \leq 1$, we have

$$\tau_{tk}^A = \gamma_{tk} P_{tk} \leq P_{tk} \text{ and } \tau_{tk}^B = (1 + \theta_k - \gamma_{tk})\, P_{tk} \leq P_{tk}.$$

Thus, we satisfy the condition $\max\left\{\tau_{tk}^A, \tau_{tk}^B\right\} \leq P_{tk} \leq \tau_{tk}^A + \tau_{tk}^B$.

Also, using (12), we get:

$$g_{tk}^A = \frac{\gamma_{tk} - \theta_k}{1 - \theta_k} Q_{tk} \text{ and } g_{tk}^B = \frac{1 - \gamma_{tk}}{1 - \theta_k} Q_{tk}.$$

15 We have used Gusek software, which combines the SCIntilla based Text Editor (SciTE) plus the linear/integer programming solver GNU Linear Programming Kit (GLPK).

Now, assume an observed data set with the same cyclical structure as in Cherchye *et al.* (2011, Example 1):

$$P_s = \begin{bmatrix} \alpha \ \eta \ \eta \end{bmatrix}, P_v = \begin{bmatrix} \eta \ \delta \ \eta \end{bmatrix}, P_w = \begin{bmatrix} \eta \ \eta \ \beta \end{bmatrix},$$
$$Q_s = \begin{bmatrix} 1 \\ \varepsilon \\ \varepsilon \end{bmatrix}, Q_v = \begin{bmatrix} \varepsilon \\ 1 \\ \varepsilon \end{bmatrix}, Q_w = \begin{bmatrix} \varepsilon \\ \varepsilon \\ 1 \end{bmatrix},$$

where α, β, δ, η are all positive parameters and $\varepsilon \in (0, 1)$, not necessarily small.

In order to rationalize the observed data set in the unitary model, let us suppose WLOG that $Q_s \mathcal{R}\ Q_v \mathcal{R} Q_w$, implying by GARP:

$$\begin{aligned} &P_s\left(Q_s - Q_v\right) \geq 0,\ P_v\left(Q_s - Q_v\right) \geq 0,\\ &P_s\left(Q_s - Q_w\right) \geq 0,\ P_w\left(Q_s - Q_w\right) \geq 0,\\ &P_v\left(Q_v - Q_w\right) \geq 0,\ P_w\left(Q_v - Q_w\right) \geq 0, \end{aligned}$$

that is,

$$\begin{aligned} &(\alpha - \eta)(1 - \varepsilon) \geq 0,\ \ (\eta - \delta)(1 - \varepsilon) \geq 0,\\ &(\eta - \beta)(1 - \varepsilon) \geq 0,\ \ (\delta - \eta)(1 - \varepsilon) \geq 0. \end{aligned}$$

These conditions reduce to:

$$\beta \leq \eta = \delta \leq \alpha.$$

Hence, the unitary model is rejected as soon as these conditions are violated, as in the example of Cherchye et al. (2011), where $\eta = 4$ and $\beta = \delta = 7$.

So, let us assume in the context of our comprehensive collective model that $Q_s \mathcal{R}^A Q_v \mathcal{R}^A Q_w$ and $Q_w \mathcal{R}^B Q_v \mathcal{R}^B Q_s$, implying by GARP:

$$\begin{aligned} &\tau_s^A\left(Q_s - Q_v\right) \geq 0, \tau_v^A\left(Q_s - Q_v\right) \geq 0, \tau_s^B\left(Q_s - Q_v\right) \leq 0, \tau_v^B\left(Q_s - Q_v\right) \leq 0,\\ &\tau_s^A\left(Q_s - Q_w\right) \geq 0, \tau_w^A\left(Q_s - Q_w\right) \geq 0, \tau_s^B\left(Q_s - Q_w\right) \leq 0, \tau_w^B\left(Q_s - Q_w\right) \leq 0,\\ &\tau_v^A\left(Q_v - Q_w\right) \geq 0, \tau_w^A\left(Q_v - Q_w\right) \geq 0, \tau_v^B\left(Q_v - Q_w\right) \leq 0, \tau_w^B\left(Q_v - Q_w\right) \leq 0. \end{aligned}$$

We thus get the following system of inequalities:

$$\begin{aligned} &\eta \max\left\{\gamma_{s2}, \gamma_{s3}\right\} \leq \alpha\gamma_{s1}, \eta\gamma_{v3} \leq \delta\gamma_{v2} \leq \eta\gamma_{v1}, \beta\gamma_{w3} \leq \eta \min\left\{\gamma_{w1}, \gamma_{w2}\right\},\\ &\alpha\left(1 + \theta_1 - \gamma_{s1}\right) \leq \eta \min\left\{1 + \theta_2 - \gamma_{s2}, 1 + \theta_3 - \gamma_{s3}\right\},\\ &\eta\left(1 + \theta_1 - \gamma_{v1}\right) \leq \delta\left(1 + \theta_2 - \gamma_{v2}\right) \leq \eta\left(1 + \theta_3 - \gamma_{v3}\right),\\ &\eta \max\left\{1 + \theta_1 - \gamma_{w1}, 1 + \theta_2 - \gamma_{w2}\right\} \leq \beta\left(1 + \theta_3 - \gamma_{w3}\right). \end{aligned}$$

Unfortunately from an identification point of view, this system of inequalities admits in general multiple solutions, as we will show. However, we may at least derive for each θ_k bounds within which the observed data set is rationalizable. In order to derive the largest intervals allowing rationalizability, we take the values of the γ_{tk}'s which are the most favorable to verifying these inequalities, namely $\gamma_{s1} = 1$, $\gamma_{s2} = \theta_2$, $\gamma_{s3} = \theta_3$, $\gamma_{v1} = 1$, $\gamma_{v3} = \theta_3$, $\gamma_{w1} = \gamma_{w2} = 1$, $\gamma_{w3} = \theta_3$. Notice that these values correspond to a regime of separate spheres for observations s and w, and to a regime of separate spheres up to one public good for observation v. With these values, the system of inequalities becomes:

$$\begin{aligned}
&\max\{\theta_2, \theta_3\} \leq \alpha/\eta \leq 1/\theta_1, \\
&\max\{\theta_1, \theta_2\} \leq \beta/\eta \leq 1/\theta_3, \\
&\frac{\theta_1 + \theta_3}{1 + \theta_2} \leq \delta/\eta \leq \frac{2}{1 + \theta_2}, \\
&\max\{(1 + \theta_2)\,\delta/\eta - 1, \theta_3\} \leq \gamma_{v2}\,(\delta/\eta) \leq \min\{(1 + \theta_2)\,\delta/\eta - \theta_1, 1\}.
\end{aligned}$$

Let us observe that:

1. For θ-rationalizability, all degrees of autonomy are upper-bounded, possibly below one:

$$\begin{aligned}
0 \leq \theta_1 \leq \min\{\eta/\alpha, \beta/\eta, 2\delta/\eta\}, 0 \leq \theta_2 \leq \min\{\alpha/\eta, \beta/\eta, 2\eta/\delta - 1\}, 0 \\
\leq \theta_3 \leq \min\{\alpha/\eta, \eta/\beta, 2\delta/\eta\}.
\end{aligned}$$

   The admissible interval for θ_2 is non-empty only if $\delta \leq 2\eta$. If we suppose $\eta < \min\{\alpha, \beta, \delta\}$, as in Cherchye et al. (2011), where $\eta = 4$ and $\alpha = \beta = \delta = 7$, θ-rationalizability prevails only for degrees of autonomy, uniform across the spouses, such that

$$(0, 0, 0) \leq (\theta_1, \theta_2, \theta_3) \leq (\eta/\alpha, 2\eta/\delta - 1, \min\{\eta/\beta, 2\delta/\eta\}),$$

   hence $(\theta_1, \theta_2, \theta_3) \leq (4/7, 1/7, 4/7)$ in the example of Cherchye et al.
2. The collective model ($\theta_1 = \theta_2 = \theta_3 = 0$) is clearly rationalizable, under the constraint $\delta \leq 2\eta$, whatever the positive parameters α, β, δ and η.
3. Noncooperative rationalizability ($\theta_1 = \theta_2 = \theta_3 = 1$) requires the very special condition that all prices are the same ($\alpha = \beta = \delta = \eta$), a condition which is still stronger in this example than the condition allowing unitary rationalizability ($\beta \leq \eta = \delta \leq \alpha$). It requires in addition that $\gamma_{v2} = 1$, so that joint contribution to some public good is incompatible with noncooperative rationalizability in this example.

6 Conclusion

Introducing a comprehensive concept of household equilibrium leads to a fundamental change of perspective in the empirical analysis of household behavior. Instead of opposing two different kinds of household decision regimes, cooperative or noncooperative, we have now a continuum of semi-cooperative regimes with the two former as limit cases. In terms of marginal willingness to pay for each public good, instead of focusing on the equality of the observed prices either to the maximum or to the sum of the MWTPs of the spouses, we should rather consider an interval between these two extremes. For empirical application, this new perspective is more demanding. In the revealed preference approach, given a set of observations to be rationalized, it should be expected that if the set is rationalized for some regime, it will also be rationalizable for other close regimes. By falsifying the rationalization of more distant regimes through MIP methods one may expect to establish bounds on the admissible values of the degrees of autonomy.

In this new perspective, though, previous results are still valid. They can be embedded in more general statements. In particular, our results appear to be a natural extension of the results in Cherchye et al. (2011). Importantly, this is verified, from a theoretical point of view, for the theorem characterizing θ-rationalizability, and from an empirical point of view, for the applicability of the MIP algorithm.

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