

College curriculum, diverging selectivity, and enrollment expansion

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Abstract We analyze the heterogeneous impact of expansion of higher education on student outcomes in the context of competition among colleges, which differentiate themselves horizontally by setting curricular standards. Our analysis is based on a novel model of human capital production where a student's outcome of studies at a college depends on the match between the student's aptitude and the standard of the college's curriculum. We find that when public or economic pressures compel less selective colleges to lower their curricular standards, low-ability students benefit at the expense of medium-ability students. This reduces competitive pressure faced by more selective colleges, which therefore adopt more demanding curricula to better serve their most able students. This model of curricular product differentiation in higher education offers an explanation for the diverging selectivity trends of American colleges.

Keywords Curricular standard · College selectivity · Enrollment expansion · College market competition · Higher education

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1 Introduction

During the last several decades, the landscape of postsecondary education in the U.S. has changed significantly. College education, once a gateway to the elite, has become increasingly accessible to the general public. As shown in Table 1, between 1959 and 2008, enrollment in postsecondary education has increased from 3.64 million to 19.10 million, or 525%. This growth was mainly driven by enrollment in public colleges, which has increased from 2.18 million to 13.97 million (641%). During the same period, enrollment in not-for-profit private colleges has increased from 1.46 million to 3.66 million (251%).¹

Furthermore, the steady increase in college enrollment far outpaced the growth of population. As shown in Table 2, after controlling for population size, the enrollment rates within given age cohorts have shown similar patterns of multifold increases.² Part of the increase in the enrollment rate reflects better “access” to higher education, driven by public policies such as the G.I. bill and the Higher Education Act of 1965.

How did the dramatic expansion of college-bound population affect the landscape of higher education on the supply side? Empirical evidence suggests that the growth of demand over the last several decades was accompanied by a pattern of diverging selectivity among the American colleges (Hoxby 2009), whereby “selective” colleges have been shown to have become more selective, and vice versa. This implied a progressively “better sorting” of students across colleges, i.e., strengthening of de facto specialization of colleges, especially at the high end of selectivity, in their chosen segments in the distribution of students in terms of performance on standardized pre-college tests.

This paper aims to analyze the phenomenon of diverging selectivity of colleges that accompanied the expansion of higher education and to examine its implications for student outcomes. Our theory is based on an innovative model of human capital production whose main novel feature is *curricular standard*, a discretionary characteristic of education technology, which is strategically chosen (as an instrument in the competition for students) and therefore potentially differs across educational institutions. In our model, the curricular standard chosen by a college is the expression of its selectivity. We argue that a student’s outcome of studies at a college depends on the match between the student’s aptitude (pre-college preparation) and the college’s curricular standard. Each student chooses his/her best match among the available options, whereby lower-ability students are better served by a less challenging curriculum and vice versa. Thus, each curriculum has a comparative advantage among certain segments of student population, and different curricular choices by colleges can be viewed as horizontally differentiated product offerings in a framework similar to Hotelling’s spatial competition.

¹ Source: National Center for Education Statistics, Digest of Education Statistics 2009, Tables 003 and 189 (<http://nces.ed.gov/programs/digest/d09>. Accessed April 13, 2011).

² Source: National Center for Education Statistics, Digest of Education Statistics 2009, Table 007.

Table 1 Enrollment in postsecondary degree-granting institutions (in thousands)

Year	Total	Public	Private		
			All	Not-for-profit	For-profit
1959	3640	2181	1459	n/a	n/a
1969	8005	5897	2108	2088	20
1979	11,570	9037	2533	2461	71
1989	13,539	10,578	2961	2731	229
1999	14,791	11,309	3482	3052	430
2008	19,103	13,972	5131	3662	1469

Table 2 Enrollment in postsecondary education by age group (in %)

Year	18–19 years old			20–24 years old			25–29 years old	30–34 years old
	Total	In basic edu.	In higher edu.	All	20–21	22–24		
1959	36.8	n/a	n/a	12.7	n/a	n/a	4.9*	2.4*
1969	50.2	n/a	n/a	23.0	34.1	15.4	7.9	4.8
1979	45.0	10.3	34.6	21.7	30.2	15.8	9.6	6.4
1989	56.0	14.4	41.6	27.0	38.5	19.9	9.3	5.7
1999	60.6	16.5	44.1	32.8	45.3	24.5	11.1	6.2
2008	66.0	17.4	48.6	36.9	50.1	28.2	13.2	7.3

*Data for 1959 unavailable; reported for 1960

In our model, colleges differ in their exogenously (historically) established relative priorities over the quality, in terms of human capital outcomes, vs. the quantity of their graduates. These exogenous priorities determine relative positions of colleges in the selectivity rankings. A high priority that less selective colleges give to the number of students implies that such colleges can also be more exposed to additional incentives to expand enrollments further, stemming for instance from political pressure from state governments to ensure greater access to higher education, expressed directly or through financial incentives.³ This interpretation would well fit the case of less selective public colleges most subject to such government policies, imposed by statutory means or financial channels. More broadly, although the financial dimension of colleges' operation is not explicitly present in our model for the sake of analytical

³ The case of for-profit colleges in the American higher education, which substantially rely on students receiving public subsidies (see, for example, Cellini 2010), can be viewed as occupying the extreme end of this spectrum in terms of their exclusive interest in the quantity of students, to the point, within our highly stylized framework, of not imposing any curricular requirements and accordingly, not delivering educational value added to their students. However, one must draw a conceptual distinction between such case and that of the so-called open-enrollment colleges (e.g., some community colleges), which do not impose explicit admission requirements. Despite relying on tuition revenue and hence placing high weight on the quantity of students, such community colleges do have a mandate for delivering educational value added to students and therefore set certain curricular standards, which serve as effective barriers to entry and/or graduation, the nominally open-enrollment feature notwithstanding.

tractability, allusions to it offer realistic motivation of the model's assumptions. In particular, a bias of less selective colleges, public or private, in favor of quantity of their students has much to do with the colleges' increased reliance on tuition revenues (which are not, however, explicitly featured in our model). Indeed, the public policies to expand access to higher education are often expressed in the U.S. via tuition subsidies, either through direct appropriation for public colleges, or through financial aid to students. On the other hand, the business model of elite private as well as some top public colleges (whose lucid formalization is offered by Hoxby 2014) is based in part on operating a private endowment which allows a college to balance its budgets inter-temporally while banking on future contributions by graduates commensurate with their career earnings, whose expected levels can be deemed proportionate to the attained human capital. This allows selective colleges to play a "long game," such as focusing on quality of students more than their quantity, a policy that may entail running budget deficits, with cost of instruction exceeding tuition revenues, but bringing rewards in the form of alumni contributions in the long run, with endowments playing a self-fulfilling role in this business model. On the other hand, less selective colleges have little or no reserves and thereby must meet short-run budget constraints. Therefore, tuition revenues, including government subsidies tied to the quantity of students, play a more dominant role in their business model.

Our analysis shows that curricular standards of colleges diverge as the college-bound population expands. The diverging selectivity in terms of curricular standards in the higher education market will obviously affect the quality of matches available to individual students. It will improve for some and worsen for others. For example, the downward adjustment in curricular standards of less selective colleges can manifest itself in more remedial course work offered, fewer challenging topics, and a slower pace of learning in general. This will benefit lower-ability students, who would otherwise struggle to keep up, at the expense of medium-ability students who are ready to learn, but are not sufficiently challenged. On the other side of the spectrum, more selective colleges will respond by elevating their curricula due to reduced pressure in competition for medium-ability students with the less selective colleges, whose "watered down" curricula make them a less appealing alternative for those students. As a result, high-ability students will benefit, again at the expense of their medium-ability peers. Thus, our results suggest overall that students in the mid-section of the aptitude distribution of college-bound population will face deteriorating quality of their best matches, as selective colleges become too challenging for them, while the less selective colleges will offer an insufficient educational challenge. These results, therefore, imply that the endogenous divergence of educational standards and hence of the quality of educational products accessible for segments of student population, which accompany the expansion of higher education, can contribute to the evolution of human capital and income inequality. Specifically, our model suggests that this distributional effect can follow a U-shaped pattern, with larger gains in the tails and relative stagnation in the middle of the distribution of college graduates.

The remainder of the paper is organized as follows: Sect. 2 relates this paper to the existing literature on the college education market and labor market outcomes. Section 3 develops a theoretical model of college education technologies characterized by college-specific curricula and derives students' optimal college choices given the

curricula of the colleges. Section 4 endogenizes the colleges' curriculum choice strategies, defines their Nash equilibrium, and obtains our main comparative statics results, which characterizes how equilibrium college curricula and the economy's human capital distribution respond to a policy of increased college access. Section 5 examines the conditions for and the impact of the entry of new colleges into the higher education market. Section 6 concludes. All proofs are in Appendix.

2 Literature review

This paper's main focus is on the heterogeneous human capital gains in college. It builds on a growing literature that emphasizes the hierarchical structure of the education process, one of whose important new insights is that the benefits from investing in superior quality of education at a given stage may critically depend and even be contingent upon sufficient preparation at its prior stage. Driskill and Horowitz (2002), Su (2004, 2006), Blankenau (2005), Blankenau et al. (2007), Cunha and Heckman (2007), and Gilpin and Kaganovich (2012) model education as a sequence of stages, where human capital output from lower stages acts as an input in the education technology at higher stages. In particular, the models of Su (2004, 2006) and Gilpin and Kaganovich (2012) feature a curricular threshold standard at the higher education stage, which sets the minimum pre-college preparation level necessary for making educational gains in college. The present paper takes student outcomes at the basic education stage as given and focuses instead on curricular choices by colleges as discussed in Introduction. We underscore an important distinction between our concept of curricular standard, which is intrinsic to education production technology and affects students' human capital gains depending on prior aptitude, and the concept of educational (admission or graduation) standards in the literature pioneered by Costrell (1994, 1997) and Betts (1998). According to the latter, college standards are sorting devices having no effect on human capital gains in college, the idea which builds on Spence (1973) concept of college education's role as purely a signal of a graduate's aptitude.

This paper also contributes to the literature on inter-school competition. Rothschild and White (1995) (see also a review by Winston 1999), Epple and Romano (1998), and Epple et al. (2006) model segmentation of the higher education market based on students' ability to study and to pay. This literature assumes that all schools use the same curriculum. That is, schools may differ in the levels of their educational inputs, particularly the peer effects, but not in their education production technologies. When peer effects are present, all students would benefit from attending school with high-ability peers and therefore are willing to pay higher tuition fees for such a benefit. Heterogeneity in both student ability and their family income then generates a stratification of school quality in equilibrium and the sorting of students across schools according to learning ability and the ability to pay. Fraja and Iossa (2002) add a geographic dimension to intercollegiate duopoly competition where students incur mobility as well as tuition costs. They demonstrate the emergence of two types of equilibria where colleges are either stratified in quality or offer identical quality and serve as unique regional providers, with the level of mobility costs determining which of the outcomes will obtain. In their model, school quality is signaled by its admission standard, so any

student admitted to a school will benefit from a higher standard, as long as he continues to pass it. MacLeod and Urquiola (2015) analyze a model, also in Spence (1973) tradition, with no peer effects in learning where information about individual ability is imperfect. Returns to education, however, do feature such effects through the factor of school reputation which is based on *ex post* mean ability signal of its graduates, hence an incentive for students to gain admission to colleges with as able peers as possible and for colleges to select accordingly. Thus, a common feature of the above literature is that the gain of *any* individual student, if admitted to a school, will grow if the school's quality characteristics, particularly the quality of its student body, are increased.

In our paper, each school adopts its optimal curriculum, its defining quality characteristic, which is thus endogenous and school specific. Our model differs from the above literature in the following important respect: not all students in a school would gain in terms of human capital if its curricular standard were raised. Instead, there will be winners and losers from such a change. This is an essential and arguably realistic feature of our model. It compels students to self-segregate, across different colleges, by ability based on the best match between it and a college's curriculum, rather than due to peer effects. Indeed, here, if a lower-ability student were to attend a school with predominantly high-ability peers, then instead of benefiting from a peer-group effect, he would find the school's curriculum geared toward them too challenging, i.e., offering a suboptimal educational product, in terms of maximizing his academic achievement. This model feature is supported by a growing body of empirical evidence (e.g., Light and Strayer 2000); it is most relevant, for instance, to understanding the problems arising in the implementation of affirmative action programs in education. Arcidiacono and Lovenheim (2016), in their survey of the related empirical literature on the subject, refer to this feature as a "match effect" (in terms of a fit between pre-college academic preparation of a student and school selectivity reflected in the "pitch" level of its instruction) in contrast to the "quality effect." The latter, which includes peer-group effect, benefits students of all ability levels. While the two types of effects interact in real educational systems,⁴ the theoretical models discussed above focus on "quality effect." In contrast, our model underscores the "match effect." Since it brings forth students' preference (given adequate information) for a curriculum befitting one's aptitude, this model is well suited to capture the fact that the influx of less prepared students associated with the expanded access to higher education necessarily swells the demand for seats in less selective colleges. Moreover, this model offers a rigorous framework demonstrating, consistent with facts, that the less selective colleges respond to the demand by further reducing their curricular standards, triggering a strategic response in the opposite direction by selective colleges, which results in the overall phenomenon of diverging selectivity of colleges.

Furthermore, our model helps in observing an additional channel through which expanding access to education, even when universal and cost-free, can magnify rather than shrink human capital and income gaps. Indeed, we show that endogenous divergence of curricular standards offered by educational institutions, that is, expanding

⁴ This is reflected, for instance, in the complexity of comparative benefits of educational "tracking," i.e., separating students by ability, vs. pooling them in terms of applying universal curricular programs and standards, as shown, for example, by Duflo et al. (2011).

product differentiation resulting from the optimizing behavior of their providers, can exacerbate unequal student outcomes. Therefore, our model and results can offer theoretical insights relevant to understanding the effects of the expansion of higher education on the changes in the earnings distribution within the group of college-educated workers. There is an extensive literature linking students' academic achievements to their labor market outcomes. Its main focus is on the college premium, i.e., the wage differential between the groups of college-educated workers (with an adjustment for workers with "some college" education) and those with high school education at most. Changes in the college premium over the recent decades have been linked to the changes on the demand side of the labor market, namely the skill-biased technological improvements (see an extensive survey of this literature provided by Acemoglu and Autor 2011). There is also a substantial body of literature analyzing the evolution and recent growth of variance of earnings within the group of college-educated workers attributable to its observed or unobserved heterogeneity. Some results point to growth, over recent decades, of this within-group residual inequality due to the variation in learning ability in particular (see, for example, Taber 2001; Lochner and Shin 2014). Some theoretical models (Galor and Moav 2000; Gould et al. 2001) offer an explanation for these results within the labor demand-side directed technological change paradigm as they argue that the change is biased toward innate ability, including the ability to adjust to change.⁵ According to this "ability-bias" concept, however, the magnitude of wage growth should exhibit monotone rise along the ability distribution. One might expect, therefore, that it will be the highest in the right tail of the wage distribution of college graduates and the lowest in its left tail. In contrast, our model illuminates a potential labor supply-side mechanism for a non-monotone pattern in the distribution of educational gains across the college-bound population, particularly with the least gains in the mid-section of student ability distribution, during the period of expansion of higher education.

3 The model

In this section, we introduce a model of a higher education system with a continuum of potential students and a finite number of colleges. The education production technology of colleges is characterized by college-specific parameters which we define as *curriculum*. The key feature of our model is that the human capital a particular student obtains at a particular college is a function of the relationship between this student's pre-college aptitude and the curriculum offered by the college.⁶ Thus, each student's

⁵ Laitner (2000) analyzes a model where individual return to investment in education is enhanced by higher individual ability as well as exogenous unbiased technological change. He notes, however, that the overall variance of income inequality within the higher education group is lowered due to composition effect, as this group expands being joined by less able agents.

⁶ This model, while advancing the concept of curricular product differentiation across colleges, is highly stylized, for the sake of ensuring its analytical tractability, in positing that each college is characterized by its singular curricular standard. In actuality, of course, each college typically offers multiple curricular products such as majors, which differ in their selectivity, and even differentiated curricula within majors (e.g., their Honors versions). Thus on the one hand, in reality, students' choices are multi-dimensional, such

choice is about finding the best match of a college (if any) in terms of maximizing his human capital value added. In this section, we analyze students' decisions about choosing a college given curricula of the colleges and each student's pre-college aptitude. Section 4 will characterize each college's strategic choice of a curriculum in competition with other colleges and equilibrium outcomes of the competition.

3.1 Education technology

A *curriculum* of a college's education technology is defined by two parameters: curricular standard c , which sets the threshold of prerequisite level of preparation to the course of study in this college, and the progress rate $A(c)$, which determines students' learning gains while in college. Thus, under curriculum $(A(c), c)$, a student's (value-added) human capital h is produced according to

$$h(q) = \begin{cases} 0 & \text{if } q \leq c, \\ A(c)(q - c) & \text{if } q > c, \end{cases} \quad (1)$$

where $q \geq 0$ denotes the student's pre-college ability. Student's ability is the only input in his human capital production. According to (1), a student will benefit from learning under curriculum $(A(c), c)$, if and only if his pre-college ability level exceeds the curricular threshold c .⁷

The curricular threshold c represents the prerequisite knowledge or skills required to study at a college under this curriculum. For example, if a course in intermediate microeconomics has algebra as prerequisite, a student not possessing such background will not benefit from learning in this course for lack of required skills, even if he attends classes. On the other hand, if a part of the course is devoted to studying the necessary math, we interpret this as lowering the curricular threshold. The student in question will then derive benefit from such a course albeit to a lesser extent than a student with superior prior preparation.⁸

Footnote 6 continued

as between college–major combinations. On the other hand, in US higher education in particular, students continue making choices upon entering college as more information is revealed, potentially changing majors or dropping out altogether. This means that the informational frictions in inter-college sorting of students are compensated by means of their intra-college sorting. Some recent empirical studies (e.g., Arcidiacono et al. 2016) suggest that the general proposition of the monotonicity of optimal sorting of better prepared students (in terms of returns to their education) toward more selective colleges remains applicable even after the cross-major dimension is taken into account. The monotonicity of the best match between student's preparation level and the "right" college is the point of departure for this theoretical analysis, which leaves informational frictions and multi-dimensionality of student choices outside of its framework.

⁷ A student's pre-college ability can be interpreted as his human capital level reached prior to college. This in turn can be modeled as the output of the basic education stage, where inputs may include the student's innate ability, learning effort, family inputs, as well as school inputs such as funding, teacher quality, and class size. More importantly, the production technology at the basic education stage may also be subject to different curricular choices. In this paper, we abstract from inter-temporal decisions across different education stages and treat a student's pre-college ability as exogenous.

⁸ Note that the presence of a threshold c implies that there are increasing marginal returns to a student's pre-college ability level: For any $q' > q > c$, we have $(q' - c)/(q - c) > q'/q > 1$. In other words, high-

The progress rate $A(c)$ represents the rate at which students can advance their knowledge in the course of study. As the notation suggests, we posit that the progress rate depends on curricular standard c . It is realistic to assume, specifically, that $A(c)$ is an increasing function, which means that the higher level of presumed prior preparation allows those who possess it to make a more significant progress with this curriculum than with one requiring less from a student. Put differently, this assumption implies a trade-off: High progress rate of education requires one to meet higher curricular standards. This also means that a curriculum that is accessible to a very broad population of students can only yield modest progress rate. A curriculum with high progress rate can benefit only a smaller group of highly prepared students. The trade-off is transparent in our expression (1) for the human capital production function. Specifically, we express it through the following simplifying assumption:

Assumption 1 Curriculum progress rate $A(c) = Ac$, where $A > 0$ is a given constant.

As described further in this section, students choose a college while taking curricula of the colleges as given. In Sect. 4, we will model the colleges' choices of their curricula as endogenous outcomes of intercollegiate competition for students.

3.2 Colleges

There are N colleges, indexed by $s \in \{1, 2, \dots, N\}$. The curriculum of college s is (Ac_s, c_s) according to Assumption 1. Let the colleges be ordered according to their curricular standards as follows:

$$c_1 > c_2 > \dots > c_N. \quad (2)$$

That is, college 1 has the most challenging curriculum with the highest threshold and fastest progress rate and can be thought of as a highly selective, elite college. The selectivity decreases as one moves from college 1 to college N , with college N being the least selective, i.e., offering the least challenging curriculum. For now, we take the number of colleges N as exogenously given. We shall address the issue of potential entry later.

Of course, many factors can affect the educational progress rate of students at any given college. For example, more experienced teachers can better motivate students and allow them to learn faster than other teachers. Similarly, a small class size may allow the instructor to provide more individualized feedback to students. Colleges may differ in these aspects of education quality and therefore require different levels of funding to provide them. An improvement in such quality characteristics will benefit all students studying under the same curriculum. We, however, do not explicitly incorporate the financial aspect of education quality, including tuition and other sources of college funding in the model, noting only that these variables tend to correlate with

Footnote 8 continued

ability students benefit disproportionately more from a challenging curriculum, compared to low-ability students, i.e., they enjoy a "talent premium" as also discussed by Gilpin and Kaganovich (2012) whose model features a similar education production function.

curricular standards of colleges. We thus assume that all differences between colleges are captured by the differences in the parameters of their curricula.

We shall now define college objectives. We posit that college s chooses its curriculum c_s to maximize a linear combination of the quality and quantity measures as follows:

$$O_s = H_s + \gamma_s N_s, \quad (3)$$

where H_s is the aggregate human capital value added by the college s students, which we take as the aggregate measure of quality, N_s is the number of students who enroll in the college, and γ_s is the school-specific weight which college s places on quantity relative to quality.

Assumption 2 Colleges are ranked in the order of declining selectivity, such that

$$\gamma_1 < \gamma_2 < \dots < \gamma_N. \quad (4)$$

As will become clear in our analysis in Sect. 4, in equilibrium, colleges with larger γ_s choose lower curricular standard c_s ; therefore, college ranking posited in Assumption 2 leads to inequalities (2) in equilibrium, i.e., preference ranking (4) implies the ranking of curricular choices (2) by colleges.

As we discussed in Introduction, the stylized facts of higher education finance, though not explicitly incorporated in our model for the sake of tractability, help motivate the model's features. Specifically, the weight γ_s a college places on the quantity of students in its objective (3) can be motivated by and serve as a proxy for the weight of tuition and other revenues tied to the number of students in the current balance sheet of a college's financial operation. We shall detail such interpretation below.

For small s , i.e., for relatively highly selective colleges, the argument is based on the fact that the aggregate human capital of the cohort of college s ' graduates H_s correlates with their aggregate lifetime income, which, for a typical selective American college, serves as a basis for expected future alumni contributions. If the college were able to charge a full tuition payment from each student, based on willingness to pay under frictionless credit market conditions, i.e., commensurate with the student's expected lifetime return, the aggregate tuition revenue would be an increasing function of H_s . If the college is unable to charge thus differentiated tuition, as is the case in reality, or charges none at all, as assumed in this model, the college will be arguably motivated by the future contributions of its alumni, which would tend to be proportionate to their human capital value added while in college.⁹ We note that the value of γ_s is likely to be small and in fact even negative for the most selective colleges. Indeed, a negative γ_s is meaningful because the aggregate human capital H_s already combines the quantity of students with their quality, so $\gamma_s = 0$ in formula (3) would merely signify a parity between quality and quantity in the college's objectives, while a negative γ_s shifts the priorities in favor of quality.

⁹ This understanding, already mentioned in Introduction, is well aligned with Hoxby's (2014) analysis of the business model of elite private colleges and arguably to some extent applies to more selective public colleges as well.

For a large s , i.e., a less selective college, besides the aggregate human capital value added of its student body, the college is directly concerned about the size of enrollment, hence its likely large positive value of γ_s . This assumption helps reflect the realities of a combination of direct pressures and financial incentives from state legislatures as well as the greater budgetary reliance of less selective, public as well as private, colleges on tuition revenues, including its components funded through public subsidies to students. The less selective college's main tool to pursue enrollment expansion is lowering the curricular threshold c_s . This gives a larger fraction of the population access to this college, but according to (1), it comes at a cost of also lowering the curriculum's progress rate Ac_s , i.e., at the expense of the college's education quality goal. As mentioned in Introduction, many of the for-profit colleges, which arguably occupy the lowest position in the higher education quality distribution, can be characterized in our framework as giving the quantity of students extremely high weight γ_s in their objective. This would imply, in our framework, that such colleges set very low curricular standards and thereby provide, according to (1), a negligible human capital value added to their students. This makes the choice of attending such colleges near-equivalent to not attending a college altogether, which could have made the former choice of questionable rationality, if one assumed away, as we do, all informational frictions and distortions, such as those expressed in the job-market "credentialing" requirements or stemming from government subsidies.¹⁰

The implicit motivation offered above for the weight γ_s colleges place on the quantity of students in their objective (3) as the degree of their dependence on current revenue flows associated with the number of students can be used similarly to incorporate the possibility of college entry and exit into the model. In a highly stylized description, the cost of setting up and operating a college includes a substantial "fixed" component whose magnitude does not depend or depends little on the number of students. The presence of this fixed cost imposes a minimum requirement on the number of students with which the college can operate, particularly in the case of a less selective college for which tuition and other funds that follow individual students are the main sources of revenue. One can further argue that the fixed cost of setting up a new highly selective college is substantially higher (one can refer, for instance, to the cost of establishing a reputation), so the entry at high level of selectivity is cost prohibitive. Therefore, it is reasonable to assume that potential entry can only occur at the bottom of the selectivity distribution of existing colleges.

Specifically, we posit that there is a potential entrant, college $N + 1$, whose objective parameter $\gamma_{N+1} > \gamma_N$ implicitly reflects the cost calculation, to which we alluded above: the need to enroll sufficient number of students to cover the fixed cost of entry. Based on our analysis of such college entry decision in Sect. 5, the fact that only N colleges exist results from the choice, by the potential college $N + 1$, of corner solution $c_{N+1} = 0$ for its curricular standard, at which no student will enroll. This is

¹⁰ Chung (2012) documents that the probability of a student choosing to attend a for-profit rather than a community college is strongly associated with factors indicative of low access to information as well as low non-cognitive skills. Cellini (2012) finds that the rate of default on federal loans is substantially higher among students attending the for-profit colleges and offers some evidence of inferior job-market returns of education at for-profits.

equivalent to college $N + 1$ not entering given the high value of its coefficient γ_{N+1} . In this framework, the effective entry, with $c_{N+1} > 0$, can occur if the coefficient gets exogenously sufficiently reduced. According to the above discussion, this can be a result of lowered fixed cost of entry, which can happen, for instance, thanks to the emergence of additional sources of funding to cover the fixed cost, e.g., an increase in government grant aid to students (see Cellini 2010). Another potential source of fixed cost reductions could be associated with technological changes, such as those allowing to move a portion of instruction online, which may reduce the cost of brick-and-mortar component required for the college's operation. In Sect. 5, we analyze the effect of entry of college $N + 1$, resulting from such a negative shock to γ_{N+1} , on the pre-entry equilibrium positions of N colleges. We note that a potential case of college exit is symmetric to that of entry in this framework.

3.3 Students

There is a continuum of students of measure 1. Pre-college aptitude or ability level of student ω is denoted by $q(\omega)$. Students are heterogeneous in their pre-college ability levels; specifically, we assume that student ability q follows a triangular distribution on $[0, Q]$ with density $f(q) = 2/Q - 2q/Q^2$. Note that the triangular distribution, while facilitating analytical tractability of the model, realistically implies that there are few high-ability students, more middle-ability students, and even more low-ability college-bound students, i.e., there is a quality–quantity trade-off between the student ability and the number of students.¹¹ Students know their own ability and observe the curriculum offered at each college, which they take as given.

A student faces only one choice: which school to attend. We assume that there is no capacity constraint in any of the colleges and that attending a college is free.¹²

A student's objective is to maximize his (value-added) human capital from college education. We denote student ω 's enrollment decision by $s^*(\omega) \in \{1, 2, \dots, N, \infty\}$, where $s^*(\omega) = \infty$ means that the student does not attend college. The following result proven in Appendix characterizes this choice:

Lemma 1 *Given the curricula (A_{c_s}, c_s) offered by colleges $s \in \{1, 2, \dots, N\}$, student ω 's optimal enrollment decision is given by:*

¹¹ As a convenient shortcut, we exclude the rising portion of the density function of a standard triangular distribution which would correspond to the population with the ability below that we are explicitly considering. For example, one could consider a standard symmetric triangular density where the function $f(q) = 1/Q - q/Q^2$ represents just the declining portion of the triangle corresponding to the above-median-ability population. This would correspond to the above setting under the realistic assumption, implicit in the above, that college education can serve (be meaningful for) only the individuals with higher than the median ability.

¹² Introducing (potentially different) tuition payments at the colleges will not change our main results qualitatively and will only affect the identities of the marginal students who are indifferent between two neighboring colleges (i.e., their net benefits are the same from attending two different colleges). If tuition levels are given and fixed, one can show that the model will yield qualitatively similar results obtained here for the free tuition model.

$$s^*(\omega) = \begin{cases} 1 & \text{if } q(\omega) \geq a_2 = c_1 + c_2, \\ s & \text{if } q(\omega) \in [a_{s+1}, a_s] \text{ for } 1 < s < N, \text{ where } a_s = c_{s-1} + c_s \\ N & \text{if } q(\omega) \in [a_{N+1}, a_N], \text{ where } a_{N+1} = c_N \\ \infty & \text{if } q(\omega) \leq a_{N+1}. \end{cases}$$

So $a_s = c_{s-1} + c_s$ for $1 < s \leq N$ is the cutoff ability level of a student indifferent between attending college $s - 1$ or s (where we define $a_1 = Q$), while $a_{N+1} = c_N$ is the cutoff ability of a student indifferent between attending the least selective college or none at all.

Thus, students are stratified across colleges by their ability levels,¹³ with the highest ability students attending the most selective college 1, the next highest ability segment of students going to college 2, and so forth, while the lowest ability segment of the population does not pursue higher education. The cutoffs between these subgroups are determined by the curricular standards offered by two neighboring colleges.

We conclude this section with offering a flavor of the results to come. Note that, because there is a finite number of colleges, curricula cannot be individually designed to best serve each student's needs. Instead, each college enrolls students of different pre-college ability levels pooled together to be educated using the same curriculum. For all but a measure zero of students, this will not be an ideal learning technology. For student ω , his most preferred curriculum, the one that maximizes his human capital output, is $c(\omega) = \frac{q(\omega)}{2}$. Thus, high-ability students prefer curricula with higher thresholds (curricular standards) and, accordingly, faster progress rates, while low-ability students prefer curricula with lower thresholds and slower progress rates.

Now consider a student whose pre-college ability is such that

$$c_{s+1} < c(\omega) < c_s.$$

Relative to this student's individually optimal curriculum, college $s + 1$ is too easy and college s is too demanding. Of course, if either c_{s+1} or c_s is not far from $c(\omega)$, student ω will be able to study in an "almost ideal" learning environment, and the fact that no college offers exactly ω 's ideal curriculum will not affect this student's learning outcomes much. If, however, c_{s+1} is located substantially below $c(\omega)$, and c_s is substantially above $c(\omega)$, then student ω will find himself "stuck in the middle," i.e., placed in a suboptimal learning environment regardless of which college he chooses.

According to Lemma 1, given $c_1 > c_2 > \dots > c_N$, students with ability $q(\omega) \in [a_{s+1}, a_s]$ will enroll in college s , where $a_s = c_{s-1} + c_s$ for $1 < s \leq N$, $a_1 = Q$, and $a_{N+1} = c_N$. Thus, the aggregate human capital of college s ' student body is given by

¹³ Here we assume that each student is perfectly informed about his ability. When such information is imperfect, derived for instance from imprecise signals such as standardized test scores, it can over- or underestimate one's true ability and lead students to make ex post suboptimal college choices—too "hard" or too "easy" for a particular student, and result in some deviation from perfect sorting of students across colleges in terms of true ability. However, if the informational frictions are unbiased and students are risk-neutral (such is the case, for example, considered by MacLeod and Urquiola 2015), this will not affect the validity of our analysis qualitatively. On similar grounds, our model presumes that students choose a college (if any) based solely on the goal to maximize their human capital outcome, excluding considerations such as prestige of college-specific credentials. Indeed, such signaling value of colleges does not exist in the absence of informational asymmetries, as presumed here.

$$H_s = \int_{a_{s+1}}^{a_s} A c_s (q - c_s) f(q) dq, \quad (5)$$

and the number of students enrolled at college s is

$$N_s = \int_{a_{s+1}}^{a_s} f(q) dq.$$

These expressions determine the components of college s ' objective defined in (3).

4 Equilibrium curricula

In the previous section, the curricula of colleges were taken as given and used to derive students' enrollment choices and human capital outcomes. In this section, we focus on the choices of curricula by the colleges. We model the colleges' curricular choices as a Nash equilibrium outcome of a game played among the schools.

4.1 Nash equilibrium

We now examine equilibrium curricular choices by the colleges, given their objectives described above. Each college s chooses a feasible curricular standard c_s , and accordingly, the progress rate $A c_s$, to maximize (3), taking curricula chosen by other colleges as given. Thus, curricular choices at equilibrium in this game played by N colleges constitute a (pure strategy) Nash equilibrium.

For the least selective college N , the differentiation of (3) yields first-order condition as

$$-\frac{c_{N-1}}{3Q^2} (A c_{N-1} (6c_N + 2c_{N-1} - 3Q) + 6\gamma_N) = 0,$$

so the solution for c_N is given by

$$c_N = Q/2 - c_{N-1}/3 - \frac{\gamma_N}{A c_{N-1}}. \quad (6)$$

One can directly verify that the second-order sufficient condition is always satisfied since $-\frac{2A c_{N-1}^2}{Q^2} < 0$.

For college s with $1 < s < N$, the first-order condition is

$$\begin{aligned} & -\frac{c_{s-1} - c_{s+1}}{3Q^2} (A(6c_s(c_{s-1} + c_{s+1}) \\ & + 2(c_{s-1}^2 + c_{s+1}^2 + c_{s-1}c_{s+1}) - 3Q(c_{s-1} + c_{s+1})) + 6\gamma_s) = 0. \end{aligned}$$

The solution to this equation, given $c_{s-1} > c_{s+1}$, is thus

$$c_s = Q/2 - \frac{c_{s-1}^2 + c_{s+1}^2 + c_{s-1}c_{s+1}}{3(c_{s-1} + c_{s+1})} - \frac{\gamma_s}{A(c_{s-1} + c_{s+1})}. \quad (7)$$

The second-order sufficient condition is satisfied, $-\frac{2A(c_{s-1}^2 - c_{s+1}^2)}{Q^2} < 0$, because $c_{s-1} > c_{s+1}$.

Lastly, for the most selective college 1, the first-order condition is

$$\frac{Q - c_1 - c_2}{3Q^2} \left(A(-4c_1^2 + 4c_1c_2 + 2c_2^2 + 5Qc_1 - Qc_2 - Q^2) + 6\gamma_1 \right) = 0.$$

Since $c_1 + c_2 < Q$ is always true, the solution to this equation is

$$c_1 = \frac{5Q + 4c_2 - \sqrt{48c_2^2 + 24Qc_2 + 9Q^2 + 96\gamma_1/A}}{8}. \quad (8)$$

The second-order sufficient condition for college 1 is given by

$$-\frac{2}{Q^2} \left(A(2c_1^2 - c_2^2 - 3Qc_1 + Q^2) - \gamma_1 \right) < 0,$$

which is satisfied if γ_1 is negative and sufficiently large by absolute value.

The Nash equilibrium is given by the solution of the N first-order conditions as defined in (6), (7), and (8). The existence, uniqueness, and interiority of such a solution hinges on parametric conditions on these equations, which are not explicitly tractable in the general case of an arbitrary number of colleges N . In the next subsection, we will start by establishing the existence of interior equilibrium for the case $N = 2$. This will motivate the extension of our analysis to the general multi-college case in Sect. 4.3.¹⁴

4.2 The case of two colleges

When $N = 2$, the curricular response c_1 of the selective college 1, given the curricular standard c_2 of the less selective college 2, is the solution of the equation

$$4c_1^2 - 4c_1c_2 - 2c_2^2 - 5Qc_1 + Qc_2 + Q^2 - 6\gamma_1/A = 0. \quad (9)$$

The curricular response c_2 of college 2, given c_1 , is

$$c_2 = Q/2 - c_1/3 - \gamma_2/(Ac_1). \quad (10)$$

¹⁴ The above first-order conditions presume that solutions are, in fact, interior. As will be seen in the next subsection, under some parameter values corner solutions can occur where the most selective college will set $c_1 = Q$, i.e., will admit measure zero of students, and/or the least selective college will choose $c_N = 0$, making it accessible to everyone. Further, we note that the first-order conditions, (6), (7), and (8), apply to unilateral moves by each college that preserve the ordering of college curricula $c_1 > c_2 > \dots > c_N$. In our proofs of existence of Nash equilibrium placed in Appendix, we consider all possible unilateral deviations by a college, including those that are not order preserving.

A Nash equilibrium is then a pair of curricular thresholds (c_1^*, c_2^*) which are mutual best responses—that is, $c_1^* = c_1(c_2^*)$ and $c_2^* = c_2(c_1^*)$. The equilibrium is *locally stable*, if a small perturbation of the equilibrium results in best response dynamics converging back to it (Moulin 1986). The following result provides a sufficient condition for the existence, uniqueness, and stability of an interior Nash equilibrium in the curricular choices in the game of two colleges.

Proposition 1 (Existence and Stability) *There are bounds $\underline{\gamma}_1 < \bar{\gamma}_1 < 0$ and $0 < \underline{\gamma}_2 < \bar{\gamma}_2$ such that when $\gamma_1 \in (\underline{\gamma}_1, \bar{\gamma}_1)$ and $\gamma_2 \in (\underline{\gamma}_2, \bar{\gamma}_2)$, a unique locally stable Nash equilibrium exists with $c_1^* > c_2^* > 0$ and $c_1^* + c_2^* < Q$.*

Proposition 1 shows that when the selective college (college 1) is moderately biased toward quality, and likewise, when the less selective college (college 2) is moderately biased toward quantity, they will optimally choose to differentiate their curricular offerings to students. More specifically, since the selective college cares more about quality than quantity, it chooses a more challenging curriculum to attract the high-ability students. On the other hand, the less selective college cares more about quantity than quality, so it chooses a less challenging curriculum to attract a large number of less able students. The bounds on the parameter values ensure that both colleges' curricular choices are interior solutions that, furthermore, preserve the order of colleges according to their preferences for student quantity given by (4).¹⁵

We now proceed to the comparative statics analysis of the equilibrium responses of both colleges to a positive shock to the less selective college's priority to admit more students. We obtain the following

Proposition 2 (Diverging Selectivity) *At the Nash equilibrium, $\frac{\partial c_2^*}{\partial \gamma_2} < 0$, $\frac{\partial c_1^*}{\partial \gamma_2} > 0$, and $\frac{\partial(c_1^* + c_2^*)}{\partial \gamma_2} < 0$. In other words, while college 2 will lower its curricular threshold, college 1 will do the opposite; the absolute enrollments will expand in both colleges.*

Proposition 2 shows that when the less selective college 2 experiences a shock compelling it to increase the weight it places on the quantity of its students (an increase in γ_2), it lowers its curricular threshold to pursue enrollment expansion. Such exogenous change in college 2 priorities may stem from budgetary or political pressures. Recall that under the triangular distribution, the population is denser at lower ability levels. Thus, lowering the curricular threshold makes college education attractive for an additional densely populated segment of less prepared students for whom this was not the case before. However, such a lower curriculum threshold comes at the expense of human capital attainment of the top segment of the students originally bound for college 2. For these medium-ability students, the downward adjustment of curriculum makes college 2 less appealing an option, so some may shift their college choice toward college 1. In other words, the competitive pressure faced by the more selective college 1 will somewhat weaken, and as a strategic response, it will be able

¹⁵ Specifically, condition $\gamma_1 > \underline{\gamma}_1$ ensures that college 1 will want to admit a positive measure of students. Likewise, condition $\gamma_2 < \bar{\gamma}_2$ prevents the situation where college 2 gives extreme priority to the quantity of students over their quality and as a result will completely eliminate curricular standard, i.e., choose the corner solution $c_2 = 0$.

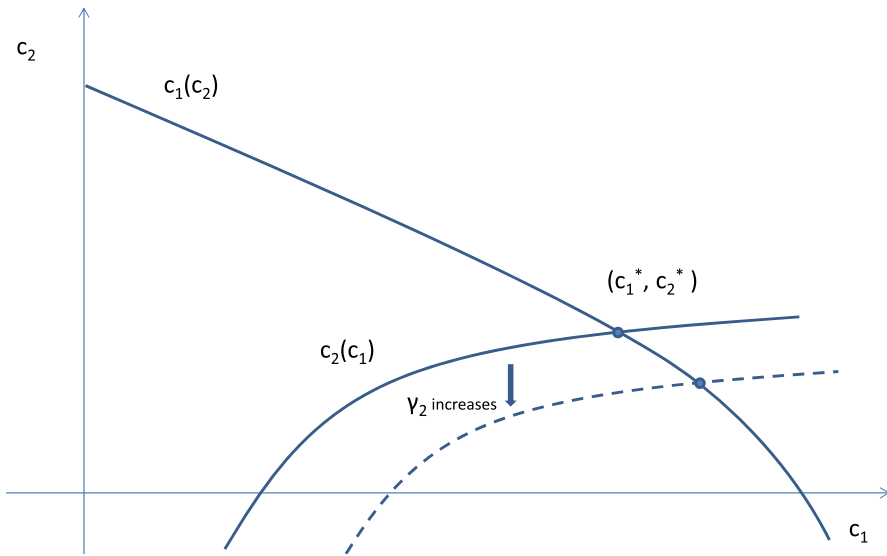


Fig. 1 Best responses and Nash equilibrium

to afford giving less attention to the human capital gains of its lower-end students. Thus, college 1 will find it optimal to raise its curricular standard to the benefit of its better prepared students, because the human capital loss of its lower-end students is more than offset by the increase in human capital of its high-ability students, due to the increased marginal returns to ability (see Footnote 8). In sum, the pursuit of enrollment expansion therefore causes the less selective college 2 to make its curriculum less demanding, eliciting the optimal response from the selective college to further elevate its curriculum.

Figure 1, which depicts the appropriately sloped (as can be seen in the proof of Proposition 1 in Appendix) best response curves of the selective college $c_1(c_2)$ and its less selective counterpart $c_2(c_1)$, also illustrates the comparative statics adjustments discussed above. As γ_2 increases, college 2 best response curve shifts downward (the dotted line). College 1's best response curve, on the other hand, is independent of γ_2 and stays fixed. Thus, as γ_2 increases, the equilibrium slides down along college 1's best response curve. This implies that c_1^* increases and c_2^* decreases, as stated in Proposition 2. Furthermore, the stability of the Nash equilibrium implies that the best response curve for college 1 is steeper than the best response curve for college 2, so an increase in γ_2 leads to a smaller increase in c_1^* and a larger decrease in c_2^* , with the overall effect on the cutoff $a_2 = c_1^* + c_2^*$ to be a net decrease. Thus, student enrollment in both the less selective *and* the more selective college increases when γ_2 increases. This happens despite the fact that the selective college becomes even more challenging (c_1^* increases). The reason is that students of relatively high ability who previously would have enrolled in the less selective college will switch to the selective college when the former's curriculum becomes less demanding, because the relative learning gain of attending college 1 now becomes more significant.

We shall now examine how increased access to higher education affects the welfare and human capital outcomes of students. Consider an increase in γ_2 , and let $(c_1^{\text{old}}, c_2^{\text{old}})$ and $(c_1^{\text{new}}, c_2^{\text{new}})$ denote the equilibrium curricula before and after the change in γ_2 . According to Proposition 2,

$$c_2^{\text{new}} < c_2^{\text{old}} < c_1^{\text{old}} < c_1^{\text{new}}.$$

That is, the “wedge” between the less selective and the more selective colleges’ curricular thresholds widens. The following result describes how students’ human capital is affected by this.

Corollary 1 (Distributional Effects) *At the Nash equilibrium, a positive shock to γ_2 will have the following effects on students’ human capital:*

- (a) *Students with ability levels $q \in (c_2^{\text{new}}, c_2^{\text{old}} + c_2^{\text{new}})$ and $q \in (c_1^{\text{old}} + c_1^{\text{new}}, Q]$ accumulate more human capital.*
- (b) *Students with ability levels $q \in (c_2^{\text{old}} + c_2^{\text{new}}, c_1^{\text{old}} + c_1^{\text{new}})$ accumulate less human capital.*
- (c) *Students with ability levels $q \in [0, c_2^{\text{new}}]$ do not attend college before or after the change in γ_2 . They accumulate the same amounts of human capital, before and after the change.*

Thus, the changes in equilibrium college curricula affect students differently, depending on their initial ability. Corollary 1 characterizes the distributional impacts of curricular adjustments. If γ_2 increases, medium-ability students lose out, while high-ability and low-ability college enrollees are made better off. As the gap between the curricular standards of the less selective and the more selective colleges widens, the curriculum of the more selective college moves closer to the ideal curriculum of the most able students. Similarly, the less selective college’s curriculum moves closer to the ideal curriculum of the least able students. Both curricula, however, move away from the ideal curriculum of medium-ability students.

Overall, the endogenous strategic curricular responses of both colleges resulting from an exogenous shock to the less selective college’s priority to expand admission have a non-monotone impact on students with respect to their ability. It is important to emphasize that the presence of strategic interaction between colleges plays an essential role in this non-monotonicity phenomenon. If the selective college did not have an incentive to adjust its curriculum in response to the less selective college’s move, enrollment expansion at the latter would have increased the human capital of low-ability students and negatively affected medium-ability students (some of whom would have switched to the selective college as a result), without affecting the high-ability students who are already enrolled in the selective college. It is the selective college’s strategic “beefing up” of its curriculum—in response to the less selective college’s “watering down” of its curriculum—that enhances the gains of the most able students. But now, the selective college’s curriculum adjustment adds to the losses suffered by the medium-ability students by making the education at the selective college also a less adequate match for them.

4.3 The general case: multiple colleges

In the previous subsection, we considered the special two-college case. We first proved that there are meaningful parametric conditions under which locally stable interior Nash equilibrium of curricular choices exists (Proposition 1), and then went on to obtain the main comparative statics result demonstrating the diverging selectivity of the colleges when the overall college access expands (Proposition 2). We first note that the task of closed form characterization of the parametric conditions under which a *locally stable interior Nash equilibrium* of curricular choices exists, as provided in Proposition 1, is generally not tractable in the general case of multiple colleges. We shall therefore proceed with our theoretical analysis under the assumption that a locally stable interior Nash equilibrium exists under given parameter values for a given $N > 2$. We shall then provide numerical examples, for the cases $N = 3$ and $N = 4$, featuring locally stable interior Nash equilibria, along with their comparative statics analysis, under meaningful parameter values.

We also make an additional simplifying assumption that in a Nash equilibrium, for $j \in \{1, 2, \dots, N-1\}$, if one or several of the least selective colleges $s = N-j+1, \dots, N$ were to fix their curricular standards, i.e., stopped responding to external changes, then the Nash equilibrium for the subgame among the rest of the colleges, $s = 1, \dots, N-j$, would remain locally stable. We assume the same for the subgame among colleges $s = j+1, \dots, N$ when the j most selective colleges, $s = 1, \dots, j$, fix their curricular positions.

Thus, we take the above existence and local stability conditions (i.e., standard sufficient conditions for comparative statics analysis) in the multi-college case for granted, having established them under appropriate parametric conditions for the two-college case, and focus, in the remainder of this section, on carrying out the comparative statics analysis. Specifically, we focus on extending the diverging selectivity result of Proposition 2 to the multi-college case, based on the above assumptions.

Recall that if Nash equilibrium is interior, it satisfies the system of first-order conditions given by (6), (7), and (8). We begin by making two observations: (1) Parameter γ_s only shows up in the first-order condition for college s ; (2) the best response of college s , namely c_s , depends only on the curricular choices of its immediate neighboring colleges, namely c_{s-1} and c_{s+1} . Therefore, when there is a shock to γ_m , the priority college m gives to the quantity of its students, one can separate the analysis of its impact on the Nash equilibrium into the direct impact of this exogenous change on the curriculum of college m itself, c_m , the sequential indirect impact on the lower-ranked colleges $c_{m+1}, c_{m+2}, \dots, c_N$, and the sequential indirect impact on the higher-ranked colleges $c_{m-1}, c_{m-2}, \dots, c_1$.

According to Assumption 2 colleges are ranked according to their selectivity, with the more selective colleges characterized by smaller weight γ they put on the quantity of students. As we explained in the discussion following Assumption 2, a bias in college's priorities in favor of quality is appropriately characterized by a negative value of γ_s . One can therefore consider college s to be "selective" if $\gamma_s < 0$, and "less selective" otherwise. We will assume that our list of N colleges contains both types. We also note that since triangular distribution of student ability is denser at lower ability levels, strategic interaction of colleges in equilibrium implies that a college with

larger γ (giving more weight to the quantity of students) will choose lower curricular standard c than its counterpart with the next smaller value of γ . This implies that college selectivity ranking according to Assumption 2 yields, in a Nash equilibrium, a corresponding their ranking in terms of curricular standards $c_1 > c_2 > \dots > c_N$.

The following proposition establishes the comparative statics result for the general multi-college case, a counterpart to Proposition 2 for the two-college case.

Proposition 3 (Diverging Selectivity) *Assume that N colleges with $\gamma_1 < \dots < 0 < \dots < \gamma_N$ are in a Nash equilibrium and thereby $c_1^* > c_2^* > \dots > c_N^*$. There exist two cutoffs K and M , where $1 \leq K \leq N - 1$ and $M \geq K + 1$, such that for any m satisfying $m \geq M$, when college m experiences a positive shock to γ_m , then*

- (a) $\frac{\partial c_s^*}{\partial \gamma_m} < 0$ for all $s \geq K + 1$, i.e., college $K + 1$ and all colleges ranked below it become less selective by reducing their curricular thresholds;
- (b) $\frac{\partial c_K^*}{\partial \gamma_m} > 0$, i.e., college K becomes more selective by means of raising its curricular threshold, i.e., the group of colleges ranked at K or above becomes collectively more selective. Within this group, a pairwise clustering pattern emerges: colleges K , $K - 2$, etc., raise their selectivity, while colleges $K - 1$, $K - 3$, etc., lower theirs.

Proposition 3 divides the set of colleges into two subgroups, the less selective colleges and the more selective ones. It shows that when one of the less selective colleges (specifically, colleges ranked at M or below) experiences an exogenous shock compelling it to increase the priority it gives to the size of its enrollment, the entire group of less selective colleges (namely all colleges ranked at $K + 1$ or below) becomes more accessible to students, i.e., all of these colleges lower their curricular standards. On the other hand, the group of more selective colleges, namely those ranked K and above, collectively becomes less accessible. Interestingly, the impact is not monotonic within the group of more selective colleges: As college K raises its curricular standard c_K^* , college $K - 1$ lowers its curriculum c_{K-1}^* (albeit still remaining above c_K^*), college $K - 2$ raises c_{K-2}^* , and college $K - 3$ lowers c_{K-3}^* , and so forth. In other words, there is a pairwise clustering pattern of changing selectivity within the group of more selective colleges. Nonetheless, despite this pairwise clustering effect, the entire group becomes more selective overall, as evidenced by the increased gap between the curricular choices of college $K + 1$ and college K . It is also worth noting that the above result remains true if several of the less selective colleges, i.e., not just college m , experience simultaneous positive shocks to the weights they assign to the quantity of their students.

Recall that Corollary to Proposition 2 in the previous subsection derived the taxonomy of the human capital gains across student population resulting from the phenomenon of the “diverging selectivity” of colleges, i.e., the comparative statics effects established in Proposition 2 for the case of two colleges. Namely, it demonstrated the U-shaped effect on human capital value added with relative gains at its ends and relative losses in the middle. Since Proposition 3 establishes the phenomenon for the multi-college case whereby the two groups of colleges (the more and the less selective) diverge, a similar analysis applies.

Table 3 N -college Nash equilibrium

$N = 3$							
r_1	r_2	r_3		x_1	x_2	x_3	
-0.08	0.04	0.09		0.3320	0.2883	0.0917	
-0.08	0.04	0.10		0.3344	0.2827	0.0520	
-0.08	0.05	0.09		0.3441	0.2601	0.0673	
-0.08	0.05	0.10		0.3511	0.2440	0.0088	
$N = 4$							
r_1	r_2	r_3	r_4	x_1	x_2	x_3	x_4
-0.12	0.03	0.055	0.08	0.4084	0.2862	0.2535	0.1000
-0.12	0.03	0.055	0.085	0.4078	0.2872	0.2467	0.0732
-0.12	0.03	0.06	0.08	0.4068	0.2889	0.2360	0.0823
-0.12	0.03	0.06	0.085	0.4055	0.2910	0.2208	0.0414

Indeed, since the curricular standard of college $K + 1$ and all the colleges below it will be adjusted downward, the overall quality of the student–college matches in terms of educational gains will improve for the lower-ability fraction of the student population and worsen for those in the medium-ability range. In particular, the relatively high-ability students among those attending college $K + 1$ will suffer. Likewise, since as we have shown, the curricular standard of college K will be adjusted upward, the quality of the student–college match will suffer for the relatively lower-ability fraction of its student body, unlike their higher-ability peers who will benefit. Together, two segments of student population in the medium-ability range, namely those with relatively high ability among the attendees of the less selective colleges and the relatively low ability ones among students attending the more selective colleges, form the “sagging middle” in the distribution of the effects of diverging selectivity on the educational gains across the population, as compared to the very low and the very high ability students who benefit from the improved student–college matches.¹⁶

The numerical examples presented in Table 3 illustrate the diverging selectivity results of Proposition 3 along with their implications discussed above. Nash equilibrium we obtain in each of the examples satisfies all the provisions of Proposition 3. Below, we use the same simplifying notation helping to streamline the set of relevant parameters that we apply in the proofs of Propositions 1–3 in Appendix: $r_s = \gamma_s/(AQ^2)$ and $x_s = c_s/Q$ for $s = 1, 2, \dots, N$.

¹⁶ We note that such effects will have intervals of non-monotonicity, where the curricular change at each of the less selective colleges benefits its lower-ability fraction of the students while hurting the higher-ability fraction of its students. Likewise, while the quality of student–college matches improves overall at the higher end of the ability distribution (since such students benefit from higher standards), such effects will have intervals of non-monotonicity according to our result on the pairwise clustering of selective colleges. Indeed, the curricular change at the colleges ranked at K or above by an even number ($K, K - 2, K - 4$, etc.) benefits its higher-ability fraction of the students while hurting the lower-ability fraction, and the opposite is true for the colleges above K by an odd number ($K - 1, K - 3$, etc.).

In the upper panel of Table 3, we present a set of numerical examples for $N = 3$. Here $K = 1$ and $M = 2$, so college 1 is the selective college while colleges 2 and 3 form the group of less selective colleges. Accordingly, when there is a positive shock to r_3 (or r_2 or both), both x_3^* and x_2^* decrease, and x_1^* increases, resulting in a pattern of diverging selectivity between the two groups.

In the lower panel, we present a set of numerical examples for $N = 4$. Here $K = 2$ and $M = 3$, so colleges 1 and 2 form the group of selective colleges and colleges 3 and 4 form the group of less selective colleges. Accordingly, when there is a positive shock to r_4 (or r_3 or both), both x_4^* and x_3^* decrease, but x_2^* increases and x_1^* decreases, resulting in a pattern of diverging selectivity between the two groups.

5 The impact of college entry

The previous section analyzed equilibrium where the number of colleges N was considered to be exogenously given. As discussed in Sect. 3, such situation can be an outcome in a broader framework allowing for an endogenous entry of colleges, most naturally at the lower end of the spectrum of college selectivity. We shall now consider such potential college entry along with the effect it will have on the equilibrium outcome. Specifically, based on the motivation offered in Sect. 3, we examine the effect of entry of college $N + 1$ at the bottom end of college selectivity spectrum, which occurs as a result of a negative shock to the weight γ_{N+1} it placed on the quantity of students in its objective function (3) previously, while holding off entering. Similar to our comparative statics analysis in Sect. 4.3, we proceed from the assumption that a locally stable interior Nash equilibrium exists in an N -college environment for given $\gamma_1 < \gamma_2 < \dots < \gamma_N$, and furthermore, a locally stable (but not necessarily interior) Nash equilibrium also exists when another college with $\gamma_{N+1} > \gamma_N$ is added to the group, although this college's optimum may feature the corner solution $c_{N+1} = 0$. We are then able to characterize the impact of the entry of college $N + 1$ on the college market equilibrium, resulting from an exogenous negative shock to γ_{N+1} , as follows.

Proposition 4 *Let $c_1^* > c_2^* > \dots > c_N^* > 0$ be the locally stable interior N -college equilibrium for given $\gamma_1 < \gamma_2 < \dots < \gamma_N$, and let $c_1^{**} > c_2^{**} > \dots > c_N^{**} > c_{N+1}^{**} \geq 0$ be the locally stable $N + 1$ -college Nash equilibrium with $\gamma_{N+1} > \gamma_N$. There exists $\bar{\gamma}_{N+1} > \gamma_N$ such that*

- (i) *If $\gamma_{N+1} \geq \bar{\gamma}_{N+1}$, then no new college entry occurs, i.e., $c_{N+1}^{**} = 0$ and $c_s^{**} = c_s^*$ for $s = 1, 2, \dots, N$.*
- (ii) *If $\bar{\gamma}_{N+1} - \delta < \gamma_{N+1} < \bar{\gamma}_{N+1}$ for sufficiently small positive δ , then $c_{N+1}^{**} > 0$, i.e., college $N + 1$ does enter the market.*

Furthermore, there exists a cutoff K where $1 \leq K \leq N - 1$ such that

- (a) *$c_s^{**} > c_s^*$ for all $K + 1 \leq s \leq N$, i.e., college $K + 1$ and all colleges ranked below it become more selective by increasing their curricular thresholds;*
- (b) *$c_K^{**} < c_K^*$, i.e., college K becomes less selective by lowering its curricular threshold, i.e., the group of colleges ranked at K or above becomes collectively less selective. Within this group, a pairwise clustering pattern emerges: Colleges $K, K - 2$, etc., lower their selectivity, while colleges $K - 1, K - 3$, etc., raise theirs.*

Table 4 Impact of college entries

$N = 2$							
r_1	r_2			x_1	x_2		
-0.12	0.03			0.4055	0.2908		
$N = 3$							
r_1	r_2	r_3		x_1	x_2	x_3	
-0.12	0.03	0.06		0.4037	0.2939	0.1979	
$N = 4$							
r_1	r_2	r_3	r_4	x_1	x_2	x_3	x_4
-0.12	0.03	0.06	0.08	0.4068	0.2889	0.2360	0.0823

This Proposition shows that depending on the magnitude of the negative shock to γ_{N+1} , the entry of a new college at the bottom of the selectivity spectrum may either have no impact on the selectivity of existing colleges, or generate a pattern of converging selectivity among the pre-existing colleges (that is, the opposite to that established in Proposition 3). The corner solution $c_{N+1} = 0$ presented in part (i) represents the scenario where college $N + 1$ effectively holds off on entering, as explained in our Sect. 3 discussion. The scenario presented in part (ii) of the Proposition characterizes the impact of an effective entry by college $N + 1$, resulting from a sufficient negative shock to γ_{N+1} . We illustrate the results of Proposition 4 with the following numerical examples summarized in Table 4.

We start the analysis with the two-college equilibrium, $r_1 = -0.12$ and $r_2 = 0.03$. Proceeding from it, we consider a potential entry of a third college. We show that $r_3 \geq \bar{r}_3 = 0.1172$ would yield the corner solution $x_3^* = 0$, which is equivalent to college 3 not entering, so this has no impact on the curricular decisions of the two pre-existing colleges. However, when $r_2 < r_3 < \bar{r}_3$, e.g., $r_3 = 0.06$ (i.e., if r_3 receives a sufficient negative shock), we see that $x_3^* > 0$. Furthermore, the entry of college 3 will produce converging selectivity among the pre-existing colleges 1 and 2: The less selective college 2 will increase its selectivity by raising x_2^* , while selective college 1 will respond by lowering its selectivity, i.e., reducing x_1^* . Thus, the situation corresponds to $K = 1$ in the notation of Proposition 4.

Following the attainment of three-college equilibrium, we examine the question of potential entry of college 4 and calculate the cutoff value $\bar{r}_4 = 0.0859$. For a potential entrant with $r_4 \geq 0.0859$, the corner solution $x_4^* = 0$ is optimal, which is equivalent to not entering and thereby not having an impact on the selectivity levels of pre-existing colleges. On the other hand, when $r_3 < r_4 < \bar{r}_4$, e.g., $r_3 = 0.08$, we see that $x_4^* > 0$, i.e., college 4 does enter the market. Furthermore, the pattern of converging selectivity emerges in this case as well, where college 3, forming with college 4 the group of less selective colleges, will increase its selectivity by raising x_3^* , while in the group of more selective colleges comprised of colleges 1 and 2, college 2 reduces its selectivity (x_2^* is reduced) and college 1 raises it by increasing x_1^* . This corresponds to $K = 2$ in Proposition 4.

It is important to note the complementary relation between the results of Proposition 4 and those of Propositions 2 and 3. The latter characterize the changes in the higher education market when less selective colleges become more quantity oriented, while there is no entry of new colleges. The former, on the other hand, characterizes the effects of entry of a new college at the bottom of the selectivity spectrum, assuming that the preferences of its pre-existing competitors are not subject to any changes. The overall evolution of the higher education market is likely to combine both phenomena: Some colleges enter at the very bottom, while at the same time pre-existing colleges in the less selective group become more quantity oriented. The former phenomenon makes the low end of the market more crowded, forcing some to increase their curricular standards. The latter phenomenon, due to the fact that the pre-existing less selective colleges are themselves subject to exogenous pressures (modeled here as preference shocks), puts downward pressure on their curricular standards, which in turn drives the diverging selectivity in the higher education market. The empirical evidence presented by Hoxby (2009) suggests that this diverging selectivity phenomenon prevails over the opposite effect that could be potentially produced by the entry of new colleges at the bottom.

6 Concluding remarks

This paper introduces the concept of an educational institution's curriculum as a characteristic of its human capital production function, which posits that the value added attained by an individual student depends on the relationship between his aptitude (prior preparation) and the curricular standard set by the institution. Our model also suggests that colleges' curricula are set endogenously and moreover strategically, as an instrument in their competition over the relevant populations of students. This paper makes a first step toward understanding colleges' strategies to determine their curricular choices and the effects of these choices on the distribution of human capital attainment by heterogeneous students looking for the best curricular match for their aptitude. Our model offers an explanation for the evidence of diverging selectivity of American colleges that accompanied the expansion of access to higher education in the U.S., particularly a downward adjustment in the curricular standards of historically less selective colleges and an upward adjustment of the curricula in the more selective ones.

We argue that the distributional impact of these changes is non-monotone: While low- and high-ability students gain in terms of human capital value added, medium-ability students lose out. To be sure, the model we use is highly stylized, which is necessitated by keeping the focus on the key insights associated with the impact of curricular changes while preserving analytical tractability. One of the substantial simplifications which deserve future investigation is the assumption of a singular curricular product per college, whereas in reality students' choices are multi-dimensional, such as between college–major combinations. Another substantial simplification in our model is the exclusion of an explicit account for the financial side of college economics. An extension of our analysis to incorporate education expenditure in the education production technology could help capture school quality aspects besides their curricula,

such as teacher quality, class size, classroom and laboratory technology, and so forth. While different students prefer different curricula, all students can benefit from higher levels of the aforementioned characteristics of quality. Allowing for interactions across different school quality characteristics can help produce novel insights into their roles in educational attainment in different segments of student population and the trade-offs involved, and explore the question of optimal costs associated with different curricular standards. These questions are especially important when examining policies which allocate public funds to achieve college access objectives.

Appendix

Proof of Lemma 1 Note that staying out of college is equivalent to learning under curriculum with $c = 0$. A student of ability q prefers curricular standard c to c' whenever $c(q - c) > c'(q - c')$. Now suppose $c_1 > c_2 > \dots > c_N > 0$. A student with ability q prefers staying out of college to attending college N if $q < c_N$. He prefers college n over college $n + 1$ for $n = 1, 2, \dots, N - 1$ if $c_n(q - c_n) > c_{n+1}(q - c_{n+1})$. This implies the lemma's result. \square

Proof of Proposition 1 To streamline notation, we denote $x = c_1/Q$, $y = c_2/Q$, $r = \gamma_1/(AQ^2)$, and $t = \gamma_2/(AQ^2)$. Then the first-order condition for college 1 becomes

$$4x^2 - 4xy - 2y^2 - 5x + y + 1 - 6r = 0.$$

Solving for y in terms of x , we have

$$y = \phi(x) = \frac{1 - 4x + \sqrt{48x^2 - 48x + 9 - 48r}}{4}.$$

Similarly, the first-order condition for college 2 becomes

$$y = g(x) = 1/2 - x/3 - t/x.$$

To establish sufficient conditions, we show below that when $r \in (-\frac{1}{16}, -\frac{1}{24})$ and $t \in (\frac{1}{12}, \frac{5}{48})$, a unique stable and order-preserving Nash equilibrium exists with $x^* \in (1/4, 1/2)$ and $y^* \in (0, 1/6)$.

First, regarding the function $g(\cdot)$, we note that when $t \in (\frac{1}{12}, \frac{5}{48})$ and $x \in (1/4, 1/2)$, it is guaranteed that $0 < g(x) < x$. Furthermore, $g'(x) = -1/3 + \frac{t}{x^2} > 0$, and $g''(x) = -\frac{2t}{x^3} < 0$, namely $g(x)$ is upward sloping and concave.

Next, regarding the function $\phi(\cdot)$, we note that when $r \in (-\frac{1}{16}, -\frac{1}{24})$, $\phi(x)$ is not defined for $x \in (1/2 - \sqrt{1/16 + r}, 1/2 + \sqrt{1/16 + r})$, so its relevant domain is $x \in (0, 1/2 - \sqrt{1/16 + r}]$. Furthermore, $\phi'(x) = -1 + \frac{12x - 6}{\sqrt{48x^2 - 48x + 9 - 48r}} < 0$, and $\phi''(x) = -\frac{3 + 48r}{(48x^2 - 48x + 9 - 48r)^{3/2}} < 0$, namely $\phi(x)$ is downward sloping and concave.

Third, we establish the single crossing between $\phi(x)$ and $g(x)$ for $x \in (1/4, 1/2)$. When $r \in (-\frac{1}{16}, -\frac{1}{24})$, it is guaranteed that $\phi(1/4) > g(1/4)$ and $\phi(1/2 -$

$\sqrt{1/16+r} < g(1/2 - \sqrt{1/16+r})$, so the single-crossing property ensures that a solution exists with $1/4 < x^* < 1/2 - \sqrt{1/16+r} < 1/2$. Substituting x^* into the function $g(\cdot)$, it is easy to check that $0 < y^* < 1/6$.

Fourth, we verify that x^* and y^* indeed represent Nash equilibrium in that no unilateral deviation can improve a college’s payoff, holding the other college’s choice as given. More specifically, when college 2’s strategy is y^* , x^* is college 1’s best response and therefore represents the maximum among all the order-preserving strategies, i.e., for all $x > y^*$. To ensure that x^* represents the global maximum, i.e., without an *a priori* limitation on strategy, we also need to consider $O_1(x|x < y^*) = A Q^2(\int_x^{x+y^*} x(q-x)f(q)dq + r \int_x^{x+y^*} f(q)dq)$ and $O_1(x|x = y^*) = A Q^2(\int_{y^*}^1 y^*(q-y^*)f(q)dq + r \int_{y^*}^1 f(q)dq)/2$, since the two colleges equally split the students if they have the same curricular standard. Note that $\frac{\partial O_1(x|x < y^*)}{\partial x} = A Q^2/3y^*((3-6x+2y^*)y^*-6r) > 0$ since $x < y^* < 1/6$ and $r < 0$, so $O_1(x|x < y^*) < \lim_{x \rightarrow y^*-} O_1(x|x < y^*)$. It can be verified that when $r \in (-\frac{1}{16}, -\frac{1}{24})$ and $t \in (\frac{1}{12}, \frac{5}{48})$, $O_1(x^*|x^* > y^*) > \lim_{x \rightarrow y^*-} O_1(x|x < y^*)$, and furthermore, $O_1(x^*|x^* > y^*) > O_1(x|x = y^*) = (\lim_{x \rightarrow y^*-} O_1(x|x < y^*) + \lim_{x \rightarrow y^*+} O_1(x|x > y^*))/2$. Thus, given y^* , college 1’s decision x^* indeed leads to the global maximum for college 1. Similarly, given x^* , it can be verified that y^* leads to the global maximum for college 2. Thus, x^* and y^* represent the unique Nash equilibrium characterized by $x^* > y^*$, i.e., it is order preserving.

Finally, to establish local stability of the Nash equilibrium, it is sufficient to show that $-\phi'(x) > g'(x)$, so that the mutual best response dynamics upon a small perturbation of the equilibrium (x^*, y^*) converges back to the equilibrium. Recall that $\phi''(x) < 0$ and $g''(x) < 0$ for $x \in [1/4, 1/2 - \sqrt{1/16+r}]$, so it is sufficient to show that $-\phi'(1/4) > g'(1/4)$. This inequality is indeed satisfied since $-\phi'(1/4) = 1 + \frac{3}{\sqrt{-48r}} > g'(1/4) = -1/3 + 16t$ for the parameter values $r \in (-\frac{1}{16}, -\frac{1}{24})$ and $t \in (\frac{1}{12}, \frac{5}{48})$. □

Proof of Proposition 2 Keeping the above notation, it is easy to see that at a given equilibrium (x^*, y^*) , when t is marginally increased, the reaction curve $\phi(x)$ is unaffected, but $g(x)$ shifts down. So the change in equilibrium results in a movement along the $\phi(x)$ curve. This implies that $dy^* < 0$, and since, as established in the proof of Proposition 1, $y = \phi(x)$ is downward sloping, it follows that $dx^* > 0$. Furthermore, since $-\phi'(x^*) > -\phi'(1/4) = 1 + \frac{3}{\sqrt{-48r}} > 1$, we also know $dx^* < |dy^*|$, and therefore, $dx^* + dy^* < 0$. It then follows that $sign(\frac{\partial c_1^*}{\partial \gamma_2}) = sign(\frac{\partial x^*}{\partial t}) > 0$, $sign(\frac{\partial c_2^*}{\partial \gamma_2}) = sign(\frac{\partial y^*}{\partial t}) < 0$, and $sign(\frac{\partial (c_1^*+c_2^*)}{\partial \gamma_2}) = sign(\frac{\partial (x^*+y^*)}{\partial t}) < 0$. □

Proof of Corollary 1 According to Proposition 2, when γ_2 increases, curricular thresholds adjust from (c_1^{old}, c_2^{old}) to (c_1^{new}, c_2^{new}) such that

$$c_2^{new} < c_2^{old} < c_1^{old} < c_1^{new},$$

and

$$c_1^{new} + c_2^{new} < c_1^{old} + c_2^{old}.$$

According to Lemma 1, student population can be partitioned into the following five subgroups:

1. Students with $q \in [0, c_2^{\text{new}}]$ do not attend college after the policy change.
2. Students with $q \in (c_2^{\text{new}}, c_2^{\text{old}}]$ do not attend college before the policy change, but attend college 2 after the change.
3. Students with $q \in (c_2^{\text{old}}, c_2^{\text{new}} + c_1^{\text{new}}]$ attend college 2 before and after the change.
4. Students with $q \in (c_2^{\text{new}} + c_1^{\text{new}}, c_2^{\text{old}} + c_1^{\text{old}}]$ attend college 2 before the policy change, but switch to college 1 after the change.
5. Students with $q \in (c_2^{\text{old}} + c_1^{\text{old}}, Q]$ attend college 1 before and after the change.

By Assumption 1, a student accumulates more human capital and hence prefers curriculum c over $c' > c$, iff

$$Ac(q - c) > Ac'(q - c') \text{ or equivalently, } q < c + c'.$$

Students in the first subgroup are obviously unaffected by the policy change. Students in the second subgroup gain in human capital after the policy change.

A student in the third group (who attends college 2 before and after the policy change) prefers c_2^{new} to c_2^{old} , if $q < c_2^{\text{old}} + c_2^{\text{new}}$; otherwise, he prefers c_2^{old} to c_2^{new} .

All students in the fourth group (who attend college 2 before, and college 1 after the change) prefer c_2^{old} to c_1^{new} because for these students $q < c_2^{\text{old}} + c_1^{\text{old}} < c_2^{\text{old}} + c_1^{\text{new}}$.

Any student in the fifth group (who attends college 1 before and after the change) prefers c_1^{old} to c_1^{new} if $q < c_1^{\text{old}} + c_1^{\text{new}}$; otherwise, he prefers c_1^{new} to c_1^{old} .

Putting these results together, after the policy change, students with $q \in (c_2^{\text{new}}, c_2^{\text{old}} + c_2^{\text{new}})$ or $q \in (c_1^{\text{old}} + c_1^{\text{new}}, Q]$ acquire more human capital, but students with $q \in (c_2^{\text{old}} + c_2^{\text{new}}, c_1^{\text{old}} + c_1^{\text{new}})$ acquire less human capital. \square

Proof of Proposition 3 To streamline notation, similarly to that used in the proof of Proposition 1, we denote $x_i = c_i/Q$ and $r_i = \gamma_i/(AQ^2)$ for $i = 1, \dots, N$. Then a Nash equilibrium is a solution $(x_1^*, x_2^*, \dots, x_N^*)$ of the system of equations below, given parameter values r_1, r_2, \dots, r_N :

$$\begin{aligned}
 x_1 &\equiv f_1(x_2, r_1) = \frac{5 + 4x_2 - \sqrt{48x_2^2 + 24x_2 + 9 + 96r_1}}{8}, & (a.1) \\
 x_s &\equiv f_s(x_{s-1}, x_{s+1}, r_s) = 1/2 - \frac{x_{s-1}^2 + x_{s+1}^2 + x_{s-1}x_{s+1}}{3(x_{s-1} + x_{s+1})} \\
 &\quad - \frac{r_s}{x_{s-1} + x_{s+1}}, \quad 1 < s < N \\
 x_N &\equiv f_N(x_{N-1}, r_N) = 1/2 - x_{N-1}/3 - r_N/x_{N-1}.
 \end{aligned}$$

Now consider a small change in the parameter values. The resulting new equilibrium can be written as $(x_1^* + dx_1, x_2^* + dx_2, \dots, x_N^* + dx_N)$. Linearizing system (a.1) around the original equilibrium, we can write

$$dx_1 = f_{12}dx_2 + D_1dr_1, \tag{a.2}$$

$$dx_s = f_{s,s-1}dx_{s-1} + f_{s,s+1}dx_{s+1} + D_s dr_s, \quad 1 < s < N$$

$$dx_N = f_{N,N-1}dx_{N-1} + D_N dr_N,$$

where $f_{s,s\pm 1} = \frac{\partial f_s}{\partial x_{s\pm 1}}$ and $D_s = \frac{\partial f_s}{\partial r_s}$ for $s = 1, \dots, N$ are direct derivatives with respect to the parameters. According to the equations in (a.1), $D_s < 0$ for all s .

For tractability, we break down the rest of the proof into several steps as follows.

Step 1.

Consider the signs of $f_{s,s+1}$ for $s = 1, \dots, N - 1$. Note that $f_{12} = \frac{1}{2} - \frac{12x_2+3}{2\sqrt{48x_2^2+24x_2+9+96r_1}}$, so $f_{12} < \frac{1}{2}(1 - \sqrt{\frac{144x_2^2+72x_2+9}{48x_2^2+24x_2+9}}) < 0$ whenever $r_1 < 0$.

Likewise, for any $1 < s < N$, $f_{s,s+1} = \frac{-2x_{s-1}x_{s+1}-x_{s+1}^2+3r_s}{3(x_{s-1}+x_{s+1})^2}$, so $\text{sign}(f_{s,s+1}) = \text{sign}(-2x_{s-1}x_{s+1} - x_{s+1}^2 + 3r_s)$. Since $x_1 > x_2 > \dots > x_N$ and $r_1 < r_2 < \dots < r_N$, the numerator $-2x_{s-1}x_{s+1} - x_{s+1}^2 + 3r_s$ is strictly increasing. Thus, there exists $1 \leq K \leq N - 1$ such that $f_{s,s+1} < 0$ for all $s \leq K$, and $f_{s,s+1} > 0$ for all $s > K$. Note that $K = N - 1$ would imply that $f_{s,s+1} < 0$ for all possible $s = 1, \dots, N - 1$.

Similarly, consider the signs of $f_{s,s-1}$ for $s = 2, \dots, N$. Note that for any $1 < s < N$, $f_{s,s-1} = \frac{-2x_{s-1}x_{s+1}-x_{s-1}^2+3r_s}{3(x_{s-1}+x_{s+1})^2}$, and $f_{N,N-1} = \frac{-x_{N-1}^2+3r_N}{3x_{N-1}^2}$. So overall, if we denote $x_{N+1} = 0$, $\text{sign}(f_{s,s-1}) = \text{sign}(-2x_{s-1}x_{s+1} - x_{s-1}^2 + 3r_s)$ for all $s = 2, \dots, N$. Since $x_1 > x_2 > \dots > x_N > x_{N+1} = 0$ and $r_1 < r_2 < \dots < r_N$, the numerator $-2x_{s-1}x_{s+1} - x_{s-1}^2 + 3r_s$ is again strictly increasing. Thus, there exists $1 \leq L \leq N$ such that $f_{s,s-1} < 0$ for all $s \leq L$, and $f_{s,s-1} > 0$ for all $s > L$. Note that $L = 1$ would imply that $f_{s,s-1} > 0$ for all possible $s = 2, \dots, N$, while $L = N$ would imply that $f_{s,s-1} < 0$ for all possible $s = 2, \dots, N$.

Finally, we observe that for any $1 < s < N$, we have $f_{s,s+1} > f_{s,s-1}$ since $c_{s+1} < c_{s-1}$. It then follows that $K \leq L$.

Step 2.

Rewrite the local linearization equations (a.2) in the matrix format:

$$\mathbf{A} \cdot \mathbf{dx} = \mathbf{b}, \tag{a.3}$$

where \mathbf{A} is the $N \times N$ tridiagonal matrix with nonzero elements $a_{s,s} = 1$, $a_{s,s-1} = -f_{s,s-1}$, and $a_{s,s+1} = -f_{s,s+1}$, \mathbf{dx} is an $N \times 1$ vector with elements dx_s , and \mathbf{b} is also an $N \times 1$ vector with elements $D_s dr_s$.

Further, the solution of the linearized system (a.3) is the equilibrium of the following dynamic system, with the superscript as the time period index,

$$\mathbf{dx}^{t+1} = \mathbf{Bdx}^t + \mathbf{b}, \tag{a.4}$$

where $\mathbf{B} = \mathbf{A} - \mathbf{I}$, i.e., the tridiagonal matrix which differs from \mathbf{A} in that \mathbf{B} has all zeros on its main diagonal. It is easy to see that system (a.4) represents a linearization of the system of best responses of colleges for which the original nonlinear system (a.1) defines the Nash equilibrium. To ensure local stability of the Nash equilibrium, we follow Moulin (1986) and impose a sufficient condition that all eigenvalues of

matrix \mathbf{B} lie strictly within the unit circle. As we will show immediately below, this implies that matrix \mathbf{A} is non-singular, so the solution of the linearized system (a.3) is uniquely defined and comparative statics analysis is applicable, similar to the case under $N = 2$ in Proposition 2. (As mentioned above, having established the sufficient conditions for comparative statics under feasible parametrization for the two-college case, we assume that appropriate feasible ranges for parameters exist for $N > 2$ and focus on applying comparative statics analysis.)

Lemma 2 (i) Matrix \mathbf{A} is non-singular. Moreover, its determinant $|\mathbf{A}| > 0$. (ii) The same is true for all the leading and trailing principal minors of matrix \mathbf{A} , namely $|\mathbf{A}^j| > 0$ and $|\mathbf{A}_j| > 0$ for the leading and trailing principal minors of order $j = 1, \dots, N - 1$, respectively.

Proof Consider all eigenvalues of the matrix \mathbf{B} : λ_k for $k = 1, \dots, m$. Since they lie strictly within the unit circle, function $h(\lambda_k) = (1 - t\lambda_k)^{-1}$ is well defined for all real numbers $t \in [0, 1]$. Then, according to Gantmacher (1959), the matrix function $h(\mathbf{B}) = (\mathbf{I} - t\mathbf{B})^{-1}$ is also well defined by a matrix all of whose eigenvalues are given by $h(\lambda_k) = (1 - t\lambda_k)^{-1}$, $k = 1, \dots, m$ with their respective multiplicities. This means that matrix $\mathbf{I} - t\mathbf{B}$ is invertible so $|\mathbf{I} - t\mathbf{B}| \neq 0$ for all $t \in [0, 1]$. Note that the determinant $|\mathbf{I} - t\mathbf{B}|$ is a continuous function of t and that it equals 1, and hence positive when $t = 0$. Since, as we have just shown, $|\mathbf{I} - t\mathbf{B}| \neq 0$ for all $t \in [0, 1]$, it must also be positive for $t = 1$. We can thus conclude that $|\mathbf{A}| = |\mathbf{I} - \mathbf{B}| > 0$, which completes the proof of part (i) of the Lemma.¹⁷

The proof of part (ii) is completely analogous to the above. Note that matrices \mathbf{A}^j and \mathbf{B}^j , the leading principal submatrices of order j of matrices \mathbf{A} and \mathbf{B} , respectively, correspond to the system of linearized equations similar to (a.3) and the dynamical system of mutual best responses similar to (a.4), which arise in the subgame of colleges 1, 2, ..., j when the positions of the $N - j$ least selective colleges ($s = j + 1, \dots, N$) are fixed at the equilibrium levels. As assumed above, the sufficient condition for the stability of Nash equilibrium in this subgame is satisfied, i.e., all eigenvalues of \mathbf{B}^j lie strictly within the unit circle. Thus, following the footsteps of part (i) proof, we obtain $|\mathbf{A}^j| > 0$. The result for the trailing principal minors $|\mathbf{A}_j| > 0$ obtains similarly, arising from the analysis of the subgame played by colleges $N - j + 1, \dots, N$ when positions of the $N - j$ most selective colleges ($s = 1, \dots, N - j$) are fixed at the equilibrium levels. \square

Step 3.

Denote $M \equiv \max\{L, K + 1\}$. Now we derive the comparative statics when there is a positive shock to γ_m (hence r_m), the weight college m attaches to the quantity of its students, where $m \geq M$.

First, since $L \leq N$ and $K \leq N - 1$, we know $M = \max\{L, K + 1\} \leq N$, so we can always consider a positive shock to r_N , i.e., vector \mathbf{b} has only one nonzero element $b_N < 0$. Using Cramer's Rule, we have $dx_N = \frac{b_N |\mathbf{A}^{N-1}|}{|\mathbf{A}|} < 0$. Similarly, $dx_{N-1} = \frac{f_{N-1,N} b_N |\mathbf{A}^{N-2}|}{|\mathbf{A}|}$, so $dx_{N-1} < 0$ if $f_{N-1,N} > 0$, and $dx_{N-1} > 0$ if $f_{N-1,N} < 0$. In

¹⁷ The idea for this last step of the proof is borrowed from Bellman (1960).

general, $dx_s = \frac{f_{s,s+1} \dots f_{N-1,N} b_N |A^{s-1}|}{|A|}$, so $dx_s < 0$ if $s > K$ (see Step 1), $dx_s > 0$ if $s = K$, $dx_s < 0$ if $s = K - 1$, $dx_s > 0$ if $s = K - 2$, and so forth.

Next, if $L < N$ and $K < N - 1$, we have $M \leq N - 1$, so we can also consider a positive shock to γ_{N-1} (hence r_{N-1}), i.e., vector \mathbf{b} has only one nonzero element $b_{N-1} < 0$. Using Cramer’s rule, we have $dx_{N-1} = \frac{b_{N-1} |A^{N-2}|}{|A|} < 0$. For college N , i.e., the only college ranked below $N - 1$, since $L < N$ implies $f_{N,N-1} > 0$, we have $dx_N = \frac{f_{N,N-1} b_{N-1} |A^{N-2}|}{|A|} < 0$. For colleges ranked above $N - 1$, we have $dx_{N-2} = \frac{f_{N-2,N-1} b_N |A^{N-3}|}{|A|}$, so $dx_{N-2} < 0$ if $f_{N-2,N-1} > 0$, and $dx_{N-2} > 0$ if $f_{N-2,N-1} < 0$. In general, $dx_s = \frac{f_{s,s+1} \dots f_{N-2,N-1} b_{N-1} |A^{s-1}|}{|A|}$, so again $dx_s < 0$ if $s > K$, $dx_s > 0$ if $s = K$, $dx_s < 0$ if $s = K - 1$, $dx_s > 0$ if $s = K - 2$, and so forth.

Lastly, if $L < N - 1$ and $K < N - 2$, we have $M \leq N - 2$ and we can consider in general a positive shock to r_m for any $m \geq M$. Using Cramer’s rule, we have $dx_m = \frac{b_m |A^{m-1}| |A_{N-m}|}{|A|} < 0$. For colleges ranked below m , we have $dx_N = \frac{f_{m+1,m} \dots f_{N,N-1} b_m |A^{m-1}|}{|A|} < 0$, and $dx_{N-1} = \frac{f_{m+1,m} \dots f_{N-1,N-2} b_m |A^{m-1}|}{|A|} < 0$. More generally, for $m < s \leq N - 2$ (when $m < N - 2$), we have $dx_s = \frac{f_{m+1,m} \dots f_{s,s-1} b_m |A_{N-s}| |A^{m-1}|}{|A|} < 0$. For colleges ranked above m , we have $dx_1 = \frac{f_{12} \dots f_{m-1,m} b_m |A_{N-m}|}{|A|}$, so $dx_1 > 0$ if K is odd (so $K - 1$ is even), and $dx_1 < 0$ if K is even. Next, if $m > 2$, we have $dx_2 = \frac{f_{23} \dots f_{m-1,m} b_m |A_{N-m}|}{|A|}$, so $dx_2 < 0$ if $K = 1$, $dx_2 > 0$ if $K = 2$, $dx_2 < 0$ if $K = 3$, and so forth. More generally, for $3 \leq s < m$ (when $m > 3$), we have $dx_s = \frac{f_{s,s+1} \dots f_{m-1,m} b_m |A^{s-1}| |A_{N-m}|}{|A|}$, so $dx_s < 0$ if $s > K$, $dx_s > 0$ if $s \leq K$ and $K - s$ is even, and $dx_s < 0$ if $s < K$ and $K - s$ is odd.

Overall, we have shown that when there is a positive shock to the relative weight γ_m that a relatively low-ranked college $m \geq M$ attaches to the quantity of its students, the equilibrium curriculum choices of all colleges change in a divergent pattern: All less selective colleges $s \geq K + 1$ lower their curricular thresholds and become less selective; colleges $s \leq K$ become collectively more selective, while a pairwise clustering pattern occurs within, namely college K increases its curricular threshold, college $K - 1$ lower its curricular threshold, and so forth. □

Proof of Proposition 4 We continue to use the notation $x_i = c_i/Q$ and $r_i = \gamma_i/(AQ^2)$. The N -college equilibrium $x_1^* > x_2^* > \dots > x_N^* > 0$ for given $r_1 < r_2 < \dots < r_N$ can be technically viewed as an $N + 1$ -college equilibrium with $x_{N+1}^* = 0$, where $r_{N+1} = (1/2 - x_N^*/3)x_N^* \equiv \bar{r}_{N+1}$. Indeed, in the $N + 1$ -college setting, under the above condition, it can be verified that $x_1^* > x_2^* > \dots > x_N^* > x_{N+1}^* = 0$ is the solution to the system of $N + 1$ FOCs, where FOCs for colleges 1, 2, ..., N are the same as those in the N -college setting, and FOC for college $N + 1$ holds true by construction. The presence of college $N + 1$ with $x_{N+1}^* = 0$ is fictitious, as it will not be attended by any students.

To prove part (i), we observe that if $r_{N+1} \geq \bar{r}_{N+1}$, then given $x_s^{**} = x_s^*$ for $s = 1, 2, \dots, N$, the objective function for college $N + 1$ is decreasing in its choice variable and therefore, in light of the nonnegativity constraint, the optimal solution is $x_{N+1}^{**} = 0$. Furthermore, given $x_{N+1}^{**} = 0$, $x_s^{**} = x_s^*$ for $s = 1, 2, \dots, N$ are also best responses for colleges 1 through N .

To prove part (ii), it suffices to note that when $\bar{r}_{N+1} - \delta < r_{N+1} < \bar{r}_{N+1}$ for sufficiently small positive δ , the experiment in question can be viewed as a response to a negative shock to r_{N+1} starting at the fictitious $N + 1$ -college equilibrium with $x_{N+1}^* = 0$ defined above, allowing one to apply the results of Proposition 3 for $dr_{N+1} < 0$. \square

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