Econ Theory (2016) 61:365–392
DOI 10.1007/s00199-015-0897-4

SYMPOSIUM

Deposit insurance and bank liquidation without commitment: Can we sleep well?

Russell Cooper[1] · **Hubert Kempf**[2,3]

Received: 30 November 2014 / Accepted: 16 July 2015 / Published online: 5 August 2015

Abstract This paper assesses the effects of the orderly liquidation of a failing bank and the *ex post* provision of deposit insurance on the prospect of bank runs. Assuming that the public institutions in charge of these policies lack commitment power, these interventions, both individually and jointly, are chosen and undertaken *ex post*. The costs of liquidation and redistribution across heterogeneous households play key roles in these decisions. If investment is sufficiently illiquid, a credible liquidation policy will deter runs. Despite the lack of commitment, deposit insurance, funded by an *ex post* tax scheme, will be provided unless it requires a (socially) undesirable redistribution of consumption that outweighs insurance gains. If taxes are set optimally *ex post*, runs are prevented by deposit insurance without costly liquidation. If not, a combination of the two policies will prevent runs.

Russell Cooper is grateful to the NSF for financial support. Comments from Todd Keister, Antoine Martin, Jonathan Willis and seminar participants at the Banque de France, the University of Bologna, the Central Bank of Turkey, Koc University, the RMM Conference 2010 at the University of Toronto, Washington University at St. Louis, the Riksbank, the University of Pennsylvania, Rice University, the University of Iowa, the Tinbergen Institute and the Federal Reserve Bank of Kansas City are appreciated. We are grateful to the editor and referee for valuable suggestions.

✉ Russell Cooper
russellcoop@gmail.com

Hubert Kempf
hubert.kempf@ens-cachan.fr

1 Economics Department, Pennsylvania State University, University Park, PA, USA

2 Centre d’Economie de la Sorbonne, Ecole Normale Superieure Cachan, Universite Paris-Saclay, 61 boulevard du president Wilson, 94230 Cachan, France

3 Paris School of Economics, Paris, France

Keywords Bank runs · Commitment · Deposit insurance · Optimal liquidation · Redistribution

JEL Classification G21 · G28 · E61

1 Introduction

> Sleep well, knowing that since the creation of the FDIC in 1934, no depositor has ever lost one penny of insured deposits.[1]

This quote captures the widely held belief that a government institution, such as the FDIC, will support the depositors of failed banks through deposit insurance. Within the framework of Diamond and Dybvig (1983), the implications of deposit insurance are well understood.[2] If agents believe that deposit insurance will be provided, then bank runs, driven by beliefs, will not occur. In equilibrium, the government need not act: Deposit insurance is never provided and costly liquidations are avoided. Instead, deposit insurance works through its effects on beliefs, supported by the commitment of a government to its provision.

Yet, recent events during the financial crisis lead one to question the capacity of governments to commit. In many countries, such as the USA, the parameters of deposit insurance were adjusted during the crisis period. In other countries, such as UK, ambiguities about the deposit insurance program contributed to banking instability. In yet other countries, such as China, the exact nature of deposit insurance is not explicit. And, in Europe, the combination of a common currency, the commitment of the ECB not to bailout member governments and fiscal restrictions, cast some doubt upon the ability of individual countries to finance deposit insurance as needed. The March 2013 Cypriot banking crisis is a telling example of commitment problems and the ensuing political and economic difficulties a country faces in funding deposit insurance and managing failing banks.

There is also the question of how broadly to define a bank and thus the types of financial arrangements deposit insurance (in some cases interpretable as an *ex post* bailout) might cover.[3] As argued in Cipriani et al. (2014), money market funds, though outside the FDIC system, operate in a very similar manner as traditional banks. Further, the bailout of AIG, for example, along with the choice not to bail out Lehman Brothers, makes clear that some form of deposit insurance is possible *ex post* for some, but not all, financial intermediaries.

This background motivates our study of two types of interventions, the liquidation of bank assets and the provision of deposit insurance, without *any* commitment. While the no-commitment assumption is extreme, it highlights what measures will be undertaken without *ex ante* commitment by public authorities.

[1] From http://www.fdic.gov/deposit/deposits/penny/.

[2] See Martin (2006) for a thorough presentation and discussion of deposit insurance with commitment.

[3] This was brought out clearly in a presentation, http://www.federalreserve.gov/newsevents/speech/bernanke20100924a.htm, by Ben Bernanke at Princeton University in September 2010.

This distinction between types of intervention are motivated by the Dodd–Frank Act which established a Orderly Liquidation Authority (OLA) alongside deposit insurance (DI).[4] The model contains very stylized versions of these two types of intervention. In the case of OLA, a regulator intervenes as soon as a run is underway, takes control of the bank, decides on the liquidation of long-term assets and recontracts with non-served depositors. In the case of DI, the bank fully liquidates its long-term assets and serves a fraction of early withdrawers. A treasury provides funds to the non-served withdrawers to fulfill the deposit contract, and finances this outflow by taxes.

The central question of the paper is: In the absence of commitment, does the *ex post* provision of deposit insurance and an optimal liquidation policy deter runs? The analysis provides sufficient conditions for runs preventing interventions in a heterogenous agent economy.

There are two central building blocks for our analysis: (i) a model of banking along the lines of Diamond and Dybvig (1983) and (ii) the lack of commitment leading governments to make *ex post* decisions about the liquidation and deposit insurance. In this setting, a trade-off emerges between the gains from transfers to depositors who were not served in a bank run and the potential costs of redistribution and liquidation to fund these transfers.

The standard argument about gains to deposit insurance, as in Diamond and Dybvig (1983), is present in the *ex post* choice of providing deposit insurance since agents face the risk of obtaining a zero return on deposits in the event of a run. But there are potential costs of redistribution across heterogeneous households. This depends on the social objective function. If the social objective favors equality of consumption, then the provision of deposit insurance may lead to less equal consumption distributions and thus be socially costly.[5] Whether this trade-off leads to the provision to deposit insurance *ex post* depends, in part, on the tax system used to fund this insurance. If the taxes needed to fund deposit insurance are determined *ex post* along with the decision on deposit insurance itself, we find that deposit insurance will always be provided.

As in Ennis and Keister (2009), the government is unable to commit to a liquidation policy. Instead, it is determined *ex post*. Inefficient liquidations may be used instead of deposit insurance, for both insurance and redistributive gains. The crucial parameter is the cost of liquidation. If illiquid investments are not easily convertible into current consumption goods, then *ex post*, the regulator will choose to protect the assets of the bank and thus prevent runs. Else, liquidation will occur *ex post* and runs may not be prevented.

Interestingly, this result contrasts sharply with the condition for runs in Diamond and Dybvig (1983), extended in Cooper and Ross (1998). In those papers, a high cost of liquidation implied that banks are more susceptible to runs. This is because the condition for runs assumed full liquidation of bank investments, regardless of the liquidation cost. But, once the liquidation policy is chosen optimally, large liquidation

4 Recent US legislation, the so-called Dodd–Frank Act, provides a process, termed an "Orderly Liquidation Authority" to deal with failed financial institutions outside of the FDIC system. In fact, this regulation was partly motivated by the need to make explicit the government's role in the event of financial failures.

5 These costs of redistribution play a key role in the Cooper et al. (2008) study of bailout of one region by others in a fiscal federation.

costs imply a bank's long-term investments will be preserved for patient households, thus avoiding runs.

Finally, our analysis looks jointly at liquidation policy and deposit insurance in an environment without commitment to either form of intervention.[6] These policies interact. The effect of deposit insurance on runs depends, in part, on whether illiquid assets are liquidated. Further, the liquidation decision depends on the form of deposit insurance as well as the taxation used to finance it. If the tax system to finance deposit insurance is set optimally *ex post*, then deposit insurance will be provided *ex post*. In this case, runs are prevented without costly liquidation. The tax system is sufficiently flexible to provide insurance without costly redistribution. Inefficient liquidations are not needed and, in fact, a policy which suspends withdrawals to stop a run is credible.

The model assumes a particular structure of deposit contracts between banks and depositors. In particular, these contracts do not condition on an agent's place in line (as in a model with explicit sequential services). Nor does the contract reflect the possibility of bank runs. The analysis rationalizes these restrictions: if *ex post* interventions prevent runs, then the simple contract is sufficient to uniquely support the full-information allocation.

These results build upon the bank runs literature starting with the contribution of Diamond and Dybvig (1983). With few exceptions, policy in this environment are studied under the assumption of full commitment. Ennis and Keister (2009) focus on *ex post* interventions in the form of a "deposit freeze" and payment rescheduling. An important feature of that analysis is the lack of commitment: The decision on the policy intervention arises during the run.[7] Ennis and Keister (2010) analyze the case of "limited commitment" where the banking authority has the capacity to allow discounted withdrawals when observing a fraction of excessive withdrawals and prove that runs develop in "waves" of successive discounts. Keister (2010) studies the trade-off between the *ex ante* incentive effects and *ex post* gains to a bailout. Here the attention is on the design of *ex ante* measures given the prospect of a bailout *ex post*.

None of those papers focus on the heterogeneity across households and thus the redistributive aspects of deposit insurance and liquidation. The redistributive effects of different forms of bailouts are surely present in the ongoing political debate. These effects are central to the contribution of this paper.

2 Model

We first present the contracting environment, detailing feasible contracts and the role of sequential service. We then turn to the optimization problems of households and banks.

[6] This paper subsumes Cooper and Kempf (2011) which focused more narrowly on deposit insurance assuming a liquidation policy.

[7] Our analysis of orderly liquidation is similar to the rescheduling of payments studied in this paper. However our environment differs from Ennis and Keister's so as to generate different results, as will be clearer below.

2.1 Environment

The model is a version of Diamond and Dybvig (1983) with heterogeneity across the endowments and hence deposits of agents. The model is structured to highlight a tension across agents based upon their claims on the financial system.

There are three periods, with $t = 0, 1, 2$. Households receive endowments in periods 0 and 1 and can consume in periods 1 and 2.

Households are differentiated by their period 0 endowment, denoted α^0. We index households by their period 0 endowment and refer to them as "endowment type" α^0, or simply agent type when there is no ambiguity. Let $f(\alpha^0)$ be the pdf and $F(\alpha^0)$ the cdf of the period 0 endowment distribution with $F(\alpha_-) \geq 0$ and $F(\alpha_+) = 1$ at the lower (α_-) and upper (α_+) supports of the distribution. This heterogeneity reflects both innate differences across agents as well as the outcome of *ex ante* redistributive government policies.[8] All households receive an endowment of $\bar{\alpha}$ in period 1. As made precise below, this endowment is sufficiently large to fund the provision of deposit insurance in the various scenarios we study.

Households consume in either period 1, or in period 2. In the former case, households are called "early" consumers, in the latter case, "late" consumers. Hereafter we term this the "taste type" of the household. The fraction of early consumers for *each* household endowment type is π. The preferences of households are determined at the start of period 1. Utility in periods 1 and 2 is given by $v(c^{\mathrm{E}})$ if the household is an early consumer and by $v(c^{\mathrm{L}})$ if the household is a late consumer. Assume $v(\cdot)$ is strictly increasing and strictly concave. Some results require that $\zeta v'(\zeta c + \bar{\alpha})$ is increasing in ζ. This is a restriction on the curvature of $v(\cdot)$ and is similar to assuming that substitution effects dominate income effects in a static labor supply problem.

There are three important assumptions about the environment. First, π is independent of α^0. Second, there is no aggregate uncertainty in π. Third, while the endowment type of a household is observed by the bank, her taste type is private information.

There are two storage technologies available in the economy. There is a one period technology which generates a unit of the good in period $t + 1$ from each unit stored in period t. Late households can store their period 1 endowment using this technology.

There is also a two period technology which yields a return of $R > 1$ in period 2 for each unit stored in period 0. This technology is illiquid though: It has a return of $\epsilon \leq 1$ if it is interrupted in period 1. If $\epsilon < 1$, there is a non-trivial choice between investing in the two technologies.

The economy is assumed to be competitive. Households and banks act as price takers. Neither households nor banks are large enough to impact the choices of the government.[9]

[8] We do not study this *ex ante* problem explicitly but rather focus on *ex post* redistribution following a bank run.

[9] As in Chari and Kehoe (1990), the government is the only large player in the game.

2.2 Full-information allocation

The full-information allocation assumes both endowment and taste types are observed. The allocation maximizes $U_{\alpha^0}(\chi^{\mathrm{E}}(\alpha^0), \chi^{\mathrm{L}}(\alpha^0))$ for each endowment type subject to a feasibility constraint:

$$\phi\left(\alpha^0\right)\alpha^0 \geq \chi^{\mathrm{E}}\left(\alpha^0\right)\pi \quad \text{and} \quad \left(1-\phi\left(\alpha^0\right)\right)\alpha^0 R \geq \chi^{\mathrm{L}}\left(\alpha^0\right)(1-\pi) \tag{1}$$

where $\phi(\alpha^0)$ is the fraction of the deposit placed into the liquid technology. The constraints hold for each α^0. The first-order condition is:

$$v'\left(\bar{\alpha}+\chi^{\mathrm{E}}\left(\alpha^0\right)\right) = Rv'\left(\bar{\alpha}+\chi^{\mathrm{L}}\left(\alpha^0\right)\right). \tag{2}$$

Denote the allocation satisfying these conditions by $(\chi^{*\mathrm{E}}(\alpha^0), \chi^{*\mathrm{L}}(\alpha^0))$. It will serve as a benchmark in the analysis that follows. In particular, consumptions of both early and late consumers are increasing in the level of endowment, α^0.

Proposition 1 *Both $\chi^{*\mathrm{E}}(\alpha^0)$ and $\chi^{*\mathrm{L}}(\alpha^0)$ are strictly increasing in α^0.*

Proof From the first-order condition, (2), $\chi^{*\mathrm{E}}(\alpha^0)$ is strictly increasing in α^0 iff $\chi^{*\mathrm{L}}(\alpha^0)$ is increasing in α^0. So either both consumption levels increase in α^0 or both decrease. Since the contract for each α^0 satisfies (1), households with higher α^0 and hence larger deposits receive strictly higher levels of both early and late consumption compared to households with a lower α^0. A contract supplying less to both early and late households would be suboptimal. □

In this solution, $R > 1$ implies that $\bar{\alpha} + \chi^{*\mathrm{E}}(\alpha^0) < \bar{\alpha} + \chi^{*\mathrm{L}}(\alpha^0)$. That is, the consumption of late consumers exceeds that of early consumers for each type α^0. Thus there is no incentive for late consumers to pretend to be early consumers, given that all other late consumers are patient and wait until period 2 to withdraw from any of the banks.

2.3 Contracts and sequential service

Using this benchmark, we study decentralized allocations in which household taste types are not observed. To focus on the role of private information over liquidity needs, the assumption of observed endowment types is maintained.

In the decentralized economy, banks offer contracts to depositors. Given the informational assumptions, a contract stipulates a return on deposits in the two periods, $(r^{\mathrm{E}}(\alpha^0), r^{\mathrm{L}}(\alpha^0))$, for type α^0. As is well understood in the literature and discussed further below, this contract has the virtue of simplicity and supports the full-information consumption allocation. But it also supports other equilibria, i.e., 'bank runs.'

This is clearly a restricted constraint. By assumption, it is not possible to make consumption allocations dependent on their place in line, as in Green and Lin (2000), or dependent on how many depositors have been served before them, as in Peck and Shell (2003).

This simplicity is by design. Overall, our problem is structured to address a particular question: assuming banks offer the simple contract associated with the full-information allocation, will the government have an incentive to intervene *ex post* through orderly liquidation and/or deposit insurance? If the government chooses to do so, then the choice of the simple contract, along with placing zero probability on a run, is rationalized. If the government does not intervene and prevent runs, then of course the contracting problem has to be restructured to take this into account.

2.3.1 Bank optimization and a competitive equilibrium

Competitive banks offer contracts to households. Through this competition, the equilibrium outcome will maximize household utility subject to a zero expected profit constraint. Since household types are observable, the contracts will be dependent on α^0.

We assume that banks contract with a single endowment type. This allows us to separate issues of redistribution within banks from redistribution across households through DI.

The bank chooses a contract and an investment plan, $(r^{\mathrm{E}}(\alpha^0), r^{\mathrm{L}}(\alpha^0), \phi(\alpha^0))$ to maximize household utility, subject to a zero expected profit constraint for each type α^0. By assumption, it assigns zero probability to a run occurring.[10] The bank places a fraction of deposits, $\phi(\alpha^0)$, into the liquid storage technology which yields a unit in either period 1 per unit deposited in period 0 or in period 2 per unit deposited in period 1. The remainder is deposited into the illiquid technology.

The zero expected profit condition for a type α^0 contract is:

$$r^{\mathrm{E}}\left(\alpha^0\right)\pi\alpha^0 + r^{\mathrm{L}}\left(\alpha^0\right)(1-\pi)\alpha^0 = \phi\left(\alpha^0\right)\alpha^0 + \left(1-\phi\left(\alpha^0\right)\right)\alpha^0 R. \tag{3}$$

To guarantee the bank can meet the needs of customers, the following constraints on its portfolio must hold as well:

$$\phi\left(\alpha^0\right)\alpha^0 \geq r^{E}\left(\alpha^0\right)\alpha^0\pi \quad \text{and} \quad \left(1-\phi\left(\alpha^0\right)\right)\alpha^0 R \geq r^{\mathrm{L}}\left(\alpha^0\right)(1-\pi)\alpha^0. \tag{4}$$

Clearly if the two constraints in (4) hold with equality, then the zero expected profit condition is met. Note that these conditions hold for any level of deposits.

The expected utility of a household given a contract is given by $U_{\alpha^0}(\cdot) = \pi v(\bar{\alpha} + \chi^{\mathrm{E}}(\alpha^0)) + (1-\pi)v(\bar{\alpha} + \chi^{\mathrm{L}}(\alpha^0))$. Here $\chi^i(\alpha^0) \equiv r^i(\alpha^0)\alpha^0$ is the total return to a deposit with endowment α^0 in period $i = E, L$. It is more direct and without loss of generality to characterize the contract in terms of the total return $\chi^i(\cdot)$ instead of the return per unit of endowment.

As is standard, the full-information allocation, characterized by (1) and (2), is a decentralized equilibrium. In equilibrium, household welfare is maximized for each endowment type, the allocation is feasible and banks earn zero profits. Further, households have an incentive to truthfully reveal their taste types given that other households do so as well.

[10] As runs are not contemplated, there is no liquidation policy in place at the bank.

2.3.2 Responding to a run

As is well understood in the literature, there can be another equilibrium associated with a bank run. Since agents' taste types are private information, late households may choose to attempt to withdraw in period 1. They may have an incentive to do so if other late households are withdrawing as well.

In the model of Diamond and Dybvig (1983), if the amount owed to all agents when each claimed to be an early household exceeds the amount of resources available to the bank, including liquidation of the illiquid investment, then there is an equilibrium with a run. That is, if

$$\chi^{*E}\left(\alpha^{0}\right) \geq \left(\phi\left(\alpha^{0}\right) + \left(1 - \phi\left(\alpha^{0}\right)\right)\epsilon\right)\alpha^{0} \tag{5}$$

then the contract for endowment type α^0 has a runs equilibrium in which all late households pretend to be early households and attempt to withdraw in period 1.

Diamond and Dybvig (1983) provide conditions on preferences such that (5) holds assuming $\epsilon = 1$. Cooper and Ross (1998) extend that analysis to look at conditions on both household risk aversion and liquidation costs for runs.

As long as there are no aggregate shocks to the fraction of early households, runs can be averted by a commitment to suspend convertibility once the fraction of withdrawers exceeds π. Further, if deposit insurance is promised *ex ante*, then runs can be avoided.

But (5) and the implementation of these policies to avoid runs assume *ex ante* government commitment. As in Ennis and Keister (2009), our analysis does not assume the government can credibly promise an intervention once a run has begun. Instead, the government is required to respond optimally *ex post*.

We ask two questions. First, given its available tools, what is the response of the government to a run? Second, anticipating this intervention, is a run prevented?

To be clear, to say a run is prevented means that late households have no strict incentive to misrepresent and pretend to be early households. Else, there is an equilibrium with a run.[11] The exact condition such that runs are prevented is made clear in the analysis that follows.

The paper is organized around two distinct forms of government intervention in response to a run. The first is orderly liquidation and entails the (partial) liquidation of bank assets, undertaken by a *regulator* that manages the bank in the event of a run. As soon as a run is underway, i.e., the number of depositors served exceeds π, the regulator intervenes. The regulator controls the liquidation of bank assets as well as payments to non-served depositors. These payments are limited by the information of the regulator and the ability of depositors to misrepresent their taste types. The timing of decisions and choice problem of the regulator are made precise in Sect. 3.

The second government action, deposit insurance, denoted "DI," is undertaken by a *Treasury* that finances its provision through taxation. In this case, a bank liquidates all of its illiquid assets, serves a fraction of early withdrawers and the Treasury eventually provides deposit insurance to the fraction of non-served depositors, funded by means

[11] This is equivalent to the terminology in Ennis and Keister (2009) of banking fragility.

of taxation. As developed in Sect. 4, this taxation creates the basis for redistribution and thus a cost to the provision of deposit insurance. After understanding these two forms of intervention, the provision of deposit insurance along with optimal liquidation is presented in Sect. 5.

Throughout, there are two restrictions imposed on the contracting problem: the simple contract and the ignorance of the possibility of runs in the determination of that contract. That is, the period 0 contract is given by $(\chi^{*E}(\alpha^0), \chi^{*L}(\alpha^0))$. This is consistent with the focus of the paper: determining sufficient conditions for *ex post* interventions that prevent bank runs.

3 Orderly liquidation

This section studies the intervention by the regulator. Once the number of withdrawals exceeds π, the regulator is contacted and informed that a run is in process. The bank does not suspend convertibility unilaterally but instead effectively hands control of its operations to the regulator. Thus the decision to suspend convertibility is made optimally *ex post*, as in Ennis and Keister (2009). The analysis characterizes this intervention and provides conditions under which runs are prevented.

This public authority has the same capacity as the "courts" in Ennis and Keister (2009) of rescheduling payments due to the unserved depositors as well as the power to liquidate part of or all the long-term assets. A difference with Ennis and Keister (2009) is that our regulator does not know a priori the taste types of these depositors. Thus it offers a contract menu to the unserved depositors, subject to incentive compatibility, such that depositors reveal their type by choosing one contract.

Our regulator is not allowed to redistribute consumption across endowment types.[12] By assumption, there is no deposit insurance.

But there are some limits to this intervention. First, the regulator is unable to tax households to finance transfers to depositors. Thus any redistribution within a bank must be feasible without the infusion of outside resources. Second, the regulator is also unable to recapture the deposits received from the household served in the bank run. In this sense, the regulator is constrained by the sequential service of the bank.

The timing and the regulator's problem are summarized as follows.

- The bank contacts the regulator when total withdrawals exceed the liquid investment.
- The regulator chooses
  - the optimal liquidation policy;
  - the allocation of the bank assets to early and late depositors.
- The regulator is
  - unable to transfer from depositors who have already been served;
  - must offer an incentive compatible allocation.

[12] This issue does not arise in Ennis and Keister (2009) as their agents are homogenous except for tastes.

3.1 Optimal liquidation policy

The optimization problem of the regulator at the time of its intervention is to choose a (nonnegative) liquidation policy, $Z(\alpha^0) \geq 0$, as well as allocations to early and late households, denoted by $(\tilde{\chi}^{\mathrm{E}}(\alpha^0), \tilde{\chi}^{\mathrm{L}}(\alpha^0))$, to maximize

$$(1-\pi)\left[\int \omega\left(\alpha^0\right)\pi v\left(\bar{\alpha}+\tilde{\chi}^{\mathrm{E}}\left(\alpha^0\right)\right)\mathrm{d}F\left(\alpha^0\right)\right.$$
$$\left.+\int \omega\left(\alpha^0\right)(1-\pi)v\left(\bar{\alpha}+\tilde{\chi}^{\mathrm{L}}\left(\alpha^0\right)\right)\mathrm{d}F\left(\alpha^0\right)\right]. \tag{6}$$

The first term is the welfare of the early households and the second is for the late households. The fraction already served prior to the intervention of the regulator is π. So the objective and constraints are multiplied by the fraction not yet served. The weight for household of type α^0, given by $\omega(\alpha^0)$, implies that some households receive relatively more weight than others in the social objective, perhaps reflecting political power.

For each α^0, equivalently each bank, there are two resource constraints:

$$(1-\pi)\pi\tilde{\chi}^{\mathrm{E}}\left(\alpha^0\right)=\epsilon Z\left(\alpha^0\right), \quad \text{and} \tag{7}$$

$$(1-\pi)(1-\pi)\tilde{\chi}^{\mathrm{L}}\left(\alpha^0\right)=\left[(1-\phi)\alpha^0-Z\left(\alpha^0\right)\right]R. \tag{8}$$

The incentive compatibility condition is:

$$\tilde{\chi}^{\mathrm{L}}\left(\alpha^0\right)\geq\tilde{\chi}^{\mathrm{E}}\left(\alpha^0\right) \tag{9}$$

so that late households prefer to reveal their type rather than claim to be early households.[13] Finally, there are two nonnegativity conditions:

$$\tilde{\chi}^{\mathrm{L}}\left(\alpha^0\right)\geq 0, \quad \tilde{\chi}^{\mathrm{E}}\left(\alpha^0\right)\geq 0 \tag{10}$$

In this problem, the regulator does not redistribute across wealth groups, despite the potential desirability of doing so. This is a consequence of imposing that the

[13] More formally, consider a direct revelation mechanism in which the regulator stipulates $(\tilde{\chi}^{\mathrm{E}}(\alpha^0), \tilde{\chi}^{\mathrm{L}}(\alpha^0))$. If a fraction π or less agents announce they are early types, they each obtain $\tilde{\chi}^{\mathrm{E}}(\alpha^0)$. If more than a fraction π announce they are early types, then the allocation is random and agents obtain $\tilde{\chi}^{\mathrm{E}}(\alpha^0)$ with a probability <1. The same rule applies for the allocation to late households: as long as a fraction $(1-\pi)$ or less announce they are late households they obtain $\tilde{\chi}^{\mathrm{L}}(\alpha^0)$. Else, only a fraction of those announcing late get $\tilde{\chi}^{\mathrm{L}}(\alpha^0)$. This creates feasible allocations for all feasible announcements. Clearly, as long as (10) holds, truthtelling is a Nash equilibrium. Under this allocation mechanism, late households have no incentive to misrepresent their types regardless of the announcement of other late households since obtaining $\tilde{\chi}^{\mathrm{L}}(\alpha^0)$ by telling the truth is always feasible. In particular, there is no runs equilibrium in which late households pretend to be early households. Further, there are no other equilibria since early households will never pretend to be late households.

constraints (7) and (8) hold for each α^0. So, in effect, (6), is solved for each α^0, given (7) and (8). The incentive compatibility condition, (10), also holds for each α^0.

The allocation to late households, $\tilde{\chi}^{\mathrm{L}}(\alpha^0)$ is critical for determining if runs will occur given the optimal liquidation policy. As long as $\tilde{\chi}^{\mathrm{L}}(\alpha^0) > \chi^{*\mathrm{E}}(\alpha^0)$ a run will be prevented as late households have no strict incentive to misrepresent. With commitment, this condition is easy to meet. Without commitment, it requires restrictions so as to ensure the feasibility of liquidation and the desirability of the resulting redistribution from late to early households.

Our first result studies the extreme case of $\epsilon = 0$ so that liquidation is not feasible. In this case, liquidation does not occur *ex post* and consequently runs are prevented. This is similar to the outcome with commitment in Diamond and Dybvig (1983).

The second result assumes $\epsilon > 0$ and shows that liquidation depends on the level of household wealth: Liquidation occurs for high-wealth but not low-wealth households. In this case, runs may not be prevented.

Proposition 2 *If ϵ is zero, then the optimal response of the regulator entails no liquidation of the illiquid investment. This intervention will prevent runs.*

Proof Assume $\epsilon = 0$, so there is no liquidation value. Then the solution to (6) must be $Z(\alpha^0) = 0$. In this case, the left side of (8) is the total payout to the late households, $(1-\pi)^2\tilde{\chi}^{\mathrm{L}}(\alpha^0)$. This equals the fraction of their deposits to the bank, multiplied by the return: $(1-\phi)\alpha^0 R$. From the full-information allocation, $(1-\pi)\chi^{*\mathrm{L}}(\alpha^0) = (1-\phi)\alpha^0 R$. Hence $\tilde{\chi}^{\mathrm{L}}(\alpha^0) > \chi^{*\mathrm{L}}(\alpha^0)$. Since $\chi^{*\mathrm{L}}(\alpha^0) > \chi^{*\mathrm{E}}(\alpha^0)$ from the competitive equilibrium, $\tilde{\chi}^{\mathrm{L}}(\alpha^0) > \chi^{*\mathrm{E}}(\alpha^0) > 0$. This last inequality implies that (10) holds.

Further, $\tilde{\chi}^{\mathrm{L}}(\alpha^0) > \chi^{*\mathrm{E}}(\alpha^0)$ implies that a late consumer would prefer not to be served and obtain the allocation coming from the solution to (6). Thus late households have no incentive to run. □

This is an important and perhaps surprising result. Though the "illiquidity" of banks induced by $\epsilon = 0$ may seem to limit the ability of the financial system to respond to a crisis, in fact this illiquidity is stabilizing. In effect, the illiquidity of the banks substitutes for its inability to commit.[14] The resulting allocation is identical to that from Diamond and Dybvig (1983): The period 0 contract of $(\chi^{*\mathrm{E}}(\alpha^0), \chi^{*\mathrm{L}}(\alpha^0))$ is supported as a unique equilibrium by a regulator that optimally sets liquidation to zero.

This result holds in the extreme case of $\epsilon = 0$. The following proposition argues that the liquidation policy depends on the depositor's wealth levels and the ratio of the return on the long-term investment relative to the liquidation value, $\frac{R}{\epsilon}$, in the case of $\epsilon > 0$. The critical values of these objects are specified in the proposition and proof.

Proposition 3 *For sufficiently large values of $\frac{R}{\epsilon}$, the solution to (6) entails no liquidation for all α^0. For sufficiently small values of $\frac{R}{\epsilon}$, the solution to (6) entails liquidation*

[14] This is reminiscent of Villamil (1991) which separates the function of collecting deposits from the investment decision by separating the intermediary from an entrepreneur with whom it contracts and who has the power to liquidate. It is as if the bank's capacity of liquidating was nil: $\epsilon = 0$. Importantly, Villamil (1991) assumes *ex ante* commitment by the entrepreneur to liquidation.

for all α^0. For intermediate values of $\frac{R}{\epsilon}$, the solution to (6) entails no liquidation for low values of α^0 and positive liquidation for sufficiently high values of α^0.

Proof Substituting the constraints into (6), the first-order condition with respect to $Z(\alpha^0)$ is

$$\epsilon v'\left(\bar{\alpha} + \tilde{\chi}^{\mathrm{E}}\left(\alpha^0\right)\right) + \lambda^Z\left(\alpha^0\right) = Rv'\left(\bar{\alpha} + \tilde{\chi}^{\mathrm{L}}\left(\alpha^0\right)\right). \tag{11}$$

Here $\lambda^Z(\alpha^0)$ is the multiplier on the constraint $Z(\alpha^0) \geq 0$. This condition ignores the incentive compatibility condition which is checked later.

Define two critical levels of $\frac{R}{\epsilon}$ by the condition that (11) holds at $\lambda^Z(\alpha^0) = 0$ and $Z(\alpha^0) = 0$ at the two limits of the distribution of period 1 endowments, α^0_- and α^0_+. These two levels, denoted by $(\frac{R}{\epsilon})_-$ and $(\frac{R}{\epsilon})_+$, are defined by the following conditions:

$$v'\left(\bar{\alpha}\right) = \left(\frac{R}{\epsilon}\right)_- \quad v'\left(\bar{\alpha} + \frac{\chi^{*\mathrm{L}}\left(\alpha^0_-\right)}{1-\pi}\right) \tag{12}$$

and

$$v'\left(\bar{\alpha}\right) = \left(\frac{R}{\epsilon}\right)_+ \quad v'\left(\bar{\alpha} + \frac{\chi^{*\mathrm{L}}\left(\alpha^0_+\right)}{1-\pi}\right) \tag{13}$$

Since $\alpha^0_- < \alpha^0_+$, $\left(\frac{R}{\epsilon}\right)_+ > \left(\frac{R}{\epsilon}\right)_-$.

For sufficiently small values of $\frac{R}{\epsilon}$, i.e. $\frac{R}{\epsilon} \leq \left(\frac{R}{\epsilon}\right)_-$ the solution to (6) entails some liquidation for all α^0. This follows from (12), the strict concavity of $v(\cdot)$ and the monotonicity in $\chi^{*\mathrm{L}}(\alpha^0)$ established in Proposition 1. Using a parallel argument, for sufficiently large values of $\frac{R}{\epsilon}$, i.e. $\frac{R}{\epsilon} \geq \left(\frac{R}{\epsilon}\right)_+$ the solution to (6) entails no liquidation for all α^0.

Finally, for $\left(\frac{R}{\epsilon}\right)_- < \frac{R}{\epsilon} < \left(\frac{R}{\epsilon}\right)_+$, the solution to (6) entails liquidation for values of $\alpha^0 \geq \tilde{\alpha}$ where $\tilde{\alpha}$ solves:

$$v'\left(\bar{\alpha}\right)\epsilon = Rv'\left(\bar{\alpha} + \frac{\chi^{*\mathrm{L}}\left(\tilde{\alpha}\right)}{1-\pi}\right). \tag{14}$$

Here $\tilde{\alpha}$ is such that the nonnegatively constraint on liquidation does not bind in the solution to (11) and liquidation equals zero: $\lambda^Z(\tilde{\alpha}) = 0$ and $Z(\tilde{\alpha}) = 0$. Clearly $\tilde{\alpha}$ is monotonically increasing in $\frac{R}{\epsilon}$.

For $\alpha^0 < \tilde{\alpha}$, the right side of (11) will be higher than the left side at $\alpha^0 = \tilde{\alpha}$ since, from Proposition 1, the consumption of late households is lower for lower α^0. Thus $\lambda^Z(\alpha^0) > 0$ for $\alpha^0 < \tilde{\alpha}$ in order for (11) to hold.

If $\alpha^0 > \tilde{\alpha}$, then the right side of (11) will be lower than the left side at $\alpha^0 = \tilde{\alpha}$. For (11) to hold, $Z(\alpha^0) > 0$ and thus $\lambda^Z(\alpha^0) = 0$ for $\alpha^0 > \tilde{\alpha}$.

The incentive compatibility constraint, (10), holds for all allocations with $Z(\alpha^0) > 0$. This follows directly from (11) with $\lambda^Z(\alpha^0) = 0$ and using the strict concavity of $v(\cdot)$. Further, when there is no liquidation, (10) holds as $\tilde{\chi}^{\mathrm{L}}(\alpha^0) > 0$ from (8). □

Proposition 3, in contrast to Proposition 2, does not make a statement about preventing runs. Whether or not the *ex post* liquidation prevents runs depends on the magnitude of α^0 through $Z(\alpha^0)$.

For low values of α^0, there is no liquidation. In that case, as shown in the proof to Proposition 3, $\tilde{\chi}^{\mathrm{L}}(\alpha^0) = \frac{\chi^{*\mathrm{L}}(\alpha^0)}{1-\pi}$ which, from the full-information solution, is bigger than $\chi^{*\mathrm{E}}(\alpha^0)$. Thus for the low α^0 households, the lack of liquidation protects investment for late households and thus prevents runs.

But for sufficiently high α^0 households, the liquidation may reduce their consumption below the level, $\chi^{*\mathrm{E}}(\alpha^0)$, obtained in a run. This depends on the magnitude of the liquidation which, from (11), depends on ϵ.

This result for low values of α^0 contrasts with Corollary 3 of Ennis and Keister (2009) which argues that banks are fragile for sufficiently high liquidation costs.[15] The difference in findings can be traced to the period 1 endowment of $\bar{\alpha}$. In Ennis and Keister (2009), $\bar{\alpha} = 0$ and $v(c) = \frac{c^{1-\gamma}}{1-\gamma}$. Hence, if no investment is liquidated, the remaining impatient depositors will receive zero consumption and obtain an infinitely negative utility when $\gamma > 1$. To avoid this, some illiquid investment will be liquidated even if liquidating is very costly. The fact that liquidation is costly means the resources available for patient depositors can fall dramatically, which evidently gives these depositors an incentive to run.[16]

As we assume $\bar{\alpha} > 0$, the marginal utility of impatient depositors is bounded since they will always have positive consumption. For this reason, it is not efficient to liquidate investment when ϵ is small (Proposition 3) and, since no investment is liquidated, the payment available to patient agents is large enough to avoid runs.

3.2 An example

The following example illustrates how the magnitude of ϵ and γ jointly determine if the liquidation prevents a run. It complements Proposition 3 since the example fixes α^0.

Assume preferences are represented by a CRRA utility function, $v(c) = \frac{c^{1-\gamma}}{1-\gamma}$. The fraction of impatient households is 50 %. The return on the illiquid investment is 1.1. The period 0 and period 1 endowments, α^0 and $\bar{\alpha}$ respectively, both equal 1.

Given these parameters, the full-information allocation is determined. Then, assuming a run, the optimal liquidation policy is characterized. Finally, the condition for no runs, $\tilde{\chi}^{\mathrm{L}} > \chi^{*\mathrm{E}}$, is evaluated (Fig. 1).

The (red) curve with "+" marks in the figure shows the combinations of γ and ϵ such that the intervention of the regulator prevents a run: i.e., the late households consumption level after the reallocation from liquidation equals the consumption of the early households: $\tilde{\chi}^{\mathrm{L}} = \chi^{*\mathrm{E}}$. Below this curve, the combination of ϵ and γ implies that runs are prevented: $\tilde{\chi}^{\mathrm{L}} > \chi^{*\mathrm{E}}$.

[15] We are grateful to the referee for encouraging this comparison.

[16] As in our analysis, there is no liquidation and no run if $\epsilon = 0$. But for Ennis and Keister (2009) there is a discontinuity since there is a run for $\epsilon > 0$.

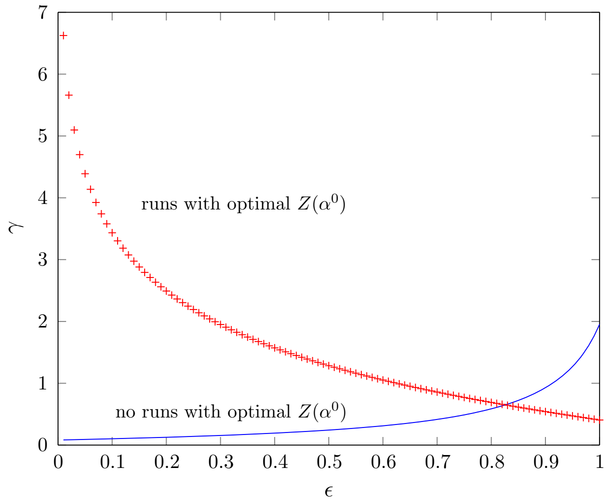

Fig. 1 Optimal liquidation and runs. This figure shows two curves. The first, indicated by *red* +, shows combinations of γ and ϵ such that $\tilde{\chi}^{\mathrm{L}} = \chi^{*\mathrm{E}}$. Below this curve, the intervention prevents runs. The *blue solid curve* shows combinations of γ and ϵ such that (15) holds with equality. *Above* this curve, bank runs occur under the full-information contract assuming complete liquidation (color figure online)

Intuitively, for a given value of ϵ, as γ increases, $\chi^{*\mathrm{E}}$ will increase as well to reduce $\frac{\chi^{*\mathrm{L}}}{\chi^{*\mathrm{E}}}$ in the full-information allocation. This requires that ϕ increase to fund the higher consumption of early households. Consequently, the investment in the illiquid technology is lower and hence, all else the same, $\tilde{\chi}^{\mathrm{L}}$ is lower. This combination of a higher $\chi^{*\mathrm{E}}$ and a lower $\tilde{\chi}^{\mathrm{L}}$ means that runs are not prevented for sufficiently high values of γ given ϵ.

This is one point where commitment clearly enters the analysis. When there is no commitment to the magnitude of liquidation, the incentive to liquidate may be strong enough to drive the consumption of late households below that of households who are served at the start of the run. Thus the liquidation is optimal *ex post* but does not prevent a run. *Ex ante* the regulator might have chosen to commit to less liquidation in order to prevent runs. That policy is not followed in the absence of commitment.

The solid (blue) curve in the figure returns to the conditions for bank runs assuming full liquidation. This was given earlier as

$$\chi^{*\mathrm{E}}\left(\alpha^0\right) \geq \left(\phi\left(\alpha^0\right) + \left(1 - \phi\left(\alpha^0\right)\right)\epsilon\right)\alpha^0 \tag{15}$$

where $\chi^{*\mathrm{E}}(\alpha^0)$ and thus $\phi(\alpha^0)$ comes from the full-information contracting problem and thus reflect the risk aversion of households. Along the curve, (15) holds as an equality.

These curves illustrate the significance of allowing the regulator to choose an optimal liquidation policy *ex post* on the possibility of runs. Consider the case of large liquidation costs, i.e., a small value of ϵ. In this case, for a point below the red line there are no runs. With ϵ small, the optimal liquidation provides a low value of $\tilde{\chi}^{\mathrm{E}}$ and hence $\tilde{\chi}^{\mathrm{L}} > \chi^{*\mathrm{E}}$.

But in the standard Diamond and Dybvig (1983) analysis, a small value of ϵ has just the opposite affect. When ϵ is small, the right side of (15) is small as the bank does not have a large amount of resources *ex post*. Consequently, runs are likely for small ϵ unless the household is only slightly risk averse.

Hence when ϵ is small the *ex post* flexibility of the regulator to optimally choose the amount of liquidation is stabilizing. The alternative of forcing complete liquidation leads to bank fragility.

4 Deposit insurance

In the previous section, a regulator controlled the liquidation decision of the bank and was able to redistribute the bank's resources only across depositors of the same endowment type at that bank. The regulator had no access to tax revenues to supplement the illiquid investment of the bank.

In this section, we study the response to a run from a very different perspective. Thus we assume (15) holds so that there is a bank run given the optimal period 0 contract of $(\chi^{*E}(\alpha^0), \chi^{*L}(\alpha^0))$. Our focus is on the conditions under which deposit insurance is provided *ex post*, that is after a run, and funded through the taxation power of a Treasury. Thus there are resources available here that were not present in the regulator's problem.

As before, when a run is underway, the bank informs the Treasury that it has served π early households and others are demanding their deposits. In contrast to the OLA discussion, the Treasury is unable to control the liquidation decision of the bank. Further, there is, by assumption, no suspension at the bank level. If there is a run, the bank meets the demands of depositors until its resources, both liquid and liquidated illiquid investments, are exhausted. At this point, the Treasury has the authority to provide deposit insurance.

We ask: Will deposit insurance be provided by the Treasury? Whether deposit insurance is provided will depend on the tax system that is used to finance these flows. A key issue is the redistributive effects of deposit insurance and the taxation needed to fund these transfers.

We first describe household welfare when deposit insurance is provided and when it is not. We then characterize the conditions under which deposit insurance increases social welfare and thus when it will be provided *ex post*.

Throughout, the provision of deposit insurance is assumed to cover all depositors. However, the structure of the tax system to fund these transfers is an object of the analysis. In this way, the net flow to the depositors is endogenous.

The timing and the Treasury's problem are summarized as follows.

- The bank contacts the Treasury when total withdrawals exceeds the liquid investment.
- The Treasury.
  - does not limit the liquidation of the bank's assets;
  - decides whether to provide deposit insurance or not;
  - taxes households to fund the provision of any deposit insurance.

4.1 Household welfare

If *ex post* the government provides deposit insurance in the event of a run by all households, leading to full liquidation of the illiquid investment, social welfare is:

$$W^{\mathrm{DI}} = \int \omega\left(\alpha^0\right) v\left(\bar{\alpha} + \chi^{*\mathrm{E}}\left(\alpha^0\right) - T\left(\alpha^0\right)\right) f\left(\alpha^0\right) \mathrm{d}\alpha^0 \tag{16}$$

where $\chi^{*\mathrm{E}}(\alpha^0)$ is the total promised by the bank to an early household of type α^0. In this expression, $\omega(\alpha^0)$ is again the weight in the social welfare function given to a type α^0 household.

If $\omega(\alpha^0)$ is a constant, then the objective of the government is just a population weighted average of household expected utility. In general, the structure of $\omega(\alpha^0)$ will be relevant for gauging the costs and benefits of the redistribution associated with DI.

Another key element in the redistribution is the tax system used to pay for DI. In (16), $T(\alpha^0)$ is the tax paid by an agent of type α^0. Government budget balance requires

$$\int\left(T\left(\alpha^0\right)+\left[\phi\left(\alpha^0\right)+\left(1-\phi\left(\alpha^0\right)\right)\epsilon\right]\alpha^0\right) f\left(\alpha^0\right)\mathrm{d}\alpha^0=\int\chi^{*\mathrm{E}}\left(\alpha^0\right) f\left(\alpha^0\right)\mathrm{d}\alpha^0. \tag{17}$$

The left-hand side of this expression is the total tax revenues collected by the government plus the sum of the liquid investment and the liquidated value of the illiquid investment. The right-hand side is the total paid to depositors.

This specification of the tax burden assumes that all households are taxed, not just those who were not served during the bank run. While the banking activities of households may be constrained by sequential service, the government is assumed to have the ability to directly tax the period 1 endowment of households, as in $T(\alpha^0)$. Assume $\bar{\alpha} \geq \alpha_+$ so that this endowment is sufficient to meet the taxation needs of the government regardless of the deposit contract offered by banks.[17] The government does not condition taxation on the household taste type. In this sense, it has the same informational constraints as the banks.

If, *ex post*, there is a bank run without any government intervention, then social welfare is given by:

$$W^{\mathrm{NI}} = \int \omega\left(\alpha^0\right)\left[\zeta v\left(\bar{\alpha} + \chi^{*\mathrm{E}}\left(\alpha^0\right)\right) + (1-\zeta) v(\bar{\alpha})\right] \mathrm{d}F\left(\alpha^0\right). \tag{18}$$

Here ζ is the probability a household obtains the full return on its deposit at a representative bank under sequential service. As all households are assumed to be served with equal probability,

$$\zeta = \frac{\int\left[\phi\left(\alpha^0\right) + \left(1-\phi\left(\alpha^0\right)\right)\epsilon\right]\alpha^0 f\left(\alpha^0\right)\mathrm{d}\alpha^0}{\int \chi^{*\mathrm{E}}\left(\alpha^0\right) f\left(\alpha^0\right)\mathrm{d}\alpha^0}. \tag{19}$$

[17] With this restrictions on $\bar{\alpha}$, if the entire period 0 endowment is used to finance $\chi^{*\mathrm{E}}(\alpha^0)$, the government has a sufficiently large tax base to fund DI.

The welfare values with and without DI are both calculated at the start of period 1. This is because the government lacks the ability to commit to DI before agents make withdrawal decisions. The government can only react to an actual bank run in period 1.

4.2 Welfare effects of DI

The government has an incentive to provide deposit insurance iff $\Delta \equiv W^{\mathrm{DI}} - W^{\mathrm{NI}} \geq 0$. We can write this difference as

$$\begin{aligned}\Delta = &\int \omega\left(\alpha^0\right)\left[v\left(\chi^{*\mathrm{E}}\left(\alpha^0\right)+\bar{\alpha}-T\left(\alpha^0\right)\right)-v\left(\chi^{*\mathrm{E}}\left(\alpha^0\right)+\bar{\alpha}-\bar{T}\right)\right] f\left(\alpha^0\right) \mathrm{d}\alpha^0\\ &+\int \omega\left(\alpha^0\right)\left[v\left(\chi^{*\mathrm{E}}\left(\alpha^0\right)+\bar{\alpha}-\bar{T}\right)-v\left(\zeta\chi^{*\mathrm{E}}\left(\alpha^0\right)+\bar{\alpha}\right)\right] f\left(\alpha^0\right) d\left(\alpha^0\right)\\ &+\int \omega\left(\alpha^0\right)\left[v\left(\zeta\chi^{*\mathrm{E}}\left(\alpha^0\right)+\bar{\alpha}\right)-\zeta v\left(\chi^{*\mathrm{E}}\left(\alpha^0\right)+\bar{\alpha}\right)-(1-\zeta)v\left(\bar{\alpha}\right)\right] f\left(\alpha^0\right)\mathrm{d}\alpha^0\end{aligned} \tag{20}$$

where $\bar{T} = \int T(\alpha^0) f(\alpha^0)\mathrm{d}\alpha^0$.

Here there are three terms. The first two terms capture the two types of redistribution through deposit insurance. One effect is through differences in tax obligations and the other effect comes from differences in deposit levels across types. The third term is the insurance effect of deposit insurance.

Specifically, the first term captures the redistribution from taxes. It is the utility difference between consumption with deposit insurance and type dependent taxes and consumption with deposit insurance and type independent taxes, $\bar{T}$.

The second term captures the effects of *redistribution* through deposit insurance. The term $v(\chi^{*\mathrm{E}}(\alpha^0) + \bar{\alpha} - \bar{T}) - v(\zeta\chi^{*\mathrm{E}}(\alpha^0) + \bar{\alpha})$ is the difference in utility between the consumption allocation if a type α^0 household gets his promised allocation and bears a tax of $\bar{T}$ and the allocation obtained if all households received a fraction ζ of their promised allocation. This second part is the utility of the expected consumption if there are runs without deposit insurance.

The third term captures the *insurance* gains from DI. It is clearly positive if $v(c)$ is strictly concave. These gains are independent of the shape of $\omega(\alpha^0)$.

The key trade-off to the provision of DI *ex post* is whether the insurance gains dominate the redistribution effects. The insurance gains are apparent if there is no heterogeneity across households, so $F(\alpha^0)$ is degenerate. In this case, deposit insurance is valued as it provides risk sharing across households of the uncertainty coming from sequential service.

Proposition 4 *If $F(\alpha^0)$ is degenerate, $v(c)$ is strictly concave, then the government will have an incentive to provide deposit insurance. Runs are prevented.*

Proof In this case, the first two terms of (20) are zero. The third term is strictly positive since $v(\cdot)$ is strictly concave. Hence $\Delta > 0$. □

This is parallel to the standard interpretation of the Diamond and Dybvig (1983) model although it obtains here without commitment. It highlights the insurance gain from DI when there are no costs of redistribution. Here the insurance benefit is enough to motivate the provision of deposit insurance without commitment.

When there is heterogeneity across households, these insurance gains may be offset by redistribution costs. In the next two subsections, we consider two situations. In the first one, the tax system to fund DI is set at the same time the decision is made to provide DI or not. In this case, there is enough flexibility in the tax system to offset any redistribution effects of DI. In the second scenario, we take the tax system as given and explore the incentives to provide DI.

4.3 Optimal taxation: DI will be provided

Consider a government which can choose the tax system used to finance DI at the same time it is choosing to provide insurance or not. This can be interpreted as the choice of a supplemental tax to fund DI.

In this setting, W^{DI} is the solution to an optimal tax problem:

$$W^{\mathrm{DI}} = \max_{T(\alpha^0)} \int \omega\left(\alpha^0\right) v\left(\chi^{*\mathrm{E}}\left(\alpha^0\right) + \bar{\alpha} - T\left(\alpha^0\right)\right) f\left(\alpha^0\right) \mathrm{d}\alpha^0 \tag{21}$$

subject to a government budget constraint (17). The first-order condition implies that $\omega(\alpha^0)v'(\chi^{*\mathrm{E}}(\alpha^0)+\bar{\alpha}-T(\alpha^0))$ is independent of α^0. This creates a connection between $\omega(\alpha^0)$ and $T(\alpha^0)$ which can be used to evaluate the gains to DI.

Proposition 5 *If $T(\alpha^0)$ solves the optimization problem* (21), *then deposit insurance is always provided.*

Proof Using the first-order condition from (21), we rewrite (20) as:

$$\begin{aligned}\Delta = &\int \frac{\left[v\left(\chi^{*\mathrm{E}}\left(\alpha^0\right) + \bar{\alpha} - T\left(\alpha^0\right)\right) - v\left(\zeta\chi^{*\mathrm{E}}\left(\alpha^0\right) + \bar{\alpha}\right)\right]}{v'\left(\chi^{*\mathrm{E}}\left(\alpha^0\right) + \bar{\alpha} - T\left(\alpha^0\right)\right)} f\left(\alpha^0\right) d\left(\alpha^0\right)\\ &+ \int \omega\left(\alpha^0\right)\left[v\left(\zeta\chi^{*\mathrm{E}}\left(\alpha^0\right) + \bar{\alpha}\right) - \zeta v\left(\chi^{*\mathrm{E}}\left(\alpha^0\right) + \bar{\alpha}\right)\right.\\ &\left.-(1-\zeta)v\left(\bar{\alpha}\right)\right] f\left(\alpha^0\right) \mathrm{d}\alpha^0\end{aligned}$$

The second term is positive as argued previously. The first term can be shown to be positive as well.

To see this, do a second-order approximation of the second part of the first term, $v(\zeta\chi^{*\mathrm{E}}(\alpha^0) + \bar{\alpha})$, around the first part, $v(\chi^{*\mathrm{E}}(\alpha^0) + \bar{\alpha} - T(\alpha^0))$. Using the fact that $\int T(\alpha^0) f(\alpha^0)\mathrm{d}\alpha^0 = (1-\zeta)\int \chi^{*\mathrm{E}}(\alpha^0) f(\alpha^0)\mathrm{d}\alpha^0$, the first term reduces to

$$\int \frac{-\left((1-\zeta)\chi^{*\mathrm{E}}\left(\alpha^0\right) - T\left(\alpha^0\right)\right)^2 v''\left(\chi^{*\mathrm{E}}\left(\alpha^0\right) + \bar{\alpha} - T\left(\alpha^0\right)\right)}{v'\left(\chi^{*\mathrm{E}}\left(\alpha^0\right) + \bar{\alpha} - T\left(\alpha^0\right)\right)} f\left(\alpha^0\right) d\left(\alpha^0\right) \tag{22}$$

which is positive as $v(\cdot)$ is strictly concave. Thus $\Delta > 0$. □

Why is there always a gain to deposit insurance here? Because with this *ex post* tax scheme, the current government can undo any undesirable redistribution coming from DI. Thus the redistribution costs are not present.

This intervention will (weakly) prevent a run. Along this path, all agents receive $\chi^{*E}(\alpha^0)$, whether they ran and were served by the bank directly or were later served by the deposit insurance authorities.

This result is important for the design of policy. As governments strive to make clear the conditions under which deposit insurance and other financial bailouts will be provided *ex post*, they ought to articulate how revenues will be raised to finance those transfers. If a government says it will not rely on existing tax structures but instead will, in effect, solve (21), then private agents will know that the government will have enough flexibility in taxation to overcome any redistributive costs of deposit insurance. This will enhance the credibility of a promise to provide deposit insurance *ex post*.

4.4 Type independent taxes: DI may not be provided

In some cases, a government may not have the flexibility due to time lags and political obstacles to levy a special tax to fund a bailout. Instead, the use of general tax revenues may be required, leading to additional taxation to fund existing government spending. If the tax system to fund DI is not set *ex post*, costly redistribution may arise. Then the trade-off between insurance gains and redistribution costs emerges. As we shall see, these redistribution effects can be large enough to offset insurance gains.

To study these issues, we return to (20) which cleanly distinguishes the redistribution and insurance effects. We start with a case in which taxes are independent of type to gain some understanding of the trade-off and then return to the more general case where taxes depend on agent types.

To focus on one dimension of the redistributive nature of deposit insurance, assume that taxes are type independent: $T(\alpha^0) = \bar{T}$ for all α^0. Under this tax system, (20) simplifies to:

$$\begin{aligned}\Delta = &\int \omega\left(\alpha^0\right)\left[v\left(\chi^{*E}\left(\alpha^0\right) - \bar{T} + \bar{\alpha}\right) - v\left(\zeta\chi^{*E}\left(\alpha^0\right) + \bar{\alpha}\right)\right] f\left(\alpha^0\right) d\left(\alpha^0\right)\\ &+ \int \omega\left(\alpha^0\right)\left[v\left(\zeta\chi^{*E}\left(\alpha^0\right) + \bar{\alpha}\right) - \zeta v\left(\chi^{*E}\left(\alpha^0\right) + \bar{\alpha}\right)\right.\\ &\left. - (1-\zeta)v\left(\bar{\alpha}\right)\right] f\left(\alpha^0\right) \mathrm{d}\alpha^0. \end{aligned} \tag{23}$$

If taxes are independent of type, then the government budget constraint implies

$$\bar{T} = \int \left(\chi^{*E}\left(\alpha^0\right) - \left[\phi\left(\alpha^0\right) + \left(1-\phi\left(\alpha^0\right)\right)\epsilon\right]\alpha^0\right) f\left(\alpha^0\right) \mathrm{d}\alpha^0. \tag{24}$$

With type independent taxes, redistribution arises solely from differences in deposits across types. In some cases, this redistribution can be costly to society. The next two propositions require the additional restriction on $v(\cdot)$, mentioned earlier, that $\zeta v'(\zeta c + \bar{\alpha})$ is increasing in ζ.

Proposition 6 *If $\omega(\alpha^0)$ is weakly decreasing in α^0, then the redistribution effect of deposit insurance reduces social welfare.*

Proof The effect of redistribution is captured by the first term in (23). Using ζ from (19), $\bar{T} = (1-\zeta)\int \chi^{*\mathrm{E}}(\alpha) f(\alpha)\mathrm{d}\alpha$. Letting $\hat{c}(\alpha^0) = \zeta\chi^{*\mathrm{E}}(\alpha^0) + \bar{\alpha}$ and $\bar{c} \equiv \int \hat{c}(\alpha^0) f(\alpha^0)\mathrm{d}\alpha^0$, the first term in (23) becomes

$$\int \omega\left(\alpha^0\right)\left[v\left(\frac{1}{\zeta}\left(\hat{c}\left(\alpha^0\right)-\bar{c}\right)+\bar{c}\right)-v\left(\hat{c}\left(\alpha^0\right)\right)\right] f\left(\alpha^0\right)\mathrm{d}\alpha^0. \tag{25}$$

The first consumption allocation, $\frac{1}{\zeta}(\hat{c}(\alpha^0)-\bar{c})+\bar{c}$, is a mean-preserving spread of the second, $\hat{c}(\alpha^0)$. Both have the same mean of $\bar{c}$ and since $1 > \zeta$ the variance of the first consumption allocation is larger. From the results on mean-preserving spreads, if $v(c)$ is strictly concave

$$\int\left[v\left(\chi^{*\mathrm{E}}\left(\alpha^0\right)-\bar{T}+\bar{\alpha}\right)-v\left(\zeta\chi^{*\mathrm{E}}\left(\alpha^0\right)+\bar{\alpha}\right)\right] f\left(\alpha^0\right)d\left(\alpha^0\right)<0. \tag{26}$$

Using the fact that the welfare weights integrate to one, we can write the first term in (23) as

$$\begin{aligned}\int&\left[v\left(\chi^{*\mathrm{E}}\left(\alpha^0\right)-\bar{T}+\bar{\alpha}\right)-v\left(\zeta\chi^{*\mathrm{E}}\left(\alpha^0\right)+\bar{\alpha}\right)\right] f\left(\alpha^0\right)d\left(\alpha^0\right)\\&+\operatorname{cov}\left(\omega\left(\alpha^0\right), v\left(\chi^{*\mathrm{E}}\left(\alpha^0\right)-\bar{T}+\bar{\alpha}\right)-v\left(\zeta\chi^{*\mathrm{E}}\left(\alpha^0\right)+\bar{\alpha}\right)\right).\end{aligned} \tag{27}$$

From the discussion above, the first term is negative. If $\omega(\alpha^0)$ is independent of α^0, then the covariance term in (27) is zero and so (27) is negative. This corresponds to costly redistribution.

If $\omega(\alpha^0)$ is decreasing in α^0, then social welfare puts less than the population weight on high α^0 agents. The difference, $v(\chi^{*\mathrm{E}}(\alpha^0)-\bar{T}+\bar{\alpha})-v(\zeta\chi^{*\mathrm{E}}(\alpha^0)+\bar{\alpha})$ in (27) is increasing in α^0. This follows from the assumption that $\zeta v'(\zeta c+\bar{\alpha})$ is increasing in ζ and the fact that $\chi^{*\mathrm{E}}(\alpha^0)$ is monotonically increasing from Proposition 1. As a consequence, the covariance term in (27) is negative. Thus the redistribution effect reduces welfare if either $\omega(\alpha^0)$ is either independent of, or decreasing in, α^0. □

Proposition 6 makes clear that the provision of DI may entail distribution effects that are socially undesirable. There are two key pieces of the argument. First, if welfare weights are type independent, then the provision of deposit insurance financed by a lump-sum tax creates a mean-preserving spread in consumption. This is welfare reducing. Second, if welfare weights are decreasing so that the rich are valued less than the poor in the social welfare function, then the redistribution from poor to rich from the provision of deposit insurance reduces social welfare further. This second influence is captured by the covariance term in (27).

This result contrasts with Proposition 4 which eliminates the redistribution issue and thus highlights the gains from the provision of deposit insurance. One important factor in the trade-off between insurance and redistribution is the underlying distribution of social (political) weights.

Proposition 7 *If $\omega(\alpha^0)$ is sufficiently increasing in α^0, then deposit insurance is provided. Further, if $\omega(\alpha^0)$ is sufficiently decreasing in α^0 and households are not too risk averse, then deposit insurance is not provided.*

Proof Let $\Omega(\alpha^0) \equiv v(\chi^{*\mathrm{E}}(\alpha^0) - \bar{T} + \bar{\alpha}) - v(\zeta\chi^{*\mathrm{E}}(\alpha^0) + \bar{\alpha})$. This is contained in the first term of (23). By the assumption that $\zeta v'(\zeta c + \bar{\alpha})$ is increasing in ζ, using $\bar{T} > 0$ and $\chi^{*\mathrm{E}}(\alpha^0)$ is increasing in α^0, $\Omega'(\alpha^0) = [v'(\chi^{*\mathrm{E}}(\alpha^0) - \bar{T} + \bar{\alpha}) - \zeta v'(\zeta\chi^{*\mathrm{E}}(\alpha^0) + \bar{\alpha})]\chi^{*\mathrm{E}'}(\alpha^0) > 0$.

Using the construction in the proof of Proposition 6, there exists $\tilde{\alpha}$ such that $\zeta\chi^{*\mathrm{E}}(\tilde{\alpha}) + \bar{\alpha} = \bar{c}$ and therefore $\Omega(\tilde{\alpha}) = 0$. Thus for $\alpha^0 > \tilde{\alpha}$, $\Omega(\alpha^0) > 0$.

If $\omega(\alpha^0)$ is sufficiently large for $\alpha^0 \geq \tilde{\alpha}$ and sufficiently close to zero for $\alpha^0 < \tilde{\alpha}$, then the first term in (23) is positive and thus Δ is positive. In this case, there are both insurance and redistributive gains from the provision of DI.

By a similar logic, for $\alpha^0 < \tilde{\alpha}$, $\Omega(\alpha^0) < 0$. If $\omega(\alpha^0)$ is sufficiently large for $\alpha^0 \leq \tilde{\alpha}$ and sufficiently small for $\alpha^0 \geq \tilde{\alpha}$, then the first term in (23) is negative. If households are not too risk adverse insurance gains will be small. Given these small insurance gains, there will exist welfare weights such that Δ will be negative. □

These propositions highlight the redistributive effects of deposit insurance. Proposition 6 provides sufficient conditions for redistribution to be costly. Proposition 7 makes clear that the effects of redistribution depend on the distribution of weights in the social objective function. If the rich are weighted heavily enough, the redistribution from deposit insurance is a gain rather than a cost. Then deposit insurance will be provided.

Else, as in the second part of Proposition 7, the redistribution may be so undesirable that it dominates insurance gains. In this case, deposit insurance will not be provided.

5 Combining deposit insurance and orderly liquidation

As discussed in Diamond and Dybvig (1983), a key piece of government intervention is avoiding inefficient liquidation of long-term illiquid investments. As they put it, "What is crucial is that deposit insurance frees the asset liquidation policy from strict dependence on the volume of withdrawals." Thus it is important that not only deposit insurance be provided in some form but that fire sale liquidations be prevented as well. This section does so by combining the OLA and the provision of DI. Are these policies substitutes or complements? Does credibility in the provision of deposit insurance support liquidation policies that might not have otherwise been credible?

We supplement the analysis to allow both optimal liquidation and a deposit insurance scheme funded through taxation. The regulator and Treasury jointly choose the liquidation policy as well as the taxation needed to finance DI. As before, DI entails complete insurance to early households.

The authorities jointly intervene as soon as a run is under way. Successful withdrawers in period 1 of type α^0 receive $\chi^{*\mathrm{E}}(\alpha^0)$ directly from the bank. Unsuccessful withdrawers receive $\chi^{*\mathrm{E}}(\alpha^0)$ if they are of the early type and $\tilde{\chi}^{\mathrm{L}}(\alpha^0)$ if they are late consumers. The financing of the transfers received by unsuccessful withdrawers is through taxation and (partial) liquidation of the illiquid investment. All agents are taxed according to their endowment. Using this framework, we study the optimal joint liquidation and taxation policy and whether the resulting allocation prevents a run. Finally, we return to the question of whether DI will be provided.

Given that a run has occurred and the authorities have been contacted, the optimization problem is to choose $(\tilde{\chi}^{\mathrm{L}}(\alpha^0), Z(\alpha^0), Q(\alpha^0), T(\alpha^0))$ to maximize:

$$\begin{aligned} W = \max(1-\pi)\Big[\int \omega\left(\alpha^0\right)\Big[\pi v\left(\bar{\alpha} + \chi^{*\mathrm{E}}\left(\alpha^0\right) - T\left(\alpha^0\right)\right) \\ + (1-\pi)v\left(\bar{\alpha} + \tilde{\chi}^{\mathrm{L}}\left(\alpha^0\right) - T\left(\alpha^0\right)\right)\Big]\mathrm{d}F\left(\alpha^0\right)\Big] \\ + \pi\int \omega\left(\alpha^0\right) v\left(\bar{\alpha} + \chi^{*\mathrm{E}}\left(\alpha^0\right) - T\left(\alpha^0\right)\right)\mathrm{d}F\left(\alpha^0\right). \end{aligned} \tag{28}$$

The optimization is constrained by two resource constraints, written in per capita terms:

$$(1-\pi)\pi\chi^{*\mathrm{E}}\left(\alpha^0\right) = \epsilon Z\left(\alpha^0\right) + Q\left(\alpha^0\right) \tag{29}$$

and

$$(1-\pi)^2\tilde{\chi}^{\mathrm{L}}\left(\alpha^0\right) = \left[\left(1-\phi\left(\alpha^0\right)\right)\alpha^0 - Z\left(\alpha^0\right)\right]R \tag{30}$$

for each α^0. To guarantee incentive compatibility,

$$\tilde{\chi}^{L}\left(\alpha^0\right) \geq \chi^{*\mathrm{E}}\left(\alpha^0\right) \tag{31}$$

for each α^0. Finally, liquidation must be nonnegative

$$Z\left(\alpha^0\right) \geq 0 \tag{32}$$

for each α^0.

The government budget constraint holds across the households, rather than for each α^0, and is

$$\int Q\left(\alpha^0\right) f\left(\alpha^0\right) d\left(\alpha^0\right) = \int T\left(\alpha^0\right) f\left(\alpha^0\right) d\left(\alpha^0\right). \tag{33}$$

This condition guarantees that the total amount of transfers to support the consumption of early households is financed by taxes levied on all agents, including those already served. The tax can depend on endowment type but not on the realized taste type, consistent with sequential service.

In (29), the payment to early households of $\chi^{*\mathrm{E}}(\alpha^0)$ is financed either through liquidations, $Z(\alpha^0)$, or transfers, $Q(\alpha^0)$. The cost of liquidations is indicated in (30). The cost of transfers is through the collection of taxes on all households, as in (33).

The incentive compatibility constraint, (31), insures that late household's have an incentive to reveal their type. Note that the early households all receive the full-information allocation of $\chi^{*\mathrm{E}}(\alpha^0)$. Hence (31) not only is an incentive compatibility condition but implies that the intervention must prevent runs.

The presence of DI creates a redistributive dimension to liquidation policy. Though liquidations are still, by assumption, bank specific, they now have a redistributive element across endowment types through the tax system. That is, liquidations at, say, a high endowment type bank can be offset by lower transfers to that bank. Instead the transfers can be used to finance the consumption of households at low endowment type banks.

We first study the optimal allocation with $F(\cdot)$ degenerate in order to understand the trade-off between liquidation and taxation without redistribution across income groups. Building on this, we then study the problem with heterogenous households.

Proposition 8 *If $F(\cdot)$ is degenerate, then there exists a critical $\left(\frac{R}{\epsilon}\right)^{\mathrm{c}}$ such that the solution to (28) sets $Z > 0$ if $\frac{R}{\epsilon} < (\frac{R}{\epsilon})^{\mathrm{c}}$ and $Z = 0$ otherwise. Runs are prevented.*

Proof When $F(\cdot)$ is degenerate, the tax on each household is $\bar{T} = \chi^{*\mathrm{E}}(1-\pi)\pi - \epsilon Z$ from (29). Inserting this into the consumption levels of early and late consumers there is a single first-order condition associated with the choice of *Z*:

$$\frac{\lambda^z}{\epsilon} = (1-\pi)^2 v'\left(c^{\mathrm{L}}\right)\left(\frac{R}{\epsilon(1-\pi)^2} - 1\right) - (2-\pi)\pi v'\left(c^{*\mathrm{E}}\right) \tag{34}$$

where λ^z is the multiplier associated with (32), $c^{*\mathrm{E}} = \bar{\alpha} + \chi^{*\mathrm{E}}(\alpha^0) - \chi^{*\mathrm{E}}(1-\pi)\pi + \epsilon Z$ and $\tilde{c}^{\mathrm{L}} = \bar{\alpha} + \tilde{\chi}^{\mathrm{L}}(\alpha^0) - \chi^{*\mathrm{E}}(1-\pi)\pi + \epsilon Z$.

Define $(\frac{R}{\epsilon})^{\mathrm{c}} \equiv (1-\pi)^2 + (2-\pi)\frac{v'(c^{*\mathrm{E}})}{v'(\tilde{c}^{\mathrm{L}})}$ with $Z = 0$. Note that $c^{*\mathrm{E}}$ and $\tilde{c}^{\mathrm{L}}$ are independent of (R, ϵ) when $Z = 0$.

If $\frac{R}{\epsilon} > (\frac{R}{\epsilon})^{\mathrm{c}}$, $\lambda^z > 0$ for (34) to hold. In this case, $\tilde{c}^{\mathrm{L}} > c^{*\mathrm{L}} > c^{*\mathrm{E}}$. Hence, the allocation is both incentive compatible and runs-proof. If $\frac{R}{\epsilon} \le (\frac{R}{\epsilon})^{\mathrm{c}}$, $\lambda^z = 0$ for (34) to hold and there are liquidations. In this case, (34) becomes

$$v'\left(c^{\mathrm{L}}\right)\left(\frac{R}{\epsilon} - (1-\pi)^2\right) = (2-\pi)\pi v'\left(c^{*\mathrm{E}}\right). \tag{35}$$

As $(2-\pi)\pi = 1-(1-\pi)^2$, $c^{\mathrm{L}} > c^{*\mathrm{E}}$ for (34) to hold. Thus the allocation is both incentive compatible and there is no run. □

The degenerate case indicates that liquidation is suboptimal when its cost, $\frac{R}{\epsilon}$, is too large. Instead, redistribution arises through the tax system. However, if $\frac{R}{\epsilon}$ is small enough, liquidation is used for redistribution since it is a more direct way to transfer resources from the late households to the early ones not served during the run.

As we shall see, this is not necessarily the case when there are heterogeneous households. The system of lump-sum taxes is regressive, redistributing from poor to rich households. Consequently, liquidations may coexist along with the use of the tax system to finance DI.

To see this interaction, assume that taxes are lump-sum and independent of the endowment type: $T(\alpha^0) = \bar{T}$ for all α^0. Moreover assume that all agents are given equal weight. The following proposition characterizes the optimal liquidation policy:

Proposition 9 *If taxes are independent of* α^0, $\omega(\alpha^0)$ *is constant and* $\epsilon > 0$, *the solution to* (28) *entails no liquidations for low values of* α^0 *and positive liquidation for sufficiently high values of* α^0. *Runs are prevented.*

Proof When $F(\cdot)$ is not degenerate, the lump-sum tax on each household is

$$\bar{T} = \int \left[(1-\pi)\pi\chi^{\mathrm{E}}\left(\alpha^0\right) - \epsilon Z\left(\alpha^0\right)\right] \mathrm{d}F\left(\alpha^0\right). \tag{36}$$

Substituting this into the consumption levels of early and late consumers, the first-order condition associated with the choice of $Z(\alpha^0)$ is:

$$\begin{aligned} &\int \left[(2-\pi)\pi v'\left(c^{*\mathrm{E}}(x)\right) + (1-\pi)^2 v'\left(\tilde{c}^{\mathrm{L}}(x)\right)\right] \mathrm{d}F(x) \\ &\quad - v'\left(\tilde{c}^{\mathrm{L}}\left(\alpha^0\right)\right)\frac{R}{\epsilon} + \lambda^{\mathrm{IC}}\left(\alpha^0\right) + \frac{\lambda^z\left(\alpha^0\right)}{\epsilon} = 0 \end{aligned} \tag{37}$$

where $\lambda^z(\alpha^0)$ is the multiplier associated with (32) and $\lambda^{\mathrm{IC}}(\alpha^0)$ is the multiplier associated with (31) for type α^0, $c^{*\mathrm{E}}(x) = \bar{\alpha} + \chi^{*\mathrm{E}}(x) - \bar{T}$ and $\tilde{c}^{\mathrm{L}}(x) = \bar{\alpha} + \tilde{\chi}^{\mathrm{L}}(x) - \bar{T}$ where x indicates an arbitrary level of endowment. The first term is common to all of the first-order conditions with respect to $Z(\cdot)$. The other terms are type α^0 specific. The solution is continuous in α^0 from the maximum theorem.

We initially study the solution when the incentive compatibility constraint is not binding. Later we return to this initial stipulation.

If $\lambda^Z(\hat{\alpha}^0) > 0$, then for any $\alpha^0 < \hat{\alpha}^0$, $Z(\alpha^0) = 0$ as well in order for (38) to hold. To see this, suppose $\alpha^0 < \hat{\alpha}^0$ but $Z(\alpha^0) > 0$. From (30), $Z(\alpha^0) > 0$ implies that $\tilde{\chi}^{\mathrm{L}}(\alpha^0) < \frac{\chi^{*\mathrm{L}}(\alpha^0)}{1-\pi}$ while $Z(\alpha^0) = 0$ implies $\tilde{\chi}^{\mathrm{L}}(\hat{\alpha}^0) = \frac{\chi^{*\mathrm{L}}(\hat{\alpha}^0)}{1-\pi}$.

From Proposition 1, $\hat{\alpha}^0 > \alpha^0$ implies $\chi^{*\mathrm{L}}(\hat{\alpha}^0) > \chi^{*\mathrm{L}}(\alpha^0)$. Using this, $\tilde{\chi}^{\mathrm{L}}(\hat{\alpha}^0) > \tilde{\chi}^{\mathrm{L}}(\alpha^0)$. For (38) to hold, $\lambda^Z(\hat{\alpha}^0) > 0$ and $\lambda^Z(\alpha^0) = 0$ implies $\tilde{\chi}^{\mathrm{L}}(\alpha^0) > \tilde{\chi}^{\mathrm{L}}(\hat{\alpha}^0)$. This is a contradiction.

By this same argument, if $Z(\hat{\alpha}^0) > 0$ and $\lambda^Z(\hat{\alpha}^0) = 0$, then for any $\alpha^0 > \hat{\alpha}^0$, $Z(\alpha^0) > 0$ as well. Thus there is a critical value of α^0 such that there are liquidations only for endowment types above this level.

By continuity, there exists $\tilde{\alpha}^0$ such that $Z(\tilde{\alpha}^0) = 0$ and $\lambda^Z(\tilde{\alpha}^0) = 0$. Here there are no liquidations and the constraint is not binding. The first-order condition with respect to liquidation for this type would be

$$\int \left[(2-\pi)\pi v'\left(c^{*\mathrm{E}}(x)\right) + (1-\pi)^2 v'\left(\tilde{c}^{\mathrm{L}}(x)\right)\right] \mathrm{d}F(x) = v'\left(\tilde{c}^{\mathrm{L}}\left(\tilde{\alpha}^0\right)\right)\frac{R}{\epsilon}. \tag{38}$$

From the arguments above, for $\alpha^0 < \tilde{\alpha}^0$, $Z(\alpha^0) = 0$ and $\tilde{\chi}^{\mathrm{L}}(\alpha^0)$ will be increasing in α^0. From $\alpha^0 > \tilde{\alpha}^0$, $Z(\alpha^0) > 0$. If the distribution of α^0 is sufficiently dispersed, $\tilde{\alpha}$ will be interior.

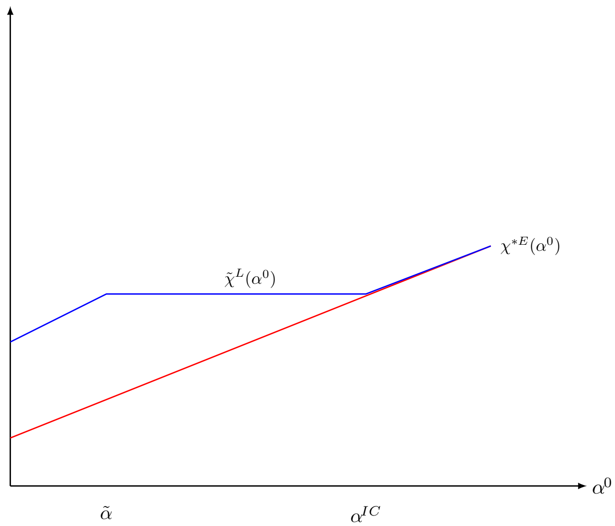

Fig. 2 Optimal liquidation with deposit insurance

To see that the allocations are incentive compatible, for $\alpha^0 < \tilde{\alpha}$ there are no liquidations and $\tilde{\chi}^{\mathrm{L}}(\alpha^0) > \chi^{*\mathrm{L}}(\alpha^0) > \chi^{*\mathrm{E}}(\alpha^0)$ so late households have higher consumption than early ones. When there is liquidation, (37) holds with $\lambda^z(\alpha^0) = 0$. If the incentive compatibility constraint is not binding, then $\tilde{\chi}^{\mathrm{L}}(\alpha^0)$ will be independent of α^0.

As noted above, we stipulated that the incentive compatibility condition was not binding. But it can be that in the solution without this constraint, the consumption levels of late high α^0 households will fall below $\chi^{*\mathrm{E}}(\alpha^0)$. In this event, the solution to (37) must be modified to allow $\tilde{\chi}^{\mathrm{L}}(\alpha^0) \geq \chi^{*\mathrm{E}}(\alpha^0)$ to bind for sufficiently large α^0. In (37), this implies that $\lambda^{\mathrm{IC}}(\alpha^0) > 0$. Consequently, the liquidations from the high α^0 will be modified so that the outcome is incentive compatible. Note that there must still be liquidations for the high α^0 households. Else, they will receive $\tilde{\chi}^{\mathrm{L}}(\alpha^0) > \chi^{*\mathrm{L}}(\alpha^0) > \chi^{*\mathrm{E}}(\alpha^0)$ and the incentive constraint will not be binding.

In either case, the contract satisfies the incentive compatibility condition. Since all early consumers obtain the same level of consumption whether they were served initially or not, the incentive compatibility condition, $\tilde{\chi}^{\mathrm{L}}(\alpha^0) > \chi^{*\mathrm{E}}(\alpha^0)$, implies the contract is runs-proof. □

Figure 2 illustrates the solution. There are two critical levels of α^0. For $\alpha^0 \leq \tilde{\alpha}$, there are no liquidations and $\tilde{\chi}^{\mathrm{L}}(\alpha^0)$ is larger than $\chi^{*\mathrm{L}}(\alpha^0)$ and hence larger than $\chi^{*\mathrm{E}}(\alpha^0)$. For $\alpha^0 > \tilde{\alpha}$ there are optimal liquidations. Over this region, $\tilde{\chi}^{\mathrm{L}}(\alpha^0)$ is independent of α^0 as liquidations increase in α^0. For $\alpha^0 > \alpha^{\mathrm{IC}}$, the incentive constraint binds so that $\tilde{\chi}^{\mathrm{L}}(\alpha^0) = \chi^{*\mathrm{E}}(\alpha^0)$.

This result shows the power of an integrated regulator and Treasury. The liquidation policy is optimal: Relatively poor late households' illiquid investment is left intact to fund their late consumption. For relatively rich households, there is some liquidation of the illiquid investment. This liquidated investment along with the revenues from the lump-sum tax are used to support the consumption of early households.

The proposition assumes a constant welfare weight. If $\omega(\alpha^0)$ is not constant, then of course the relative gains and costs of redistribution will enter in the analysis. These effects are apparent from earlier results, such as Proposition 6.

The main finding is that even with a lump-sum tax, the joint intervention of providing deposit insurance along with an optimal liquidation policy is sufficient to eliminate bank runs. And these policies are all implemented without commitment.

With these results, we return to one of the central questions of the paper: will DI be provided? Compared to the difference in social welfare with and without deposit insurance, recall (23), the gains to deposit insurance must be larger since the liquidation policy is optimal. As indicated in Proposition 10, the optimal policy is to liquidate the investment of high α^0 types while the liquidation underlying (23) was total. Hence the gains to deposit insurance must be larger when there is optimal liquidation compared to a setting with total liquidation.

Still, this does not imply that deposit insurance will be provided. As before, the redistribution costs may outweigh insurance gains.

These results assume that taxes are lump sum. This was intended to highlight the resolution of the resulting tension between redistribution through DI and efficient liquidation. Both costly liquidation and taxation are used to finance DI and redistribute across heterogeneous agents. Liquidation arises because the tax system is not sufficiently flexible to offset the undesirable aspects of DI.

From Proposition 5, if taxes are set optimally *ex post* and are dependent on α^0, then deposit insurance is provided and runs are prevented. That result will clearly carry over to the case of optimal liquidation along with deposit insurance since the allocation from Proposition 5 is nested in the version of (28) allowing type dependent taxes. This leads us to a final result on the provision of DI along with optimal liquidation.

Proposition 10 *If taxes depend on the endowment type, the solution to* (28) *entails no liquidations, no runs and the intervention is welfare improving.*

Proof The result that the intervention is welfare improving comes from the fact that in the solution to (28) total liquidation is feasible. Thus the welfare from the solution to (28) cannot be less than the outcome with total liquidation, (21), which, as shown in Proposition 5, dominates the bank runs outcome. So the provision of DI along with optimal liquidation is welfare improving. Since the solution to (28) is incentive compatible, it must be runs-proof.

The first-order conditions for (28) are

$$\epsilon\lambda^{\mathrm{E}}\left(\alpha^0\right) - R\lambda^{\mathrm{L}}\left(\alpha^0\right) + \lambda^{Z}\left(\alpha^0\right) = 0, \tag{39}$$

$$\lambda^{\mathrm{E}}\left(\alpha^0\right) = \mu, \tag{40}$$

$$(1-\pi)^2\left[v'\left(c^{\mathrm{L}}\left(\alpha^0\right)\right) - \lambda^{\mathrm{L}}\left(\alpha^0\right)\right] + \lambda^{\mathrm{IC}}\left(\alpha^0\right) = 0, \tag{41}$$

and

$$\pi(2-\pi)v'\left(c^{\mathrm{E}}\left(\alpha^0\right)\right) + (1-\pi)^2 v'\left(c^{\mathrm{L}}\left(\alpha^0\right)\right) = \mu. \tag{42}$$

From the incentive compatibility constraint, for (42) to hold,

$$v'\left(c^{\mathrm{E}}\left(\alpha^0\right)\right) \geq \mu \geq v'\left(c^{\mathrm{L}}\left(\alpha^0\right)\right). \tag{43}$$

Using this in (41) implies

$$(1-\pi)^2\left[\mu - \lambda^{\mathrm{L}}\left(\alpha^0\right)\right] + \lambda^{\mathrm{IC}}\left(\alpha^0\right) \geq 0. \tag{44}$$

Use (39) and (40) to solve for μ and substitute into (44) to yield:

$$(1-\pi)^2\left[\left(\frac{R}{\varepsilon}-1\right)\lambda^{\mathrm{L}}\left(\alpha^0\right) - \frac{\lambda^z\left(\alpha^0\right)}{\varepsilon}\right] + \lambda^{\mathrm{IC}}\left(\alpha^0\right) \geq 0. \tag{45}$$

Since $\frac{R}{\varepsilon} - 1 > 0$, $\lambda^z(\alpha^0) > 0$ in order for (45) to hold. Thus there are no liquidations. □

The flexible tax system facilitates redistribution across households by their wealth types. The liquidation scheme which redistributes across households by their tastes, early or late, within a wealth type is dominated by the tax system. Thus costly liquidation is avoided.

In fact, the *ex post* provision of DI is sufficient to make the suspension of contractual withdrawals (which corresponds to the absence of liquidation) credible. The commitment problem on liquidation, highlighted by Ennis and Keister (2009), is not present when DI is credibly provided. That is, a policy to suspend withdrawals once π households have been served is credible once it is backed by a DI system funded through flexible taxation. Runs are avoided.

6 Conclusion

This paper studies liquidation policy and the provision of deposit insurance in the absence of commitment. The goal is to characterize these interventions and to see if they are sufficient to prevent bank runs.

Intervention through the control of liquidation policy is appropriate for considering the regulation of financial intermediaries not covered by a deposit insurance scheme. For these institutions, if liquidation is very costly, then the optimal amount of liquidation is limited. This protects illiquid assets and helps to avoid runs. In other circumstances, the illiquid assets of wealthy investors will be partially liquidated to transfer resources to households with liquidity needs. These transfers can be large enough *ex post* that this intervention may not prevent a run.

We interpret deposit insurance broadly to encompass a variety of forms of *ex post* bailout of financial intermediaries. While steps taken recently to support the financial system in a number of countries may have been warranted, these *ex post* interventions have a consequence: agents will now realize that governments will make *ex post* decisions on deposit insurance.

If so, it is natural to investigate the conditions under which deposit insurance will be supplied *ex post*. Initially we do so assuming full liquidation prior to the provision of DI. In our environment with differences in deposit levels, a trade-off emerges between risk sharing and the redistribution created by the funding of the transfers inherent in a deposit insurance system. In some cases, these redistribution costs may be large enough to offset insurance gains. These costs are reflected in the ongoing discussion of bailouts in the US and other countries insofar as those policies entail a regressive redistribution.

From our analysis, the tax system used to finance payments to depositors plays a crucial role in determining whether deposit insurance will be provided. If the tax system is set *ex post* along with deposit insurance, then the government can optimally choose the net transfer and avoid the conflict between insurance and redistribution. But if the deposit insurance must be financed by an *ex ante* tax system that allows for redistributions from the poor to the rich through the provision of deposit insurance, then the credibility of deposit insurance is weakened. This was illustrated through our discussion of lump-sum taxes.

When optimal liquidation and the provision of deposit insurance are put together, the optimal arrangement may involve some liquidation of illiquid assets held by wealthy investors. If taxes are optimally set *ex post* and are dependent on the endowment type, then no liquidations arise. In either case, the resulting allocation is runs-proof.

Throughout the paper, we assumed a simple bank contract that ignores the prospect of runs. Further, the contract did not link the return to an agent to its “place in line”. Nor did we allow the suspension of withdrawals unilaterally by a bank. Instead, suspension was handled through the regulator. These restrictions on the optimal contract support the full-information solution. As long as the combination of optimal regulation and deposit insurance is provided *ex post*, this simple contract is privately optimal and runs-proof as well.

References

Chari, V., Kehoe, P.: Sustainable plans. J. Polit. Econ. **98**(4), 783–802 (1990)

Cipriani, M., Martin, A., McCabe, P.E., Parigi, B.M.: Gates, Fees, and Preemptive Runs. Federal Reserve Bank of New York, working paper no. 2014-30 (2014)

Cooper, R., Kempf, H.: Deposit Insurance Without Commitment: Wall St. Versus Main St. NBER working paper no. 16752 (2011)

Cooper, R., Kempf, H., Peled, D.: Is it is or it is ain’t my obligation? Regional debt in a fiscal federation. Int. Econ. Rev. **49**, 1469–1504 (2008)

Cooper, R., Ross, T.: Bank runs: liquidity costs and investment distortions. J. Monet. Econ. **41**(1), 27–38 (1998)

Diamond, D., Dybvig, P.: Bank runs, deposit insurance and liquidity. J. Polit. Econ. **91**, 401–419 (1983)

Ennis, H., Keister, T.: Bank runs and institutions: the perils of intervention. Am. Econ. Rev. **99**(4), 1588–1607 (2009)

Ennis, H.M., Keister, T.: Banking panics and policy responses. J. Monet. Econ. **57**(4), 404–419 (2010)

Green, E., Lin, P.: Diamond and Dybvig’s classic theory of financial intermediation: what’s missing? Fed. Reserv. Bank Minneap. Q. Rev. **24**(1), 3–13 (2000)

Keister, T.: Bailouts and Financial Fragility. Federal Reserve Bank of New York, staff report no. 473 (2010)

Martin, A.: Liquidity provision vs. deposit insurance: preventing bank panics without moral hazard. Econ. Theory **28**(1), 197–211 (2006)

Peck, J., Shell, K.: Equilibrium bank runs. J. Polit. Econ. **111**(1), 103–123 (2003)

Villamil, A.P.: Demand deposit contracts, suspension of convertibility, and optimal financial intermediation. Econ. Theory **1**(3), 277–288 (1991)