

Is there a plausible theory for decision under risk? A dual calibration critique

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Abstract Can any prominent theory of decision under risk rationalize both small-stakes risk aversion and large-stakes risk aversion? Do some prominent theories fail to rationalize patterns of same-stakes risk aversion? How do reference payoffs enter in the answer to these questions? What would be the characteristics of a theory of decision under risk that would be immune to calibration critique? We offer a theoretical duality analysis that addresses these questions. We report dual propositions and corollaries that calibrate the implications of nonlinear transformation of probabilities or payoffs (or both). We also report several experiments that provide data on the empirical relevance of the two types of calibration patterns.

Keywords Risk aversion · Calibration · Duality · Reference dependence · Experiments

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1 Introduction

Models of risk averse decision-makers are applied in a wide variety of contexts. For example, expected utility models have recently been applied in research on principle-agent theory (Chade and de Serio 2002), risk aversion and prudence (Lajeri and Nielsen 2000; Eichner and Wagener 2003), bounds on utility (LiCalzi 2000; Zambrano 2008), non-location scale distributions (Boyle and Conniffe 2008), and frequentist perspective (Hu 2009). Rank-dependent models have recently been applied to research on savings decisions (Bleichrodt and Eeckhoudt 2005), monotone risk aversion (Chateauneuf et al. 2005), and medical insurance (Ryan and Vaithianathan 2003). Dual theory of expected utility and cumulative prospect theory have, respectively, recently been applied in research on incomplete preferences (Maccheroni 2004) and the St. Petersburg paradox (Rieger and Wang 2006). Our paper reports research on internal coherence problems that can arise in applications of such models.

Models of decision under risk represent risk preferences with utility functionals that are nonlinear in payoffs or nonlinear in probabilities or nonlinear in both. For example, expected utility theory represents risk aversion with concave utility of payoffs. The dual theory of expected utility (Yaari 1987) represents risk aversion with convex transformation of decumulative probabilities. Rank-dependent utility theory (Quiggin 1993) and cumulative prospect theory (Tversky and Kahneman 1992) represent risk attitudes with nonlinear transformations of both payoffs and probabilities. The nonlinear transformations suggest questions about the internal coherence of the theoretical models.

Rabin (2000) demonstrated how modeling risk aversion with concave utility of payoffs can fail to provide a coherent theory of both small-stakes risk aversion and large-stakes risk aversion.¹ Although Rabin's statement of the critique applies only to the expected utility of terminal wealth model, subsequent authors extended this *payoffs* calibration critique to a class of theories of decision under risk that have utility functionals that are nonlinear in payoffs.² But the *payoffs* calibration arguments have no implications for nonlinear transformations of *probabilities*, which is an alternative way to model risk aversion. Sadiraj (2012) presents a *probabilities* calibration that demonstrates the implausible implications of nonlinear transformation of probabilities. Each of the distinct types of calibration (of nonlinear payoffs transformation or nonlinear probabilities transformation) by itself has implications of implausible risk aversion for theories of decision under risk that transform both payoffs and probabilities.

Together, the *payoffs* calibrations and the *probabilities* calibrations provide an answer to a central question raised by the calibration literature: What would be the characteristics of a theory of risk-avoiding behavior that is immune to both *payoffs* calibration and *probabilities* calibration arguments? This paper offers insights into

¹ See Hansson (1988) for an earlier critique of expected utility theory with similar arguments.

² Several studies report *payoffs* calibration patterns that apply to models defined on (1) terminal wealth or (2) income. Studies that focus on terminal wealth models include Hansson (1988), Rabin (2000), Neilson (2001) and Safra and Segal (2008, 2009). Varying-payoffs calibrations for models defined on income are reported by Barberis et al. (2006), Cox and Sadiraj (2006), and Rubinstein (2006).

answering this question with an analysis of decision theories based on duality. We show that the *payoffs* calibration patterns conform to the linearity in payoffs property of the utility functional for the dual theory of expected utility (Yaari 1987). In contrast, the *probabilities* calibration patterns conform to the linearity in probabilities property of the utility functional for expected utility theory (von Neumann and Morgenstern 1947). Our dual analysis reveals that Sadiraj's (2012) probabilities calibration patterns, together with Rabin's (2000) payoffs calibration patterns, provide an answer to the central question about the properties that would characterize a theory of risk-avoiding preferences that would *not* be called into question by calibration critique. A theory of risk preferences with functional that is linear in probabilities would be immune to the probabilities calibration critique. A theory of risk preferences with functional that is linear in payoffs or assumes variable reference payoffs would be immune to the payoffs calibration critique. Hence, a theoretical model characterized by linearity in probabilities *and* variable reference payoffs would be immune to both of the dual calibration critiques. A version of the vintage model in Markowitz (1952) has the requisite properties to survive the dual critiques unscathed. In contrast, currently-popular models are vulnerable to one or both types of calibration critique.

The fundamentality of the calibration literature ultimately rests on empirical validity of the patterns of risk aversion supposed in the two types of calibration propositions. To date, however, there has been argument about the "reasonableness" of calibration suppositions, but no data from real-payoff controlled experiments to inform the issue. This paper reports several experiments conducted in three countries (India, Germany, and the United States) with idiosyncratic opportunities for implementing a variety of experimental designs and protocols covering both payoffs calibration patterns and probabilities calibration patterns that, together, have broad implications for plausibility of theories of decision under risk.

2 Dual calibration patterns and dual paradoxes

For any given integer m , let I_m denote the set of positive integers not larger than m , that is $I_m = \{1, 2, \dots, m\}$. Let $\{y_k, p_k\}_{k \in I_m}$ denote an m -outcome lottery, L that pays y_k with probability p_k , $k \in I_m$, where $\sum_{k=1}^m p_k = 1$. We use the convention $y_{j+1} \geq y_j$, for $j \in I_{m-1}$. Whenever the smallest payoff is zero (i.e., $y_1 = 0$), we use the simpler notation $\{y_m, p_m, \dots, y_2, p_2\}$.

In this paper, we focus on a class of theories of decision under risk that includes all of the more prominent ones. The most familiar such theory is expected utility theory (EU) that represents the utility of a lottery, $L = \{y_k, p_k\}_{k \in I_m}$ with the functional

$$U(L) = \sum_{k=1}^m v(y_k) p_k, \quad (\text{EU-1})$$

where $v(\cdot)$ is a continuous positively monotonic function that transforms payoffs. As is well known, EU represents risk aversion solely by concavity of the utility function $v(\cdot)$. The distinguishing property of functional (EU-1) is its linearity in probabilities,

which follows from the Independence Axiom.³ With the terminal wealth model of EU, $v(y_k) = u(w + y_k)$, $\forall k \in I_m$, where w is the amount of initial wealth, whereas in case of the income model of EU, the utility function $v(\cdot)$ is invariant to w .

In order to facilitate exposition of less familiar models, we will first rewrite (EU-1) in an alternative, logically equivalent form. Let $P_k, k \in I_m$ denote the (decumulative) probability that the lottery L pays y_k or more, that is $P_k = \sum_{i=k}^m p_i, k \in I_m$. Using this notation, (EU-1) can be rewritten as

$$U(L) = v(y_m)P_m + \sum_{k=1}^{m-1} v(y_k)[P_k - P_{k+1}]. \tag{EU-2}$$

The class of theories of decision under risk that we consider have functionals that can be written in forms similar to (EU-2) but that relax the assumption of linearity in probabilities. The utility of a lottery L for this class of theories is given by

$$U(L) = v(y_m)f(P_m) + \sum_{k=1}^{m-1} v(y_k)[f(P_k) - f(P_{k+1})] \tag{NL-1}$$

where $f(\cdot)$ is a continuous, positively monotonic function that transforms decumulative probabilities and $v(\cdot)$ is a continuous, positively monotonic function that transforms payoffs. Of course, the EU functional is the special case of (NL-1) in which the probability transformation function is the identity mapping, $f(P) = P$.

Theories with functionals that are nonlinear in both payoffs and probabilities include rank-dependent utility theory (Quiggin 1993) and cumulative prospect theory (Tversky and Kahneman 1992). We subsequently refer to this class of theories as NLPP theories.

In the case of dual theory of expected utility (DU), the functional is the special case of (NL-1) that is linear in payoffs. The DU functional takes the simple form

$$U(L) = y_m f(P_m) + \sum_{k=1}^{m-1} y_k [f(P_k) - f(P_{k+1})] \tag{DU-1}$$

that is dual to (EU-2). The distinguishing property of (DU-1), linearity in payoffs, follows from the Dual Independence Axiom (Yaari 1987).⁴ Recall that, for DU, risk aversion is equivalent to $f(\cdot)$ being convex (see Yaari 1987, p. 107).

³ This axiom was introduced by von Neumann and Morgenstern (1947). The statement of the axiom by Yaari (1987, p. 98) is: (IA) For all lotteries A and B , if lottery A is preferred to lottery B then for all lotteries C , and all $\alpha \in [0, 1]$, an α -probability mixture of lotteries A and C is preferred to an α -probability mixture of lotteries B and $C : \alpha A + (1 - \alpha)C \geq \alpha B + (1 - \alpha)C$. That is getting lottery A with probability α and lottery C otherwise is preferred to the lottery that offers lottery B with the same probability α and lottery C otherwise.

⁴ The dual independence axiom (Yaari 1987, p. 99) is: (DIA) For all lotteries A and B , if lottery A is preferred to lottery B , then for all lotteries C , and all $\alpha \in [0, 1]$, α -payoff mixture of lotteries A and C is preferred to α -payoff mixture of lotteries B and $C : (\alpha P_A^{-1} + (1 - \alpha)P_C^{-1})^{-1} \geq (\alpha P_B^{-1} + (1 - \alpha)P_C^{-1})^{-1}$ where $(\cdot)^{-1}$ is a (generalized) inverse operator, whereas P_A and P_B denote decumulative probability distribution functions of lotteries A and B .

We begin with two examples that illustrate payoffs calibrations and probabilities calibrations. For ease of exposition, the examples build on EU and DU because of the simplicity that follows from linearity in either probabilities or payoffs. The propositions reported in Sects. 3 and 4, however, show that *each* of the dual patterns of risk aversion *by itself* has implausible implications for NLPP theories, such as rank-dependent utility theory (Quiggin 1993) and cumulative prospect theory (Tversky and Kahneman 1992), that are characterized by functionals that are nonlinear in both payoffs and probabilities. As shown by Corollary 2.2 in Sect. 4, the probabilities calibration patterns described in Sect. 2.2 also have implausible risk aversion implications for rank-dependent models with *endogenous* reference amounts of payoff.

The first example, which illustrates *payoffs* calibration, builds on a pattern of small-stakes risk aversion that appears in Rabin (2000). This pattern conforms to the dual theory of expected utility because of linearity of the DU functional in payoffs, as explained below. The second example, which illustrates *probabilities* calibration, builds on a pattern of risk aversion introduced in Sadiraj (2012). This second pattern conforms to expected utility theory because of linearity of the EU functional in probabilities, as explained below.

2.1 Example 1: John Doe's risk preferences conform to the dual theory of expected utility

Suppose that John Doe, whose initial wealth is \$125, is observed to reject a 50/50 bet in which he could lose \$100 or gain \$110. Does decision theory make any predictions about what choice John would make at other wealth levels? Expected utility theory makes no prediction based on this one observation unless one adds special-case assumptions about risk attitudes such as constant or decreasing absolute risk aversion. In contrast, dual theory of expected utility (Yaari 1987) makes a general prediction about John's choices. According to the DU functional [as in (DU-1)], John's rejection of the 50/50 lose \$100 or gain \$110 bet at initial wealth level \$125 reveals $125 \geq U_{DU}(bet) = (125 + 110)f(0.5) + (125 - 100)(1 - f(0.5)) = 125 + 210f(0.5) - 100$; thus $f(0.5) \leq 100/210$. From the last inequality, it follows that $w \geq U_{DU}(bet)$ for all initial wealth $w \geq 125$ because $w \geq U_{DU}(bet) = (w + 110)f(0.5) + (w - 100)(1 - f(0.5)) = w + 210f(0.5) - 100$ is true if and only if $f(0.5) \leq 100/210$. Therefore, according to DU, John will (weakly) reject the bet at all wealth levels if he does so at wealth level \$125.

What are the implications for expected utility theory if one actually observes rejection of the bet or indifference for all initial wealth levels in some finite interval? Consider the pairs of lotteries in Table 1. The first row shows the alternative options discussed above, a choice between certain payoff $x = 125$ in the option B column and a 50/50 bet, with outcomes $\$(x - 100)$ or $\$(x + 110)$ in the option A column. Now suppose that (as predicted by dual theory of expected utility) additional observations of John's decisions under risk show him choosing option B or indifference in all rows of Table 1. What are the implications of these observations for EU? EU implies that, if John weakly prefers the certain payoff to the lottery in all rows of Table 1, then he will also prefer a certain payoff of \$3,000 to a 50/50 bet with payoffs of

Table 1 Example 1: Varying-payoffs calibration pattern

Row	Option A $\{x - 100, 0.5; x + 110\}$	Option B $\{x, 1\}$
1	25 or 235	125
2	235 or 445	335
3	445 or 655	545
...
t	$25 + 210(t - 1)$ or $25 + 210t$	$125 + 210(t - 1)$
...
114	23,755 or 23,965	23,855
115	23,965 or 24,175	24,065

\$125 or \$3.2 million (see Proposition 1 in Sect. 3). What accounts for this implausible implication of EU?

According to EU, choice of option B in the first row in Table 1 reveals that $v(125) \geq 0.5v(235) + 0.5v(25)$, which together with the (weak) concavity of the continuously differentiable function,⁵ $v(\cdot)$ imply that $110v'(235) \leq v(235) - v(125) \leq v(125) - v(25) \leq 100v'(25)$. Therefore $v'(25 + 210) \leq (10/11)v'(25)$. Choice of option B in the second row of Table 1 reveals that $v'(25 + 2 \times 210) \leq (10/11)v'(25 + 210)$, which together with the penultimate inequality imply $v'(25 + 2 \times 210) \leq (10/11)^2v'(25)$. Similarly, it can be verified that $v'(25 + 210t) \leq (10/11)^t v'(25)$ follows from the weak preference for option B in all rows one to t in Table 1. So, if option B is not rejected in all rows of Table 1, then the (weak) concave utility of payoffs explanation of these choices implies that

$$v'(24,175) = v'(25 + 115 \times 210) \leq (10/11)^{115}v'(25) < 0.00002 \times v'(25).$$

Such extreme diminishing marginal utility produces ridiculously low marginal utilities for large payoffs that leads to implausible large-stakes risk aversion such as the one stated at the end of the preceding paragraph.

2.2 Example 2: Jane Doe’s risk preferences conform to expected utility theory

Now suppose that Jane Doe has been observed to reject a 50/50 bet that pays \$30 or \$0 in favor of a bet that pays \$30 with probability 0.45, \$10 with probability 0.1, or \$0 with probability 0.45. Does decision theory make any predictions about what choice Jane would make when given options with higher or lower probability for the high payoff (of \$30) but with the same 0.1 probability of the middle outcome (of \$10)? DU makes no prediction based on this one observation unless one adds special-case assumptions

⁵ Recall that for the terminal wealth model $v(y_k) = u(w + y_k)$, whereas for the income model $v(y_k) = u(y_k)$.

⁶ This illustrative example uses a differentiable utility function for simplicity. Concavity calibration does not require differentiability; see Appendix 8.1 or Rabin (2000).

Table 2 Example 2: Varying-probabilities calibration pattern

Row	Option A {30, p ; 0}		Option B {30, $p - 1/20$; 10, 2/20; 0}		
	Payoff 30	Payoff 0	Payoff 30	Payoff 10	Payoff 0
1	1/20	19/20	0/20	2/20	18/20
2	2/20	18/20	1/20	2/20	17/20
...
10	10/20	10/20	9/20	2/20	9/20
...
i	$i/20$	$(20 - i)/20$	$(i - 1)/20$	2/20	$(19 - i)/20$
...
18	18/20	2/20	17/20	2/20	1/20
19	19/20	1/20	18/20	2/20	0/20

about risk attitudes. In contrast, EU makes a general prediction about Jane’s choices. Using the functional in statement (EU-1), one infers that Jane’s rejection of the two-outcome bet reveals that $0.5 \times v(30) \leq 0.45 \times v(30) + 0.1 \times v(10)$ if, without any loss of generality, the utility of outcome 0 is normalized to 0. The last inequality simplifies to $0 \leq -0.05v(30) + 0.1 \times v(10)$. Adding $p \times v(30)$ to both sides of the immediately preceding inequality, one has $p \times v(30) \leq (p - 0.05) \times v(30) + 0.1 \times v(10)$. Hence, EU predicts that Jane will (weakly) reject the two-outcome lottery that pays \$30 with probability p and \$ 0 otherwise, in favor of the three-outcome lottery that pays \$30 with probability $p - 0.05$, \$ 10 with probability 0.1 and \$0 otherwise, for all $p \in \{0.05, 0.1, \dots, 0.95\}$, if she does so for $p = 0.5$.

What are the implications for DU if one actually observes the choices that conform to EU, (weak) rejection of the two-outcome lottery in favor of the three-outcome lottery for all $p \in \{0.05, 0.1, \dots, 0.95\}$? Consider the pairs of lotteries in Table 2. Row 10 shows the first pair of options discussed above, a choice between a 50/50 bet that pays \$30 or \$0 in the option A column and a bet that pays \$30 with probability 9/20, \$10 with probability 2/20, or \$0 with probability 9/20 in the option B column. Suppose that (as predicted by expected utility theory) observations of Jane’s decisions under risk show her choosing option B or indifference in all rows of Table 2. Probabilities calibration (see Sect. 4) shows that these observations have the implausible implication that Jane will also prefer a certain payoff of \$3,000 to a 50/50 bet that pays \$3 million or \$0. What accounts for this implausible implication of DU?

Consider row 18 of Table 2. According to DU, the dual expected utilities of option A_{18} and option B_{18} are $U(A_{18}) = 30f(18/20)$ and $U(B_{18}) = 30f(17/20) + 10[f(19/20) - f(17/20)]$. Subtraction of $10f(18/20)$ from both $U(A_{18})$ and $U(B_{18})$ and rearrangement of terms show that $U(B_{18}) \geq U(A_{18})$ if and only if $[f(19/20) - f(18/20)] \geq 2[f(18/20) - f(17/20)]$. It follows from the last inequality and convexity of the continuously differentiable function,⁷ $f(\cdot)$ that $f'(19/20) \geq 2f'(17/20)$.

⁷ The proofs of Proposition 2 and its corollaries do not require either differentiability or convexity. To see that extreme implications follow for cases in which the probability transformation is not everywhere convex

Next, rejection of the two-outcome lottery in favor of the three-outcome lottery in row 16 implies $f'(17/20) \geq 2f'(15/20)$. Hence, rejection of the two-outcome lottery in rows 16 and 18 reveals $f'(19/20) \geq 2^2 f'(15/20)$. Similarly, rejection of the two-outcome lottery in all of the even-numbered rows of Table 2 reveals that

$$\begin{aligned} f' \left(\frac{19}{20} \right) &= f' \left(\frac{1 + 2 \times 9}{20} \right) \geq 2f' \left(\frac{1 + 2 \times 8}{20} \right) \geq \dots \geq 2^9 f' \left(\frac{1}{20} \right) \\ &= 512 f' \left(\frac{1}{20} \right). \end{aligned}$$

Such extreme increasing marginal probability transformation produces implausible risk aversion such as the one stated above.

2.3 Postulated preferences for John and Jane Doe are paradoxical

The postulated risk preferences for John and Jane Doe imply a double paradox: (1) John's pattern, which conforms to the dual theory of expected utility theory, has implausible risk aversion implications for expected utility theory; and (2) Jane's pattern, which conforms to expected utility theory, has implausible risk aversion implications for the dual theory of expected utility theory.

In Sects. 3 and 4, we offer a dual analysis of implications of patterns of risk aversion that (P.1) conform to the dual theory of expected utility or (P.2) conform to expected utility theory. The analysis shows that *both* patterns of type (P.1) and patterns of type (P.2) have implausible risk aversion implication for theories, such as rank-dependent utility theory (Quiggin 1993) and cumulative prospect theory (Tversky and Kahneman 1992), with functionals that are nonlinear in both payoffs and probabilities.

The empirical relevance of these thought exercises with patterns of risk aversion rests on empirical validity of (weaker versions of) the patterns of risk aversion assumed in the above examples. We address this issue with the experiments reported in Sects. 5 and 6. We next turn our attention to formal statements of calibration propositions and corollaries in Sects. 3 and 4.

3 Payoffs calibrations

Calibration propositions for theories with nonlinear utility of money payoffs have been reported in several papers (cited above in the introduction). In order to provide a foundation for our payoffs calibration experiments, we report a calibration proposition for expected utility theory and a corollary that applies to rank-dependent theories on finite domains. The focus is on finite domains because of our intention to apply the theory to data from experiments. Design of experiments reported in Sect. 5 is based on the calibration patterns discussed here.

Footnote 7 continued

assume it has an inverted S-shape with an inflection point at $1/3$; in that case, one still gets extreme risk aversion as revealed by: $f'(19/20) = f'((7 + 2 \times 6)/20) \geq 2^6 f'(7/20)$.

3.1 Calibration for nonlinear payoff transformation functions

Consider payoff calibration patterns like those in Example 1 of Sect. 2.1. For bounded intervals of income, Proposition 1 states a payoff calibration result for expected utility theory with weakly concave utility of money payoff function $v(\cdot)$.⁸

The following standard notation is used: \geq indicates weak preference; $>$ indicates strong preference; $\{z, p; y\}$ denotes a binary lottery that pays z and y with probabilities p and $1 - p$; $\lfloor x \rfloor$ is the largest integer smaller than x .

Let the domain of monetary prizes be a closed interval, $[m, M] \subset (0, \infty)$. For any given positive payoffs ℓ, g such that $0 < \ell < g$ let $z_* = m + (\ell + g) \times (2 - \ln 2 / \ln(\ell/g))$. Consider statements

$$x \geq \{x + g, 0.5; x - \ell\}, \text{ for all integers } x \in [m, M] \tag{P.1}$$

$$\{z + G, 0.5; m\} > z, \tag{Q.1}$$

for some $z \in (z_*, M)$ and sufficiently large $G > (z - m)g/\ell$.

Proposition 1 *Let prizes ℓ, g such that $0 < \ell < g$ and positive integer m be given. Then*

- a. *DU predicts that statement P.1 is equivalent to $0 \geq \{g, 0.5; -\ell\}$,*
- b. *For all $z > z_*$ and all $G > (z - m)g/\ell$,*
 - (i) *for all $M > m$ both statements P.1 and Q.1 are true for DU with*

$$f(0.5) \in \left(\frac{z - m}{z - m + G}, \frac{\ell}{g + \ell} \right]$$

- (ii) *there exists $M > m$ such that P.1 and Q.1 cannot both be true for EU.*

Proof Parts (a) and (b.i) are straightforward; for part (b.ii) see Appendix 8.1. □

Part (a) of this proposition says that any DU agent who rejects lottery $\{g, 0.5; -\ell\}$ will satisfy pattern P.1; the inverse is also true. Part (b) says, that for G as large as one wants it to be, there are DU agents but no EU agents who satisfy both patterns P.1 and Q.1. The proof is constructive with respect to M . The following expression that relates M to G and z [see the proof in Appendix 8.1, inequality (8.9)] will be useful in our numerical illustrations:

$$G/(g + \ell) < N - K - 1 + r^{-N}A(r, K), \tag{*}$$

where $K = \lfloor (z - m)/(g + \ell) \rfloor$, $N = \lfloor (M - m)/(g + \ell) \rfloor$, $A(r, K) = r - r^{1+K} - r^K$ and $r = \ell/g$. We use statements (*) and P.1 in Proposition 1 to construct the illustrative examples that are reported in Table 3.

⁸ See Rabin (2000) and Cox and Sadiraj (2006) for concavity calibrations on unbounded domains.

Table 3 Calibrations for varying-payoffs patterns: $3,000 \succ \{G^*, 0.5; 100\}$

Rejection intervals [100, M]	Calibration for $g = 110, \ell = 100$	Calibration for $g = 90, \ell = 50$	Calibration for $g = 30, \ell = 20$
M	G^*	G^*	G^*
6,000	6,690	0.22×10^{13}	0.13×10^{23}
8,000	9,913	0.85×10^{16}	0.14×10^{30}
10,000	15,298	0.31×10^{20}	0.16×10^{37}
30,000	0.47×10^8	0.10×10^{57}	0.44×10^{107}
50,000	0.40×10^{12}	0.32×10^{93}	0.12×10^{178}

Suppose that an agent weakly prefers the certain amount of income x to the binary lottery $\{x + 110, 0.5; x - 100\}$, for all integers $x \in [100, M]$, where values of M are given in the “Rejection Intervals” column of Table 3. In that case, all three expected utility (of terminal wealth, income, and initial wealth and income) models⁹ predict that the agent prefers receiving the amount of income 3,000 for sure to a risky lottery $\{z + G, 0.5; 100\}$, where the values of $z + G (= G^*)$ are given in the second column of Table 3. For example, if $[m, M] = [100, 50,000]$, as in the last row of Table 3, then $G^* = 0.4 \times 10^{12}$; that is, expected utility theory implies that the agent will prefer 3,000 for sure to a 50/50 lottery that pays 100 or 0.40×10^{12} . According to the entry in the third column and $M = 30,000$ row of Table 3, expected utility theory predicts that if an agent prefers certain payoff in amount x to lottery $\{x + 90, 0.5; x - 50\}$, for all integers x between 100 and 30,000, then such an agent will prefer 3,000 for sure to the 50/50 lottery with positive outcomes of 100 or 0.10×10^{57} .

3.2 Calibration for fixed reference payoff models

The generalization of Proposition 1 is straightforward for the NLPP class of theories that includes cumulative prospect theory with zero-income reference point (Tversky and Kahneman 1992) and rank-dependent utility theory (Quiggin 1993). We use $R(p)$ to denote the following function $R(p) = (1 - h(p))\ell/h(p)g, p \in (0, 1]$. One has:

Corollary 1.1 *Suppose that the value function is (weakly) concave. For any positive prizes ℓ, g such that $R(0.5) < 1$, NLPP theories predict that for all $z > z_* = m + (\ell + g)(2 - \ln 2 / \ln(R(0.5)))$ and any given large G there exists M such that z, M and G satisfy inequality (*) with $r = R(0.5)$ and statements P.1 and Q.1 cannot both be true.*

Proof See Appendix 8.1. □

Recall that for expected utility theory, with a functional that is linear in probabilities, Proposition 1 reveals implausible large-stakes risk aversion if $g > \ell$. In the corollary, this implication holds when $h(0.5)g > [1 - h(0.5)]\ell$, which is equivalent to

⁹ See Cox and Sadiraj (2006) for discussion of these three expected utility models.

$r = R(0.5) < 1$. Examples that illustrate the implications of Corollary 1.1 are similar to those in Table 3.

A reference-dependent theory can incorporate *variable* reference amounts of money payoff. Wakker (2005, 2010) explains that variable reference payoffs can immunize prospect theory to payoffs calibration arguments based on the small-stakes risk aversion pattern (Rabin 2000). A straightforward extension of Wakker’s arguments implies that the Markowitz (1952) model can be immunized to payoffs calibration by a suitable interpretation of its reference points.

Empirical validity of the P.1 pattern of small-stakes risk aversion is testable. Section 5 reports experiments on this question.

4 Probabilities calibrations

We now consider varying probabilities, fixed payoffs patterns of risk aversion like those in Example 2 of Sect. 2.2. We report a calibration proposition for the dual theory of expected utility and corollaries that apply to NLPP theories that have functionals that are nonlinear in both probabilities and payoffs. The design of experiments reported in Sect. 6 is based on patterns of risk preferences discussed here.

4.1 Calibration for nonlinear probability transformation functions

Consider $2n - 1$ pairs of lotteries A_i and $B_i, i \in I_{2n-1}$. Lottery $A_i = \{y, p_i\}$ pays a positive amount of money y with probability p_i or the amount 0 with probability $1 - p_i$. Denote $\delta = 1/2n$ and let $p_i = i/2n, i \in I_{2n-1}$. Lottery $B_i = \{y, p_i - \delta; x, 2\delta\}$ pays the amount y with probability $p_i - \delta$ or the amount $x, x \in (0, y)$ with probability 2δ or the amount 0 with probability $1 - (p_i - \delta + 2\delta)$.

Suppose that an agent (weakly) prefers the three-outcome lottery B_i to the two-outcome lottery A_i , for all $i \in I_{2n-k^*}$, where k^* is a positive integer not larger than n . Note that by linearity in probabilities (see EU-1) any expected utility maximizer who prefers x for sure to the 50/50 lottery that pays y or 0 satisfies this supposition. Discussion following the statement of Proposition 2, however, shows that if the high outcome y is larger than twice the intermediate outcome x , then this supposition implies implausible risk aversion for DU agents.

Let prizes x, y such that $y > 2x > 0$, and positive integer k^* be given. For any integer $n, n \geq k^*$ consider the following statements

$$\{y, (i - 1)/2n; x, 1/n\} \succeq \{y, i/2n\}, \text{ for all } i \in \{1, 2, \dots, 2n - k^*\} \tag{P.2}$$

$$\{zG, 0.5\} \succ \{z, 1 - (k^* - 1)/2n\}, \text{ for some positive } z \text{ and large } G. \tag{Q.2}$$

Proposition 2 *Let prizes x, y such that $y > 2x > 0$, and positive integer k^* be given. Then*

- a. For EU, statement P.2 is equivalent to $x \succeq \{y, 0.5\}$.
- b. For any given sufficiently large G

Table 4 Calibrations for varying-probabilities patterns: $1,000 > \{\bar{G}, 0.5\}$

Rejection intervals n	Calibrated values of \bar{G} for different values of n and C				
	$C = 2.5$	$C = 3$	$C = 3.5$	$C = 4.0$	$C = 5.0$
5	8,593	33,000	98,000	244,000	1,025,000
10	58,665	1,025,000	9,530,000	0.59×10^8	0.10×10^{10}
20	3,326,256	0.10×10^{10}	0.90×10^{11}	0.34×10^{13}	0.10×10^{16}
50	0.63×10^{12}	0.11×10^{19}	0.78×10^{23}	0.71×10^{27}	0.12×10^{34}
100	0.40×10^{21}	0.12×10^{34}	0.62×10^{43}	0.51×10^{51}	0.16×10^{64}
200	0.16×10^{39}	0.16×10^{64}	0.38×10^{83}	0.26×10^{99}	0.25×10^{124}
500	0.11×10^{92}	0.32×10^{154}	0.93×10^{202}	0.36×10^{242}	0.10×10^{305}

- (i) for all integers $n \geq k^*$ both statements P.2 and Q.2 are true for EU with $v(y) < 2v(x)$ and $2v(1/G) < v(1)$.
- (ii) there exist integers $n \geq k^*$ such that statements P.2 and Q.2 cannot both be true for DU.

Proof Parts (a) and (b.i) are straightforward; for part (b.ii) see Appendix 8.2. □

The proof is constructive with respect to n and G . The following expression that relates n to G [see the proof in the Appendix 8.2, inequality (8.12)] will be useful in our numerical illustrations:

$$G < T(y/x, n - k^*, n - 1), \tag{**}$$

where $T(t, m, n) = 1 + \sum_{j=0}^m (t - 1)^{j+1} / \sum_{i=0}^n (t - 1)^{-i}$. We use statements (**), and P.2 in Proposition 2 to construct the illustrative examples that are reported in Table 4 for the special case of $k^* = 1$. In the table, C denotes the ratio of the highest payoff to the second highest payoff in the three prize lottery, $C = y/x$. With $C = 2.5$ and $n = 20$, Proposition 2 tells us that for this P.2 pattern DU predicts that the agent prefers 1,000 for sure to a lottery that pays 3.3 million or 0 with even odds, as reported in the second column and third row of Table 4. With $C = 3.5$ and $n = 50$, the prediction is preference for 1,000 for sure over a 50/50 lottery that pays 0 or 0.78×10^{23} , as reported in the fourth column and fourth row of Table 4. Finally, with $C = 5$ and $n = 10$, the prediction is preference for 1,000 for sure to the 50/50 lottery that pays 0 or 1 billion.

4.2 Calibration for reference-dependent models

Proposition 2 is stated for the dual theory of expected utility that is characterized by a preference functional that is linear in payoffs and nonlinear in decumulative probabilities. The generalization is straightforward for the NLPP class of theories. First consider a NLPP model with fixed, zero-income reference point, as in Tversky

and Kahneman (1992) and Quiggin (1993). For nonlinear transformation of payoff functions $v(\cdot)$ that are sub-additive on positive payoffs one has:

Corollary 2.1 *For $v(y) > 2v(x)$, NLPP models with zero-income reference point predict that for any given sufficiently large G , there exist integers $n \geq k^*$ such that statements P.2 and Q.2 cannot both be true.*

Proof see Appendix 8.2. □

For NLPP models with zero-income reference point, the relation between n and G is given by $G < T(v(y)/v(x), n - k^*, n - 1)$. Implications of Corollary 2.1 for the special case in which $k^* = 1$ are given in Table 4 for the (alternative) definition $C = v(y)/v(x)$. For example, if the value of the high payoff $v(y)$ is at least 3 times as large as the value of the intermediate (positive) payoff $v(x)$, then implications of calibration pattern P.2 are given by the $C = 3$ column of Table 4, and so on.

Probabilities calibration does not apply to the Markowitz (1952) model because its functional is linear in probabilities. In contrast, probabilities calibration applies to rank-dependent models because of their nonlinear transformation of probabilities, whether or not the reference point is exogenous. The reason for this is straightforward: the calibration is constructed by varying the probabilities for which three or two payoffs are paid, not by varying the payoff amounts. Hence, it makes no difference to this calibration whether the reference amount of payoff is or is not fixed at zero payoff. Here is a formal statement of the result that the calibration applies to rank-dependent models with endogenous reference payoff. Let $\mu(\cdot) < 0$ denote the value function for negative payoffs. For $v(\cdot)$ sub-additive on positive payoffs one has:

Corollary 2.2 *Let the reference point be the intermediate payoff x and $v(y - x) > -\mu(-x)$. Rank-dependent models predict that for any given sufficiently large G , there exist integers $n \geq k^*$ such that statements P.2 and Q.2 cannot both be true.*

Proof See Appendix 8.2. □

It can be verified (see Appendix 8.2) that the relation between n and G in this case is given by $G < T(R + 1, n - k^*, n - 1)$, where $R = -v(y - x)/\mu(-x)$. Similar corollaries can be stated for cases in which the endogenous reference point is the highest payoff or the lowest payoff (or other convex combinations of the high and low payoffs) rather than the intermediate payoff. Empirical validity of the pattern P.2 is testable. Section 6 reports experiments on this question.

5 Experiments with varying payoffs

We ran three experiments with calibration patterns (P.1) for payoff transformation theories identified in Proposition 1 and Corollary 1.1 in Calcutta (India) and Magdeburg (Germany). We explain the common features and idiosyncratic lotteries used in these experiments after presenting a detailed discussion of one experiment to provide a representative example.

5.1 Experimental design: an example

Subjects in one experiment parameterization were asked to make six choices between a certain amount of money x and a binary lottery $\{x + 30, 0.5; x - 20\}$ for values of x from the set $\{100, 1K, 2K, 4K, 5K, 6K\}$, where $K = 1,000$. Subjects were asked to choose among option A (the risky lottery), option B (the certain amount of money), and option I (indifference). The choice tasks given to the subjects for this parameterization are presented in Table 5. Each row of Table 5 shows a certain amount of money and paired lottery in a choice task included in the experiment. The subjects were *not* presented with a fixed order of decision tasks, as in Table 5. Instead, each pair of a sure payoff and a risky lottery was shown on a separate (response form) page. Each subject picked up a set of response pages that were arranged in independently drawn random order. He or she could mark choices in any order desired.

5.2 Experimental design: alternative parameterizations and protocols

We conducted three experiments on empirical validity of the calibration pattern P.1 in Proposition 1. These experiments used the random decision selection payoff protocol in which one of each subject's several decisions is randomly selected for payoff at the end of the experiment. In the Calcutta 30/−20 experiment, binary lotteries $\{x + 30, 0.5; x - 20\}$ and sure payoffs x were from the set $\{100, 1K, 2K, 4K, 5K, 6K\}$, where $K = 1,000$; payoffs were in rupees. In the Calcutta 90/−50 experiment, binary lotteries $\{x + 90, 0.5; x - 50\}$ and sure payoffs x were from the set $\{50, 800, 1.7K, 2.7K, 3.8K, 5K\}$, where $K = 1,000$; payoffs were in rupees. Finally, in the Magdeburg 110/−100 experiment, binary lotteries $\{x + 110, 0.5; x - 100\}$ and sure payoffs x were from the set $\{3K, 9K, 50K, 70K, 90K, 110K\}$, where $K = 1,000$; payoffs were in *contingent* euros.

An appendix available from the authors reports the subject instructions (in English), the response forms (or pages), and detailed information on the protocol used in all of the experiments. Before presenting data, we note economic significance of rupee payoffs in Calcutta experiments and the meaning of *contingent* euro payoffs in the Magdeburg experiment.

5.3 Economic significance of payoffs

Data collected in Calcutta at the time of the first experiment show that the 50 rupee amount at risk in the Calcutta 30/−20 experiment lotteries was the monetary equivalent

Table 5 Choice alternatives in varying-payoffs experiment Calcutta 30/−20

Row	Option A	Option B
1	80 or 130	100
2	980 or 1,030	1,000
3	1,980 or 2,030	2,000
4	3,980 or 4,030	4,000
5	4,980 or 5,030	5,000
6	5,980 or 6,030	6,000

of: (1) a full day's pay for the student subjects; (2) 15 (grocery store) servings of chicken; or (3) 14 bus tickets. Rupee payoffs in the Calcutta 90/–50 experiment were even more significant.

The Magdeburg 110/–100 experiment used contingent payoffs in amounts as high as 110K euros. We could offer to pay such large amounts in *contingent* euros by using a casino payoff protocol explained in Appendix 8.3.

5.4 Data provide support for the concavity calibration Pattern P.1

Statement P.1 in Proposition 1 involves weak preference for (safe) option B over (risky) option A. Therefore, in all tests, we aggregate choices of option B with (the very small number of) choices of option I (indifference) and denote the aggregated choice category as B^I . We report tests for incidence in the data of patterns of choices that, according to Proposition 1 and Corollary 1.1, imply implausible risk aversion in the large with expected utility theory and, for the experiments in Calcutta, with original cumulative prospect theory (with zero-income reference point) and with rank dependent utility theory.

We use error rate analysis for statistical inferences on the proportion of subjects who made choices consistent with the calibration patterns.¹⁰ Choice probabilities are assumed to deviate from 1 or 0 by an error rate ε , as in Harless and Camerer (1994). Thus if B^I is preferred to A then $\Pr(\text{choose } B^I) = 1 - \varepsilon$ and if B^I is *not* preferred to A then $\Pr(\text{choose } B^I) = \varepsilon$, where $\varepsilon < 0.5$. The error rate model postulates that a subject with real preferences for B^I (respectively A) over A (respectively B^I) in all six rows could nevertheless be observed to have chosen B^I in five (or fewer) out of six rows. That is, the model allows that a subject with real underlying preferences such as $[B^I, B^I, B^I, B^I, B^I, B^I]$ could, instead, choose a different pattern, say $[B^I, B^I, B^I, A, B^I, B^I]$, an event with probability $(1 - \varepsilon)^5 \varepsilon$, where ε is an error rate.

Models I, II, and III considered here are as follows. Model I includes only choices of all B^I (corresponding to $M = 6,000$ in Proposition 1 for the Calcutta 30/–20 experiment, for example) as a calibration pattern and its mirror, all A's as the other pattern. Let the small-stakes lotteries be $\{x + 30, 0.5; x - 20\}$ for x from 100 to 6,000. According to Proposition 1, the choice pattern “all B^I ” implies that 1,000 for sure is preferred to the lottery that pays 0.13×10^{23} or 100 with equal probabilities. Model II (which corresponds to Proposition 1 with $M = 5,000$ for the Calcutta 30/–20 experiment) contains the Model I pair of (calibration and other) patterns, and one additional calibration pattern with A as the last entry (for $x = 6,000$) and its mirror image as an additional “other pattern.” According to Proposition 1, the calibration patterns in Model II imply that getting 1,000 for sure is preferred to the 50/50 lottery that pays 0.409×10^{19} or 100. Finally, Model III (which corresponds to Proposition 1 with $M = 4,000$ for the Calcutta 30/–20 experiment) contains patterns with four sequential B^I in the first four positions (for $x = 100, 1000, 2000,$ and 4000) as calibration patterns and their mirror images as other patterns. With these calibration

¹⁰ We are grateful to Nathaniel Wilcox for generous advice about this approach to data analysis and for supplying SAS code. See Wilcox (2008) for discussion of econometric methods for analysis of data from binary discrete choice under risk.

Table 6 Maximum likelihood estimates of the proportion of subjects who exhibit patterns P.1 and predictions (not Q.1) for payoffs calibration

Experiment	No. of subjects	Model I	Model II	Model III
Calcutta 90/–50 <i>m</i> = 50	40	0.82** (0.70, 0.94) <i>M</i> = 5,000: 1,000 > {0.32 × 10 ¹¹ , 0.5; 50}	0.81** (0.69, 0.93) <i>M</i> = 4,000: 1,000 > {0.53 × 10 ⁹ , 0.5; 50}	0.81** (0.69, 0.93) <i>M</i> = 3,000: 1,000 > {0.86 × 10 ⁷ , 0.5; 50}
Calcutta 30/–20 <i>m</i> = 100	30	0.43** (0.25, 0.62) <i>M</i> = 6,000: 1,000 > {0.13 × 10 ²³ , 0.5; 100}	0.48** (0.30, 0.66) <i>M</i> = 5,000: 1,000 > {0.40 × 10 ¹⁹ , 0.5; 100}	0.48** (0.30, 0.67) <i>M</i> = 4,000: 1,000 > {0.12 × 10 ¹⁶ , 0.5; 100}
Magdeburg 110/–100 <i>m</i> = 3,000	41	0.54** (0.39, 0.68) <i>M</i> = 110,000: 5,000 > {0.12 × 10 ²³ , 0.5; 3,000}	0.54** (0.39, 0.68) <i>M</i> = 90,000: 5,000 > {0.14 × 10 ¹⁹ , 0.5; 3,000}	0.54** (0.36, 0.71) <i>M</i> = 70,000: 5,000 > {0.17 × 10 ¹⁵ , 0.5; 3,000}

The Wald 90 % confidence interval is in parenthesis; ** *p*-value < 0.01

patterns, Proposition 1 implies that getting 1,000 for sure is preferred to the lottery that pays 0.12×10^{16} or 100 with equal probabilities.

The top row in Table 6 shows estimated proportions of subjects whose choices satisfy the calibration patterns for versions of Models I, II, and III using data from Calcutta 90/–50. The estimated proportion for Model I (*M* = 5,000) is 0.82, with Wald 90 % confidence interval (0.70, 0.94). The estimated proportions for all three models vary between 0.81 and 0.82; all are significant at 1 % (indicated by superscripted double asterisks, **). The estimations for Calcutta 90/–50 imply that 81–82 % of the subjects in this experiment made choices that conform to payoffs calibration patterns P.1 that are problematic for expected utility theory, rank-dependent utility theory, and cumulative prospect theory with fixed reference point. The second row of Table 6 reports estimates for data from Calcutta 30/–20. The estimated proportions vary between 0.43 and 0.48, and all are significant at 1 %. Estimates in the third row for data from Magdeburg 110/–100 are 0.54; all are significant at 1 %.

6 Experiments with varying probabilities

We ran four probabilities calibration pattern P.2 experiments in Germany, India, and the United States. We explain the common design features and idiosyncratic lotteries in these experiments and present a more detailed discussion of one experiment to provide a representative example. We begin with the example.

6.1 Experimental design: an example

Subjects in one experiment parameterization were asked to make choices for each of the nine pairs of lotteries shown in Table 7. The fractions in the rows of the table are the

Table 7 Choice alternatives in varying-probabilities experiment Atlanta 14/4

Row	Option A		Option B		
	Payoff 14	Payoff 0	Payoff 14	Payoff 4	Payoff 0
1	1/10	9/10	0/10	2/10	8/10
2	2/10	8/10	1/10	2/10	7/10
3	3/10	7/10	2/10	2/10	6/10
4	4/10	6/10	3/10	2/10	5/10
5	5/10	5/10	4/10	2/10	4/10
6	6/10	4/10	5/10	2/10	3/10
7	7/10	3/10	6/10	2/10	2/10
8	8/10	2/10	7/10	2/10	1/10
9	9/10	1/10	8/10	2/10	0/10

probabilities of receiving the prizes in the two-outcome (option A) and three-outcome (option B) lotteries. Each row of Table 7 shows a pair of lotteries included in the experiment. The subjects were *not* presented with a fixed order of lottery pairs, as in Table 7. Instead, each lottery pair was shown on a separate (response form) page. Each subject picked up a set of response pages that were arranged in independently drawn random order. He or she could mark choices in any order desired. On each decision page, a subject was asked to choose among a two-outcome lottery (option A in some row of Table 7), a three-outcome lottery (option B in the same row of Table 7), and indifference (“option I”).

6.2 Experimental design: alternative parameterizations and protocols

We conducted four experiments on empirical validity of the calibration pattern P.2 postulated in Proposition 2. One experiment parameterization uses pairs of two-outcome and three-outcome lotteries, $A_j = \{y, p_j; 0\}$ and $B_j = \{y, p_j - 0.1; x, 0.2; 0\}$, for $j \in \{1, 2, \dots, 9\}$, and $y = 14$, $x = 4$ as shown in Table 7. We also ran experiments with the parameterizations $\{y, x\} = \{40, 10\}$ and $\{400, 80\}$.

The experiments were conducted in Magdeburg (Germany), Atlanta (U.S.A.) and Calcutta (India) with payoffs, respectively, in euros, U.S. dollars, and Indian rupees. The experiments used the following parameters: in the Magdeburg 40/10 experiment, $y = 40$ euros and $x = 10$ euros; in the Atlanta 40/10 experiment, $y = 40$ dollars and $x = 10$ dollars; in the Atlanta 14/4 experiment, $y = 14$ dollars and $x = 4$ dollars; in the Calcutta 400/80 experiment, $y = 400$ rupees and $x = 80$ rupees. Economic significance of rupee payoffs is discussed in Sect. 5.3. The two experiments in Atlanta each included two treatments that used different payoff protocols. One protocol is the conventional one in which a single decision by a subject is randomly selected for payoff at the end of the experiment; this is labeled POR, for “pay one randomly,” in Table 8. The other payoff protocol is the one that is theoretically incentive compatible for dual theory of expected utility (Cox et al. 2012). In this payoff protocol, all decisions are

paid correlated at the end. In order to keep the level of incentives similar across payoff mechanisms, we used the version of the protocol in which the amounts of all payoffs are divided by the number of decisions; this is labeled PAC/9, for “pay all correlated/9,” in Table 8. An appendix available from the authors reports subject instructions (in English), response forms (or pages), and detailed information on the protocol used in all of the experiments.

6.3 Data provide support for calibration pattern P.2

In testing for the presence of choices that satisfy the calibration pattern, we aggregate choices of option B with (the very small number of) choices of option I (indifference) because statement P.2 in Proposition 2 involves weak preference for B over A. Aggregated choices of B and I are reported as B^I . Subjects’ choice patterns are recorded as sequences of nine letters, ordered according to the probability of the high outcome. For example, the pattern [A, B^I , B^I , A, B^I , B^I , B^I , B^I , A] would indicate that a subject chose A (a two-outcome lottery) when the probability of the high outcome was 1/10, 4/10 and 9/10—indexed as $j = 1, 4$ and 9—and chose B or I for all other values of the index j . For the experiment with parameterization shown in Table 7, this pattern would mean the subject chose option A on (randomly ordered) pages with the lottery pairs in rows 1, 4, and 9 in the table and chose option B or option I on all other pages.

We use error rate models to draw statistical conclusions from these data. The error rate model postulates that a subject with real preferences for B^I (respectively A) over A (respectively B^I) in all nine lottery pairs could nevertheless be observed to have chosen the other option in some rows. For example, according to this model a subject with underlying preferences [B^I , B^I , B^I , B^I , B^I , B^I , B^I , B^I , B^I] could, instead, be observed to choose a different pattern such as [B^I , B^I , A, B^I , A, B^I , B^I , B^I , B^I], an event with probability $(1 - \varepsilon)^7 \varepsilon^2$.

Stochastic choice Model I contains only the choice pattern with a sequence of nine B^I in the category “calibration pattern” and its dual (“mirror”) image with a sequence of nine A in the “other pattern.” According to Proposition 2, this calibration pattern implies that 1,000 for sure is preferred to the 50/50 lottery that pays 98,000 or 0 for the Atlanta 14/4 experiment, as reported in the third row in Table 8. For the Calcutta 400/80 experiment, Proposition 2 implies that 1,000 for sure is preferred to the 50/50 lottery that pays 1 million or 0, as reported in the bottom row in Table 8.

Model I is overly conservative in its specification of calibration patterns because other data patterns can be calibrated to imply implausible risk aversion. Stochastic choice Model II includes two patterns in the category “calibration patterns”: the pattern with choice of B^I for index $j \in \{1, 2, \dots, 8\}$ and the all B^I pattern (that is, $j \in \{1, 2, \dots, 9\}$). The mirror images of these two patterns comprise the “other patterns” for Model II. Application of Proposition 2 demonstrates that these two patterns of “no A except for index $j = 9$ ” imply that 1,000 for sure is preferred to the 50/50 lottery that pays 81,000 or 0, as reported for the Atlanta 40/10 and Magdeburg 40/10 listings in Table 8. We also consider Model III that includes the patterns “no A except for indexes $j = 8$ and/or 9” in the category of calibration patterns. The mirror images of these patterns comprise the other patterns for Model III. An implication of Proposition 2

Table 8 Maximum likelihood estimates of the proportion of subjects who exhibit patterns P.2 and predictions (not Q.2) for probabilities calibration

Experiment	No. of subjects	Model I	Model II	Model III
Atlanta 14/4	39 POR	0.74** (0.55, 0.93)	0.82** (0.68, 0.96)	0.88** (0.77, 0.99)
	34 PAC/9	0.81** (0.61, 1.0)	0.90** (0.76, 1.0)	0.93** (0.81, 1.0)
Atlanta 40/10	22 POR	0.56** (0.37, 0.75)	0.59** (0.39, 0.78)	0.59** (0.40, 0.79)
	35 PAC/9	0.76** (0.56, 0.95)	0.83** (0.66, 1.0)	0.93** (0.85, 1.0)
Magdeburg 40/10	31 POR	1,000 > {98,000, 0.5; 0}	1,000 > {39,000, 0.5; 0}	1,000 > {15,700, 0.5; 0}
		0.65** (0.49, 0.81)	0.67** (0.51, 0.83)	0.71** (0.56, 0.87)
Calcutta 400/80	40 POR	1,000 > {244,000, 0.5; 0}	1,000 > {81,000, 0.5; 0}	1,000 > {27,000, 0.5; 0}
		0.72** (0.58, 0.86)	0.72** (0.58, 0.86)	0.73** (0.59, 0.86)
		1,000 > {1 million, 0.5; 0}	1,000 > {256,000, 0.5; 0}	1,000 > {64,000, 0.5; 0}

The Wald 90 % confidence interval is in parenthesis; ** p -value < 0.01

for these calibration patterns in case of $n = 5$ and $C = 4$ is preference for 1,000 for sure to the 50/50 lottery that pays 27,000 or 0, as shown in the Atlanta 40/10 and Magdeburg 40/10 listings in the table.

Table 8 reports results from maximum likelihood estimation of the proportion of subjects who exhibit the calibration patterns for Models I, II and III. The first row of Table 8 shows results for data from the Atlanta 14/4 experiment with POR payoff protocol. For Model I, the estimated proportion of subjects who exhibited the calibration pattern is 0.74. The Wald 90 % confidence interval is (0.55, 0.93). The 0.74 estimate is significant at 1 % (as indicated by double asterisks, **). The other columns in the first row of Table 8 report the estimated proportions of subjects whose choice patterns in the Atlanta 14/4, POR protocol treatment conform to calibration patterns of Models I, II, and III. These estimates vary between 0.74 and 0.88, and they are all significant at 1 %. Results look similar for the Atlanta 14/4, PAC/9 protocol data in the second row of Table 8 except that the proportions of subjects consistent with the calibration patterns are even higher; they vary from 0.81 to 0.93.

Table 8 shows the estimated proportions of subjects whose choices are consistent with calibration patterns in experiments Atlanta 40/10, Magdeburg 40/10, and Calcutta 400/80. Depending on the model, the estimated proportion of subjects with data consistent with the calibration patterns in the Atlanta 40/10, POR protocol

treatment varies from 0.56 to 0.59, all significant at 1%. Results look somewhat different for the Atlanta 40/10, PAC/9 protocol data; here, the estimated proportions of subjects with choices consistent with the calibration patterns again are higher; they vary from 0.76 to 0.93. The estimates for data from Magdeburg 40/10 vary from 0.65 to 0.71, all significant at 1%. Estimates with data from experiment Calcutta 400/80 lie between 0.72 and 0.73; all are significant at 1%.

7 Is there a plausible theory for decision under risk?

Prominent theories of decision under risk model individuals' preferences over lotteries with nonlinear transformation of money payoffs and/or nonlinear transformation of probabilities. Previous calibration literature, sparked by Rabin (2000), has focused on the implications of nonlinear transformation of payoffs. Sadiraj (2012) offers calibrations that focus on the implications of nonlinear transformation of probabilities. Theories with functionals that are nonlinear in both probabilities and payoffs are vulnerable to both types of calibration. If one allows for variable reference amounts of payoff, the probabilities calibration is problematic for these theories, but the payoffs calibrations appearing elsewhere in the literature are not.

This paper develops a dual analysis of the calibration patterns. Taken together, the two calibration propositions provide a paradoxical insight into theories of decision under risk: a pattern of risk aversion that conforms to rational behavior for a theory with utility functional that is linear in probabilities (respectively, linear in payoffs) has implausible implications for a theory with functional that is linear in payoffs (respectively, linear in probabilities).

The internal consistency problems that follow from probabilities calibration are even more problematic than those from payoffs calibration. Whereas the payoffs calibration critique applies across "small" and "large" payoff domains, the probabilities calibration critique also holds within a ("small" or "large") payoff domain.

First, note that the scale of the payoffs in pattern Q.1 depends on both the scale of payoffs in pattern P.1 and the length of the interval where P.1 holds. The scale of payoffs in pattern P.1 involves small-stakes risk aversion, but the pattern Q.1 must characterize risk attitudes at large-stakes to make the pair paradoxical. Alternatively, P.1 must involve small-stakes risk aversion to pair paradoxically with a given plausible Q.1 in the large. Therefore, varying-payoffs, fixed-probabilities calibrations question the ability of expected utility theory, rank-dependent utility theory, and cumulative prospect theory with fixed reference point to rationalize plausible risk attitudes across different domains of risk: "small-stakes" and "large-stakes".

In contrast, the scale of the payoffs in pattern P.2 is not required to be different from the scale of payoffs in pattern Q.2.¹¹ Therefore, probabilities calibration questions the ability of dual theory of expected utility and rank-dependent theories with exogenous

¹¹ For example, Proposition 2 and its corollaries tell us that the following two statements are inconsistent for DU and NLPP models. Statement P.2e: the three-outcome lottery that pays 400 or 80 or 0 with probabilities $p - 0.1$, 0.2 and $1 - (p - 0.1 + 0.2)$ is preferred to the two-outcome lottery that pays 400 or 0 with probabilities p and $1 - p$, for all p in $\{0.1, 0.2, \dots, 0.8, 0.9\}$. Statement Q.2e: the 50/50 lottery that pays 330 or 0 is preferred to a sure payoff of 10. These statements P.2e and Q.2e involve implausible combinations of

(zero income) *or* endogenous reference point to rationalize plausible risk attitudes not only across different domains of payoffs (small vs. large) but also within the same domain of payoffs.

Previous literature has not offered real-payoff, controlled experiment data on patterns of risk aversion that appear in calibration suppositions. This paper reports data from several experiments. As explained in Sect. 6, the data provide support for empirical validity of risk aversion patterns underlying both of the dual calibrations.

Popular theories of decision under risk have been shown to be vulnerable to calibration critique. This suggests a central question: What properties would characterize a theory of risk-avoiding preferences that would not be called into question by the critique? The answer follows from the dual calibration propositions and corollaries presented in this paper. A theory of risk preferences with a functional that is linear in probabilities would be immune to the probabilities calibration critique. A theory with variable reference payoffs would be immune to the payoffs calibration critique. The vintage Markowitz (1952, pp. 154–155) model has these two properties if one identifies the second inflection point with the windfall gain offered by the sure option in the payoff calibration pattern.¹² Although a version of the Markowitz model does survive the critique, unlike all of the currently popular models, whether the former can survive other experimental tests is a question that needs to be addressed in subsequent research.

8 Appendix

8.1 Proof of Proposition 1 and Corollary 1

We state a general Proposition 8.1 that applies to a theory with utility functional U as in statement (NL-1). Proposition 1 and Corollary 1.1 follow directly from Proposition 8.1.

General result 1.¹³ Let a decision theory with “utility functional” U as in statement (NL-1) be given. Referring to statement P.1, denote $a = \ell$, $b = g + \ell$ and let N be the largest integer smaller than $(M - m)/b$. For a general probability, p of the large outcome, $x + g$ statement P.1 for a sure outcome $x + a$ becomes

Footnote 11 continued

same-stakes risk aversion. Furthermore, this same-stakes implausible risk aversion holds for all scales of payoffs because the P.2e and Q.2e statements are dimension-invariant; that is, the numbers 400, 80, 330 and 10 could refer to numbers of cents, dollars or thousands of dollars or millions of dollars or any other payoff scale. The figures 330 and 1 in statement Q.2e apply for all utility, $v(\cdot)$ of prizes that satisfy $v(400)/v(80) \geq 3$.

¹² Unlike cumulative prospect theory with loss aversion, in a neighborhood of the origin, the Markowitz model’s transformation function for payoffs is convex on positive changes and concave on negative changes. How can one get risk aversion of type P.1 with this model? As an example think of the function being x^2 on the right of the origin up to the third inflection point and being x^3 on the left of the origin down to the first inflection point, where both the first and the third inflection points are further from the origin than the gains (g) and the losses (ℓ) in pattern P.1.

¹³ This part of the proof is similar to Cox and Sadiraj (2006, p. 58).

$$x + a \succeq \{x + b, p; x\}, \text{ for all integers } x \in [m + a, M + a]. \tag{8.i}$$

Suppose that¹⁴

$$bf(p) > a, \tag{8.ii}$$

where $f(\cdot)$ is the transformation function for decumulative probabilities as stated in the main text. Let $z_* = m + b(2 - \ln 2 / \ln q)$, where $q = (1 - f(p))\ell / f(p)g$. Let K be the largest integer smaller than $(z - m)/b$, for a given $z \in (z_*, M)$, and J be the largest integer smaller than $N - K + 1 + A(q, K)q^{-N}$, where $A(q, K) = q - q^{1+K} - q^K$.

Proposition 8.1 *For all (weakly) concave functions, $v(\cdot)$ and all positive numbers a, b and $f(p)$ that satisfy (8.ii) one has:*

1. *For all $M > m$, for all $z \in (z_*, m + bN)$, statement (8.i) implies $z \succ \{z + (J - 1)b, p; m\}$;*
2. *For all positive G , for all $z > z_*$ there exists N such that statement (8.i) implies that $z \succ \{z + G, p; m\}$.*

Proof The proof of 8.1.1 consists of two steps. □

First, we show that statements (NL-1), (8.i) and (weak) concavity of $v(\cdot)$ imply that for all $x \in [m + a, m + Nb + a]$

$$v(x + jb) - v(x + (j - 1)b) \leq q^j (v(x) - v(x - b)), \quad \forall j \in \Psi_x, \tag{8.1}$$

where Ψ_x is the set of all positive integers j such that $x + (j - 1)b - a$ belongs to the interval $[m, m + Nb]$, that is $\Psi_x = \{j \in \mathbb{N} | x + (j - 1)b - a \in [m, m + Nb]\}$.

Then, we show that for any given $z \in (z_*, m + bN)$,

$$m + Kb \geq \{m + (K + J)b, p; m\}. \tag{8.2}$$

That is getting $m + Kb$ for sure is preferred to the binary lottery $\{m + (K + J)b, p; m\}$, which completes the proof of part 1 since by construction of $K, m + (K + 1)b > z \geq m + Kb$ and therefore

$$\begin{aligned} v(z) &\geq v(m + Kb) \geq f(p)v(m + (K + J)b) + (1 - f(p))v(m) \\ &> f(p)v(z + (J - 1)b) + (1 - f(p))v(m). \end{aligned}$$

It follows from statements (NL-1) and (8.i) that

$$v(x + a) \geq (1 - f(p))v(x) + f(p)v(x + b), \quad \forall x \in [m + a, m + Nb + a] \cap \mathbb{N}. \tag{8.3}$$

¹⁴ For theories that assume linearity in probabilities and $p = 0.5$, condition (8.ii) is simply $g > \ell$; it says that the expected value of risky lottery $\{b, 0.5; 0\}$ is larger than the sure amount of money a . For theories that assume nonlinearity in probabilities, condition (8.ii) says that the expected value of the risky lottery after applying the probability transformation, that is $\text{EV}(\{b, f(p); 0\})$, is larger than the sure amount of money a .

To derive (8.1) write $v(x + a) = f(p)v(x + a) + (1 - f(p))v(x + a)$ and rearrange terms in (8.3) to get

$$(1 - f(p)) [v(x + a) - v(x)] \geq f(p) [v(x + b) - v(x + a)], \quad \forall x \in [m + a, m + Nb + a] \cap \mathbb{N}. \quad (8.4)$$

Note that from (weak) concavity of $v(\cdot)$,¹⁵ $(v(x) - v(x - g))/g \geq (v(x + \ell) - v(x))/\ell$, which together with statement (8.4) and using notations $q = (1 - f(p))\ell/f(p)g$, $a = \ell$ and $b = g + \ell$ imply

$$\begin{aligned} v(x + b) - v(x) &= v(x + b) - v(x + a) + v(x + a) - v(x) \\ &\leq \left(\frac{1 - f(p)}{f(p)} + 1 \right) (v(x + a) - v(x)) \\ &\leq \frac{1}{f(p)} \frac{\ell}{g} (v(x) - v(x - g)) = \frac{q}{1 - f(p)} (v(x) - v(x - g)) \\ &= q \left(1 + \frac{f(p)}{1 - f(p)} \right) (v(x) - v(x - g)) \\ &\leq q (v(x) - v(x - g) + v(x - g) - v(x - b)) \\ &= q (v(x) - v(x - b)). \end{aligned}$$

Finally, statement (8.1) follows from applying j times the preceding derived inequality, $v(x + b) - v(x) \leq q(v(x) - v(x - b))$.

To show statement (8.2), let $y = m + Kb$ and verify that statement (8.1) implies

$$v(y) - v(y - bK) = \sum_{k=0}^{K-1} [v(y - kb) - v(y - (k + 1)b)] \geq \Delta v(y) \sum_{k=0}^{K-1} (1/q)^k, \quad (8.5)$$

where $\Delta v(y) = v(y) - v(y - b)$. Next, it can be verified that $y \geq z - b > z_* - b$ implies that $K > 1 - \ln 2 / \ln q$. Hence $A(q, K) > 0$ and therefore $J + K > N$, by construction of J . It follows from (weak) concavity of $v(\cdot)$ and statement (8.1) that

¹⁵ To see this, write x as a convex combination of $x + \ell$ and $x - g$ and apply the definition of concavity to get $v(x) = v([\ell(x - g) + g(x + \ell)]/(g + \ell)) \geq [\ell v(x - g) + g v(x + \ell)]/(g + \ell)$.

$$\begin{aligned}
 v(y + Jb) - v(y) &= \sum_{j=0}^{J-1} [v(y + (j + 1)b) - v(y + jb)] \\
 &\leq \Delta v(y) \left[q^{N-K+1}(J - N + K - 1) + \sum_{j=0}^{N-K} q^{j+1} \right]. \tag{8.6}
 \end{aligned}$$

Statements (8.5) and (8.6) imply that a sufficient condition for (8.2) is

$$(1 - f(p)) \sum_{k=0}^{K-1} (1/q^k) > f(p) \left[q^{N-K+1}(J - N + K - 1) + \sum_{j=0}^{N-K} q^{j+1} \right]. \tag{8.7}$$

Substitute $(1-q) \sum_{j=0}^{N-K} q^{j+1} = q(1-q^{N-K+1})$, $(1-q) \sum_{k=0}^{K-1} q^{-k} = q(q^{-K} - 1)$ and $(1 - f(p))/f(p) = qg/\ell$ in (8.7) to get

$$J < N - K + 1 + \left(\frac{g}{\ell} (q - q^{1+K}) - q^K + q^{1+N} \right) \frac{1}{(1 - q)q^N}. \tag{8.8}$$

To show that inequality (8.8) is indeed satisfied recall that $J \leq N - K + 1 + A(q, K)q^{-N}$ by construction, $q \in (0, 1)$ by statement (8.ii) and verify that:

$$A(q, K) = q - q^{1+K} - q^K < \left(\frac{g}{\ell} (q - q^{1+K}) - q^K + q^{N+1} \right) \frac{1}{(1 - q)}.$$

Statement A.2.2 follows from statement A.1.1. Let $G > 0$ and $z > z_*$ be given. Then, as in part 1, let K be the largest integer smaller than $(z - m)/b$. It follows from $A(q, K) > 0$ and $q \in (0, 1)$ that there exists N^* such that for all integers $N > N^*$, one has

$$N - K - 1 + A(q, K)q^{-N} > G/b \tag{8.9}$$

If statement (8.i) is true for some M such that $N = \lfloor (M - m)/b \rfloor$ satisfies (8.9) then one has: $z > \{z + (J - 1)b, p; m\} > \{z + G, p; m\}$; the first inequality follows from part 1 (8.1.1), whereas the second one follows from the construction of J and first-order-stochastic dominance.

Proof of Proposition 1 (expected utility theory) Part a and Part b.i follow directly by linearity in payoffs of the DU functional (see DU-1) and $p = 0.5$. Condition $G > (z - m)g/\ell$ implies that $0 < (z - m)/(z - m + G) < \ell/(g + \ell) < 1$. To show Part b.ii., note that statement (8.ii) is satisfied and apply part 2 of Proposition 8.1 with $p = 0.5$, $f(p) = p$ and $q = \ell/g$ to find N such that statement P.1 for $M = m + N(g + \ell)$ implies $z > \{z + G, 0.5; m\}$, hence Q.1 is not true. \square

Proof of Corollary 1.1 (rank-dependent utility theory) It is an application of Proposition 8.1.2 with $p = 0.5$, $f(p) = h(p)$ and $v(z) = v(z)$. \square

8.2 Proof of Proposition 2 and its Corollaries

General result 2.¹⁶ Let preferences over finite discrete lotteries, $L = \{x_j, p_j\}$, $j \in I_m$ be represented with utility functional as in statement (NL-1).

$$U(L) = v(x_m)P_m + \sum_{j=1}^{m-1} v(x_j) \int_{P_{j+1}}^{P_j} df \tag{8.iii}$$

where $P_j = \Pr(x : x \geq x_j)$ and $\int_{P_{j+1}}^{P_j} df = f(P_j) - f(P_{j+1})$. Without any loss of generality, we use the normalization, $v(0) = 0$.

Suppose that statement P.2 as stated above Proposition 2 holds; that is for some given $y \geq 2x > 0$, and positive integers k^* and $n \geq k^*$, an agent prefers the following three-outcome lottery to the binary one,

$$\{y, p_i - \delta; x, 2\delta\} \succeq \{y, p_i\}, \text{ for all } i \in \{1, 2, \dots, 2n - k^*\}, \tag{8.iv}$$

where $\delta = 1/2n$ and $p_i = i/2n = i\delta$. Using notation $C \equiv v(y)/v(x)$ and function $T(\cdot)$ as defined in Sect. 4.1, we first state and show a general Proposition 8.2 for sub-additive value functions of prizes. Proposition 2 and its Corollaries follow straightforwardly from Proposition 8.2.

Proposition 8.2 *Suppose that statement P.2 is true. Then there exists integer K such that $\{z, 1 - (k^* - 1)\delta\} \succ \{zK, 0.5\}$, for all $z > 0$. The last statement is true for all integers K that do not exceed $T(C, n - k^*, n - 1)$.*

Proof According to functional (NL-1), statement P.2 [i.e. Eq. (8.iv)] requires that

$$v(x) \int_{(i-1)\delta}^{(i+1)\delta} df + v(y)f((i-1)\delta) \geq v(y)f(i\delta), \quad \forall i \in \{1, \dots, 2n - k^*\},$$

for some positive integer k^* not larger than n . Adding and subtracting $v(x)f(i\delta)$ and rearranging terms in the last inequality, we get

$$\int_{i\delta}^{(i+1)\delta} df \geq (C - 1) \int_{(i-1)\delta}^{i\delta} df, \quad \forall i \in \{1, \dots, 2n - k^*\}.$$

Write the last inequality for $i + k (= 1, \dots, 2n - k^*$ and apply it k times to get

$$\int_{(i+k)\delta}^{(i+k+1)\delta} df \geq (C - 1) \int_{(i+k-1)\delta}^{(i+k)\delta} df \geq \dots \geq (C - 1)^k \int_{i\delta}^{(i+1)\delta} df,$$

¹⁶ This part of the proof is similar to Sadiraj (2012).

which generalizes as

$$\int_{j\delta}^{(j+1)\delta} df \geq (C - 1)^i \int_{(j-i)\delta}^{(j-i+1)\delta} df, \forall j \in \{i, \dots, 2n - k^*\}. \tag{8.10}$$

To complete the proof, it suffices to show that

$$\begin{aligned} f(0.5) &\leq \sum_{j=0}^{n-1} \left(\frac{1}{C-1}\right)^j \int_{0.5-\delta}^{0.5} df \text{ and } f((2n - k^* + 1)\delta) - f(0.5) \\ &\geq \sum_{i=0}^{n-k^*} (C - 1)^{i+1} \int_{0.5-\delta}^{0.5} df \end{aligned} \tag{8.11}$$

because two inequalities in (8.11) imply that

$$f((2n - k^* + 1)\delta) \geq f(0.5) \left[1 + \sum_{i=0}^{n-k^*} (C - 1)^{i+1} / \sum_{j=0}^{n-1} (C - 1)^{-j} \right].$$

Multiplying both sides of the last inequality by $v(z)$ and using the sub-additivity of $v(z)$, we get the needed result:

$$v(z)f((2n - k^* + 1)\delta) \geq f(0.5)v(z)T(C, n - k^*, n - 1) \geq f(0.5)v(z)K.$$

To show the first inequality of (8.11) verify that it follows from inequality (8.10) that

$$f(0.5) = \sum_{i=1}^n \int_{(i-1)\delta}^{i\delta} df \leq \sum_{i=1}^n \left(\frac{1}{C-1}\right)^{n-i} \int_{(n-1)\delta}^{n\delta} df = \sum_{j=0}^{n-1} \left(\frac{1}{C-1}\right)^j \int_{0.5-\delta}^{0.5} df$$

Similarly, to show the second inequality of (8.11) verify that

$$\begin{aligned} f((2n - k^* + 1)\delta) - f(0.5) &= \sum_{j=n+1}^{2n-k^*+1} \int_{(j-1)\delta}^{j\delta} df \geq \sum_{j=n+1}^{2n-k^*+1} (C - 1)^{j-n} \int_{0.5-\delta}^{0.5} df \\ &= \sum_{i=0}^{n-k^*} (C - 1)^{i+1} \int_{0.5-\delta}^{0.5} df \end{aligned}$$

□

Proof of Proposition 2 (dual theory of expected utility) Part (a) follows from the linearity in probabilities of the EU functional (see EU-1). For part (b.i) first note that $k^* \geq 1$ implies $(1 - (k^* - 1)\delta)v(z) \leq v(z)$; hence $v(z) < 0.5v(zG)$ is a sufficient condition for the inequality in statement Q.2 to be true. Letting $z = 1/G$,¹⁷ the last inequality is equivalently written as $v(1/G) < 0.5v(1)$ which is satisfied for G large enough because the right-hand side is a positive finite number, whereas the left-hand side approaches 0 when G is large enough. Next, any such EU agent, with $v(y) < 2v(x)$, clearly satisfies pattern P.2 (which follows from part a). \square

To show part (b.ii) first note that $y > 2x > 0$ imply that $T(y/x, n - k^*, n - 1)$ can be as large as one wants it to be for big enough n . So, for any given G , there exists $n \in N$ such that

$$G < T(y/x, n - k^*, n - 1). \tag{8.12}$$

Then apply Proposition 8.2 with $v(z) = z$ to show that P.2 implies $\{z, 1 - (k^* - 1)\delta\} \succ \{zG, 0.5\}$, for all positive z . Therefore statement Q.2 is not true.

Proof of Corollary 2.1 (zero-income reference-dependent preferences) The proof is similar to the proof of Proposition 2.b.ii. There is only one difference: use $v(z) = v(z)$ instead of $v(z) = z$. \square

Proof of Corollary 2.2 (endogenous reference-dependent preferences) Let the endogenous reference point be the middle prize, x and let R denote the ratio between the value of the perceived gain and the absolute value of the perceived loss, that is $R = -v(y - x)/\mu(-x) > 1$. Statement P.2 in this case implies

$$\begin{aligned} \mu(-x)f^-(1 - (i + 1)\delta) + v((y - x)f^+((i - 1)\delta)) &\geq \mu(-x)f^-(1 - i\delta) \\ + v(y - x)f^+(i\delta), \quad \forall i \in I_{2n-k^*}, \end{aligned}$$

which can be equivalently rewritten as

$$\int_{i\delta}^{(i+1)\delta} df^+ = \int_{1-(i+1)\delta}^{1-i\delta} df^- \geq \frac{v(y-x)}{-\mu(-x)} \int_{(i-1)\delta}^{i\delta} df^+, \quad \forall i \in I_{2n-k^*},$$

where the equality follows from $f^+(p) = 1 - f^-(1 - p)$. Use notation R and apply the last inequality $j - i$ times to get

$$\int_{j\delta}^{(j+1)\delta} df^+ \geq R^{j-i} \int_{i\delta}^{(i+1)\delta} df^+, \quad \forall j \in \{i, \dots, 2n - k^*\},$$

¹⁷ If one is interested in some particular positive t different from 1 that we consider here, then take $z = t/G$ and verify that the sufficient condition becomes $v(t/G) < 0.5v(t)$, which is satisfied for G large enough.

and then (as in the proof for the general result 2, Proposition 8.2) verify that the following inequality is true

$$f^+(1 - (k^* - 1)\delta) \geq f^+(0.5) \left[1 + \sum_{j=0}^{n-k^*} R^{j+1} \bigg/ \sum_{i=0}^{n-1} R^{-i} \right].$$

Finally, to complete the proof, use the last inequality and sub-additivity of $v(\cdot)$ and choose n such that $G < T(R + 1, n - k^*, n - 1)$; the existence of such n follows from $R > 1$. \square

8.3 Contingent euro payoff protocol

The Magdeburg 110/–100 experiment included amounts x that were as large as 110 K euros. We could credibly offer to pay such large amounts in *contingent* euros by using the following protocol. The experiment included two parts. In part 1, subjects made their choices between the sure amounts and the lotteries in the MAX-Lab at the University of Magdeburg. They were told that whether their payoffs would be hypothetical or real depended on a condition that would be described later in part 2. After making their decisions, the subjects were informed that real payoffs were conditional on winning gambles at the Magdeburg Casino. The payoff contingency was implemented in the following way. For each participant, the experimenter placed € 90 on the number 19 on one of the (four American) roulette wheels at the casino. The probability that this bet wins is $1/38$. If the bet wins, it pays 35 to 1. If the first bet won, then the experimenter would bet all of the winnings on the number 23. If both the first and second bet won, then the payoff would be € $(35 \times 35 \times 90) =$ € 110,250, which would provide enough money to make it feasible to pay any of the amounts involved in the part 1 decision tasks for that subject. The real payoff contingency was made credible to the subjects by randomly selecting three of them to accompany the experimenter to the casino and subsequently report to the others whether the experimenter had correctly placed the bets and recorded the outcomes.

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